B.1: Intro to Sample Complexity

B.1.a

Let's exam the statement:

$$\mathbb{P}\left[\hat{R}_n(f) = 0\right] = \mathbb{P}\left[\frac{1}{n}\sum_{i=1}^{n} \mathbf{1}\{f(x_i) \neq y_i\} = 0\right] \tag{B.1.a}$$
$$= \prod_{i=1}^{n} 1 - \mathbb{P}\left[f(x_i) \neq y_i\right]$$

Let's use the statement in the hypothesis. The statement was $R(f) > \epsilon$, which describes the event that $\mathbb{E}\left[\mathbf{1}\{f(x) \neq Y\}\right]$. so then $1 - R(f)$ describes exepected value of the event that: $1 - \mathbb{P}\left(f(x_i) \neq y\right)$. And notice that $1 - R(f) < 1 - \epsilon < \exp(epsilon)$, so then we can simplify the above expression into:

$$\prod_{i=1}^{n} 1 - \mathbb{P}\left[f(x_i) \neq y_i\right] = (1 - R(f)) \leq (\exp(\epsilon))^n = \exp(n\epsilon) \tag{B.1.a.1}$$