CSE 546 SPRING 2021: HW1

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Short Answer and "True or False" Conceptual Questions

Maximum Likelihood Estimator(MLE)

(A.1)

(A.1.a)

Objective: Find the expression for the maximum=likelihood estimate for the parameter λ for the poisson distribution, interns of the goal count. Assume idd rvs.

Here we will assume that observations obtained takes the form $x_1, x_2, \cdots x_N$, and then we derive the best estimator for λ in this much general context.

I will shut up and just show you the math:

$$\prod_{n=1}^{N} \text{Poi}(x_n|\lambda) \tag{A.1.a.1}$$

$$\sum_{n=1}^{N} \log\left(\text{Poi}(x_n|\lambda)\right)$$

$$\sum_{n=1}^{N} \left(-\lambda + x_n \ln(\lambda) + \log(x_n!)\right)$$

Notice that, only some of the terms are relevant to the parameter λ, therefore, the optimization problem we are solving is:

$$\lambda^{+} = \underset{\lambda}{\operatorname{argmax}} \left\{ -N\lambda + \ln(\lambda)\sum_{i=1}^{N}(x_n) \right\} \tag{A.1.a.2}$$

To solve it, we just take the derivative, set it to zero and then solve for λ, because this function is a function that has a single local maximum.

$$\begin{aligned} \partial_\lambda \left[-N\lambda + \ln(\lambda)\sum_{i=1}^{N}(x_n) \right] =& 0 \\ -N + \frac{\sum_{n=1}^{N} x_n}{\lambda^{+}} =& 0 \\ \implies \lambda^{+} =& \frac{\sum_{n=1}^{N} x_n}{N} \end{aligned} \tag{A.1.a.3}$$

Therefor, for this particular problem, the best estimator will be the average of all the observation, which is just:

$$\frac{2+4+6+1}{5} = 2.6$$

And that is the answer for the question.

(A.1.b)

The derivation of the best estimator in the general context is shown in A.1.a.

The numerical value for six observations is:

$$\frac{2+4+6+1+3}{7} = 2.6666666666\cdots$$

(A.1.c)

The numerical results for 5 observations has been shown in A.1.a and A.1.b respectively.

(A.2)

Objective: Find the MLE for the uniform distribution on $[o, \theta]$, where θ is the value we want to estimate.

Overfitting

Bias-Variance Tradeoff

Polynomial Regression

Ridge Regression on MNIST