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B1: Matrix Completion and Recommendation System

Note: Yi Yun Dong, a student in 546 is my collaborator for this problem. Type of collaborations:

1. Me doing the math first, and thinking about the problem and then read Yiyun Dong's code to speed up the implementaitons of the algorithm. No direct copy is involved, I write my own code, but based on my understanding of the problem and Yiyun's code.

2. We discussed the math together and look for mistakes in each other's argument.

B1.code

B1.a

Using the average of each user to predict new enries causes the MSE to just be the variance of the matrix.

1. The average rating of movies are computed by: total rating of i'th movie/number of user rated that movie, and if that movie is not rated by any user, replace the average estimation to be the averate of all rated movies.

2. Then we just use the $\epsilon(\hat{R})_{\text{test}}$ to compute the error.

B1.c

Create the Masking matrix as the following:

$$M_{i,j} = \begin{cases} 1 & (i,j,R_{i,j}) \in \text{Train} \\ 0 & \text{else} \end{cases} \tag{B1.c.1}$$

This matrix will mask out elements instances where given user j didn't rate movie i. Then, we can say the following with the cost function:

$$\begin{aligned}
\nabla_{u_k}\left[\sum_{(i,j)\in\text{Train}} (u_i^T v_j - R_{i,j})^2 + \lambda\sum_{i=1}^{d}\|u_i\|_2^2\right] &= \mathbf{0} \\
\nabla_{u_k}\left[\sum_{j\in\text{train}} (u_k^T v_j - R_{k,j})^2 + \lambda\|u_k\|_2^2\right] &= \mathbf{0} \\
\nabla_{u_k}\left[\sum_{j=1}^{n}(M_{k,j}u_k^T v_j - R_{k,j})^2\right] + 2\lambda u_k &= \mathbf{0} \\
\left(\sum_{j=1}^{n}(M_{k,j}u_k^T v_j - R_{k,j})M_{k,j}v_j\right) + 2\lambda u_k &= \mathbf{0} \\
\left(\sum_{j=1}^{n} M_{k,j}^2 v_j v_j^T u_k - M_{k,j}R_{k,j}v_j\right) + \lambda u_k &= \mathbf{0} \\
\left(\sum_{j=1}^{n} M_{k,j}v_j v_j^T + \lambda I\right)u_k &= \sum_{j=1}^{n} M_{k,j}R_{k,j}v_j \\
u_k &= \left(\sum_{j=1}^{n} M_{k,j}v_j v_j^T + \lambda I\right)^{-1}\sum_{j=1}^{n} M_{k,j}R_{k,j}v_j
\end{aligned} \tag{B1.c.2}$$

And this is the closed form solution of solving for one of the vector in the matrix U, similarly we can get the closed form solution for one of the vector in V:

$$\begin{aligned}
\nabla_{v_k}\left[\sum_{(i,j)\in\text{Train}}(u_i^T v_j - R_{i,j})^2 + \lambda\sum_{i=1}^{d}\|u_i\|_2^2\right] &= \mathbf{0} \\
\nabla_{v_k}\left[\sum_{i=1}^{m}(M_{i,k}u_i^T v_k - Ri,j)^2 + \lambda\|v_k\|_2^2\right] &= 0 \\
\sum_{i=1}^{m}(M_{i,k}^2 u_i^T v_k u_i - R_{i,k}M_{i,k}u_i) + \lambda v_k &= 0 \\
\sum_{i=1}^{m}(M_{i,k}u_i u_i^T v_k) + \lambda I v_k &= \sum_{i=1}^{m} M_{i,k}R_{i,k}u_i \\
\left(\sum_{i=1}^{m}(M_{i,k}u_i u_i^T) + \lambda I\right) v_k &= \sum_{i=1}^{m} M_{i,k}R_{i,k}u_i \\
v_k &= \left(\sum_{i=1}^{m}(M_{i,k}u_i u_i^T) + \lambda I\right)^{-1}\sum_{i=1}^{m} R_{i,k}M_{i,k}u_i
\end{aligned} \tag{B1.c.3}$$