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A1: Conceptual Questions

A1.a

True. This is true because SVD is looking for a orthogonal matrix (PCA uses SVD) U, such that $U\Sigma V^T$ minimizes that reconstruction error. In this case the rank of matrix U can have the same rank as the subspace span by the columns of the data matrix X giving us zero reconstruction errors.

A1.b

True Because:

$$\begin{aligned} X^TX &= (USV^T)^T(USV) \\ &= VS^TU^TUSV \\ &= VS^TSV \end{aligned} \tag{A1.b.1}$$

Take notice that S^TS is diagonal and V is orthogonal, and X^TX is Symmetric. By properties of Hermitian Adjoint Matrices, it has orthogonal Eigen Decomposition with unique real eigenvectors. And VS^TSV matches it, therefore V is the eigen vectors of matrix X^TX.

A1.c

False. The objective should not be choosing k to minimize the Loss because if $k = n$ is always the global minimum in that case and it doesn't provide any useful interpretations on the data.

A1.d

False. Singular values Decomposition has U, V that are the eigenvectors for XX^T and X^TX, eigen values decomposition is not unque because you can multiply eigenvector by negative one (or even worse by the complex unit $\exp(i\theta)$) to get another normalized eigen vector that still works. In the case of SVD remember to flip the u, v vector corssponds to the same singular value together to get different decomposition for the same matrix.

A1.e

False when the matrix is degenerate. In this case a eigenvalue can have a geometric multiplicity higher than it's algebaric multiplicity, then the rank of the matrix will be more than the number of eigenvalues it has.

A1.f

True, becaues Autoencoders with non-linear activation function can incooperate non-linear representation of data in the lower dimension.

A2: Basics of SVD and Subgradients

A2.a

A2.a.(a)

I will just show you the math and explain some keys step on why this is true. This one easy to show because there is a closed form solution that we can incorperate the singular value decomposition for the convariance matrix. let's consider the gradient of the objective function set to zero. s

$$\begin{aligned}\nabla\left[\|Xw-y\|_2^2+\lambda\|w\|_2^2\right]&=\mathbf{0} \\ 2X^T(Xw-y)+2\lambda w&=\mathbf{0} \\ X^T(Xw-y)+2\lambda w&=\mathbf{0} \\ X^TXw-X^Ty+\lambda w&=0 \\ (X^TX+\lambda I)w&=X^Ty\end{aligned} \tag{A2.a.a.1}$$

Using the Singular value decomposition we have:

$$\begin{aligned}X^TX&=(U\Sigma V^T)^T(U\Sigma V^T) \\ X^TX&=(V\Sigma U^T)(U\Sigma V^T) \\ X^TX&=V\Sigma^2V^T\end{aligned} \tag{A2.a.a.2}$$

We make use of the fact that, U,V are unitary matrices and the singular matrix Σ is a diagonal, containing all the singular vaues ranked in order of magnitude, and padded with zeros. Here we consider the case of Economic Singular Value Decomposition. Substituting the previous expression to the previous previous expression we have:

$$\begin{aligned}\hat{w}_R&=(X^TX+\lambda I)^{-1}X^Ty \\ \hat{w}_R&=((V\Sigma^2V)+V(\lambda I)V^T)^{-1}X^Ty \\ \hat{w}_R&=(V(\Sigma^2+\lambda I)V^T)^{-1}X^Ty \\ \hat{w}_R&=V^T(\Sigma^2+\lambda I)^{-1}VX^Ty \\ \|\hat{w}_R\|_2&=\|V^T\|_2\|(\Sigma^2+\lambda)^{-1}\|_2\|\|V\|_2\|X^T\|_2\|y\|_2 \\ \|\hat{w}_R\|_2&=\left(\sum_{i=1}^{\min(m,n)}\frac{1}{\sigma_i^2+\lambda}\right)\|X^T\|_2\|y\|\end{aligned} \tag{A2.a.a.3}$$

Taking the limit as $\lambda\rightarrow\infty$ will yield zero for the norm of $\hat{w}_R$, and in this case, here are the facts we used

1. we used the fact that the induced 2 norm of a unitary matrix is one, which will be proven in the next part.
2. And the induced 2 norm for a diagonal matrix is just the sum of all it's diagonal elements.
3. The inverse of a unitary matrix is it's Transpose, assuming it's real, and in this case, U,V are unitary matrices.
4. $(AB)^{-1}=B^{-1}B^{-1}$ assuming invertible A,B
5. $(AB)^T=B^TA^T$

Note: The bound on the summation is implicitly making the assumption that X is a $m\times n$ or $n\times m$ matrix.

A2.a.(b)

A2.a.b) From the previous part, A2.a(a), I have shown that $X^TX=V\Sigma^2V$ where $X=U\Sigma V$. And in this question, we just had U instead of X, let's use X, cause U is already involved in the SVD. Let $\Sigma=I_n$ which sets the singular values of X to be all ones, then $X^TX=VI_nV^T=I_n$ Becase V is a unitary matrix and its inverse it's its transpose, similarly for XX^T

$$\begin{aligned} XX^T &= (U\Sigma V^T)(U\Sigma V^T)^T \\ XX^T &= (U\Sigma V^T)(V\Sigma U^T) \\ XX^T &= (U\Sigma\Sigma U^T \\ XX^T &= U\underbrace{\Sigma^2}_{I_n}U^T \\ XX^T &= UU^T = I_n \end{aligned} \tag{A2.a.b.1}$$

Now, using the definition of the Norm we have:

$$\begin{aligned} \|Ux\|_2^2 &= (Ux)^T(Ux) \\ x^TUUx &= x^Tx \\ &= \|x\|_2^2 \end{aligned} \tag{A2.a.b.2}$$

A2.b

A2.0: Preliminaries

Let's denote the set of $\{g : f(y) \geq f(x) + g^T(y-x)\}$ to be $\partial[f]$, which is really a compact set in the Euclidean space. In the case of vector is going to be a cone.

From the problem statement we gather: $\exists i \in [m] : f(x) = f_i(x) \implies \partial[f_i] \subseteq \partial[f]$. Now, if we make the deliberate choice on i for a given particular x, we can make the claim that:

$$\partial[f] = (\inf\{\partial[f_i(x)] : f_i(x) = f(x)\}, \sup\{\partial[f_i(x)] : f_i(x) = f(x)\}) \tag{A2.0.1}$$

Basically, we can choose the i such to find a lower and upper bound for the sub gradient of f if $f_i(x) = f(x)$.

And when we consider the sum of a lot of convex functions $\sum_{i=1}^{n} f_i(x)$, then:

$$\begin{aligned} f_i(y) \geq f_i(x) + v^T(x-y) \quad & \forall i \in [m], v \in \partial[f_i](x) \\ \sum_{i=1}^{m} f_i(x) \geq v^T(x-y) \quad & \forall v \in \left(\sum_{i=1}^{m} \inf\{\partial[f_i]\}, \sum_{i=1}^{m} \sup\{\partial[f_i]\}\right) \end{aligned} \tag{A2.0.2}$$

And this is how summation for sub gradient works if we want to sum up several functions, we just need to sup up the supremum and infinum to get the range for the new subgradient, which is still going to be a compact set, or a cone.

A2.b.(a)

$$\begin{aligned} &\partial\left[\sum_{i=1}^{n} |x_i|\right] \\ &= \sum_{i=1}^{n} \partial[|x_i|] \\ &= \sum_{i=1}^{n} g_i \mathbf{e}_i \end{aligned} \tag{a2.b.a.1}$$

g_i is essentially:

$$g_i \in \partial[|x_i|] = \begin{cases} \{1\} & x_i \geq 1 \\ [-1, 1] & x_i = 0 \\ \{-1\} & x_i \leq 0 \end{cases} \tag{a2.b.a.2}$$

And using the hint from the next part, the sub gradient of $\|x\|_1$ is the convex combinations of all $g_i\mathbf{e}_i$:

$$\sum_{i=1}^{n} \lambda_i g_i \mathbf{e}_i \in \partial[\|x\|_1] \quad \sum_{i=1}^{n} \lambda_i \leq 1 \wedge \lambda_i \geq 0 \tag{a2.b.a.3}$$

And the span of all sub gradient for each $|x_i|$ will make up the set of sub-gradient for the original function, and hence, let v_j be the j th element of the sub gradient of $\|x\|_1$, the closed form will be:

$$v_j \in \begin{cases} \{1\} & x_j > 0 \\ [-1, 1] & x_j = 0 \\ \{-1\} & x_j \leq 0 \end{cases} \tag{A2.b.1.3}$$

A2.b.(b)

Let λ_i be the set of coefficients for a convex combinations, meaning that $\sum_{i=1}^{n} \lambda_i = 1$ and $\lambda_i \geq 0$, implying that $\lambda_i \in (0, 1)$. Using this fact and the definition of $f(x) := \max\{f_i(x)\}_i^m$, consider the following:

$$\begin{aligned} f(y) &\geq f_i(y) \quad \forall\ i \\ \lambda_i f(y) &\geq \lambda_i f_i(y) \quad \forall\ i \\ \sum_{i=1}^{m} \lambda_i f(y) &\geq \sum_{i=1}^{m} \lambda_i f_i(y) \\ \underset{(1)}{\implies} f(y) &\geq \sum_{i=1}^{m} \lambda_i f_i(y) \\ f(y) &\geq \left(\underbrace{\sum_{i=1}^{m} \lambda_i f_i(x)}_{\leq f(x)} \right) + \lambda_i \nabla[f_i](x)^T (y - x) \\ \underset{(2)}{\implies} f(y) &\geq f(x) + \lambda_i \nabla[f_i](x)^T (y - x) \quad \forall\ i \end{aligned} \tag{A2.b.b.1}$$

(1) : True because the convex combinations coefficients $\sum_{i=1}^{m} \lambda_i = 1$ and $f(y)$ is independent of the summation.

(2) : True because the $\sum_{i=1}^{m} \lambda_i f_i(x) \leq f(x)$ is already proven in (1).

Now, we are free to choose λ_i to find the bound of the all the convex combinations of the sub gradient on f_i at x. Therefore, the sub-gradient is the set defined as the following:

$$(\partial[f](x))_j = (\inf \{(\nabla[f_i](x))_j : f_i(x) = f(x)\}, \sup \{(\nabla[f_i](x))_j : f_i(x) = f(x)\}) \tag{A2.b.b.2}$$

Note: The notation of $(\bullet)_j$ is denoting the j th element of a vector, in this case, we are saying that the j th element of the sub gradient vector for f is bounded by the sup and inf of the j th element of the gradient of the smooth function f_i.

A2.c

In this case $f_i(x) = |x_i - (1 + \eta/i)|$ hence we can say v_i is a subgradient of f_i if:

$$\begin{aligned} v_i \in \partial[|x_i - (1 + \eta/i)|] = &\begin{cases} \{1\} & x > 1 + \frac{\eta}{i} \\ [-1, 1] & x_i = 1 + \frac{\eta}{i} \\ \{-1\} & x_i < 1 + \frac{\eta}{i} \end{cases} \\ \implies \forall x \in \operatorname{dom}(f), i \in [n] : &\quad -1 \leq v_i \leq 1 \\ &\implies \|v_i \mathbf{e_i}\|_\infty \leq 1 \end{aligned} \tag{A2.c.1}$$

Therefore, we know that the convex combinations will be bounded too and it's like:

$$\forall \lambda_i \geq 0 \wedge \sum_{i=1}^{n} \lambda_i \leq 1: \quad \left\Vert \underbrace{\sum_{i=1}^{n} \lambda_i v_i \mathbf{e}_i}_{\in \partial[f]} \right\Vert_\infty \in [0,1] \tag{A2.c.2}$$

Therefore, the infinity norm of the sub gradient of the function f is in the set interval $[0,1]$[1].

A3: PCA

A3.a

This is the data:

```
1 EigenValue 5.116787728342072
2 EigenValue 3.74132847886477
10 EigenValue 1.2427293764173106
30 EigenValue 0.36425572027888686
50 EigenValue 0.1697084270067159
The sum of all eigen values for the COVAR matrix is:
52.72503549512752
```

A3.b

Objective: Given the Eigenvalue Decomposition of the matrix Coveriance matrix of the sample, and the standardized data matrix, we are interested in reconstructing some samples using the eigenvalue of the covariance matrix. Before we start, I would like to change the notations a bit.

1. For notation, let's denote the covariance matrix using C instead of Σ as in the original problem statement.
2. Then, let $C = M\Lambda M^T$ be the eigen value decomposition.
3. Let the singular value decomposition of the zero mean matrix to be $X_{\text{train}} - \mathbf{1}\mu^T$ to be $U\Sigma V^T$

By definiiton of the covariance matrix we have:

$$\begin{aligned} nC &= (X_{\text{train}} - \mathbf{1}\mu^T)^T(X_{\text{train}} - \mathbf{1}\mu^T) \\ &= (U\Sigma V^T)^T(U\Sigma V^T) \\ &= (V\Sigma U^T)(U\Sigma V^T) \\ &= V\Sigma^2 V^T = M(n\Lambda)M^T \end{aligned} \tag{A3.b.1}$$

In this case, V is an ortho-normal matrix. Therefore, Σ^2/n will be equal to Λ. Which means that, given λ_i as the eigen value for the Covariance matrix, $\sigma_i = \sqrt{n\lambda_i}$.

The important thing is that, columns of V spans the row space of matrix $X_{\text{train}} - \mathbf{1}\mu$, it has all the Principal Components for rows of the matrix $X_{\text{train}} - \mathbf{1}\mu$. Therefore, given any offset vector we can project onto the matrix V:

$$\begin{aligned} \tilde{x} - \mu &= VV^T(x - \mu) \\ \tilde{x} &= MM^T(x - \mu) + \mu \end{aligned} \tag{A3.b.2}$$

[1] The infinity norm has only positive part, so it's less than one in the end

In the case when a row data matrix is given we have:

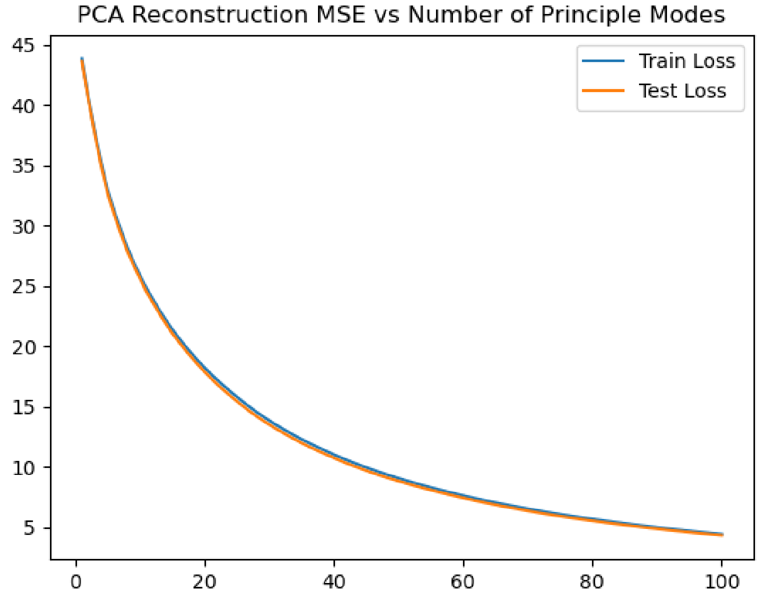
$$\begin{aligned} \tilde{X}^T - \mu\mathbf{1}^T &= VV^T(X^T - \mu\mathbf{1}^T) + \mathbf{1}\mu^T \\ \tilde{X}^T &= MM^T(X^T - \mu\mathbf{1}^T) + \mu\mathbf{1}^T \\ \tilde{X} &= (MM^T(X^T - \mu\mathbf{1}^T) + \mathbf{1}\mu^T)^T + \mathbf{1}\mu^T \end{aligned} \tag{A3.b.3}$$

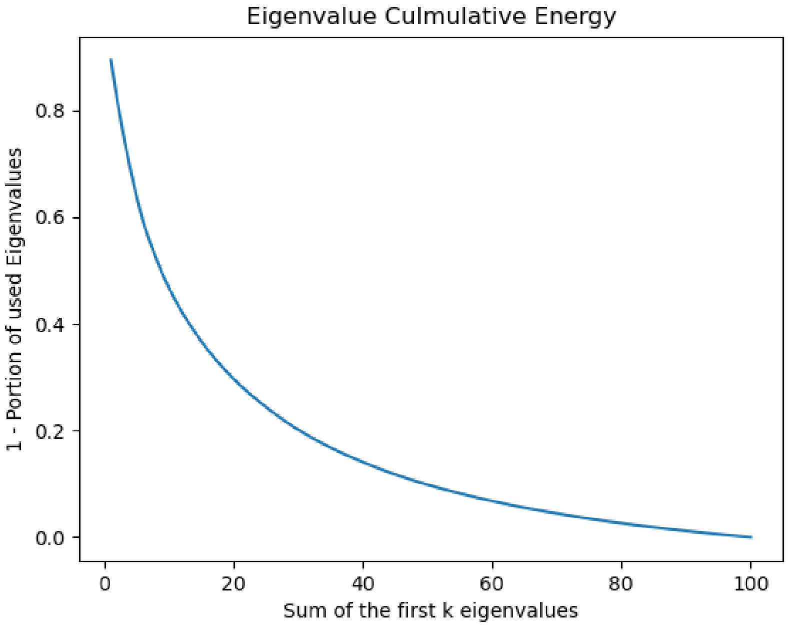
Why projecting onto principal components V? Because:

$$V = \underset{W^TW=I}{\operatorname{argmin}} \left(\left\| (X_{\text{train}} - \mathbf{1}\mu)^T - WW^T(X_{\text{train}} - \mathbf{1}\mu)^T \right\|_2^2 \right) \tag{A3.b.4}$$

The singular value decomposition matrix V as it's defined in this case, it's also the orthogonal subspace that minimizes the error of representing all the training data (Samples are rows of X_{train} in this case).

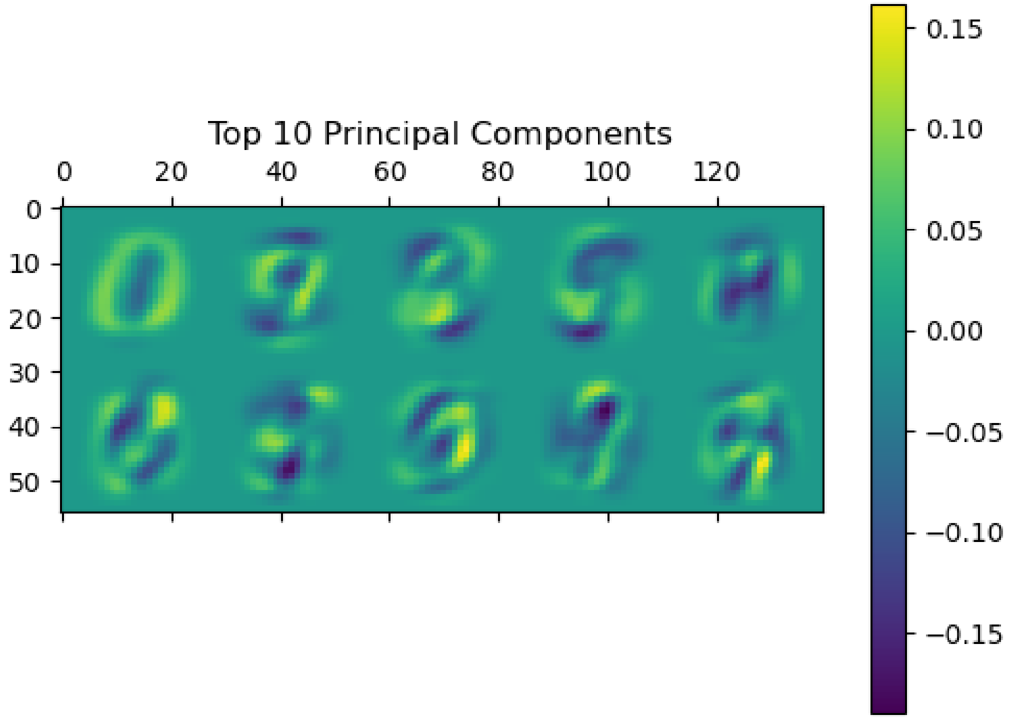
A3.c





A3.d

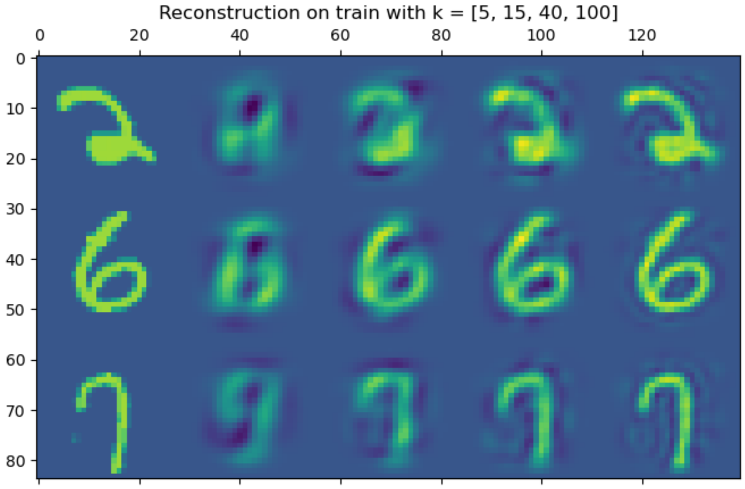
The 10 top ranking principal conponents are visualized:



From top to bottom, from left to right, modes are in decreasing order.

Each of the eigen vector of the Covariance matrix captured the pixels that exaplained most of the variance on the data set, in a manner that is, orthogonal to the subspace spanned by all previous principal mode. That is why the first principal mode looks like a zero, and then all the principle mode after it has bright spot that is non-overlapping to the previous mode (Due to the orthogonality constraint).

A3.e



The approximation of the image by the principle modes is improved as more principle modes are used. To get a recognizable reconstruction of the original image, at least 40 modes should be used, which explains over 80% of the total variance.

A3.code

This is the code.

File name: "mnist_pca.py"

```
# This is for HW4 A3
# Course: CSE 546, SPRING 2021
# Name: Hongda Li
# My code has my tyle in it please don't copy.

import numpy as np
import scipy
from scipy import linalg
from torchvision import datasets
from mnist import MNIST
import matplotlib.pyplot as plt
from tqdm import tqdm
zeros = np.zeros
randint = np.random.randint
randn = np.random.randn
eigh = linalg.eigh
norm = np.linalg.norm
cumsum = np.cumsum



if "MNIST_DATA" not in dir(): # running on interactive console will be faster
    datasets.MNIST('./data', download=True, train=True)
    MNIST_DATA = MNIST("./data/MNIST/raw/")
    TRAIN_X, _= MNIST_DATA.load_training()
    TEST_X, _ = MNIST_DATA.load_testing()
    TRAIN_X= np.array(TRAIN_X, dtype=np.float)/255
    TEST_X = np.array(TEST_X, dtype=np.float)/255
    TRAIN_Y = np.array(MNIST_DATA.train_labels)
    print("Mnist␣Dataset␣is␣ready.")

# ========================== List of Helper Functions ==========================
def Ts():
    from datetime import datetime
    SysTime = datetime.now()
    TimeStamp = SysTime.strftime("%H-%M-%S")
    return TimeStamp


def mkdir(dir):
    from pathlib import Path
    Path(dir).mkdir(parents=True, exist_ok=True)


def log(fname:str, content:str, dir):
    mkdir(dir)
    TimeStamp = Ts()
    with open(f"{dir}{TimeStamp}-{fname}.txt","w+") as f:
        f.write(content)

# ==============================================================================


class SVDEmbedding:

    def __init__(this, X):
        assert type(X) is np.ndarray
        assert X.ndim == 2
        Mu = np.mean(X, axis=0, keepdims=True)
        StdX = (X - Mu)
        n, d = X.shape
        EigenValues, V = eigh((StdX.T@StdX)/n)
        this._V = V[:, ::-1]                 # reverse the order a bit
        this.n = n
        this.d = d
        this._EigenValues = EigenValues[::-1]
        this._Mu = Mu

    @property
    def V(this):
        return this._V.copy()

    @property
    def EigenValues(this):
        return this._EigenValues.copy()

    def Reconstruct(this, X:np.ndarray, k:int):
        assert X.ndim == 2
        assert X.shape[1] == this.d
        assert k <= this.d
        V = this._V[:, :k]
        return (V @ V.T @ (X.T - this._Mu)).T + this._Mu


    def ReconstructLoss(this, X, k:int):
        return norm(X - this.Reconstruct(X, k), "fro")**2/X.shape[0]

    def GetAnalysisFor(this, X, k, loss:bool=True):
        """
            Given a list of numbers denoting the set of: number of eigenvalues
            we want to use to reconstruct this data matrix, this will return a
            map mapping the number of eigenvalues, the reconstructed row data
            matrix, and the loss on the row data matrix.
        :param X:
        :param k:
        :param loss:
            Where you want the lossm or you want the
        :return:
        """
        assert np.sum(np.array(k) <= this.d)
        Reconstructed = zeros(X.shape)
        Res = []
        for II in tqdm(range(0, np.max(k))):
            Reconstructed[:, :] += \
                (this._V[:, II:II + 1] @ this._V[:, II:II + 1].T @ (X - this._Mu).T).T
            if II + 1 in k:
                Loss =  norm((X - this._Mu) - Reconstructed, "fro")**2/X.shape[0]
                Res.append((II + 1, Loss if loss else Reconstructed.copy()))
        return Res


def main():
    OutFolder = "./A3out"
    mkdir(OutFolder)
    Instance = SVDEmbedding(TRAIN_X)

    def A3a():
        # print out specific eigen values
        with open(f"{OutFolder}/{Ts()}-A3a-eigenvalue-sum.txt", "w+") as f:
            for II in [1, 2, 10, 30, 50]:
                f.write(f"{II}␣EigenValue␣{Instance.EigenValues[II␣-␣1]}\n")
            f.write("The␣sum␣of␣all␣eigen␣values␣for␣the␣COVAR␣matrix␣is:\n")
            f.write(f"{np.sum(Instance.EigenValues)}\n")
    # A3a()

    def A3c():
        def PlotReconstructionError(X):
            Analysis = Instance.GetAnalysisFor(X, k=list(range(1, 101)))
            Ks = [Item[0] for Item in Analysis]
            MSELoss = [Item[1] for Item in Analysis]
            plt.plot(Ks, MSELoss)
        print("Getting␣Reconstruction␣graph␣for␣Train␣set")
        PlotReconstructionError(TRAIN_X)
        print("Getting␣Reconstruction␣graph␣for␣Test␣set")
        PlotReconstructionError(TEST_X)
        plt.legend(["Train␣Loss", "Test␣Loss"])
        plt.title("PCA␣Reconstruction␣MSE␣vs␣Number␣of␣Principle␣Modes")
        plt.savefig(f"{OutFolder}/{Ts()}-PCA-restruct-MSE.png")
        plt.show()
        # Accumuated Eigenvalues
        plt.plot(list(range(1, 101)), 1 - cumsum(Instance.EigenValues[:100])
                    / np.sum(Instance.EigenValues[:100]))
        plt.title("Eigenvalue␣Culmulative␣Energy")
        plt.xlabel("Sum␣of␣the␣first␣k␣eigenvalues")
        plt.ylabel("1␣-␣Portion␣of␣used␣Eigenvalues")
        plt.savefig(f"{OutFolder}/{Ts()}-PCA-restruct-Energy.png")
        plt.show()
        plt.cla()

    A3c()

    def A3d():
        ToPlot = zeros((28*2, 28*5))
        for II in range(10):
            X = (II%5)*28
            Y = (II//5)*28
            V = Instance.V[:, II].reshape(28, 28)
            ToPlot[Y: Y + 28, X: X + 28] = V

        fig = plt.figure()
        ax = fig.add_subplot(111)
        cax = ax.matshow(ToPlot)
        fig.colorbar(cax)
        plt.title("Top␣10␣Principal␣Components")
        plt.savefig(f"{OutFolder}/{Ts()}-top10-principal-modes.png")
        plt.show()
    A3d()

    def A3e():
        from random import randint
        def RandomChooseDigits():
            Chosen = []
            for II in [2, 6, 7]:
                Indices = np.argwhere(TRAIN_Y == II).reshape(-1)
                Chosen.append(Indices[randint(0, len(Indices))])
            return Chosen
        ChosenDigits = RandomChooseDigits()
        Analysis = Instance.GetAnalysisFor\
        (
            TRAIN_X[ChosenDigits],
            loss=False,
            k=[5, 15, 40, 100]
        )
        ToPlot = zeros((3 * 28, 5 * 28))
        for II in range(3):
            Original = TRAIN_X[ChosenDigits[II]]
            Original = Original.reshape((28, 28))
            ToPlot[II*28:(II + 1)*28, :28] = Original
            for JJ, (k, X) in enumerate(Analysis):
                Reconstructed = X[II].reshape((28, 28))
                ToPlot[II*28: (II + 1)*28, (JJ + 1)*28: (JJ + 2)*28] = \
                    Reconstructed
        plt.matshow(ToPlot)
        plt.title("Reconstruction␣on␣train␣with␣k␣=␣[5,␣15,␣40,␣100]")
        plt.savefig(f"{OutFolder}/{Ts()}-pca-reconstruction.png")
        plt.show()
    A3e()

    def A4d():
        from random import randint
        def RandomChooseDigits():
            Chosen = []
            for II in [2, 6, 7]:
                Indices = np.argwhere(TRAIN_Y == II).reshape(-1)
                Chosen.append(Indices[randint(0, len(Indices))])
            return Chosen

        ChosenDigits = RandomChooseDigits()
        Analysis = Instance.GetAnalysisFor \
                (
                TRAIN_X[ChosenDigits],
                loss=False,
                k=[32, 64, 128]
            )
        ToPlot = zeros((3 * 28, 4 * 28))
        for II in range(3):
            Original = TRAIN_X[ChosenDigits[II]]
            Original = Original.reshape((28, 28))
            ToPlot[II * 28:(II + 1) * 28, :28] = Original
            for JJ, (k, X) in enumerate(Analysis):
                Reconstructed = X[II].reshape((28, 28))
                ToPlot[II * 28: (II + 1) * 28, (JJ + 1) * 28: (JJ + 2) * 28] = \
                    Reconstructed
        plt.matshow(ToPlot)
        plt.title("Reconstruction␣on␣train␣with␣k␣=␣[32,␣64,␣128]")
        plt.savefig(f"{OutFolder}/{Ts()}-pca-reconstruction.png")
        plt.show()
    A4d()


if __name__ == "__main__":
    import os
    print(os.getcwd())
    print(os.curdir)
    main()
```

A4: Unsupervised Learning with Autoencoders

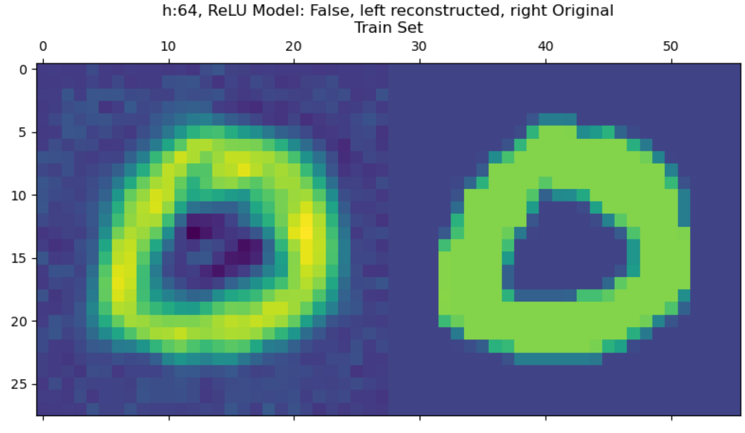
A4.a

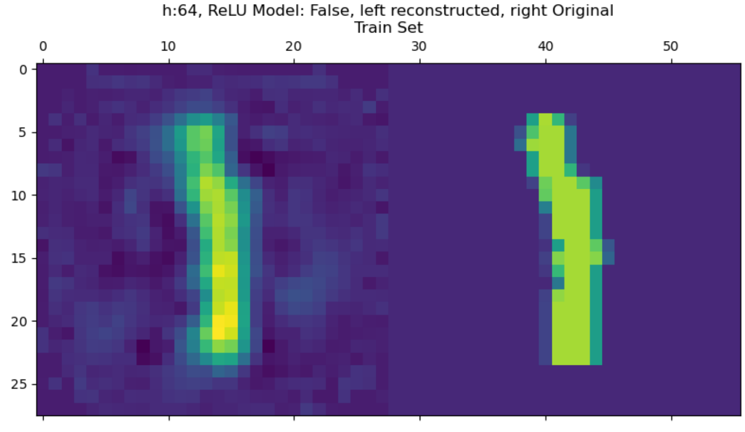
The Train Error for $h \in \{32, 64, 128\}$ for the linear models are:

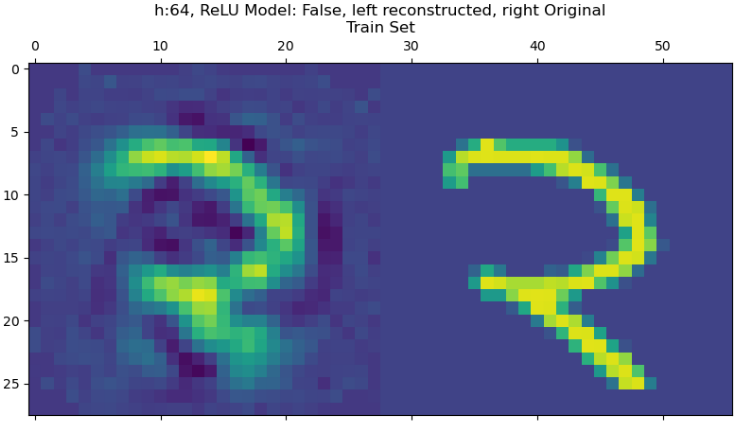
1. $h = 32$, Total Epochs: 30 Train Final MSE Loss: 0.07298633098602295
2. $h = 64$, Total Epochs: 30 Train Final MSE Loss: 0.07428810136703154
3. $h = 128$, Total Epochs: 30 Train Final MSE Loss: 0.058713519498705916

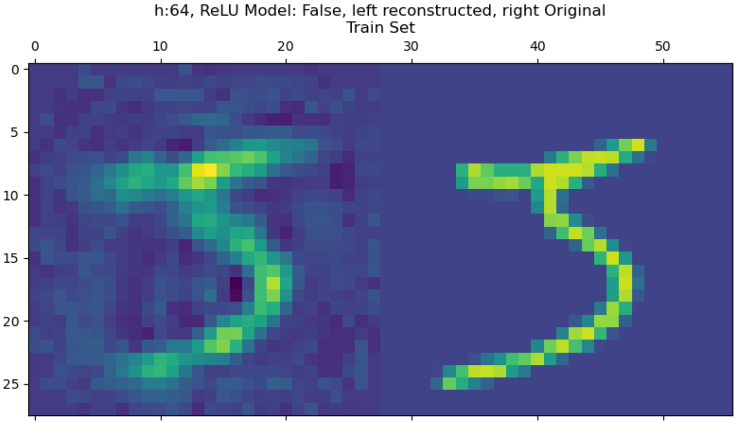
The MSE loss is computed via $\frac{1}{N}\sum_{i=1}^{N} \|f(g(x_i)) - x_i\|_2^2$, Where f, g are the encoder and decoder. The losses are divided by the total number of batches from the data loader, hence the error in the end is the squared loss on a persample basis. For code implementation, refers A4.Code.
The Autoencoder is trained on the whole training data set of MNIST.

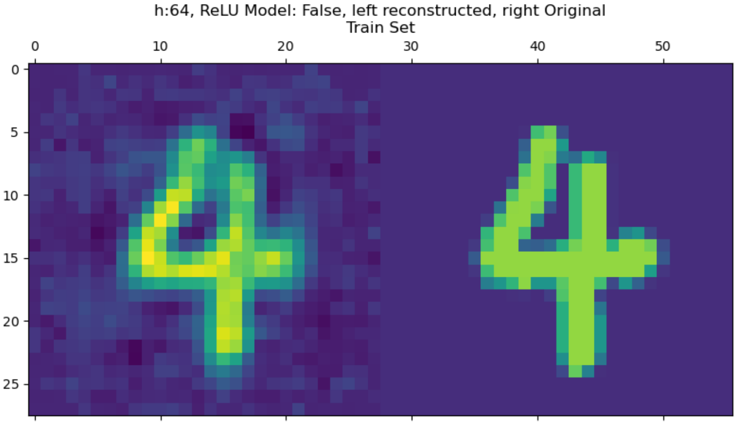
This is 10 digits reconstruction for $h = 64$ are below, and for all of the reconstruction plots for pairs of digits with $h = 32, 128$, please refer to Appendix.

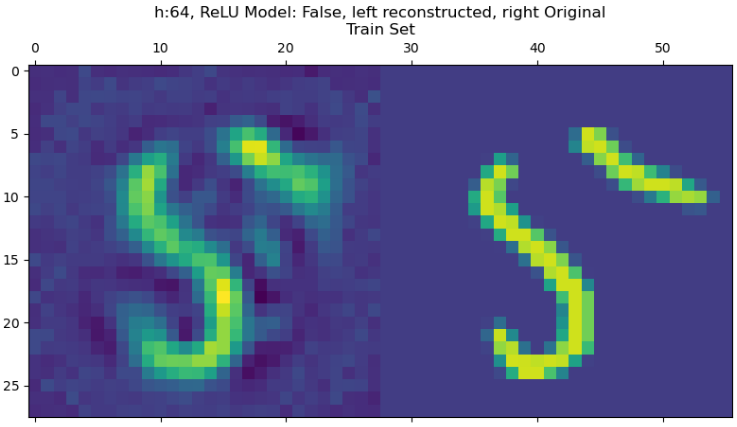


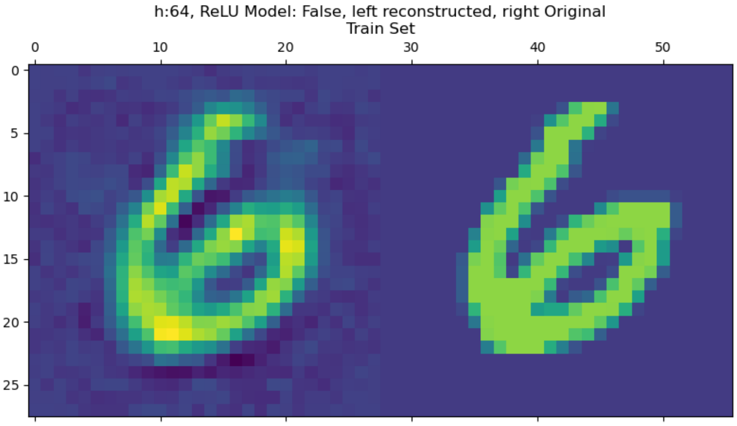


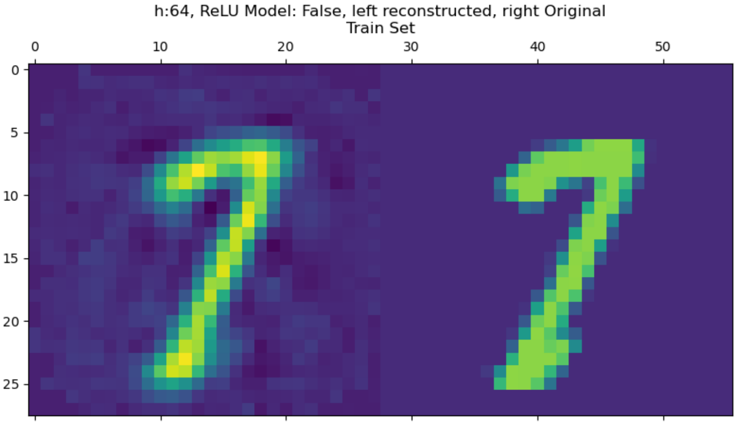

h:64, ReLU Model: False, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25

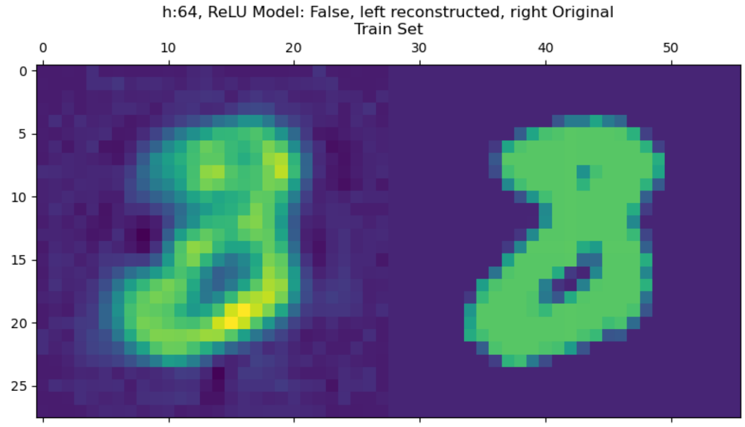

h:64, ReLU Model: False, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25

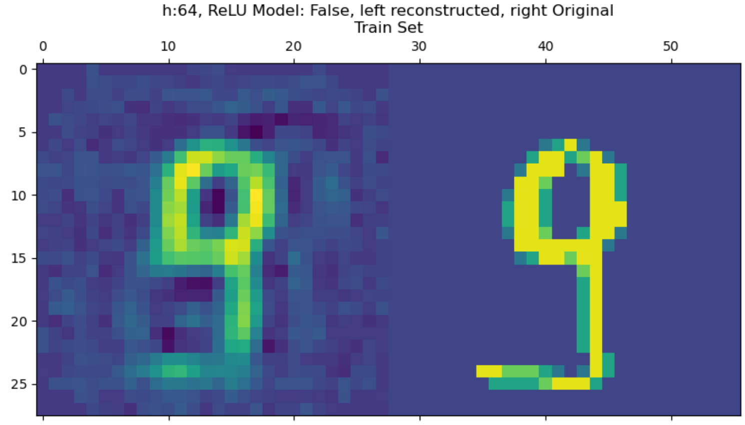

h:64, ReLU Model: False, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25


h:64, ReLU Model: False, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25


h:64, ReLU Model: False, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25


h:64, ReLU Model: False, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25



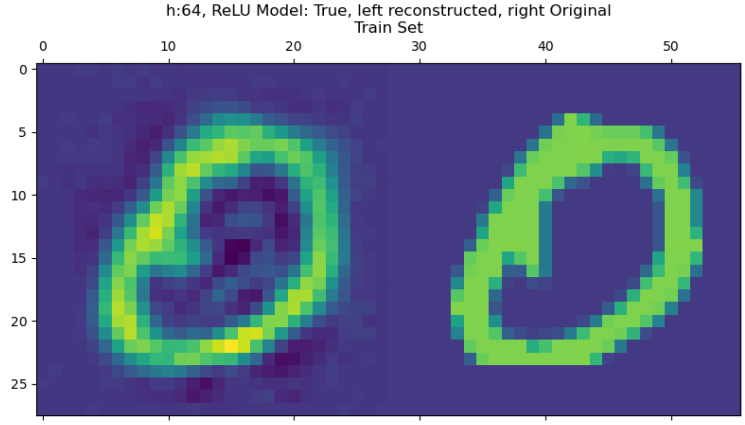


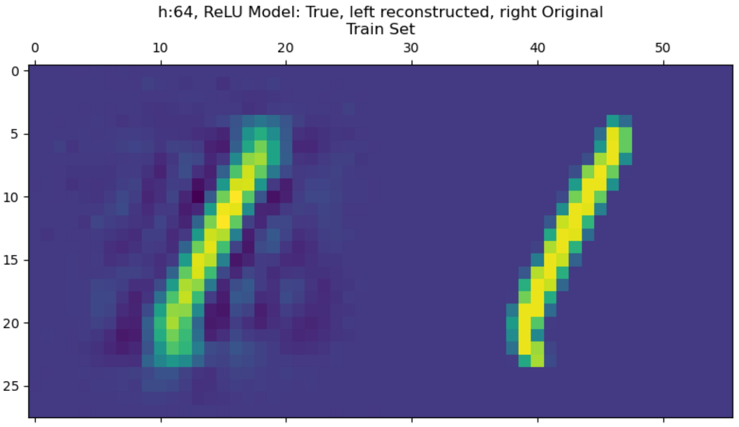
A4.b

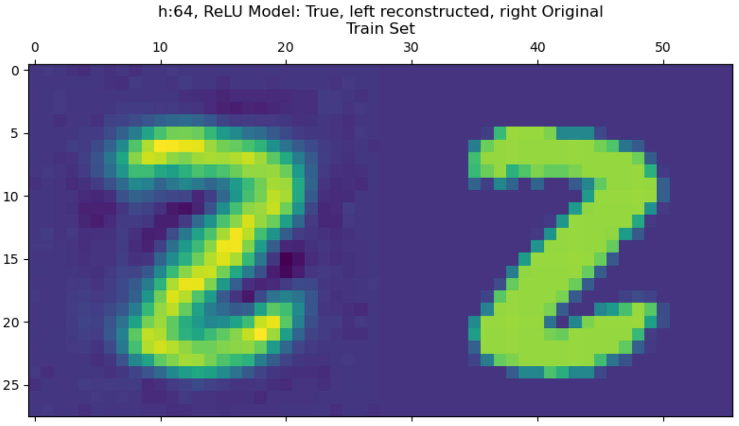
The train error for $h \in \{32, 64, 128\}$ for the non-linear model with ReLU activation is:

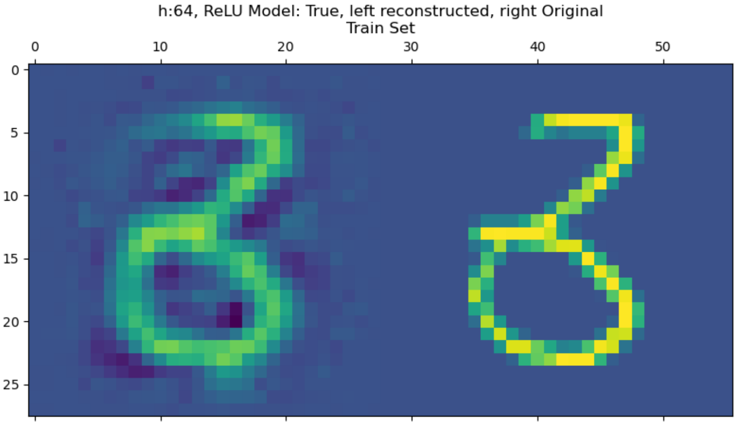
1. $h = 32$, total Epochs: 30 Train Final MSE Loss: 1.2326371114328512
2. $h = 64$, Total Epochs: 30 Train Final MSE Loss: 0.0249483520537615
3. $h = 128$, Total Epochs: 30 Test Final MSE Loss: 0.024230041910583734

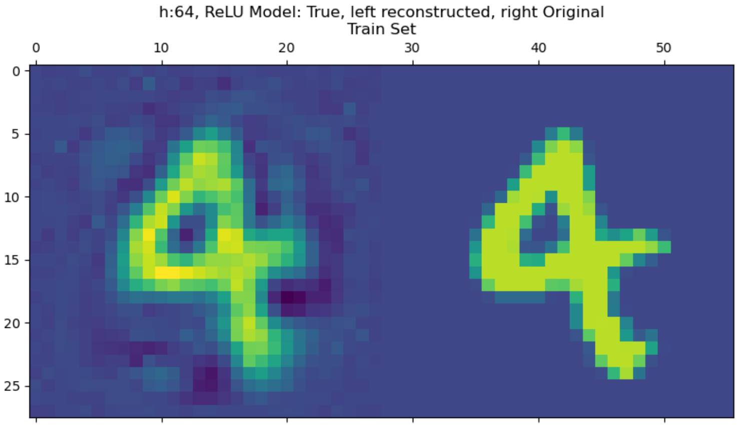
The MSE is computed the same as part A4.b. And these are some of the reconstruction images for $h = 64$:

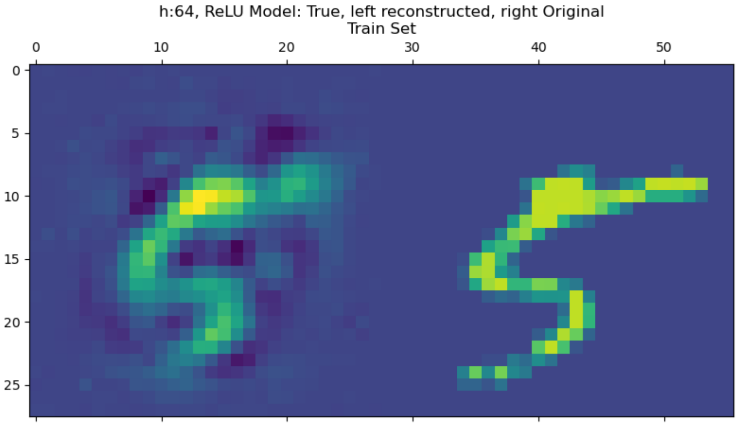


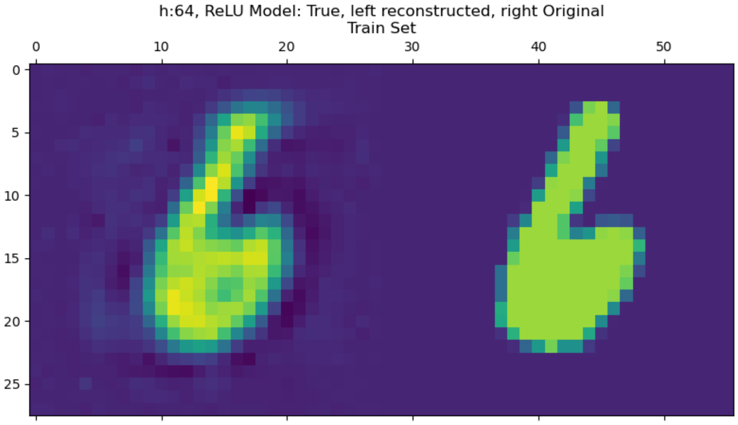

h:64, ReLU Model: True, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25

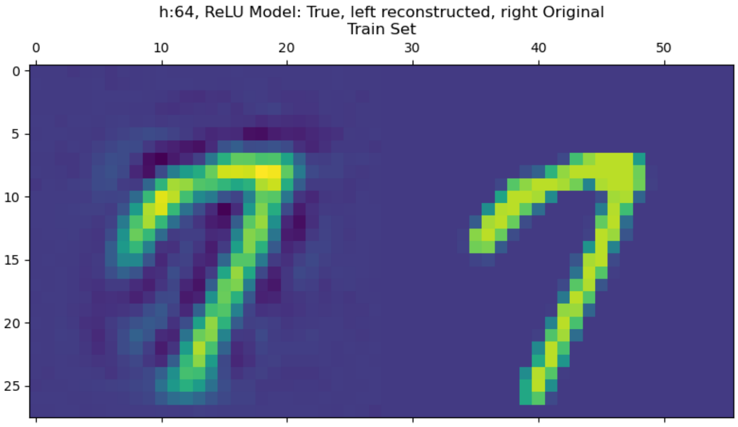

h:64, ReLU Model: True, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25

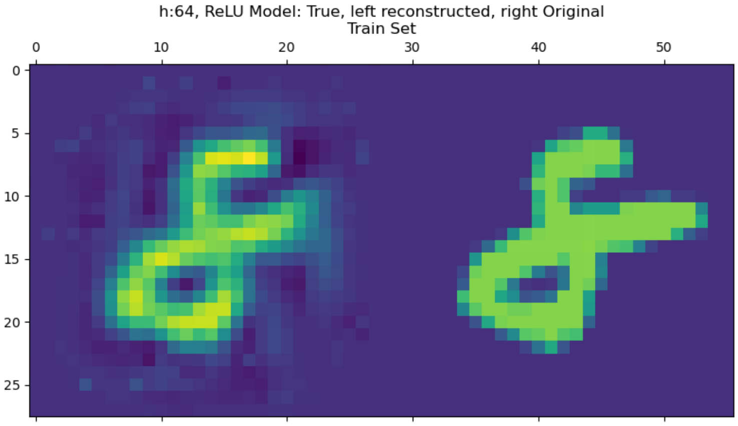

h:64, ReLU Model: True, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25

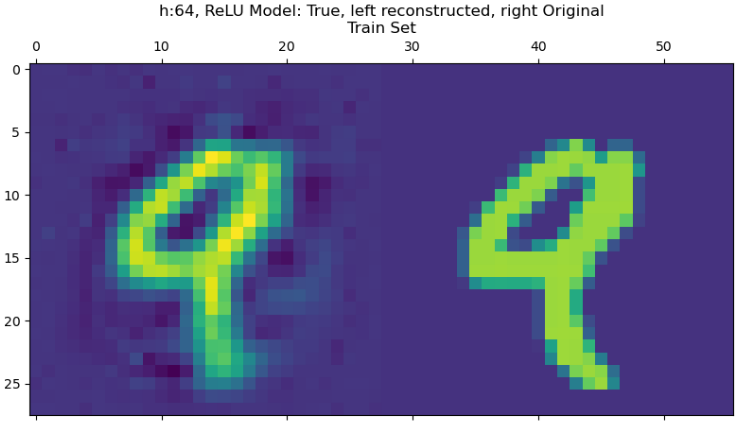

h:64, ReLU Model: True, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25


h:64, ReLU Model: True, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25


h:64, ReLU Model: True, left reconstructed, right Original
Train Set
0
10
20
30
40
50
0
5
10
15
20
25







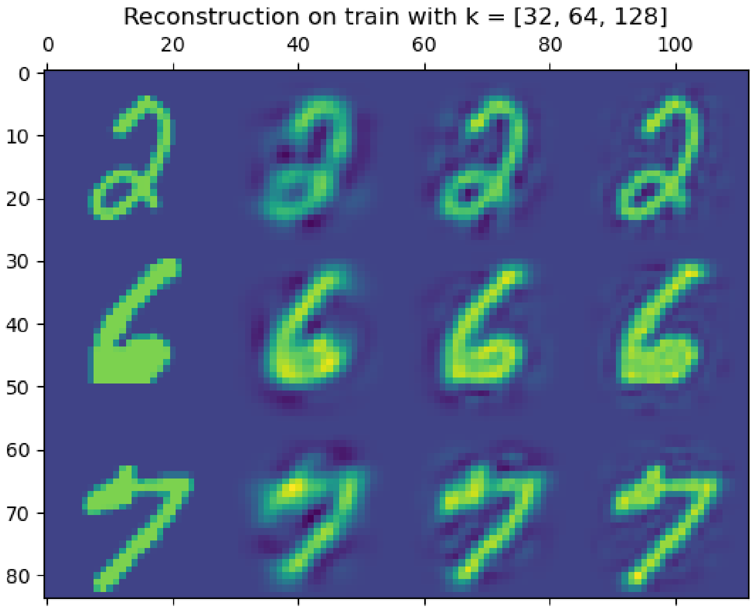
A4.c

For linear model with a hidden layer of 128, Total Epochs of 30, the final MSE loss on the test set is: 0.038117972066005104.

For the non linear model with a hidden layer of 128, total epochs of 30, the final MSE loss on the test set is: 0.024230041910583734.

A4.d

This is the reconstructed images using the first 32, 64, 128 PCA components.



The quality of the reconstructing images using PCA components is very similar to what we have for the Linear Autoencoder.

A4.Code

This is the code I used for the assignment:
Filename: "mnist_autoencoders"

```
### CLASS: CSE 546 SPRING 2021 HW4, A3
### Name: Hongda Li
### My code has my style in it don't copy.


import numpy as np
import torch
import torchvision
import matplotlib.pyplot as plt
from time import time
from torchvision import datasets, transforms
from torch import nn, optim
from tqdm import tqdm
import random as sysrandom


TRANSFORM  = transforms.Compose([transforms.ToTensor(),
                                transforms.Normalize((0.5,), (0.5,)),
                            ])
TRAIN_SET = \
    datasets.MNIST('./data', download=True, train=True, transform=TRANSFORM)
TEST_SET = \
    datasets.MNIST('./data', download=False, train=True, transform=TRANSFORM)

TRAIN_SET, TEST_SET = \
    torch.utils.data.Subset(TRAIN_SET, range(0, 1000)), \
    torch.utils.data.Subset(TEST_SET, range(0, 1000))
```

```
DEVICE = torch.device("cuda:0" if torch.cuda.is_available() else "cpu")

class A4Model(nn.Module):

    def __init__(this, h:int, non_linear:bool=False):
        """

        :param kargs:
            d: The dimension of the data.
            h: The width of the hidden layer.
            activation: TRUE, FALSE
                Whether to use ReLU activation function on hidden and output layer.
        """
        super().__init__()
        d = 28**2
        this.L1 = nn.Linear(d, h)
        this.L2 = nn.Linear(h, d)
        this.NonLin = non_linear
        this.MSE = nn.MSELoss()


    def forward(this, X):
        """
            Feed Forward without Loss function capped onto the output layer.
        :param X:
            The data matrix, row data matrix.
        :return:

        """
        x = this.L1(X)
        if this.NonLin:
            x = nn.ReLU(x)
        x = this.L2(x)
        if this.NonLin:
            x = nn.ReLU(x)
        return x

    def GetEmbeding(this, X):

        pass

    def FeedForward(this, X):
        return this.MSE(this(X), X)


def MNISTTenUniqueDigitsLoader(train=True):
    data = \
        datasets.MNIST(’./data’, download=True, train=train, transform=TRANSFORM)
    Indices = []
    for II in range(10):
        Idx = torch.where(data.targets == II)[0]
        Indices.append(Idx[sysrandom.randint(0, len(Idx))])
    Subset = torch.utils.data.Subset(data, Indices)
    return torch.utils.data.DataLoader(Subset, batch_size=10, shuffle=False)


def BatchThisModel(theModel:A4Model,
                dataLoader:torch.utils.data.DataLoader,
                optimizer:optim.Adam=None,
                transform:callable=None):
    """
        Batch this model for one epoch, give me the model optimizer and some
        extra thing, then it will collect the average loss of one epoch.
        Note:
        This one is for Regression Model, it assumes MSE loss, loss of each
        batch if divided by the total number of batches from the data loader.
    :param theModel:
```

```
    :param dataLoader:
    :param transform:
    :return:
    """
    AvgLoss = 0; theModel.to(DEVICE)
    L = len(dataLoader)
    for II, (X, _) in enumerate(tqdm(dataLoader)):
        if transform is not None: X = transform(X)
        X= X.to(DEVICE)
        if optimizer is None:
            with torch.no_grad():
                AvgLoss += float(theModel.FeedForward(X))/L
        else:
            optimizer.zero_grad()
            Loss = theModel.FeedForward(X)
            AvgLoss += Loss.item() / L
            Loss.backward()
            optimizer.step()
    with torch.no_grad():
        Loss = theModel.FeedForward(X)
        AvgLoss += Loss.item() / L
    return AvgLoss


def GetTrainTestLoaders(bs=100):
    TrainLoader = \
        torch.utils.data.DataLoader(TRAIN_SET, batch_size=bs, shuffle=True)
    TestLoader = \
        torch.utils.data.DataLoader(TEST_SET, batch_size=bs, shuffle=True)
    return TrainLoader, TestLoader


def Ts():
    from time import gmtime, strftime
    TimeStamp = strftime("%H-%M-%S", gmtime())
    return TimeStamp


def mkdir(dir):
    from pathlib import Path
    Path(dir).mkdir(parents=True, exist_ok=True)


def log(fname:str, content:str, dir):
    mkdir(dir)
    TimeStamp = Ts()
    with open(f"{dir}{TimeStamp}-{fname}.txt","w+") as f:
        f.write(content)


def main():
    def A4Run(NonLin, h):
        Bs, Epochs, lr = 100, 30, 0.01
        Losses = []
        Model = A4Model(h=h, non_linear=False)
        Optimizer = optim.Adam(Model.parameters(), lr=lr)
        TrainLoader, TestsLoader = GetTrainTestLoaders(bs=Bs)
        for Epoch in range(Epochs):
            Loss = BatchThisModel(theModel=Model,
                                  dataLoader=TrainLoader,
                                  optimizer=Optimizer,
                                  transform=lambda x: x.reshape(Bs, -1))
            Losses.append(Loss)
        log(fname=f"A4-final-train-Loss-h={h}-{'Non'␣if␣NonLin␣else␣''}lin-model",
            content=f"Total␣Epochs:␣{Epochs}␣Train␣Final␣MSE␣Loss:␣{Losses[-1]}",
            dir="./A4logs/")
        def Visualize(Train=True):
            mkdir("./A4plots")
            DigitsLoader = MNISTTenUniqueDigitsLoader(Train)
            for X, _ in DigitsLoader:
                X = X.reshape(X.shape[0], -1)
                Reconstructed = Model(X.to(DEVICE))
            for II, (Row1, Row2) in enumerate(zip(Reconstructed.data, X.data)):
                Together = torch.zeros(28, 28*2)
                Together[:, :28] = Row1.reshape(28, 28)
                Together[:,28:]  = Row2.reshape(28, 28)
                plt.matshow(Together)
                plt.title(f"h:{h},␣ReLU␣Model:␣{NonLin},␣left␣reconstructed,␣right␣Original\n"
                f"{’Train␣Set’␣if␣Train␣else␣’Test␣Set’}")
                plt.savefig(f"./A4plots/{Ts()}-h-{h}-{’non’␣if␣NonLin␣else␣’’}lin-digit-{II}.png"
                    )
                plt.show()

        Visualize()
        if h == 128:
            Loss = BatchThisModel(theModel=Model,
                                  dataLoader=TestsLoader,
                                  transform=lambda x: x.reshape(Bs, -1))
            log(fname=f"A4-test-loss-h={h}-{’Non’␣if␣NonLin␣else␣’’}lin-model",
                content=f"Total␣Epochs:␣{Epochs}␣Test␣Final␣MSE␣Loss:␣{Loss}",
                dir="./A4logs/")


        return Model, Losses

    A4Run(False, h=32)
    A4Run(False, h=64)
    A4Run(False, h=128)
    A4Run(True, h=32)
    A4Run(True, h=64)
    A4Run(True, h=128)



if __name__ == "__main__":
    import os
    print(os.curdir)
    print(os.getcwd())
    main()
```

A5: K-Mean Clustering

A5.a

This is the code I used for the whole *A*5, it contains my implementation of the Loyd’s Kmeans algorithm using numpy.

filename: “k_mean.py”

```
### CLASS CSE 564 SPRING 2021 HW4 A4
### Name: Hongda Li
### My code has my style in it don’t copy.

import numpy as np
import matplotlib.pyplot as plt
from torchvision import datasets
from mnist import MNIST
from tqdm import tqdm
zeros = np.zeros
randint = np.random.randint
randn = np.random.randn
```

```
if "MNIST_DATA" not in dir(): # running on interactive console will be faster
    datasets.MNIST('./data', download=True, train=True)
    MNIST_DATA = MNIST("./data/MNIST/raw/")
    TRAIN_X, _= MNIST_DATA.load_training()
    TEST_X, _ = MNIST_DATA.load_testing()
    TRAIN_X= np.array(TRAIN_X, dtype=np.float)/255
    TEST_X = np.array(TEST_X, dtype=np.float)/255
    print("Mnist␣Dataset␣is␣ready.")



# ======================= Helper Functions =====================================

def Ts():
    from datetime import datetime
    SysTime = datetime.now()
    TimeStamp = SysTime.strftime("%H-%M-%S")
    return TimeStamp


def mkdir(dir):
    from pathlib import Path
    Path(dir).mkdir(parents=True, exist_ok=True)


def log(fname:str, content:str, dir):
    mkdir(dir)
    TimeStamp = Ts()
    with open(f"{dir}{TimeStamp}-{fname}.txt","w+") as f:
        f.write(content)


# ==============================================================================

class KMean:

    def __init__(this, k:int, X:np.ndarray):
        """

        :param k: Number of cluster
        :param X: Row data matrix in np array type
        """
        assert k < X.shape[0] and k > 1
        assert X.ndim == 2
        n, d= X.shape[0], X.shape[1]
        this._X = X
        # this._AugX = X[:, :, np.newaxis]
        this.Assignment = {}
        this._C = np.transpose(this._X[randint(0, n, k), :][...,np.newaxis],
                               (2, 1, 0))
        this._ComputeAssignment()

    @property
    def Centroids(this):
        return np.transpose(this._C, (2, 1, 0))[..., 0].copy()
    @property
    def X(this):
        return this._X.copy()
    @property
    def AugX(this):
        return this._AugX.copy()
    @property
    def C(this):
        return this._C.copy()

    def TransferLearningFrom(this, other):
        this._C = other.C
        this._ComputeAssignment()
    def _ComputeCentroid(this):
        """
            Compute centroid using the current assignment.

        :return:
        """
        for Centroid, Idx in this.Assignment.items():
            this._C[..., Centroid] = \
                np.mean(this._X[Idx], axis=0, keepdims=True)

    def _ComputeAssignment(this, XTest=None):
        """
        Given current centroids make an assignment.
        :return:
        """

        X = this._X if XTest is None else XTest
        Distances = zeros((X.shape[0], 1, this._C.shape[2]))
        for CIdx in range(this._C.shape[2]):
            Centroid = this._C[..., CIdx]
            Distances[..., CIdx] = np.sum((X - Centroid)**2,
                                          axis=1,
                                          keepdims=True)
        AssignmentVec = np.argmin(Distances, axis=2).reshape(-1)
        NewAssignment = {}
        for Idx, Class in enumerate(AssignmentVec):
            IdxArr = NewAssignment.get(Class, [])
            IdxArr.append(Idx)
            NewAssignment[Class] = IdxArr
        if XTest is None:
            this.Assignment = NewAssignment
        del Distances
        return NewAssignment.copy()

    def Update(this):
        this._ComputeCentroid()
        this._ComputeAssignment()

    def Loss(this, Xtest=None):
        X = this._X if Xtest is None else Xtest
        TestAssignment = this._ComputeAssignment(Xtest)
        Centroids = this.Centroids
        Loss = 0
        for CentroidIdx, Idx in TestAssignment.items():
            Loss += np.sum((X[Idx] - Centroids[CentroidIdx, :])**2)
        return Loss/X.shape[0]


def main():

    def BasicTest():
        Points1 = randn(1000, 2)
        Points2 = np.array([[3, 3]]) + randn(1000, 2)
        PointsAll = np.concatenate((Points1, Points2), axis=0)
        Km = KMean(X=PointsAll, k=2)
        Losses = []
        for II in range(10):
            Km.Update()
            Losses.append(Km.Loss())
        plt.plot(Losses)
        plt.show()
        return Km

    def Learn(Km:KMean, n=None):
        Losses = []
        if n is not None:
            for _ in tqdm(range(n)):
                Km.Update()
                Losses.append(Km.Loss())
        else:
            C = Km.Centroids
            while True:
                Km.Update()
                Losses.append(Km.Loss())
                Delta = np.linalg.norm(C - Km.Centroids, np.inf)
                print(f"Delta:␣{Delta}")
                if Delta < 1e-1:
                    break
                C = Km.Centroids
        return Km, Losses

    def ClusterMnist(k=10,X=None):
        if X is None: X = TRAIN_X
        Km, Losses = Learn(KMean(X=X,k=k))
        return Km, Losses


    def A5b():
        Km, Losses = Learn(KMean(X=TRAIN_X,k=10))
        plt.plot(Losses)
        plt.title("A5(b)␣Kmean␣k=10")
        plt.xlabel("Iteration")
        plt.ylabel("Average␣Loss")
        mkdir("./A5bplots")
        plt.savefig(f"./A5bplots/{Ts()}-A5b-k=10-losses.png")
        plt.show()
        AllCentroid = zeros((28*2, 28*5))
        for Idx, Centroid in enumerate(Km.Centroids):
            Image = Centroid.reshape((28, 28))
            VerticalOffset, HorizontalOffset = (Idx//5)*28, (Idx%5)*28
            AllCentroid[VerticalOffset:VerticalOffset+28,
            HorizontalOffset:HorizontalOffset+28] = Image
        plt.matshow(AllCentroid)
        plt.title("A5(b):Cenroids␣fond␣by␣Kmean")
        plt.savefig(f"./A5bplots/{Ts()}-A5b-k=10-centroids.png")
        plt.show()

    def A5c():
        NumberOfCluster = list(map(lambda x: 2**x,range(1, 7)))
        TrainLosses, TestLosses = [], []
        for K in NumberOfCluster:
            Km, Losses = ClusterMnist(k=K, X=TRAIN_X[:5000])
            TrainLosses.append(Losses[-1])
            TestLosses.append(Km.Loss(TEST_X))
        plt.plot(NumberOfCluster, TrainLosses, ".-")
        plt.plot(NumberOfCluster, TestLosses, ".-")
        plt.legend(["Losses␣on␣Train␣Set", "Losses␣on␣Test␣Set"])
        plt.title("K-Mean␣on␣MNIST,␣Cluster␣Number␣vs␣Loss")
        plt.xlabel("Number␣of␣Cluster")
        plt.ylabel("Loss")
        plt.savefig(f"./A5bplots/{Ts()}-A5b-k-vs-loss.png")
        plt.show()

    # A5b()
    A5c()




if __name__ == "__main__":
    import os
    print(f"{os.getcwd()}")
    print(f"{os.curdir}")
    main()
```

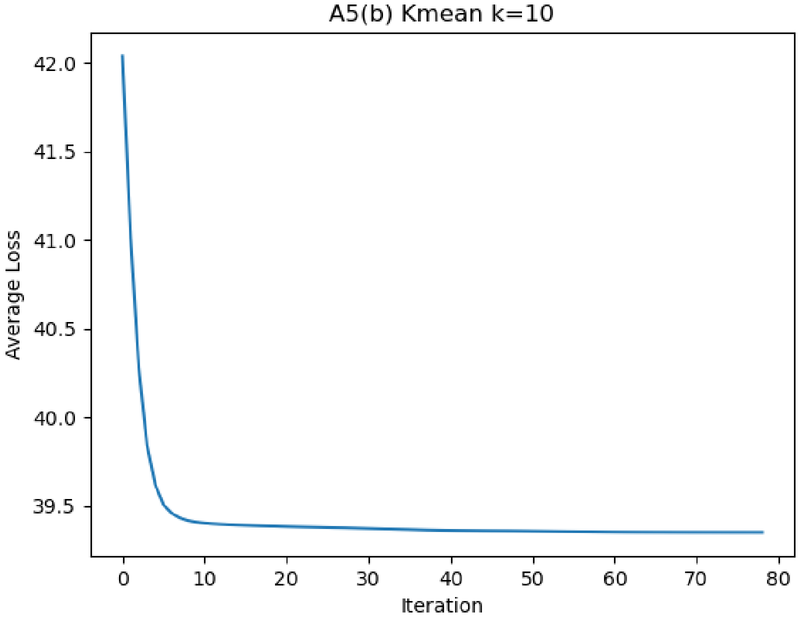
A5.b

This is the error for the k mean algorithm with $k = 10$. The algorithm iterates until the maximal centroid's position doesn't change by more than $1e - 2$.

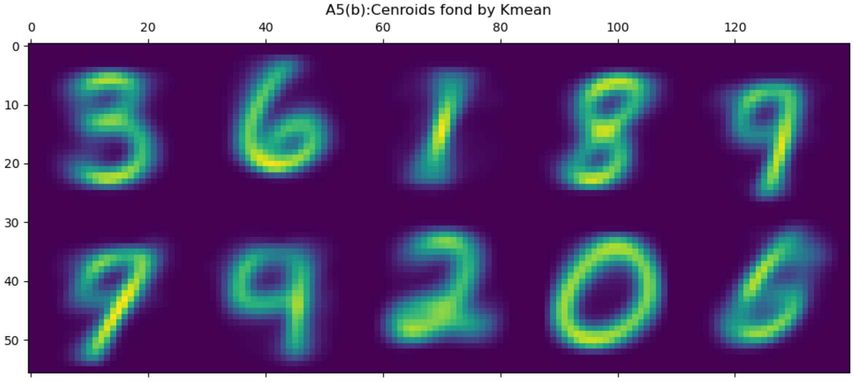
Note, here are some implementation details that might effect the average loss for each samples computed:

1. : I normalized the MNIST data by dividing it by 255 so all the piexles values are in $[0, 1]$.
2. : It's trained on the whole MNIST train datasset.

Code: A5.a

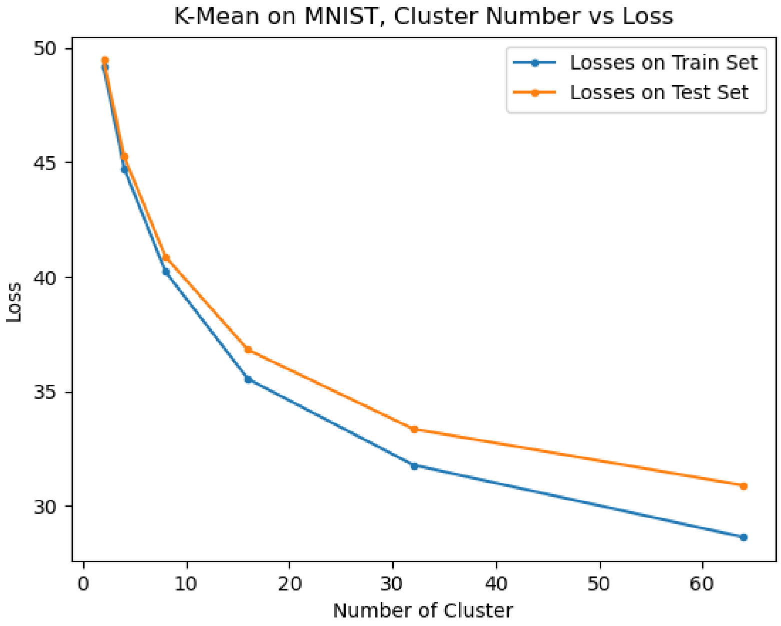


And these are 10 of the centroids identified by k-means, visualized as $28 \times 28 matrices$:



A5.c

For this part, I traied the model with values of $k \in \{2, 4, 8, 16, 32, 64\}$ on the whole normalized training MNIST dataset, and the value of the loss function after the algorithm converged are plot against each value of k, and this is the graph:
Code A5.a



A6: ML in the Real World

A6.a: Disease Susceptibility Predictor

Assuming all the data are collected correctly and filled in and “None”, “Nan” are not in the dataset.

We are going to use the lasso regression to look for the best predictors for the system. More specifically we are look for the shrinkage of difference factors to determine the most relavent factors that cause the disease.

During this stage of development, ask experts about the identified factors, and then improve models like, group lasso, or lasso ridge method etc to get more alternatives the most important group of identifiers.

After the most relavent factors are identified, we will only use those predictor and then Logistic Regression, or Neural Networks, or whatever binary classifier models that get’s the best test accuracy.

A6.b: Social Media App Facial Recognition Technology

I will look for a pretrained YOLO V3 network, this model can frame the item in the image with a label. And it’s fast. A homebrew Neural CNN can do the job, but it’s really slow, and it might not work as well since it’s just a classifier and can’t really identify the objects with its position in the photo (The bounding box with labels). If we were going to make our own neural network, then we will have to make the output of the classifier corresponds to different region of the image, and each region should output a label. And the loss function will have to be tailored for this usage, and we will be consider using the Residual Network for dimensionality reduction in this case.

Since that data is only comming from employees and their families, we might need to augment the dataset with a monochrome filters, this is an attempt to avoid biases created by the limited dataset.

Let’s hope on of the ways can work out.

A6.c: Malware Detection

For binary executable file, I will consider the usage of deep Recurrent Neural Network. However, it's possible that we have executable source code (because the question says: "**Including its contents**"), such as javascript or python script, so it's preferable to train a model for the file content for each of these above cases.

For binary file, we would need to decode it to assembly, and then train the data on Deep Recurrent Neural net, with classification loss function. The model is non-parametric so it's easy to scale. It has the challenge of vanishing and exploding gradient.

If, the above RNN doesn't work well, we are considering using BERT model as a classifier as the alternative, training on the Assembly code of course.

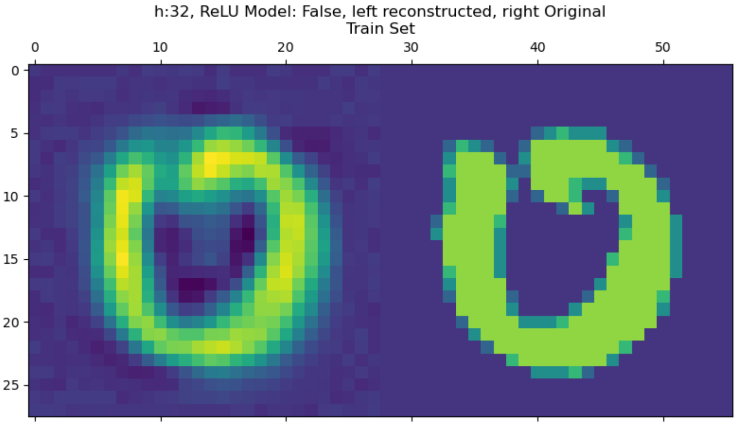
For source code, we will need to parse it into graph that keep track of all the variable flows (Decide on all the branhches the code and take and how it changes the dictionary of variables.), which should get pass obfuscated code. And then, we exract out all the API calls for the JS scripting languages, like system call, file IOs, network connection, etc. Then, we represent those elements as cannonical vectors, they are like the keywords we are interested in. Then, one vector represents a computer program. And then we are using these vector to train a Neural Net classifier.

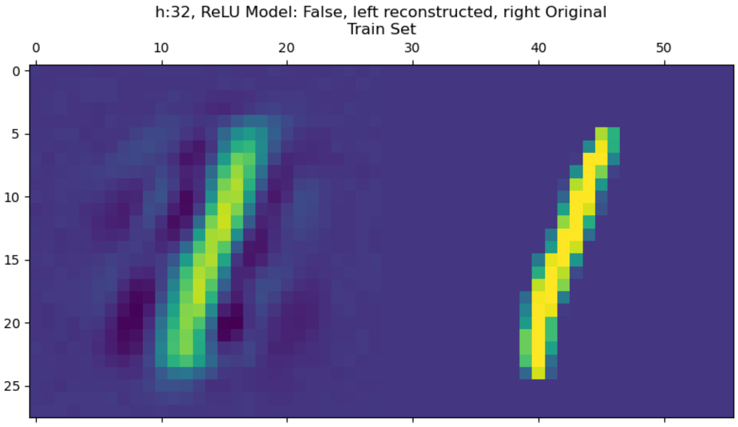
For meta data, we just include it together as an arrtibute for the computer program. If it's the binary content, then we encode that as part of the Assembly sequence. If it's source code, then we add those in as a vector as well. Both are non-parametric and accurate predictor and it should scale well.

Appendix

Extra A4Plots

Extra Plots for Linear Model

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Train Set
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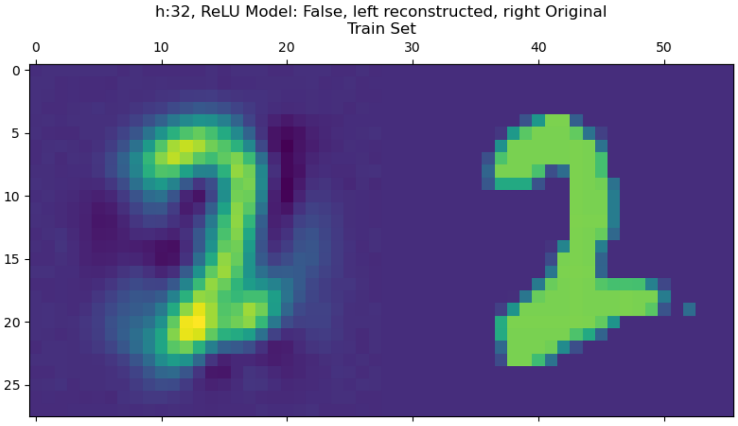
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Train Set
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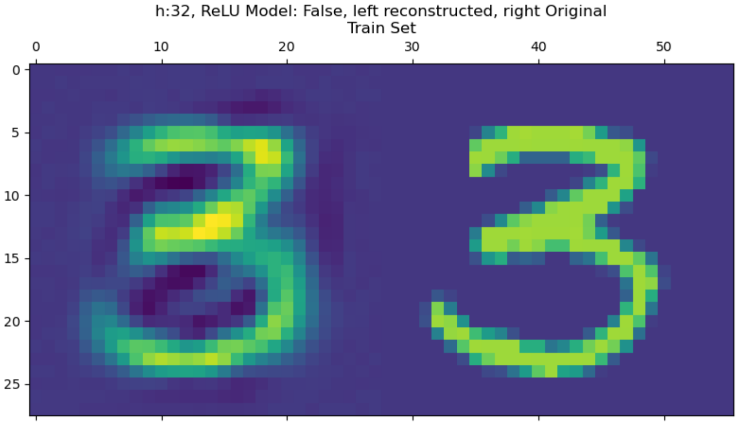
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Train Set
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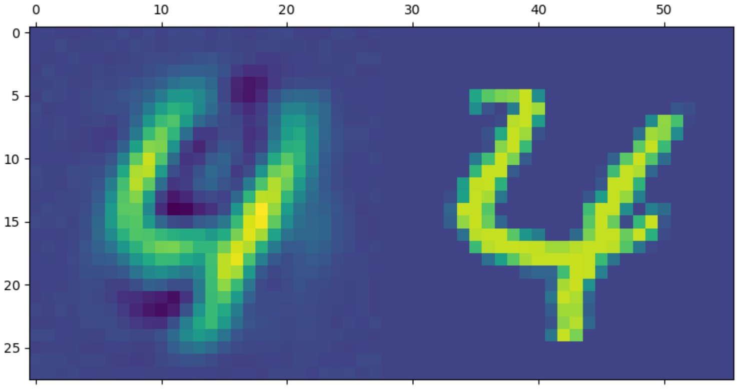
Train Set



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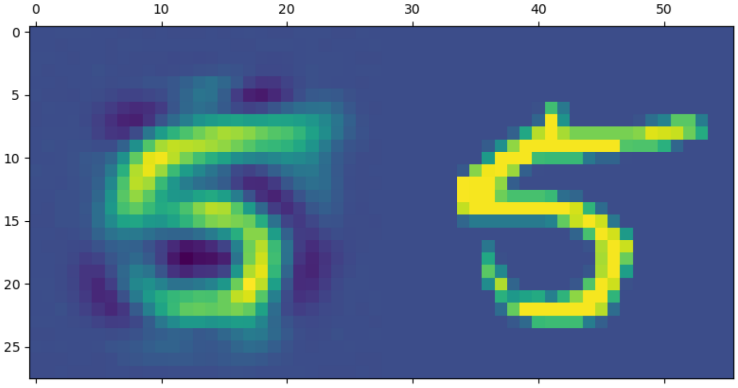
Train Set



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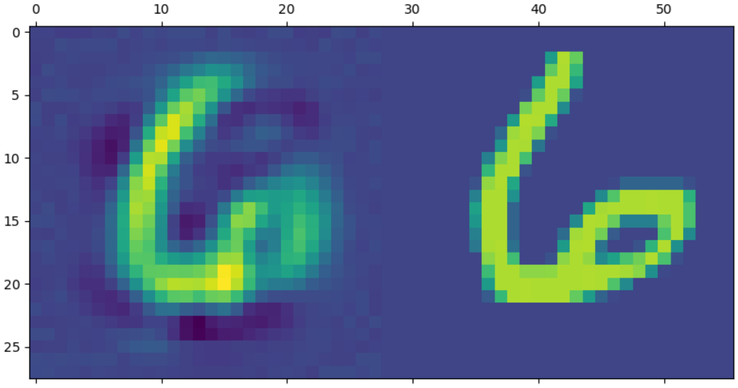
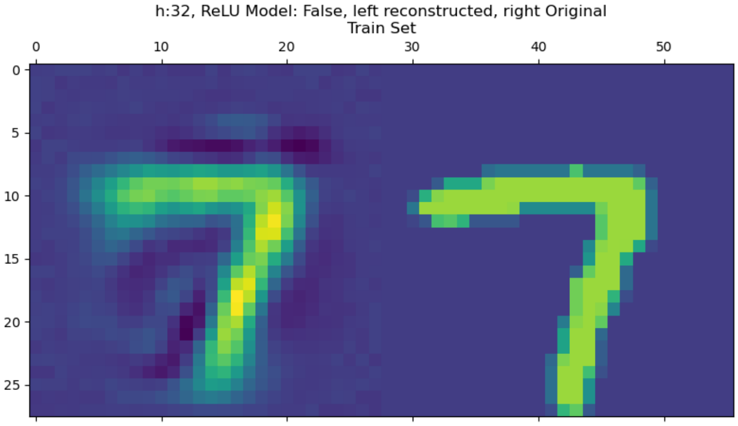
Train Set

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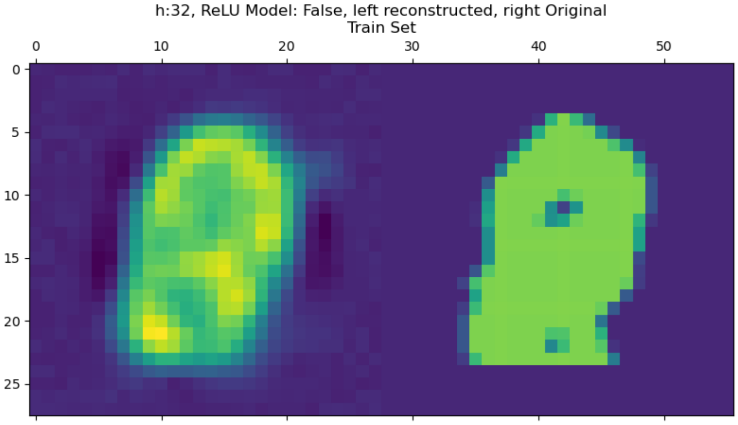
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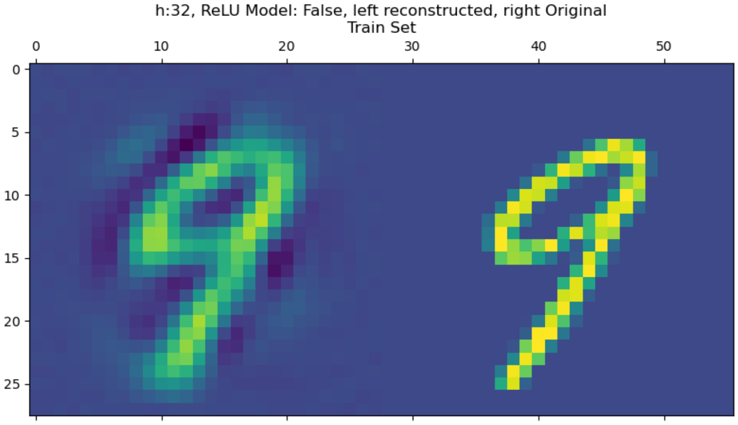
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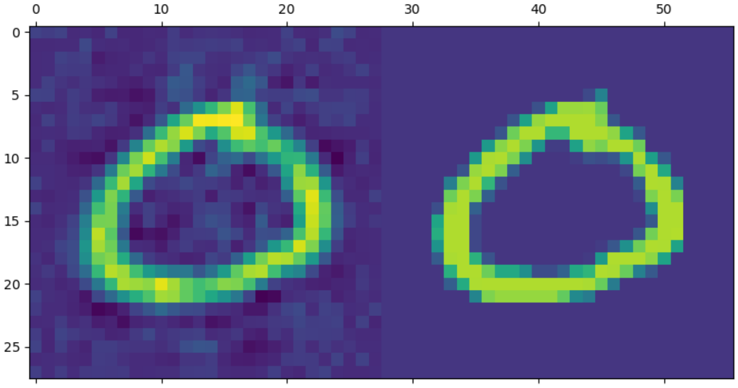
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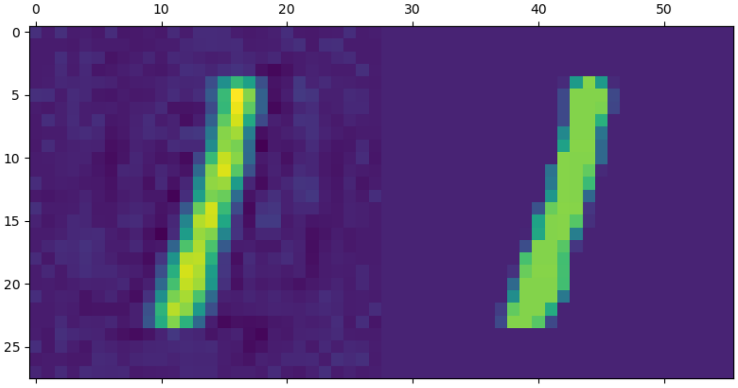
Train Set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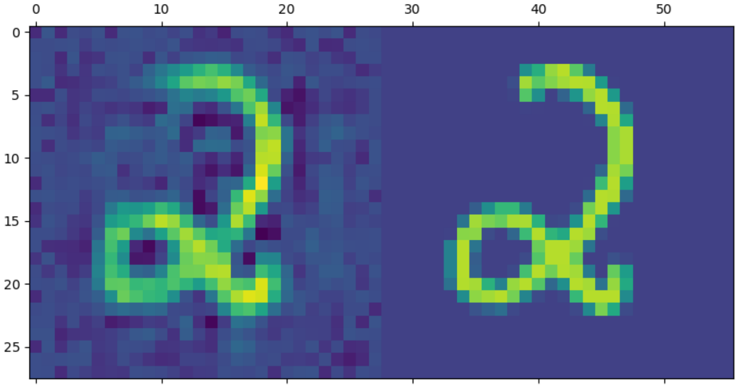
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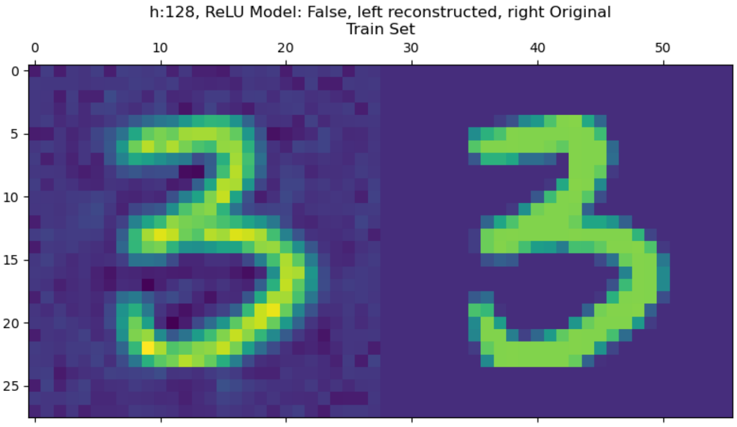
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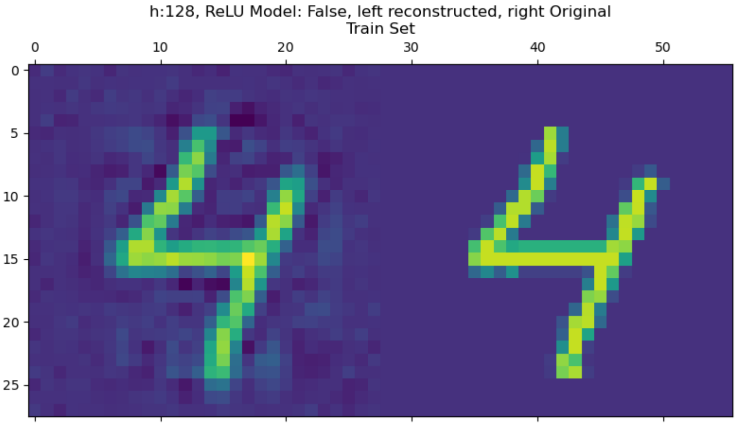
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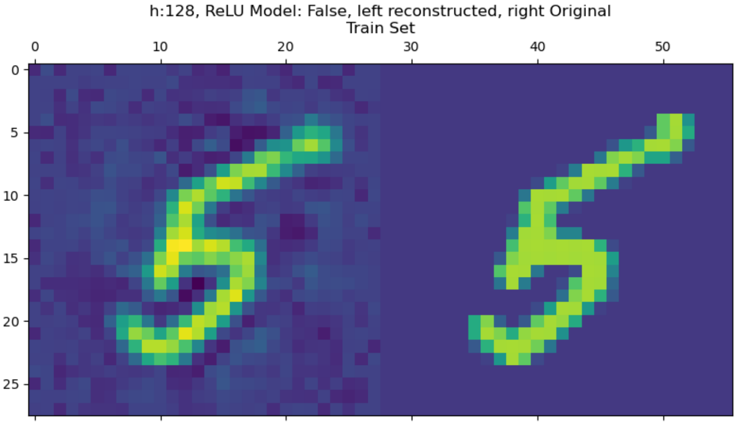
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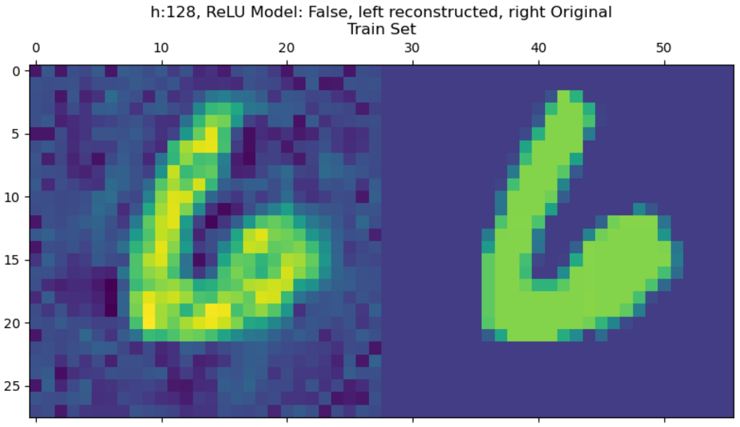
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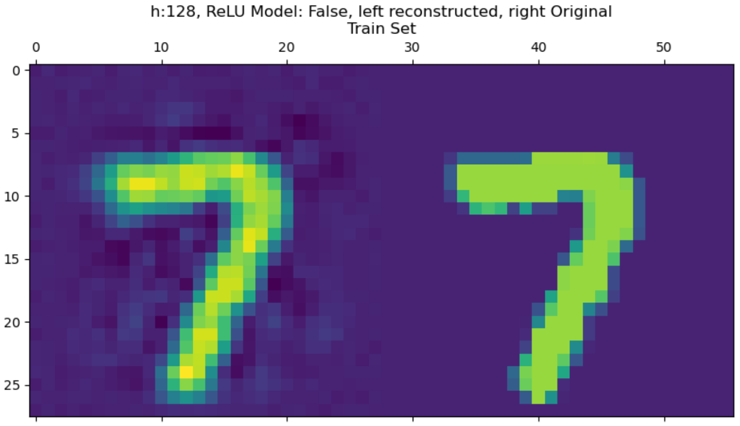
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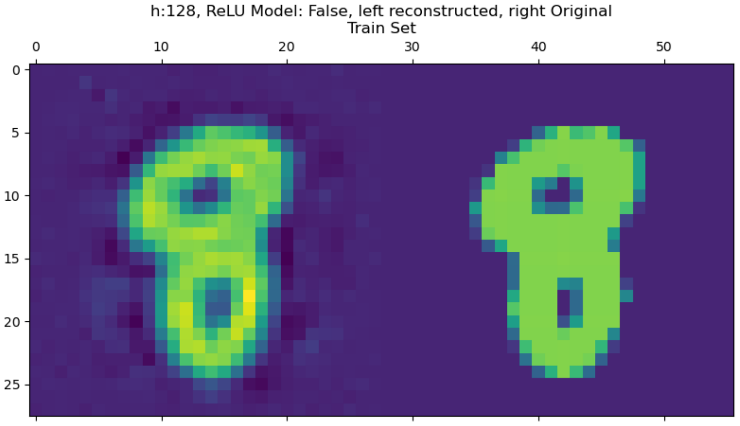
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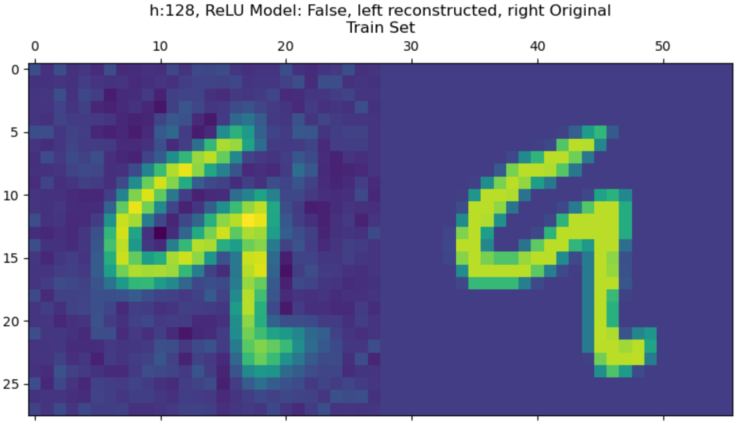
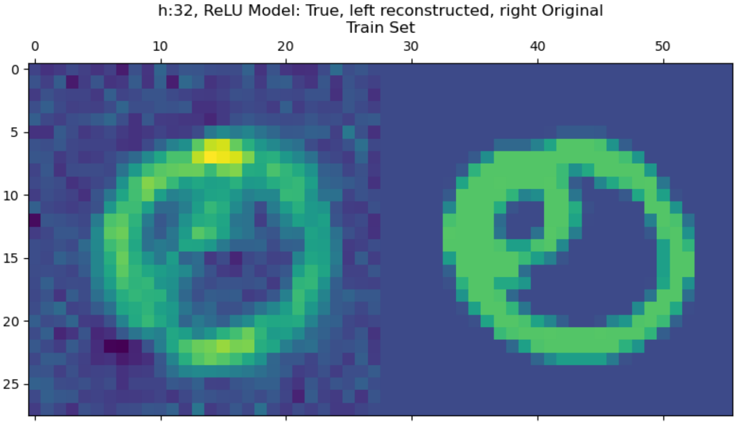
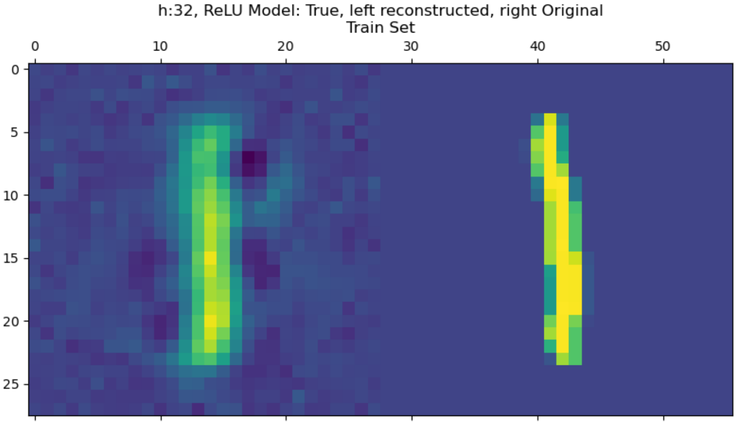
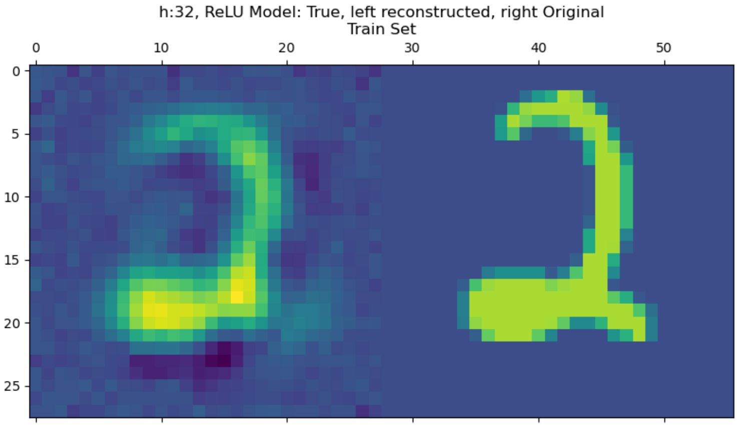


Extra Plots for Linear Model



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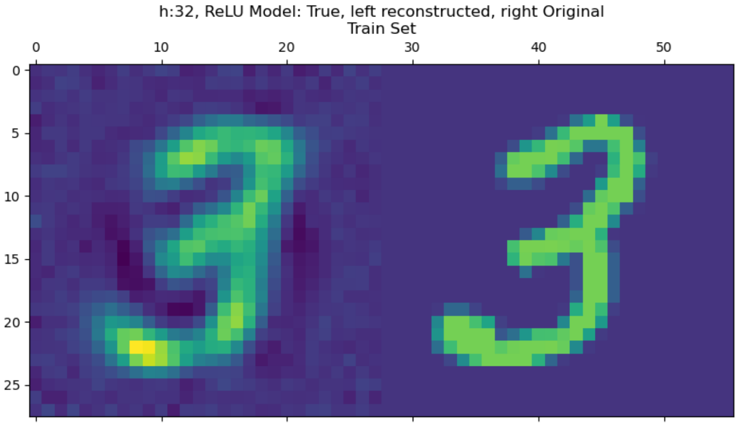
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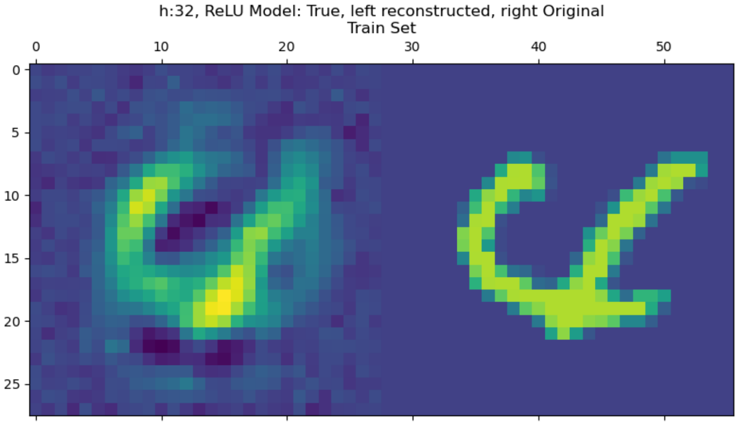
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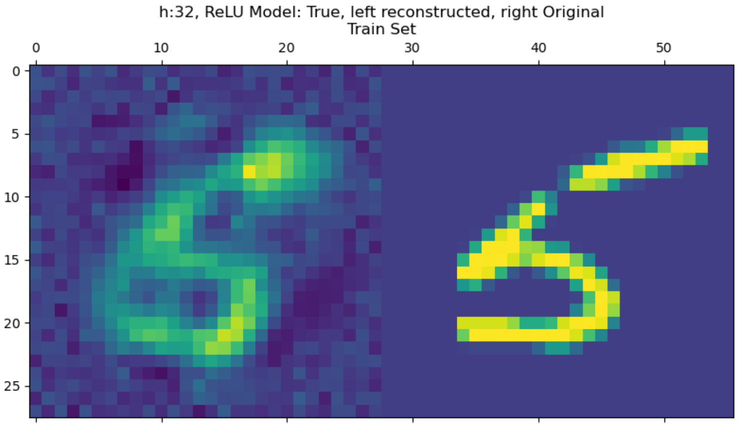
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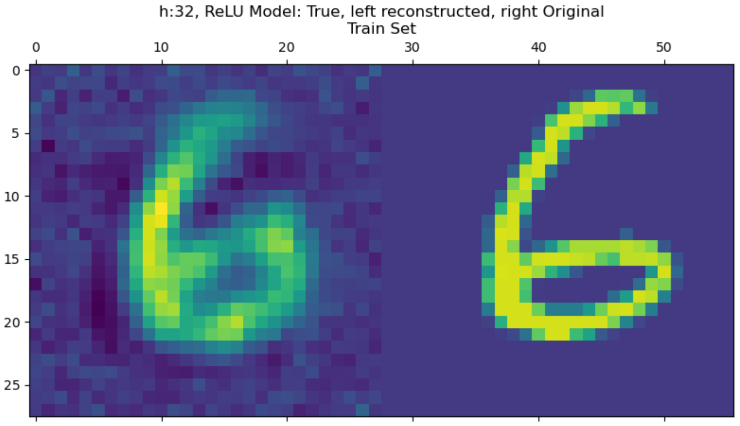
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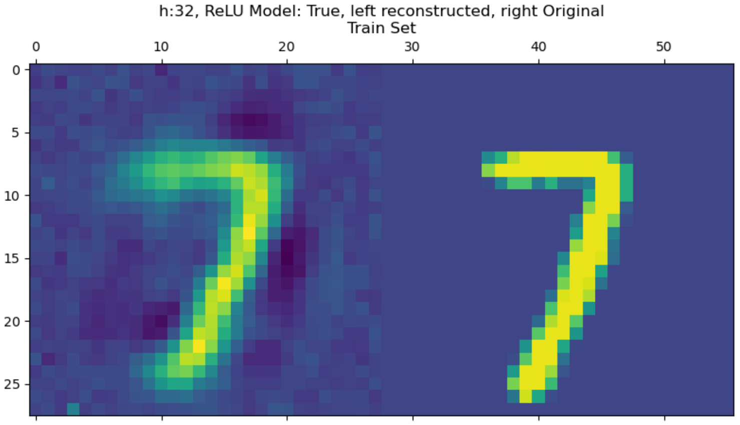
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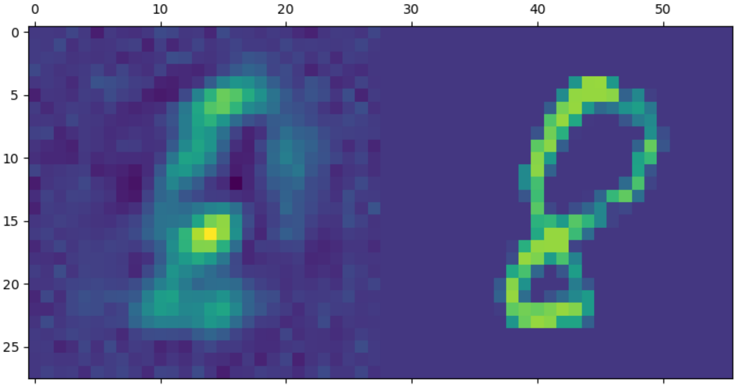
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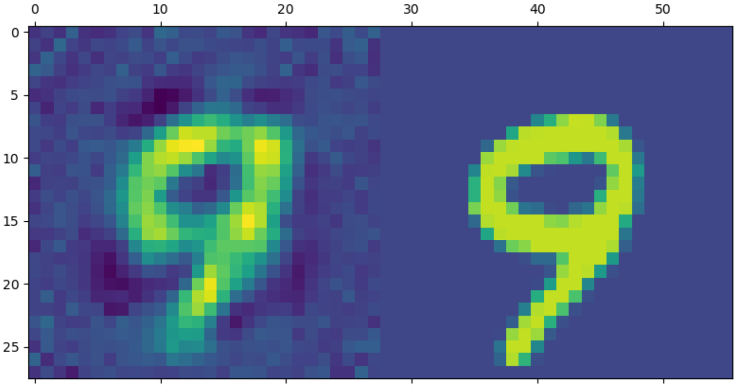
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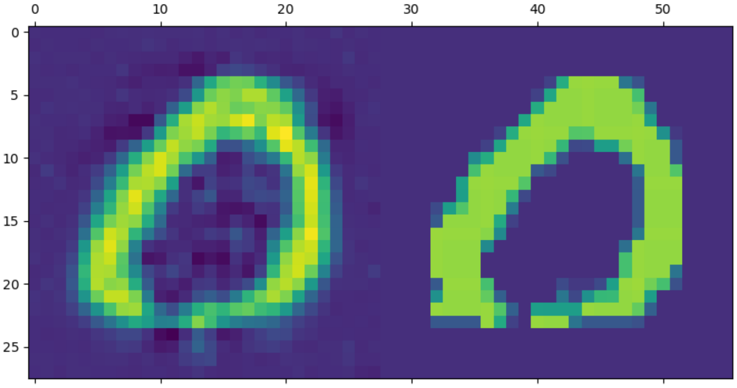
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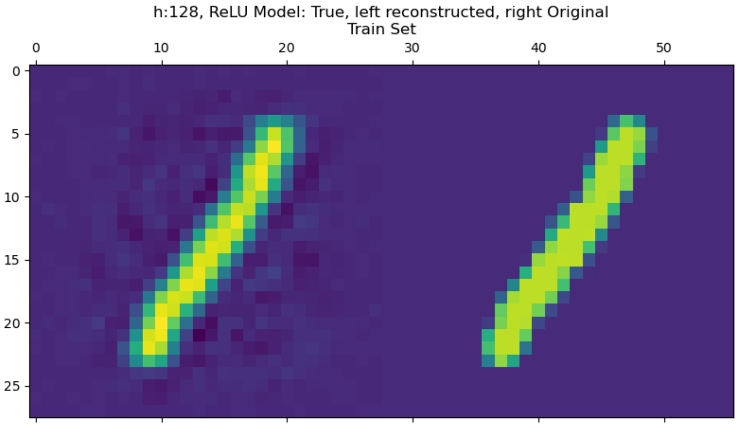
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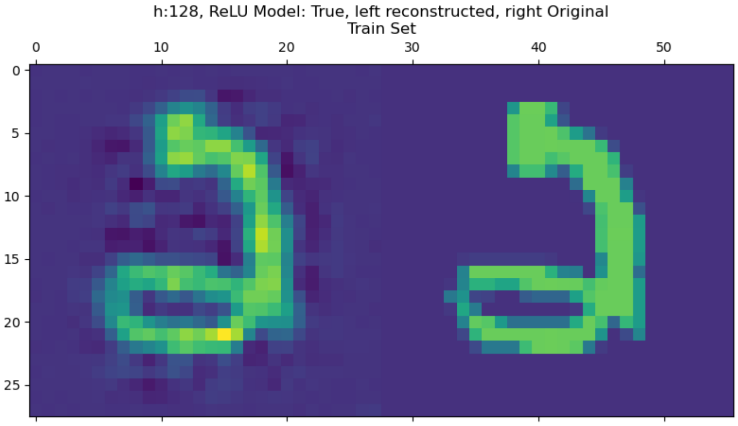
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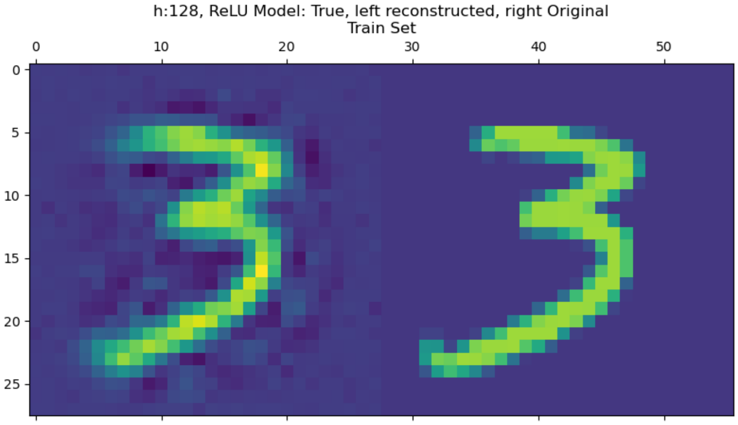
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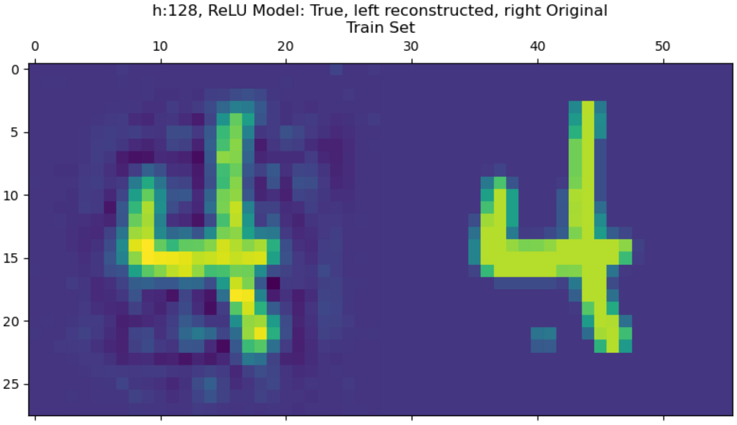
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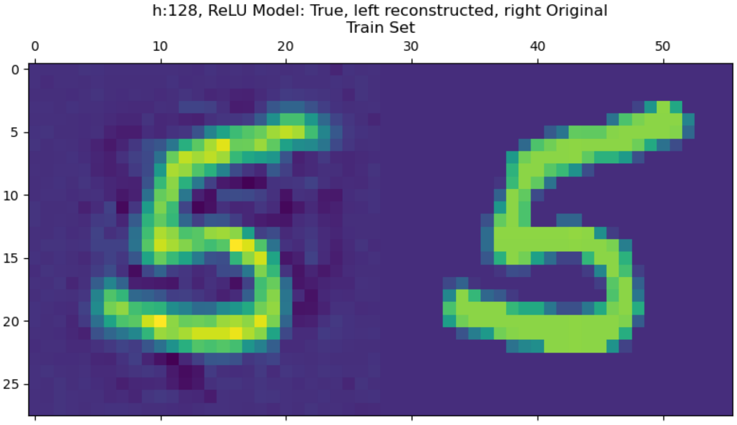
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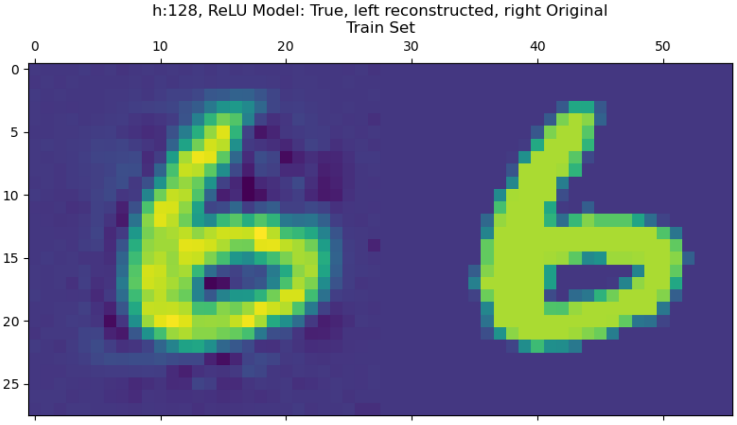
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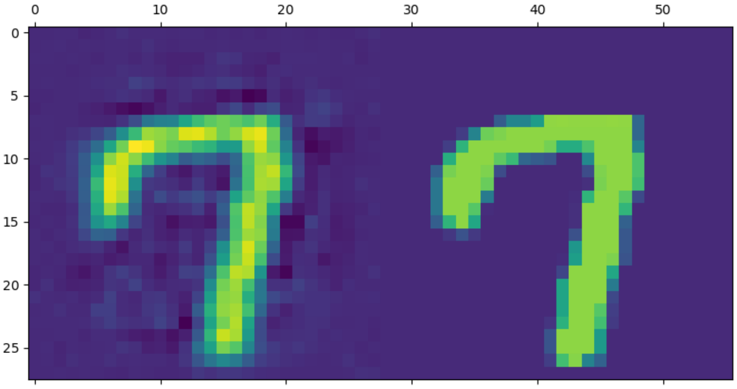

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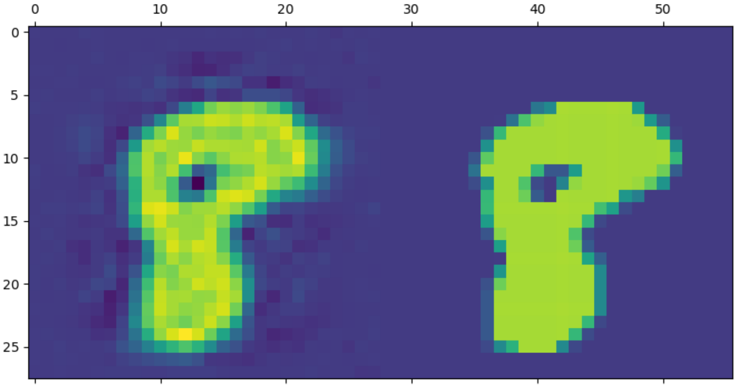

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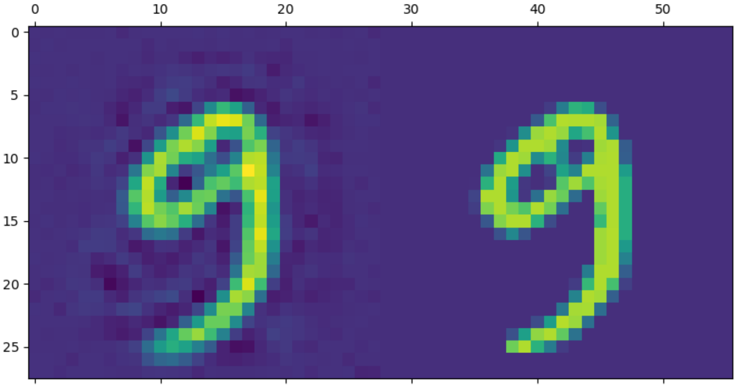
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