6.9 Summary of Prox Computations

$f(\mathbf{x})$	$\mathrm{dom}(f)$	$\mathrm{prox}_f(\mathbf{x})$	Assumptions	Reference
$\frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x} + c$	$\mathbb{R}^n$	$(\mathbf{A}+\mathbf{I})^{-1}(\mathbf{x}-\mathbf{b})$	$\mathbf{A} \in \mathbb{S}^n_+$, $\mathbf{b} \in \mathbb{R}^n$, $c \in \mathbb{R}$	Section 6.2.3
λx^3	$\mathbb{R}_+$	$\frac{-1+\sqrt{1+12\lambda[x]_+}}{6\lambda}$	$\lambda > 0$	Lemma 6.5
μx	$[0,\alpha] \cap \mathbb{R}$	$\min\{\max\{x-\mu,0\},\alpha\}$	$\mu \in \mathbb{R}$, $\alpha \in [0,\infty]$	Example 6.14
$\lambda\\|\mathbf{x}\\|$	$\mathbb{E}$	$\left(1-\frac{\lambda}{\max\{\\|\mathbf{x}\\|,\lambda\}}\right)\mathbf{x}$	$\\|\cdot\\|$—Euclidean norm, $\lambda > 0$	Example 6.19
$-\lambda\\|\mathbf{x}\\|$	$\mathbb{E}$	$\left(1+\frac{\lambda}{\\|\mathbf{x}\\|}\right)\mathbf{x}$, $\mathbf{x} \neq \mathbf{0}$, $\{\mathbf{u} : \\|\mathbf{u}\\| = \lambda\}$, $\mathbf{x} = \mathbf{0}$.	$\\|\cdot\\|$—Euclidean norm, $\lambda > 0$	Example 6.21
$\lambda\\|\mathbf{x}\\|_1$	$\mathbb{R}^n$	$\mathcal{T}_\lambda(\mathbf{x}) = [\lvert\mathbf{x}\rvert - \lambda\mathbf{e}]_+ \odot \mathrm{sgn}(\mathbf{x})$	$\lambda > 0$	Example 6.8
$\\|\boldsymbol{\omega} \odot \mathbf{x}\\|_1$	$\mathrm{Box}[-\boldsymbol{\alpha},\boldsymbol{\alpha}]$	$\mathcal{S}_{\boldsymbol{\omega},\boldsymbol{\alpha}}(\mathbf{x})$	$\boldsymbol{\alpha} \in [0,\infty]^n$, $\boldsymbol{\omega} \in \mathbb{R}^n_+$	Example 6.23
$\lambda\\|\mathbf{x}\\|_\infty$	$\mathbb{R}^n$	$\mathbf{x} - \lambda P_{B_{\\|\cdot\\|_1}[\mathbf{0},1]}(\mathbf{x}/\lambda)$	$\lambda > 0$	Example 6.48
$\lambda\\|\mathbf{x}\\|_a$	$\mathbb{E}$	$\mathbf{x} - \lambda P_{B_{\\|\cdot\\|_{a,*}}[\mathbf{0},1]}(\mathbf{x}/\lambda)$	$\\|\mathbf{x}\\|_a$—arbitrary norm, $\lambda > 0$	Example 6.47
$\lambda\\|\mathbf{x}\\|_0$	$\mathbb{R}^n$	$\mathcal{H}_{\sqrt{2\lambda}}(x_1) \times \cdots \times \mathcal{H}_{\sqrt{2\lambda}}(x_n)$	$\lambda > 0$	Example 6.10
$\lambda\\|\mathbf{x}\\|^3$	$\mathbb{E}$	$\frac{2}{1+\sqrt{1+12\lambda\\|\mathbf{x}\\|}}\mathbf{x}$	$\\|\cdot\\|$—Euclidean norm, $\lambda > 0$,	Example 6.20
$-\lambda\sum_{j=1}^n \log x_j$	$\mathbb{R}^n_{++}$	$\left(\frac{x_j+\sqrt{x_j^2+4\lambda}}{2}\right)_{j=1}^n$	$\lambda > 0$	Example 6.9
$\delta_C(\mathbf{x})$	$\mathbb{E}$	$P_C(\mathbf{x})$	$\emptyset \neq C \subseteq \mathbb{E}$	Theorem 6.24
$\lambda\sigma_C(\mathbf{x})$	$\mathbb{E}$	$\mathbf{x} - \lambda P_C(\mathbf{x}/\lambda)$	$\lambda > 0$, $C \neq \emptyset$ closed convex	Theorem 6.46
$\lambda\max\{x_i\}$	$\mathbb{R}^n$	$\mathbf{x} - \lambda P_{\Delta_n}(\mathbf{x}/\lambda)$	$\lambda > 0$	Example 6.49
$\lambda\sum_{i=1}^k x_{[i]}$	$\mathbb{R}^n$	$\mathbf{x} - \lambda P_C(\mathbf{x}/\lambda)$, $C = H_{\mathbf{e},k} \cap \mathrm{Box}[\mathbf{0},\mathbf{e}]$	$\lambda > 0$	Example 6.50
$\lambda\sum_{i=1}^k \lvert x_{\langle i\rangle}\rvert$	$\mathbb{R}^n$	$\mathbf{x} - \lambda P_C(\mathbf{x}/\lambda)$, $C = B_{\\|\cdot\\|_1}[\mathbf{0},k] \cap \mathrm{Box}[-\mathbf{e},\mathbf{e}]$	$\lambda > 0$	Example 6.51
$\lambda M_f^\mu(\mathbf{x})$	$\mathbb{E}$	$\mathbf{x} + \frac{\lambda}{\mu+\lambda}\left(\mathrm{prox}_{(\mu+\lambda)f}(\mathbf{x}) - \mathbf{x}\right)$	$\lambda,\mu > 0$, f proper closed convex	Corollary 6.64
$\lambda d_C(\mathbf{x})$	$\mathbb{E}$	$\mathbf{x} + \min\left\{\frac{\lambda}{d_C(\mathbf{x})},1\right\}(P_C(\mathbf{x}) - \mathbf{x})$	$\emptyset \neq C$ closed convex, $\lambda > 0$	Lemma 6.43
$\frac{\lambda}{2}d_C^2(\mathbf{x})$	$\mathbb{E}$	$\frac{\lambda}{\lambda+1}P_C(\mathbf{x}) + \frac{1}{\lambda+1}\mathbf{x}$	$\emptyset \neq C$ closed convex, $\lambda > 0$	Example 6.65
$\lambda H_\mu(\mathbf{x})$	$\mathbb{E}$	$\left(1-\frac{\lambda}{\max\{\\|\mathbf{x}\\|,\mu+\lambda\}}\right)\mathbf{x}$	$\lambda,\mu > 0$	Example 6.66
$\rho\\|\mathbf{x}\\|_1^2$	$\mathbb{R}^n$	$\left(\frac{v_i x_i}{v_i+2\rho}\right)_{i=1}^n$, $\mathbf{v} = \left[\sqrt{\frac{\rho}{\mu}}\lvert\mathbf{x}\rvert - 2\rho\right]_+$, $\mathbf{e}^T\mathbf{v} = 1$ ($\mathbf{0}$ when $\mathbf{x} = \mathbf{0}$)	$\rho > 0$	Lemma 6.70
$\lambda\\|\mathbf{A}\mathbf{x}\\|_2$	$\mathbb{R}^n$	$\mathbf{x} - \mathbf{A}^T(\mathbf{A}\mathbf{A}^T + \alpha^*\mathbf{I})^{-1}\mathbf{A}\mathbf{x}$, $\alpha^* = 0$ if $\\|\mathbf{v}_0\\|_2 \leq \lambda$; otherwise, $\\|\mathbf{v}_{\alpha^*}\\|_2 = \lambda$; $\mathbf{v}_\alpha \equiv (\mathbf{A}\mathbf{A}^T + \alpha\mathbf{I})^{-1}\mathbf{A}\mathbf{x}$	$\mathbf{A} \in \mathbb{R}^{m\times n}$ with full row rank, $\lambda > 0$	Lemma 6.68