

Lorum ipsum

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March 2, 2020

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Abstract

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2010 Mathematics Subject Classification: Primary 47H05, 52A41, 90C25; Secondary 15A09, 26A51, 26B25, 26E60, 47H09, 47A63. **Keywords:**

1 Schur's Cone and Its Polar

Example 1.1 Define

$$\{eqn:cone-1\} \quad Q = \left\{ x \in \mathbb{R}^n \left| \sum_{i=1}^k x_i \geq 0, \, k = 1, \dots, n-1, \, x_1 + \dots + x_n = 0 \right. \right\}. \quad (1.1)$$

Then

$$Q^\circ = \{ y \in \mathbb{R}^n | \langle y, x \rangle \geq 0 \, \forall x \in Q \} \quad (1.2)$$

$$= \{ y \in \mathbb{R}^n | y_1 \geq y_2 \geq \dots \geq y_n \}. \quad (1.3)$$

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Proof. We have $Q = H(\mathbb{R}_+^{n-1})$, where

$$H = \begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ & -1 & \ddots & \\ & & \ddots & 1 \\ & & & -1 \end{bmatrix}, \quad (1.4)$$

H is a bi-diagonal matrix with zeros on the diagonal and -1 on its sub-diagonals. Indeed, for $x = (x_1, \dots, x_{n-1}) \in \mathbb{R}_+^{n-1}$, we have

$$y = Hx \quad (1.5)$$

$$= \begin{bmatrix} 1 \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \\ -1 \\ \vdots \\ 0 \end{bmatrix} x_2 + \dots + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ -1 \end{bmatrix} x_{n-1} \quad (1.6)$$

$$= \begin{bmatrix} x_1 \\ x_2 - x_1 \\ x_3 - x_2 \\ \vdots \\ -x_{n-2} + x_{n-1} \\ -x_{n-1} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-1} \\ y_n \end{bmatrix}, \quad (1.7)$$

where $y_1 \geq 0, y_2 + y_1 \geq 0, \dots, y_{n-1} + \dots + y_1 \geq 0$, and $y_1 + \dots + y_n = 0$. So $y \in Q$. Hence $H(\mathbb{R}_+^{n-1}) \subseteq Q$. Similarly, one can write for $y \in Q$, $\exists x \in \mathbb{R}_+^{n-1}$ with $y = Hx$. In fact, $x = y_1, x_2 = y_1 + y_2, \dots, x_{n-1} = y_1 + \dots + y_{n-1}, -x_{n-1} = y_n$ because $y_1 + \dots + y_n = 0$. In particular, $x \in \mathbb{R}_+^{n-1}$.

Note that $y \in Q^\circ \iff y^\top a_i \geq 0$ for $i = 1, \dots, n-1$. Where

$$a_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ \vdots \\ 0 \end{bmatrix}, \dots, a_n = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ -1 \end{bmatrix}. \quad (1.8)$$

Now $y^\top a_i \geq 0 \iff y_i - y_{i+1} \geq 0$, for $i = 1, \dots, n-1$. Hence $Q^\circ = \{y \in \mathbb{R}^n | y_1 \geq y_2 \geq \dots \geq y_n\}$.

■

2 Lorentz Cone and its Polar

Example 2.1

$$L = \{(\xi, t) \in \mathbb{R}^n \mid \|\xi\| \leq t, \xi \in \mathbb{R}^{n-1}\}. \quad (2.1)$$

Then the polar cone of L is $L^\circ = \{(x, s) \mid \langle (x, s), (\xi, t) \rangle \geq 0\} = L$.

Proof.

Let $(x, s) \in L^\circ$. Then $\langle (x, s), (\xi, t) \rangle \geq 0, \forall (\xi, t) \in L$, i. e. $\langle x, \xi \rangle + st \geq 0$.

For $t > 0$, we have $\langle x, \xi/t \rangle \geq -s$, i.e. $\langle x, -\xi/t \rangle \leq s$. Using $\|\xi\| \leq t$ and by taking supremum over $\|\xi/t\| \leq 1$, we obtain $\|x\| \leq s$.

Hence $L^\circ = L$. ■

Corollary 2.2 *Let $A \in \mathbb{R}^{n \times n}$ be positive definite. Define*

$$L_A = \left\{ (\xi, t) \in \mathbb{R}^n \mid \sqrt{\langle \xi, A\xi \rangle} \leq t \right\}.$$

Then $(L_A)^\circ = \{(\xi, t) \mid \sqrt{\langle \xi, A^{-1}\xi \rangle} \leq t\}$.

3 Testing the references

This book by Nesterov [\[2\]](#). This book by Heinz [\[1\]](#).

4 Testing figures and images

In Figure [1](#), it has Figure [1a](#) on the left and Figure [1b](#) on the right.



(a) Alto the unicorn



(b) Alto and Minty.

{fig:two-test-fig-1}
{fig:two-test-figures}

{fig:two-test-fig-2}

Figure 1: The above 2 images are test images for the figure, subfigure latex commands.

Acknowledgments

XW were supported by NSERC Discovery grants.

References

- [1] H. H. BAUSCHKE AND P. L. COMBETTES, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, CMS Books in Mathematics, Springer International Publishing, Cham, 2017.
- [2] Y. NESTEROV, *Lectures on Convex Optimization*, vol. 137 of Springer Optimization and Its Applications, Springer International Publishing, 2018.