Appendix A

Strong Duality and Optimality Conditions

The following strong duality theorem is taken from [29, Proposition 6.4.4].

Theorem A.1 (strong duality theorem). *Consider the optimization problem*

$$
\begin{aligned}
f_{\text{opt}} = \quad \min \quad & f(\mathbf{x}) \\
\text{s.t.} \quad & g_i(\mathbf{x}) \le 0, \quad i = 1, 2, \ldots, m, \\
& h_j(\mathbf{x}) \le 0, \quad j = 1, 2, \ldots, p, \\
& s_k(\mathbf{x}) = 0, \quad k = 1, 2, \ldots, q, \\
& \mathbf{x} \in X,
\end{aligned}
\tag{A.1}
$$

where $X = P \cap C$ with $P \subseteq \mathbb{E}$ being a convex polyhedral set and $C \subseteq \mathbb{E}$ convex. The functions $f, g_i, i = 1, 2, \ldots, m : \mathbb{E} \to (-\infty, \infty]$ are convex, and their domains satisfy $X \subseteq \text{dom}(f), X \subseteq \text{dom}(g_i)$, $i = 1, 2, \ldots, m$. The functions $h_j, s_k, j = 1, 2, \ldots, p, k = 1, 2, \ldots, q$, are affine functions. Suppose there exist

(i) *a feasible solution $\bar{\mathbf{x}}$ satisfying $g_i(\bar{\mathbf{x}}) < 0$ for all $i = 1, 2, \ldots, m$;*

(ii) *a vector satisfying all the affine constraints $h_j(\mathbf{x}) \le 0, j = 1, 2, \ldots, p$, $s_k(\mathbf{x}) = 0, k = 1, 2, \ldots, q$, and that is in $P \cap \text{ri}(C)$.*

Then if problem (A.1) *has a finite optimal value, then the optimal value of the dual problem*

$$q_{\text{opt}} = \max\{q(\boldsymbol{\lambda}, \boldsymbol{\eta}, \boldsymbol{\mu}) : (\boldsymbol{\lambda}, \boldsymbol{\eta}, \boldsymbol{\mu}) \in \text{dom}(-q)\},$$

where $q : \mathbb{R}^m_+ \times \mathbb{R}^p_+ \times \mathbb{R}^q \to \mathbb{R} \cup \{-\infty\}$ is given by

$$
\begin{aligned}
q(\boldsymbol{\lambda}, \boldsymbol{\eta}, \boldsymbol{\mu}) &= \min_{\mathbf{x} \in X} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\eta}, \boldsymbol{\mu}) \\
&= \min_{\mathbf{x} \in X} \left[f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i g_i(\mathbf{x}) + \sum_{j=1}^{p} \eta_j h_j(\mathbf{x}) + \sum_{k=1}^{q} \mu_k s_k(\mathbf{x}) \right],
\end{aligned}
$$

is attained, and the optimal values of the primal and dual problems are the same:

$$f_{\mathrm{opt}} = q_{\mathrm{opt}}.$$

We also recall some well-known optimality conditions expressed in terms of the Lagrangian function in cases where strong duality holds.

Theorem A.2 (optimality conditions under strong duality). *Consider the problem*

$$(\mathrm{P}) \qquad \begin{array}{ll} \min & f(\mathbf{x}) \\ s.t. & g_i(\mathbf{x}) \le 0, i = 1, 2, \ldots, m, \\ & h_j(\mathbf{x}) = 0, j = 1, 2, \ldots, p, \\ & \mathbf{x} \in X, \end{array}$$

where $f, g_1, g_2, \ldots, g_m, h_1, h_2, \ldots, h_p : \mathbb{E} \to (-\infty, \infty]$, *and* $X \subseteq \mathbb{E}$. *Assume that* $X \subseteq \mathrm{dom}(f)$, $X \subseteq \mathrm{dom}(g_i)$, *and* $X \subseteq \mathrm{dom}(h_j)$ *for all* $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, p$. *Let* (D) *be the following dual problem:*

$$(\mathrm{D}) \qquad \begin{array}{ll} \max & q(\boldsymbol{\lambda}, \boldsymbol{\mu}) \\ s.t. & (\boldsymbol{\lambda}, \boldsymbol{\mu}) \in \mathrm{dom}(-q), \end{array}$$

where

$$q(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \min_{\mathbf{x} \in X} \left\{ L(\mathbf{x}; \boldsymbol{\lambda}, \boldsymbol{\mu}) \equiv f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i g_i(\mathbf{x}) + \sum_{j=1}^{p} \mu_j h_j(\mathbf{x}) \right\},$$
$$\mathrm{dom}(-q) = \{(\boldsymbol{\lambda}, \boldsymbol{\mu}) \in \mathbb{R}^m_+ \times \mathbb{R}^p : q(\boldsymbol{\lambda}, \boldsymbol{\mu}) > -\infty\}.$$

Suppose that the optimal value of problem (P) *is finite and equal to the optimal value of problem* (D). *Then* $\mathbf{x}^*, (\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$ *are optimal solutions of problems* (P) *and* (D), *respectively, if and only if*

(i) $\mathbf{x}^*$ *is a feasible solution of (P);*

(ii) $\boldsymbol{\lambda}^* \ge \mathbf{0}$*;*

(iii) $\lambda_i^* g_i(\mathbf{x}^*) = 0, i = 1, 2, \ldots, m$*;*

(iv) $\mathbf{x}^* \in \mathrm{argmin}_{\mathbf{x} \in X} L(\mathbf{x}; \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$.

Proof. Denote the optimal values of problem (P) and (D) by f_{opt} and q_{opt}, respectively. An underlying assumption of the theorem is that $f_{\mathrm{opt}} = q_{\mathrm{opt}}$. If $\mathbf{x}^*$ and $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$ are the optimal solutions of problems (P) and (D), then obviously (i) and

(ii) are satisfied. In addition,

$$\begin{aligned} f_{\text{opt}} = q_{\text{opt}} &= q(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) \\ &= \min_{\mathbf{x}\in X} L(\mathbf{x}, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) \\ &\le L(\mathbf{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) \\ &= f(\mathbf{x}^*) + \sum_{i=1}^m \lambda_i^* g_i(\mathbf{x}^*) + \sum_{j=1}^p \mu_j^* h_j(\mathbf{x}^*) \\ &\le f(\mathbf{x}^*), \end{aligned}$$

where the last inequality follows by the facts that $h_j(\mathbf{x}^*) = 0$, $\lambda_i^* \ge 0$, and $g_i(\mathbf{x}^*) \le 0$. Since $f_{\text{opt}} = f(\mathbf{x}^*)$, all of the inequalities in the above chain of equalities and inequalities are actually equalities. This implies in particular that $\mathbf{x}^* \in \operatorname{argmin}_{\mathbf{x}\in X} L(\mathbf{x}, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$, meaning property (iv), and that $\sum_{i=1}^m \lambda_i^* g_i(\mathbf{x}^*) = 0$, which by the fact that $\lambda_i^* g_i(\mathbf{x}^*) \le 0$ for any i, implies that $\lambda_i^* g_i(\mathbf{x}^*) = 0$ for any i, showing the validity of property (iii).

To prove the reverse direction, assume that properties (i)–(iv) are satisfied. Then

$$\begin{aligned} q(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) &= \min\nolimits_{\mathbf{x}\in X} L(\mathbf{x}, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) && [\text{definition of } q] \\ &= L(\mathbf{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) && [\text{property (iv)}] \\ &= f(\mathbf{x}^*) + \sum_{i=1}^m \lambda_i^* g_i(\mathbf{x}^*) + \sum_{j=1}^p \mu_j^* h_j(\mathbf{x}^*) \\ &= f(\mathbf{x}^*). && [\text{property (iii)}] \end{aligned}$$

By the weak duality theorem, since $\mathbf{x}^*$ and $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$ are primal and dual feasible solutions with equal primal and dual objective functions, it follows that they are the optimal solutions of their corresponding problems. □

Appendix B

Tables

Support Functions

C	$\sigma_C(\mathbf{y})$	Assumptions	Reference
$\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$	$\max_{i=1,2,\ldots,n} \langle \mathbf{b}_i, \mathbf{y} \rangle$	$\mathbf{b}_i \in \mathbb{E}$	Example 2.25
K	$\delta_{K^\circ}(\mathbf{y})$	K – cone	Example 2.26
$\mathbb{R}^n_+$	$\delta_{\mathbb{R}^n_-}(\mathbf{y})$	$\mathbb{E} = \mathbb{R}^n$	Example 2.27
Δ_n	$\max\{y_1, y_2, \ldots, y_n\}$	$\mathbb{E} = \mathbb{R}^n$	Example 2.36
$\{\mathbf{x} \in \mathbb{R}^n : \mathbf{A}\mathbf{x} \leq \mathbf{0}\}$	$\delta_{\{\mathbf{A}^T\boldsymbol{\lambda}:\boldsymbol{\lambda}\in\mathbb{R}^m_+\}}(\mathbf{y})$	$\mathbb{E} = \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{m\times n}$	Example 2.29
$\{\mathbf{x} \in \mathbb{R}^n : \mathbf{B}\mathbf{x} = \mathbf{b}\}$	$\langle \mathbf{y}, \mathbf{x}_0 \rangle + \delta_{\mathrm{Range}(\mathbf{B}^T)}(\mathbf{y})$	$\mathbb{E} = \mathbb{R}^n$, $\mathbf{B} \in \mathbb{R}^{m\times n}$, $\mathbf{b} \in \mathbb{R}^m$, $\mathbf{B}\mathbf{x}_0 = \mathbf{b}$	Example 2.30
$B_{\|\cdot\|}[\mathbf{0}, 1]$	$\|\mathbf{y}\|_*$	-	Example 2.31

Weak Subdifferential Results

Function	Weak result	Setting	Reference
$-q$ = negative dual function	$-\mathbf{g}(\mathbf{x}_0) \in \partial(-q)(\boldsymbol{\lambda}_0)$	$q(\boldsymbol{\lambda}) = \min_{\mathbf{x}\in X} f(\mathbf{x}) + \boldsymbol{\lambda}^T\mathbf{g}(\mathbf{x})$, $f : \mathbb{E} \to \mathbb{R}$, $\mathbf{g} : \mathbb{E} \to \mathbb{R}^m$, $\mathbf{x}_0$ = a minimizer of $f(\mathbf{x}) + \boldsymbol{\lambda}_0^T\mathbf{g}(\mathbf{x})$ over X	Example 3.7
$f(\mathbf{X}) = \lambda_{\max}(\mathbf{X})$	$\mathbf{v}\mathbf{v}^T \in \partial f(\mathbf{X})$	$f : \mathbb{S}^n \to \mathbb{R}$, $\mathbf{v}$ = normalized maximum eigenvector of $X \in \mathbb{S}^n$	Example 3.8
$f(\mathbf{x}) = \|\mathbf{x}\|_1$	$\mathrm{sgn}(\mathbf{x}) \in \partial f(\mathbf{x})$	$\mathbb{E} = \mathbb{R}^n$	Example 3.42
$f(\mathbf{x}) = \lambda_{\max}(\mathbf{A}_0 + \sum_{i=1}^m x_i\mathbf{A}_i)$	$(\tilde{\mathbf{y}}^T\mathbf{A}_i\tilde{\mathbf{y}})_{i=1}^m \in \partial f(\mathbf{x})$	$\tilde{\mathbf{y}}$ = normalized maximum eigenvector of $\mathbf{A}_0 + \sum_{i=1}^m x_i\mathbf{A}_i$	Example 3.56

Strong Subdifferential Results

$f(\mathbf{x})$	$\partial f(\mathbf{x})$	Assumptions	Reference
$\\|\mathbf{x}\\|$	$B_{\\|\cdot\\|_*}[\mathbf{0},1]$	$\mathbf{x}=\mathbf{0}$	Example 3.3
$\\|\mathbf{x}\\|_1$	$\left\{\sum_{i\in I_{\neq}(\mathbf{x})}\operatorname{sgn}(x_i)\mathbf{e}_i+\sum_{i\in I_0(\mathbf{x})}[-\mathbf{e}_i,\mathbf{e}_i]\right\}$	$\mathbb{E}=\mathbb{R}^n$, $I_{\neq}(\mathbf{x})=\{i:x_i\neq 0\}$, $I_0(\mathbf{x})=\{i:x_i=0\}$.	Example 3.41
$\\|\mathbf{x}\\|_2$	$\begin{cases}\left\{\frac{\mathbf{x}}{\\|\mathbf{x}\\|_2}\right\}, & \mathbf{x}\neq\mathbf{0},\\ B_{\\|\cdot\\|_2}[\mathbf{0},1], & \mathbf{x}=\mathbf{0}.\end{cases}$	$\mathbb{E}=\mathbb{R}^n$	Example 3.34
$\\|\mathbf{x}\\|_\infty$	$\left\{\sum_{i\in I(\mathbf{x})}\lambda_i\operatorname{sgn}(x_i)\mathbf{e}_i : \begin{array}{l}\sum_{i\in I(\mathbf{x})}\lambda_i=1\\ \lambda_i\geq 0\end{array}\right\}$	$\mathbb{E}=\mathbb{R}^n$, $I(\mathbf{x})=\{i:\\|\mathbf{x}\\|_\infty=\|x_i\|\}$, $\mathbf{x}\neq\mathbf{0}$	Example 3.52
$\max(\mathbf{x})$	$\left\{\sum_{i\in I(\mathbf{x})}\lambda_i\mathbf{e}_i:\sum_{i\in I(\mathbf{x})}\lambda_i=1,\lambda_i\geq 0\right\}$	$\mathbb{E}=\mathbb{R}^n$, $I(\mathbf{x})=\{i:\max(\mathbf{x})=x_i\}$	Example 3.51
$\max(\mathbf{x})$	Δ_n	$\mathbb{E}=\mathbb{R}^n$, $\mathbf{x}=\alpha\mathbf{e}$ for some $\alpha\in\mathbb{R}$	Example 3.51
$\delta_S(\mathbf{x})$	$N_S(\mathbf{x})$	$\emptyset\neq S\subseteq\mathbb{E}$	Example 3.5
$\delta_{B[\mathbf{0},1]}(\mathbf{x})$	$\begin{cases}\{\mathbf{y}\in\mathbb{E}^*:\\|\mathbf{y}\\|_*\leq\langle\mathbf{y},\mathbf{x}\rangle\}, & \\|\mathbf{x}\\|\leq 1,\\ \emptyset, & \\|\mathbf{x}\\|>1.\end{cases}$		Example 3.6
$\\|\mathbf{A}\mathbf{x}+\mathbf{b}\\|_1$	$\sum_{i\in I_{\neq}(\mathbf{x})}\operatorname{sgn}(\mathbf{a}_i^T\mathbf{x}+b_i)\mathbf{a}_i+\sum_{i\in I_0(\mathbf{x})}[-\mathbf{a}_i,\mathbf{a}_i]$	$\mathbb{E}=\mathbb{R}^n$, $\mathbf{A}\in\mathbb{R}^{m\times n}$, $\mathbf{b}\in\mathbb{R}^m$, $I_{\neq}(\mathbf{x})=\{i:\mathbf{a}_i^T\mathbf{x}+b_i\neq 0\}$, $I_0(\mathbf{x})=\{i:\mathbf{a}_i^T\mathbf{x}+b_i=0\}$	Example 3.44
$\\|\mathbf{A}\mathbf{x}+\mathbf{b}\\|_2$	$\begin{cases}\frac{\mathbf{A}^T(\mathbf{A}\mathbf{x}+\mathbf{b})}{\\|\mathbf{A}\mathbf{x}+\mathbf{b}\\|_2}, & \mathbf{A}\mathbf{x}+\mathbf{b}\neq\mathbf{0},\\ \mathbf{A}^TB_{\\|\cdot\\|_2}[\mathbf{0},1], & \mathbf{A}\mathbf{x}+\mathbf{b}=\mathbf{0}.\end{cases}$	$\mathbb{E}=\mathbb{R}^n$, $\mathbf{A}\in\mathbb{R}^{m\times n}$, $\mathbf{b}\in\mathbb{R}^m$	Example 3.45
$\\|\mathbf{A}\mathbf{x}+\mathbf{b}\\|_\infty$	$\left\{\sum_{i\in I_{\mathbf{x}}}\lambda_i\operatorname{sgn}(\mathbf{a}_i^T\mathbf{x}+b_i)\mathbf{a}_i : \begin{array}{l}\sum_{i\in I_{\mathbf{x}}}\lambda_i=1\\ \lambda_i\geq 0\end{array}\right\}$	$\mathbb{E}=\mathbb{R}^n$, $\mathbf{A}\in\mathbb{R}^{m\times n}$, $\mathbf{b}\in\mathbb{R}^m$, $I_{\mathbf{x}}=\{i:\\|\mathbf{A}\mathbf{x}+\mathbf{b}\\|_\infty=\|\mathbf{a}_i^T\mathbf{x}+b_i\|\}$, $\mathbf{A}\mathbf{x}+\mathbf{b}\neq\mathbf{0}$	Example 3.54
$\\|\mathbf{A}\mathbf{x}+\mathbf{b}\\|_\infty$	$\mathbf{A}^TB_{\\|\cdot\\|_1}[\mathbf{0},1]$	same as above but with $\mathbf{A}\mathbf{x}+\mathbf{b}=\mathbf{0}$	Example 3.54
$\max_i\{\mathbf{a}_i^T\mathbf{x}+\mathbf{b}\}$	$\left\{\sum_{i\in I(\mathbf{x})}\lambda_i\mathbf{a}_i:\sum_{i\in I(\mathbf{x})}\lambda_i=1,\lambda_i\geq 0\right\}$	$\mathbb{E}=\mathbb{R}^n$, $\mathbf{a}_i\in\mathbb{R}^n$, $b_i\in\mathbb{R}$, $I(\mathbf{x})=\{i:f(\mathbf{x})=\mathbf{a}_i^T\mathbf{x}+b_i\}$	Example 3.53
$\frac{1}{2}d_C(\mathbf{x})^2$	$\{\mathbf{x}-P_C(\mathbf{x})\}$	C = nonempty closed and convex, $\mathbb{E}$ = Euclidean	Example 3.31
$d_C(\mathbf{x})$	$\begin{cases}\left\{\frac{\mathbf{x}-P_C(\mathbf{x})}{d_C(\mathbf{x})}\right\}, & \mathbf{x}\notin C,\\ N_C(\mathbf{x})\cap B[\mathbf{0},1] & \mathbf{x}\in C.\end{cases}$	C = nonempty closed and convex, $\mathbb{E}$ = Euclidean	Example 3.49

Conjugate Calculus Rules

$g(\mathbf{x})$	$g^*(\mathbf{y})$	Reference
$\sum_{i=1}^m f_i(\mathbf{x}_i)$	$\sum_{i=1}^m f_i^*(\mathbf{y}_i)$	Theorem 4.12
$\alpha f(\mathbf{x})$ $(\alpha > 0)$	$\alpha f^*(\mathbf{y}/\alpha)$	Theorem 4.14
$\alpha f(\mathbf{x}/\alpha)$ $(\alpha > 0)$	$\alpha f^*(\mathbf{y})$	Theorem 4.14
$f(\mathcal{A}(\mathbf{x}-\mathbf{a})) + \langle \mathbf{b}, \mathbf{x}\rangle + c$	$f^*\left((\mathcal{A}^T)^{-1}(\mathbf{y}-\mathbf{b})\right) + \langle \mathbf{a}, \mathbf{y}\rangle - c - \langle \mathbf{a}, \mathbf{b}\rangle$	Theorem 4.13

Conjugate Functions

f	$\mathrm{dom}(f)$	f^*	Assumptions	Reference
e^x	$\mathbb{R}$	$y\log y - y$ $(\mathrm{dom}(f^*) = \mathbb{R}_+)$	–	Section 4.4.1
$-\log x$	$\mathbb{R}_{++}$	$-1 - \log(-y)$ $(\mathrm{dom}(f^*) = \mathbb{R}_{--})$	–	Section 4.4.2
$\max\{1-x, 0\}$	$\mathbb{R}$	$y + \delta_{[-1,0]}(y)$	–	Section 4.4.3
$\frac{1}{p}\|x\|^p$	$\mathbb{R}$	$\frac{1}{q}\|y\|^q$	$p > 1$, $\frac{1}{p} + \frac{1}{q} = 1$	Section 4.4.4
$-\frac{x^p}{p}$	$\mathbb{R}_+$	$-\frac{(-y)^q}{q}$ $(\mathrm{dom}(f^*) = \mathbb{R}_{--})$	$0 < p < 1$, $\frac{1}{p} + \frac{1}{q} = 1$	Section 4.4.5
$\frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x} + c$	$\mathbb{R}^n$	$\frac{1}{2}(\mathbf{y}-\mathbf{b})^T\mathbf{A}^{-1}(\mathbf{y}-\mathbf{b}) - c$	$\mathbf{A} \in \mathbb{S}^n_{++}$, $\mathbf{b} \in \mathbb{R}^n$, $c \in \mathbb{R}$	Section 4.4.6
$\frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x} + c$	$\mathbb{R}^n$	$\frac{1}{2}(\mathbf{y}-\mathbf{b})^T\mathbf{A}^{\dagger}(\mathbf{y}-\mathbf{b}) - c$ $(\mathrm{dom}(f^*) = \mathbf{b} + \mathrm{Range}(\mathbf{A}))$	$\mathbf{A} \in \mathbb{S}^n_{+}$, $\mathbf{b} \in \mathbb{R}^n$, $c \in \mathbb{R}$	Section 4.4.7
$\sum_{i=1}^n x_i \log x_i$	$\mathbb{R}^n_+$	$\sum_{i=1}^n e^{y_i - 1}$	–	Section 4.4.8
$\sum_{i=1}^n x_i \log x_i$	Δ_n	$\log\left(\sum_{i=1}^n e^{y_i}\right)$	–	Section 4.4.10
$-\sum_{i=1}^n \log x_i$	$\mathbb{R}^n_{++}$	$-n - \sum_{i=1}^n \log(-y_i)$ $(\mathrm{dom}(f^*) = \mathbb{R}^n_{--})$	–	Section 4.4.9
$\log\left(\sum_{i=1}^n e^{x_i}\right)$	$\mathbb{R}^n$	$\sum_{i=1}^n y_i \log y_i$ $(\mathrm{dom}(f^*) = \Delta_n)$	–	Section 4.4.11
$\max_i\{x_i\}$	$\mathbb{R}^n$	$\delta_{\Delta_n}(\mathbf{y})$	–	Example 4.10
$\delta_C(\mathbf{x})$	C	$\sigma_C(\mathbf{y})$	$\emptyset \neq C \subseteq \mathbb{E}$	Example 4.2
$\sigma_C(\mathbf{x})$	$\mathrm{dom}(\sigma_C)$	$\delta_{\mathrm{cl}(\mathrm{conv}(C))}(\mathbf{y})$	$\emptyset \neq C \subseteq \mathbb{E}$	Example 4.9
$\\|\mathbf{x}\\|$	$\mathbb{E}$	$\delta_{B_{\\|\cdot\\|_*}[\mathbf{0},1]}(\mathbf{y})$	–	Section 4.4.12
$-\sqrt{\alpha^2 - \\|\mathbf{x}\\|^2}$	$B[\mathbf{0}, \alpha]$	$\alpha\sqrt{\\|\mathbf{y}\\|_*^2 + 1}$	$\alpha > 0$	Section 4.4.13
$\sqrt{\alpha^2 + \\|\mathbf{x}\\|^2}$	$\mathbb{E}$	$-\alpha\sqrt{1 - \\|\mathbf{y}\\|_*^2}$ $(\mathrm{dom} f^* = B_{\\|\cdot\\|_*}[\mathbf{0}, 1])$	$\alpha > 0$	Section 4.4.14
$\frac{1}{2}\\|\mathbf{x}\\|^2$	$\mathbb{E}$	$\frac{1}{2}\\|\mathbf{y}\\|_*^2$	–	Section 4.4.15
$\frac{1}{2}\\|\mathbf{x}\\|^2 + \delta_C(\mathbf{x})$	C	$\frac{1}{2}\\|\mathbf{y}\\|^2 - \frac{1}{2}d_C^2(\mathbf{y})$	$\emptyset \neq C \subseteq \mathbb{E}$, $\mathbb{E}$ Euclidean	Example 4.4
$\frac{1}{2}\\|\mathbf{x}\\|^2 - \frac{1}{2}d_C^2(\mathbf{x})$	$\mathbb{E}$	$\frac{1}{2}\\|\mathbf{y}\\|^2 + \delta_C(\mathbf{y})$	$\emptyset \neq C \subseteq \mathbb{E}$ closed convex. $\mathbb{E}$ Euclidean	Example 4.11

Conjugates of Symmetric Spectral Functions over $\mathbb{S}^n$ (from Example 7.16)

$g(\mathbf{X})$	$\mathrm{dom}(g)$	$g^*(\mathbf{Y})$	$\mathrm{dom}(g^*)$
$\lambda_{\max}(\mathbf{X})$	$\mathbb{S}^n$	$\delta_{\Upsilon_n}(\mathbf{Y})$	Υ_n
$\alpha\\|\mathbf{X}\\|_F$ $(\alpha > 0)$	$\mathbb{S}^n$	$\delta_{B_{\\|\cdot\\|_F}[\mathbf{0},\alpha]}(\mathbf{Y})$	$B_{\\|\cdot\\|_F}[\mathbf{0},\alpha]$
$\alpha\\|\mathbf{X}\\|_F^2$ $(\alpha > 0)$	$\mathbb{S}^n$	$\frac{1}{4\alpha}\\|\mathbf{Y}\\|_F^2$	$\mathbb{S}^n$
$\alpha\\|\mathbf{X}\\|_{2,2}$ $(\alpha > 0)$	$\mathbb{S}^n$	$\delta_{B_{\\|\cdot\\|_{S_1}}[\mathbf{0},\alpha]}(\mathbf{Y})$	$B_{\\|\cdot\\|_{S_1}}[\mathbf{0},\alpha]$
$\alpha\\|\mathbf{X}\\|_{S_1}$ $(\alpha > 0)$	$\mathbb{S}^n$	$\delta_{B_{\\|\cdot\\|_{2,2}}[\mathbf{0},\alpha]}(\mathbf{Y})$	$B_{\\|\cdot\\|_{2,2}}[\mathbf{0},\alpha]$
$-\log\det(\mathbf{X})$	$\mathbb{S}^n_{++}$	$-n - \log\det(-\mathbf{Y})$	$\mathbb{S}^n_{--}$
$\sum_{i=1}^{n} \lambda_i(\mathbf{X})\log(\lambda_i(\mathbf{X}))$	$\mathbb{S}^n_{+}$	$\sum_{i=1}^{n} e^{\lambda_i(\mathbf{Y})-1}$	$\mathbb{S}^n$
$\sum_{i=1}^{n} \lambda_i(\mathbf{X})\log(\lambda_i(\mathbf{X}))$	Υ_n	$\log\left(\sum_{i=1}^{n} e^{\lambda_i(\mathbf{Y})}\right)$	$\mathbb{S}^n$

Conjugates of Symmetric Spectral Functions over $\mathbb{R}^{m\times n}$ (from Example 7.27)

$g(\mathbf{X})$	$\mathrm{dom}(g)$	$g^*(\mathbf{Y})$	$\mathrm{dom}(g^*)$
$\alpha\sigma_1(\mathbf{X})$ $(\alpha > 0)$	$\mathbb{R}^{m\times n}$	$\delta_{B_{\\|\cdot\\|_{S_1}}[\mathbf{0},\alpha]}(\mathbf{Y})$	$B_{\\|\cdot\\|_{S_1}}[\mathbf{0},\alpha]$
$\alpha\\|\mathbf{X}\\|_F$ $(\alpha > 0)$	$\mathbb{R}^{m\times n}$	$\delta_{B_{\\|\cdot\\|_F}[\mathbf{0},\alpha]}(\mathbf{Y})$	$B_{\\|\cdot\\|_F}[\mathbf{0},\alpha]$
$\alpha\\|\mathbf{X}\\|_F^2$ $(\alpha > 0)$	$\mathbb{R}^{m\times n}$	$\frac{1}{4\alpha}\\|\mathbf{Y}\\|_F^2$	$\mathbb{R}^{m\times n}$
$\alpha\\|\mathbf{X}\\|_{S_1}$ $(\alpha > 0)$	$\mathbb{R}^{m\times n}$	$\delta_{B_{\\|\cdot\\|_{S_\infty}}[\mathbf{0},\alpha]}(\mathbf{Y})$	$B_{\\|\cdot\\|_{S_\infty}}[\mathbf{0},\alpha]$

Smooth Functions

$f(\mathbf{x})$	$\mathrm{dom}(f)$	Parameter	Norm	Reference
$\frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x} + c$ $(\mathbf{A} \in \mathbb{S}^n, \mathbf{b} \in \mathbb{R}^n, c \in \mathbb{R})$	$\mathbb{R}^n$	$\\|\mathbf{A}\\|_{p,q}$	l_p	Example 5.2
$\langle \mathbf{b}, \mathbf{x}\rangle + c$ $(\mathbf{b} \in \mathbb{E}^*, c \in \mathbb{R})$	$\mathbb{E}$	0	any norm	Example 5.3
$\frac{1}{2}\\|\mathbf{x}\\|_p^2$, $p \in [2,\infty)$	$\mathbb{R}^n$	$p-1$	l_p	Example 5.11
$\sqrt{1+\\|\mathbf{x}\\|_2^2}$	$\mathbb{R}^n$	1	l_2	Example 5.14
$\log(\sum_{i=1}^n e^{x_i})$	$\mathbb{R}^n$	1	l_2, l_∞	Example 5.15
$\frac{1}{2}d_C^2(\mathbf{x})$ $(\emptyset \neq C \subseteq \mathbb{E}$ closed convex$)$	$\mathbb{E}$	1	Euclidean	Example 5.5
$\frac{1}{2}\\|\mathbf{x}\\|^2 - \frac{1}{2}d_C^2(\mathbf{x})$ $(\emptyset \neq C \subseteq \mathbb{E}$ closed convex$)$	$\mathbb{E}$	1	Euclidean	Example 5.6
$H_\mu(\mathbf{x})$ $(\mu > 0)$	$\mathbb{E}$	$\frac{1}{\mu}$	Euclidean	Example 6.62

Strongly Convex Functions

$f(\mathbf{x})$	$\mathrm{dom}(f)$	Strongly convex parameter	Norm	Reference
$\frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} + 2\mathbf{b}^T\mathbf{x} + c$ $(\mathbf{A} \in \mathbb{S}^n_{++}, \mathbf{b} \in \mathbb{R}^n, c \in \mathbb{R})$	$\mathbb{R}^n$	$\lambda_{\min}(\mathbf{A})$	l_2	Example 5.19
$\frac{1}{2}\|\mathbf{x}\|^2 + \delta_C(\mathbf{x})$ $(\emptyset \neq C \subseteq \mathbb{E}$ convex$)$	C	1	Euclidean	Example 5.21
$-\sqrt{1 - \|\mathbf{x}\|_2^2}$	$B_{\|\cdot\|_2}[\mathbf{0}, 1]$	1	l_2	Example 5.29
$\frac{1}{2}\|\mathbf{x}\|_p^2 \ \ (p \in (1, 2])$	$\mathbb{R}^n$	$p - 1$	l_p	Example 5.28
$\sum_{i=1}^n x_i \log x_i$	Δ_n	1	l_2 or l_1	Example 5.27

Orthogonal Projections

Set (C)	$P_C(\mathbf{x})$	Assumptions	Reference
$\mathbb{R}^n_+$	$[\mathbf{x}]_+$	–	Lemma 6.26
$\mathrm{Box}[\boldsymbol{\ell}, \mathbf{u}]$	$P_C(\mathbf{x})_i = \min\{\max\{x_i, \ell_i\}, u_i\}$	$\ell_i \leq u_i$	Lemma 6.26
$B_{\|\cdot\|_2}[\mathbf{c}, r]$	$\mathbf{c} + \frac{r}{\max\{\|\mathbf{x}-\mathbf{c}\|_2, r\}}(\mathbf{x} - \mathbf{c})$	$\mathbf{c} \in \mathbb{R}^n, r > 0$	Lemma 6.26
$\{\mathbf{x} : \mathbf{A}\mathbf{x} = \mathbf{b}\}$	$\mathbf{x} - \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}(\mathbf{A}\mathbf{x} - \mathbf{b})$	$\mathbf{A} \in \mathbb{R}^{m\times n}$, $\mathbf{b} \in \mathbb{R}^m$, $\mathbf{A}$ full row rank	Lemma 6.26
$\{\mathbf{x} : \mathbf{a}^T\mathbf{x} \leq b\}$	$\mathbf{x} - \frac{[\mathbf{a}^T\mathbf{x} - b]_+}{\|\mathbf{a}\|^2}\mathbf{a}$	$\mathbf{0} \neq \mathbf{a} \in \mathbb{R}^n, b \in \mathbb{R}$	Lemma 6.26
Δ_n	$[\mathbf{x} - \mu^*\mathbf{e}]_+$ where $\mu^* \in \mathbb{R}$ satisfies $\mathbf{e}^T[\mathbf{x} - \mu^*\mathbf{e}]_+ = 1$		Corollary 6.29
$H_{\mathbf{a},b} \cap \mathrm{Box}[\boldsymbol{\ell}, \mathbf{u}]$	$P_{\mathrm{Box}[\boldsymbol{\ell},\mathbf{u}]}(\mathbf{x} - \mu^*\mathbf{a})$ where $\mu^* \in \mathbb{R}$ satisfies $\mathbf{a}^T P_{\mathrm{Box}[\boldsymbol{\ell},\mathbf{u}]}(\mathbf{x} - \mu^*\mathbf{a}) = b$	$\mathbf{a} \in \mathbb{R}^n \backslash \{\mathbf{0}\}, b \in \mathbb{R}$	Theorem 6.27
$H^-_{\mathbf{a},b} \cap \mathrm{Box}[\boldsymbol{\ell}, \mathbf{u}]$	$\begin{cases} P_{\mathrm{Box}[\boldsymbol{\ell},\mathbf{u}]}(\mathbf{x}), & \mathbf{a}^T\mathbf{v}_{\mathbf{x}} \leq b, \\ P_{\mathrm{Box}[\boldsymbol{\ell},\mathbf{u}]}(\mathbf{x} - \lambda^*\mathbf{a}), & \mathbf{a}^T\mathbf{v}_{\mathbf{x}} > b, \end{cases}$ $\mathbf{v}_{\mathbf{x}} = P_{\mathrm{Box}[\boldsymbol{\ell},\mathbf{u}]}(\mathbf{x})$, $\mathbf{a}^T P_{\mathrm{Box}[\boldsymbol{\ell},\mathbf{u}]}(\mathbf{x} - \lambda^*\mathbf{a}) = b, \lambda^* > 0$	$\mathbf{a} \in \mathbb{R}^n \backslash \{\mathbf{0}\}, b \in \mathbb{R}$	Example 6.32
$B_{\|\cdot\|_1}[\mathbf{0}, \alpha]$	$\begin{cases} \mathbf{x}, & \|\mathbf{x}\|_1 \leq \alpha, \\ \mathcal{T}_{\lambda^*}(\mathbf{x}), & \|\mathbf{x}\|_1 > \alpha, \end{cases}$ $\|\mathcal{T}_{\lambda^*}(\mathbf{x})\|_1 = \alpha$, $\lambda^* > 0$	$\alpha > 0$	Example 6.33
$\{\mathbf{x} : \boldsymbol{\omega}^T\|\mathbf{x}\| \leq \beta,$ $-\boldsymbol{\alpha} \leq \mathbf{x} \leq \boldsymbol{\alpha}\}$	$\begin{cases} \mathbf{v}_{\mathbf{x}}, & \boldsymbol{\omega}^T\|\mathbf{v}_{\mathbf{x}}\| \leq \beta, \\ \mathcal{S}_{\lambda^*\boldsymbol{\omega},\boldsymbol{\alpha}}(\mathbf{x}), & \boldsymbol{\omega}^T\|\mathbf{v}_{\mathbf{x}}\| > \beta, \end{cases}$ $\mathbf{v}_{\mathbf{x}} = P_{\mathrm{Box}[-\boldsymbol{\alpha},\boldsymbol{\alpha}]}(\mathbf{x})$, $\boldsymbol{\omega}^T\|\mathcal{S}_{\lambda^*\boldsymbol{\omega},\boldsymbol{\alpha}}(\mathbf{x})\| = \beta, \lambda^* > 0$	$\boldsymbol{\omega} \in \mathbb{R}^n_+$, $\boldsymbol{\alpha} \in [0, \infty]^n$, $\beta \in \mathbb{R}_{++}$	Example 6.34
$\{\mathbf{x} > \mathbf{0} : \Pi x_i \geq \alpha\}$	$\begin{cases} \mathbf{x}, & \mathbf{x} \in C, \\ \left(\frac{x_j + \sqrt{x_j^2 + 4\lambda^*}}{2}\right)_{j=1}^n, & \mathbf{x} \notin C, \end{cases}$ $\Pi_{j=1}^n\left((x_j + \sqrt{x_j^2 + 4\lambda^*})/2\right) = \alpha, \lambda^* > 0$	$\alpha > 0$	Example 6.35
$\{(\mathbf{x}, s) : \|\mathbf{x}\|_2 \leq s\}$	$\left(\frac{\|\mathbf{x}\|_2 + s}{2\|\mathbf{x}\|_2}\mathbf{x}, \frac{\|\mathbf{x}\|_2 + s}{2}\right)$ if $\|\mathbf{x}\|_2 \geq \|s\|$ $(\mathbf{0}, 0)$ if $s < \|\mathbf{x}\|_2 < -s$, $(\mathbf{x}, s)$ if $\|\mathbf{x}\|_2 \leq s$.	–	Example 6.37
$\{(\mathbf{x}, s) : \|\mathbf{x}\|_1 \leq s\}$	$\begin{cases} (\mathbf{x}, s), & \|\mathbf{x}\|_1 \leq s, \\ (\mathcal{T}_{\lambda^*}(\mathbf{x}), s + \lambda^*), & \|\mathbf{x}\|_1 > s, \end{cases}$ $\|\mathcal{T}_{\lambda^*}(\mathbf{x})\|_1 - \lambda^* - s = 0$, $\lambda^* > 0$	–	Example 6.38

Orthogonal Projections onto Symmetric Spectral Sets in $\mathbb{S}^n$

set (T)	$P_T(\mathbf{X})$	Assumptions
$\mathbb{S}^n_+$	$\mathbf{U}\mathrm{diag}([\boldsymbol{\lambda}(\mathbf{X})]_+)\mathbf{U}^T$	–
$\{\mathbf{X} : \ell\mathbf{I} \preceq \mathbf{X} \preceq u\mathbf{I}\}$	$\mathbf{U}\mathrm{diag}(\mathbf{v})\mathbf{U}^T$, $v_i = \min\{\max\{\lambda_i(\mathbf{X}), \ell\}, u\}$	$\ell \le u$
$B_{\\|\cdot\\|_F}[\mathbf{0}, r]$	$\frac{r}{\max\{\\|\mathbf{X}\\|_F, r\}}\mathbf{X}$	$r > 0$
$\{\mathbf{X} : \mathrm{Tr}(\mathbf{X}) \le b\}$	$\mathbf{U}\mathrm{diag}(\mathbf{v})\mathbf{U}^T$, $\mathbf{v} = \boldsymbol{\lambda}(\mathbf{X}) - \frac{[\mathbf{e}^T\boldsymbol{\lambda}(\mathbf{X}) - b]_+}{n}\mathbf{e}$	$b \in \mathbb{R}$
Υ_n	$\mathbf{U}\mathrm{diag}(\mathbf{v})\mathbf{U}^T$, $\mathbf{v} = [\boldsymbol{\lambda}(\mathbf{X}) - \mu^*\mathbf{e}]_+$ where $\mu^* \in \mathbb{R}$ satisfies $\mathbf{e}^T[\boldsymbol{\lambda}(\mathbf{X}) - \mu^*\mathbf{e}]_+ = 1$	–
$B_{\\|\cdot\\|_{S_1}}[\mathbf{0}, \alpha]$	$\begin{cases} \mathbf{X}, & \\|\mathbf{X}\\|_{S_1} \le \alpha, \\ \mathbf{U}\mathrm{diag}(\mathcal{T}_{\beta^*}(\boldsymbol{\lambda}(\mathbf{X})))\mathbf{U}^T, & \\|\mathbf{X}\\|_{S_1} > \alpha, \end{cases}$ $\\|\mathcal{T}_{\beta^*}(\boldsymbol{\lambda}(\mathbf{X}))\\|_1 = \alpha$, $\beta^* > 0$	$\alpha > 0$

Orthogonal Projections onto Symmetric Spectral Sets in $\mathbb{R}^{m\times n}$ (from Example 7.31)

set (T)	$P_T(\mathbf{X})$	Assumptions
$B_{\\|\cdot\\|_{S_\infty}}[\mathbf{0}, \alpha]$	$\mathbf{U}\mathrm{diag}(\mathbf{v})\mathbf{V}^T$, $v_i = \min\{\sigma_i(\mathbf{X}), \alpha\}$	$\alpha > 0$
$B_{\\|\cdot\\|_F}[\mathbf{0}, r]$	$\frac{r}{\max\{\\|\mathbf{X}\\|_F, r\}}\mathbf{X}$	$r > 0$
$B_{\\|\cdot\\|_{S_1}}[\mathbf{0}, \alpha]$	$\begin{cases} \mathbf{X}, & \\|\mathbf{X}\\|_{S_1} \le \alpha, \\ \mathbf{U}\mathrm{diag}(\mathcal{T}_{\beta^*}(\sigma(\mathbf{X})))\mathbf{V}^T, & \\|\mathbf{X}\\|_{S_1} > \alpha, \end{cases}$ $\\|\mathcal{T}_{\beta^*}(\boldsymbol{\sigma}(\mathbf{X}))\\|_1 = \alpha$, $\beta^* > 0$	$\alpha > 0$

Prox Calculus Rules

$f(\mathbf{x})$	$\mathrm{prox}_f(\mathbf{x})$	Assumptions	Reference
$\sum_{i=1}^m f_i(\mathbf{x}_i)$	$\mathrm{prox}_{f_1}(\mathbf{x}_1) \times \cdots \times \mathrm{prox}_{f_m}(\mathbf{x}_m)$	–	Theorem 6.6
$g(\lambda\mathbf{x} + \mathbf{a})$	$\frac{1}{\lambda}\left[\mathrm{prox}_{\lambda^2 g}(\lambda\mathbf{x} + \mathbf{a}) - \mathbf{a}\right]$	$\lambda \ne 0, \mathbf{a} \in \mathbb{E}$, g proper	Theorem 6.11
$\lambda g(\mathbf{x}/\lambda)$	$\lambda\mathrm{prox}_{g/\lambda}(\mathbf{x}/\lambda)$	$\lambda \ne 0$, g proper	Theorem 6.12
$g(\mathbf{x}) + \frac{c}{2}\\|\mathbf{x}\\|^2 + \langle\mathbf{a}, \mathbf{x}\rangle + \gamma$	$\mathrm{prox}_{\frac{1}{c+1}g}\left(\frac{\mathbf{x}-\mathbf{a}}{c+1}\right)$	$\mathbf{a} \in \mathbb{E}, c > 0, \gamma \in \mathbb{R}$, g proper	Theorem 6.13
$g(\mathcal{A}(\mathbf{x}) + \mathbf{b})$	$\mathbf{x} + \frac{1}{\alpha}\mathcal{A}^T(\mathrm{prox}_{\alpha g}(\mathcal{A}(\mathbf{x}) + \mathbf{b}) - \mathcal{A}(\mathbf{x}) - \mathbf{b})$	$\mathbf{b} \in \mathbb{R}^m$, $\mathcal{A} : \mathbb{V} \to \mathbb{R}^m$, g proper closed convex, $\mathcal{A} \circ \mathcal{A}^T = \alpha I$, $\alpha > 0$	Theorem 6.15
$g(\\|\mathbf{x}\\|)$	$\mathrm{prox}_g(\\|\mathbf{x}\\|)\frac{\mathbf{x}}{\\|\mathbf{x}\\|}$, $\mathbf{x} \ne \mathbf{0}$; $\{\mathbf{u} : \\|\mathbf{u}\\| = \mathrm{prox}_g(0)\}$, $\mathbf{x} = \mathbf{0}$	g proper closed convex, $\mathrm{dom}(g) \subseteq [0, \infty)$	Theorem 6.18

Prox Computations

$f(\mathbf{x})$	$\mathrm{dom}(f)$	$\mathrm{prox}_f(\mathbf{x})$	Assumptions	Reference
$\frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x} + c$	$\mathbb{R}^n$	$(\mathbf{A}+\mathbf{I})^{-1}(\mathbf{x}-\mathbf{b})$	$\mathbf{A} \in \mathbb{S}^n_+$, $\mathbf{b} \in \mathbb{R}^n$, $c \in \mathbb{R}$	Section 6.2.3
λx^3	$\mathbb{R}_+$	$\frac{-1+\sqrt{1+12\lambda[x]_+}}{6\lambda}$	$\lambda > 0$	Lemma 6.5
μx	$[0,\alpha] \cap \mathbb{R}$	$\min\{\max\{x-\mu, 0\}, \alpha\}$	$\mu \in \mathbb{R}$, $\alpha \in [0,\infty]$	Example 6.14
$\lambda\\|\mathbf{x}\\|$	$\mathbb{E}$	$\left(1 - \frac{\lambda}{\max\{\\|\mathbf{x}\\|,\lambda\}}\right)\mathbf{x}$	$\\|\cdot\\|$—Euclidean norm, $\lambda > 0$	Example 6.19
$-\lambda\\|\mathbf{x}\\|$	$\mathbb{E}$	$\left(1+\frac{\lambda}{\\|\mathbf{x}\\|}\right)\mathbf{x}, \quad \mathbf{x} \neq \mathbf{0}$, $\{\mathbf{u} : \\|\mathbf{u}\\| = \lambda\}, \quad \mathbf{x} = \mathbf{0}$.	$\\|\cdot\\|$—Euclidean norm, $\lambda > 0$	Example 6.21
$\lambda\\|\mathbf{x}\\|_1$	$\mathbb{R}^n$	$\mathcal{T}_\lambda(\mathbf{x}) = [\lvert\mathbf{x}\rvert - \lambda\mathbf{e}]_+ \odot \mathrm{sgn}(\mathbf{x})$	$\lambda > 0$	Example 6.8
$\\|\boldsymbol{\omega} \odot \mathbf{x}\\|_1$	$\mathrm{Box}[-\boldsymbol{\alpha}, \boldsymbol{\alpha}]$	$\mathcal{S}_{\boldsymbol{\omega},\boldsymbol{\alpha}}(\mathbf{x})$	$\boldsymbol{\alpha} \in [0,\infty]^n, \boldsymbol{\omega} \in \mathbb{R}^n_+$	Example 6.23
$\lambda\\|\mathbf{x}\\|_\infty$	$\mathbb{R}^n$	$\mathbf{x} - \lambda P_{B_{\\|\cdot\\|_1}[\mathbf{0},1]}(\mathbf{x}/\lambda)$	$\lambda > 0$	Example 6.48
$\lambda\\|\mathbf{x}\\|_a$	$\mathbb{E}$	$\mathbf{x} - \lambda P_{B_{\\|\cdot\\|_{a,*}}[\mathbf{0},1]}(\mathbf{x}/\lambda)$	$\\|\mathbf{x}\\|_a$—norm, $\lambda > 0$	Example 6.47
$\lambda\\|\mathbf{x}\\|_0$	$\mathbb{R}^n$	$\mathcal{H}_{\sqrt{2\lambda}}(x_1) \times \cdots \times \mathcal{H}_{\sqrt{2\lambda}}(x_n)$	$\lambda > 0$	Example 6.10
$\lambda\\|\mathbf{x}\\|^3$	$\mathbb{E}$	$\frac{2}{1+\sqrt{1+12\lambda\\|\mathbf{x}\\|}}\mathbf{x}$	$\\|\cdot\\|$—Euclidean norm, $\lambda > 0$,	Example 6.20
$-\lambda\sum_{j=1}^n \log x_j$	$\mathbb{R}^n_{++}$	$\left(\frac{x_j+\sqrt{x_j^2+4\lambda}}{2}\right)_{j=1}^n$	$\lambda > 0$	Example 6.9
$\delta_C(\mathbf{x})$	$\mathbb{E}$	$P_C(\mathbf{x})$	$\emptyset \neq C \subseteq \mathbb{E}$	Theorem 6.24
$\lambda\sigma_C(\mathbf{x})$	$\mathbb{E}$	$\mathbf{x} - \lambda P_C(\mathbf{x}/\lambda)$	$\lambda > 0$, $C \neq \emptyset$ closed convex	Theorem 6.46
$\lambda\max\{x_i\}$	$\mathbb{R}^n$	$\mathbf{x} - P_{\Delta_n}(\mathbf{x}/\lambda)$	$\lambda > 0$	Example 6.49
$\lambda\sum_{i=1}^k x_{[i]}$	$\mathbb{R}^n$	$\mathbf{x} - \lambda P_C(\mathbf{x}/\lambda)$, $C = H_{\mathbf{e},k} \cap \mathrm{Box}[\mathbf{0},\mathbf{e}]$	$\lambda > 0$	Example 6.50
$\lambda\sum_{i=1}^k \lvert x_{\langle i\rangle}\rvert$	$\mathbb{R}^n$	$\mathbf{x} - \lambda P_C(\mathbf{x}/\lambda)$, $C = B_{\\|\cdot\\|_1}[\mathbf{0},k] \cap \mathrm{Box}[-\mathbf{e},\mathbf{e}]$	$\lambda > 0$	Example 6.51
$\lambda M_f^\mu(\mathbf{x})$	$\mathbb{E}$	$\mathbf{x} + \frac{\lambda}{\mu+\lambda}\left(\mathrm{prox}_{(\mu+\lambda)f}(\mathbf{x}) - \mathbf{x}\right)$	$\lambda, \mu > 0$, f proper closed convex	Corollary 6.64
$\lambda d_C(\mathbf{x})$	$\mathbb{E}$	$\mathbf{x} + \min\left\{\frac{\lambda}{d_C(\mathbf{x})}, 1\right\}(P_C(\mathbf{x}) - \mathbf{x})$	$\emptyset \neq C$ closed convex, $\lambda > 0$	Lemma 6.43
$\frac{\lambda}{2}d_C^2(\mathbf{x})$	$\mathbb{E}$	$\frac{\lambda}{\lambda+1}P_C(\mathbf{x}) + \frac{1}{\lambda+1}\mathbf{x}$	$\emptyset \neq C$ closed convex, $\lambda > 0$	Example 6.65
$\lambda H_\mu(\mathbf{x})$	$\mathbb{E}$	$\left(1 - \frac{\lambda}{\max\{\\|\mathbf{x}\\|, \mu+\lambda\}}\right)\mathbf{x}$	$\lambda, \mu > 0$	Example 6.66
$\rho\\|\mathbf{x}\\|_1^2$	$\mathbb{R}^n$	$\left(\frac{v_i x_i}{v_i+2\rho}\right)_{i=1}^n$, $\mathbf{v} = \left[\sqrt{\frac{\rho}{\mu}}\lvert\mathbf{x}\rvert - 2\rho\right]_+$, $\mathbf{e}^T\mathbf{v} = 1$ ($\mathbf{0}$ when $\mathbf{x} = \mathbf{0}$)	$\rho > 0$	Lemma 6.70
$\lambda\\|\mathbf{A}\mathbf{x}\\|_2$	$\mathbb{R}^n$	$\mathbf{x} - \mathbf{A}^T(\mathbf{A}\mathbf{A}^T + \alpha^*\mathbf{I})^{-1}\mathbf{A}\mathbf{x}$, $\alpha^* = 0$ if $\\|\mathbf{v}_0\\|_2 \leq \lambda$; otherwise, $\\|\mathbf{v}_{\alpha^*}\\|_2 = \lambda$; $\mathbf{v}_\alpha \equiv (\mathbf{A}\mathbf{A}^T + \alpha\mathbf{I})^{-1}\mathbf{A}\mathbf{x}$	$\mathbf{A} \in \mathbb{R}^{m\times n}$ with full row rank, $\lambda > 0$	Lemma 6.68

Prox of Symmetric Spectral Functions over $\mathbb{S}^n$ (from Example 7.19)

$F(\mathbf{X})$	$\mathrm{dom}(F)$	$\mathrm{prox}_F(\mathbf{X})$	Reference
$\alpha\|\mathbf{X}\|_F^2$	$\mathbb{S}^n$	$\frac{1}{1+2\alpha}\mathbf{X}$	Section 6.2.3
$\alpha\|\mathbf{X}\|_F$	$\mathbb{S}^n$	$\left(1-\frac{\alpha}{\max\{\|\mathbf{X}\|_F,\alpha\}}\right)\mathbf{X}$	Example 6.19
$\alpha\|\mathbf{X}\|_{S_1}$	$\mathbb{S}^n$	$\mathbf{U}\mathrm{diag}(\mathcal{T}_\alpha(\boldsymbol{\lambda}(\mathbf{X})))\mathbf{U}^T$	Example 6.8
$\alpha\|\mathbf{X}\|_{2,2}$	$\mathbb{S}^n$	$\mathbf{U}\mathrm{diag}(\boldsymbol{\lambda}(\mathbf{X})-\alpha P_{B_{\|\cdot\|_1}[\mathbf{0},1]}(\boldsymbol{\lambda}(\mathbf{X})/\alpha))\mathbf{U}^T$	Example 6.48
$-\alpha\log\det(\mathbf{X})$	$\mathbb{S}^n_{++}$	$\mathbf{U}\mathrm{diag}\left(\frac{\lambda_j(\mathbf{X})+\sqrt{\lambda_j(\mathbf{X})^2+4\alpha}}{2}\right)\mathbf{U}^T$	Example 6.9
$\alpha\lambda_1(\mathbf{X})$	$\mathbb{S}^n$	$\mathbf{U}\mathrm{diag}(\boldsymbol{\lambda}(\mathbf{X})-\alpha P_{\Delta_n}(\boldsymbol{\lambda}(\mathbf{X})/\alpha))\mathbf{U}^T$	Example 6.49
$\alpha\sum_{i=1}^k\lambda_i(\mathbf{X})$	$\mathbb{S}^n$	$\mathbf{X}-\alpha\mathbf{U}\mathrm{diag}(P_C(\boldsymbol{\lambda}(\mathbf{X})/\alpha))\mathbf{U}^T$, $C=H_{\mathbf{e},k}\cap\mathrm{Box}[\mathbf{0},\mathbf{e}]$	Example 6.50

Prox of Symmetric Spectral Functions over $\mathbb{R}^{m\times n}$ (from Example 7.30)

$F(\mathbf{X})$	$\mathrm{prox}_F(\mathbf{X})$
$\alpha\|\mathbf{X}\|_F^2$	$\frac{1}{1+2\alpha}\mathbf{X}$
$\alpha\|\mathbf{X}\|_F$	$\left(1-\frac{\alpha}{\max\{\|\mathbf{X}\|_F,\alpha\}}\right)\mathbf{X}$
$\alpha\|\mathbf{X}\|_{S_1}$	$\mathbf{U}\mathrm{dg}(\mathcal{T}_\alpha(\boldsymbol{\sigma}(\mathbf{X})))\mathbf{V}^T$
$\alpha\|\mathbf{X}\|_{S_\infty}$	$\mathbf{X}-\alpha\mathbf{U}\mathrm{dg}(P_{B_{\|\cdot\|_1}[\mathbf{0},1]}(\boldsymbol{\sigma}(\mathbf{X})/\alpha))\mathbf{V}^T$
$\alpha\|\mathbf{X}\|_{\langle k\rangle}$	$\mathbf{X}-\alpha\mathbf{U}\mathrm{dg}(P_C(\boldsymbol{\sigma}(\mathbf{X})/\alpha))\mathbf{V}^T$, $C=B_{\|\cdot\|_1}[\mathbf{0},k]\cap B_{\|\cdot\|_\infty}[\mathbf{0},1]$