My Ideas after Reading Papers

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Abstract

This is still a note for a draft so no abstract [1]

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1 Introduction

Necoara et al. introduced the definition of quasi strongly convex function (Q-SCNVX), Quadratic Under approximations (QUA), Quadratic Gradient Growth (QGG), Proximal Error Bound (PEB) and, Quadratic Function Growth (QFG). These conditions are relaxation of strong convexity which enables linear convergence rate of first order method, including Nesterov's accelerated variants. In this file, we showed a new perspective of their works. Our goal is to relax their definitions and, to extend the linear convergence results, using completely new ideas and perspective.

Notations. Unless specified, our ambient space is $\mathbb{R}^n$ with Euclidean norm $\|\cdot\|$. Let $C \subseteq \mathbb{R}^n$, $\Pi_C(\cdot)$ denotes the projection onto the set C, i.e: the closest point in C to another point in $\mathbb{R}^n$.

The following definitions and assumptions are their.

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{ass:necoara-linear} **Assumption 1.1 (Necoara's linear convergence assumptions)**
The following assumptions are about (f, X, X^*, L_f).

(i) $f : \mathbb{R}^n \to \mathbb{R}$ is an L_f Lipschitz smooth function.
(ii) $X \subseteq \mathbb{R}^n$ is a closed convex non-empty set.
(iii) $X^* = \underset{x \in X}{\operatorname{argmin}}\, f(x) \neq \emptyset$.

Under this assumption, the following definitions are proposed.

Definition 1.2 (Necoara's weaker characterizations of strong convexity)
Suppose that (f, X, X^*, L_f) are given by Assumption 1.1. For all $x \in X$, denote $\bar{x} = \Pi_{X^*}x$. The following definitions are relaxations of strong convexity.

(i) *f is Q-SCNVX if there exists $\kappa_f > 0$ such that $f(\bar{x}) - f(x) - \langle \nabla f(x), \bar{x} - x\rangle \geq \frac{\kappa_f}{2}\|x - \bar{x}\|^2$. Which we denote it by $f \in q\mathcal{S}(f, L_f, \kappa_f)$.*
(ii) *f is QUA if there exists $\kappa_f > 0$ such that $f(x) - f(\bar{x}) - \langle \nabla f(\bar{x}), x - \bar{x}\rangle \geq \frac{\kappa_f}{2}\|x - \bar{x}\|^2$. We denote it by $f \in \mathcal{U}(f, L_f, \kappa_f)$.*
(iii) *f is QGG if there exists $\kappa_f > 0$ such that $\langle \nabla f(x) - \nabla f(\bar{x}), x - \bar{x}\rangle \geq \frac{\kappa}{2}\|x - \bar{x}\|^2$. We denote it by $f \in \mathcal{G}(f, L_f, \kappa_f)$.*
(iv) *f is PEB if there exists $\kappa_f > 0$ such that $\|x - L^{-1}\Pi_X(x - L^{-1}\nabla f(x))\| \geq \kappa_f\|x - \bar{x}\|$. We denote it by $f \in \mathcal{E}(f, L_f, \kappa_f)$.*
(v) *f is QFG if there exists $\kappa_f > 0$ such that $f(x) - f(\bar{x}) \geq \frac{\kappa_f}{2}\|x - \bar{x}\|^2$. We denote it by $f \in \mathcal{F}(f, L_f, \kappa_f)$.*

These definitions are the keys which Necoara used to prove the linear convergence of projected gradient, and Nesterov's accelerated gradient method.

1.1 The idea of Semi Bregman Divergence

References

[1] H. H. Bauschke and P. L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, CMS Books in Mathematics, Springer International Publishing, Cham, 2017.