

First Order Nonsmooth Optimization: Algorithm Design, Analysis, Convergence, and Applications

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Abstract

The research proposal focuses on the theories and practice in solving nonsmooth optimization. The theme of proposal highlight topics of interests that emphasize the computations and applications aspect of algorithms that exhibits both practical and theoretical importance. We summarize our ongoing research in unifying Nesterov type accelerated proximal gradient method and proposes our method of spectral momentum. We survey literatures under the topic of Catalyst Meta Acceleration framework used in accelerating variance reduced methods in the settings of Data Science and Machine Learning. We present literatures and progress in topics such as Performance Estimation Problem, Inexact Proximal Point, acceleration without convexity. At the end there is a section summarizing a method we developed for tree species classifications using Sentinel-2 satellite remote sensing data using big data analytics by extract spectral signatures of ground vegetation covers.

Contents

1	Introduction	2
2	Preliminaries	3
2.1	Fundamentals in convex analysis	4
2.2	Fundamentals in non-convex analysis	4

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3	Unifying variants of Nesterov's accelerated methods	4
4	Method of spectral momentum	4
5	Catalyst accelerations and future works	4
6	Performance estimation problems	4
7	Methods of inexact proximal point	4
8	Nestrov's acceleration in the non-convex case and momentum in general	4
9	Using PostGreSQL and big data analytic method for species classification on Sentinel-2 Satellite remote sensing imagery	4

1 Introduction

Let \mathbb{R}^n be the ambient space. We consider

$$\min_{x \in \mathbb{R}^n} \{F(x) : f(x) + g(x)\}. \quad (1.1)$$

Unless specified, assume f is a smooth function with L -Lipschitz gradient operator. Unless specified, assume g is a convex function. In the literatures this type of problems are referred to as additive composite problems.

Our ongoing research concerns accelerated proximal gradient type method for solving (1). In the expository writing by Walkington [2], a variant for of accelerated gradient method for strongly convex function f is discussed. We had two lingering questions after reading it.

- (i) Do there exist a unified description for the convergence for both variants of the algorithm.
- (ii) Is it possible to attain faster convergence rate without knowledge about the strong convexity of function f .

The good news is we have definitive answers for both questions through our own efforts of research. Section 3, 4 are our ongoing research which present the answers to the questions.

In Section 3, we proposed the method of “Relaxed Weak Accelerated Proximal Gradient (R-WAPG)” as the foundation to describe several variants of Accelerated proximal gradient method in the literatures. The convergence theories of R-WAPG allows us to model convergence of accelerated proximal gradient method where the momentum sequence doesn’t strictly follow the conditions presented in the literatures. The descriptive power of R-WAPG allows convergence analysis for all the variants using one single theorem.

In Section 4 we propose a practical algorithm that exploits a specific term in the proof of R-WAPG to achieve faster convergence for solving (1) without knowing parameter L, μ in prior. Results of numerical experiments are presented.

Section 5 are results of literatures review in MATH 590. It’s based on a series of papers (CITATION HERE) in the topic of Catalyst Meta Acceleration method for First Order Variance Reduced Methods. We will point out potential future direction of research of Catalyst acceleration.

Section 6, 7, 8 preview literatures in nonsmooth optimization frontier research where progress and impacts can be made.

2 Preliminaries

Beck’s book [1].

- 2.1 Fundamentals in convex analysis
- 2.2 Fundamentals in non-convex analysis
- 3 Unifying variants of Nesterov's accelerated methods
- 4 Method of spectral momentum
- 5 Catalyst accelerations and future works
- 6 Performance estimation problems
- 7 Methods of inexact proximal point
- 8 Nesterov's acceleration in the non-convex case and momentum in general
- 9 Using PostgreSQL and big data analytic method for species classification on Sentinel-2 Satellite remote sensing imagery

References

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- [2] W. NOEL, *Nesterov's method for convex optimization*, SIAM Review, 65, pp. 539–562.