Machine Learning, a Review for Tree Segmentations Tasks

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Abstract

This is my own notes.

1 Introduction

Artificial Neural networks represents modes of numerical computations that are differentiable programs. We skips the basics and assume the reader already know something about Deep Neural Network, their components, and the automatic differentiation program on modern deep learning frameworks. For our discussion we introduce some definitions to make for a better presentation of computataional concepts occured in Artificial Neural Networks (ANNs).

Definition 1 (Component)**.** A component is a function $f(x; p|w) : \mathbb{R}^m \mapsto \mathbb{R}^n$. x is the inputs and w represent trainable parameters, usually in the form of a multi-dimensional array. And p represents parameters that are not trainable parameters.

Definition 2 (Connection)**.** Let $f : \mathbb{R}^n \mapsto \mathbb{R}^m, g : \mathbb{R}^m \mapsto \mathbb{R}^k$ be two components, then a connection between is a $\mathbb{R}^m \mapsto \mathbb{R}^m$ function $h(x; p|w)$ with trainable parameters w, and parameter p.

Example 1.0.1 (Dense Layer)**.** Let $m, n \in \mathbb{N}$, let $A \in \mathbb{R}^{n\times m}$, $b \in \mathbb{R}^n$, then a Dense layer is a $\mathbb{R}^m \mapsto \mathbb{R}^n$ functions with a list of activation functions σ_i for $i = 1, \cdots, n$. Let $x \in \mathbb{R}^m$ be the input, then a dense layer is a component. We define its computation:

$$\mathrm{DnsLyr}(x; m, n|(A, b), \{\sigma_i\}_{i=1}^n) = \left[z \mapsto \bigoplus_{i=1}^{n} \sigma_i(z_i)\right](Ax + b).$$

Where, inside of $[\cdot]$, we denote the definition of a anonymous function.

Example 1.0.2 (Multi-Layer Perceptron)**.** Let $l_1, l_2, \cdots, l_N$ be integers. We define the Multi-Layer Perceptron to be a composition of dense layer mapping from l_i to l_{i+1} for $i = 1, \cdots, N-1$. Let $\sigma_{i,j}$ represent the activation function for the j th output in the i th layer.

Then a Multi-Layer Perceptron (MLP) is a component admit representation

$$\begin{aligned}
&\mathrm{MLP}\left(x; l_1, \cdots, l_n \middle| \{(A_i, b_i)\}_{i=1}^N\right) : \mathbb{R}^{l_1} \mapsto \mathbb{R}^{l_N} \\
:= &\left[\bigodot_{i=1}^{n} \mathrm{DnsLyr}\left((\cdot); l_i, l_{i+1} \middle| (A_i, b_i), \{\sigma_{j,i}\}_{j=1}^{l_i}\right)\right](x).
\end{aligned}$$

Where $\bigodot$ is functional composition and it represents $\bigodot_{i=1}^n f_i = f_n \circ \cdots \circ f_1(x)$, and $(\cdot)$ represents the input of the anonymous function, in this case it's the dense layer.

2 Preliminaries

These concepts and components are relevant to the architecture of Vision Networks.

Definition 3 (Convolution 2D)**.** Let u, v be multi-array of dimension $m \times n$ and $k \times l$. We assume that $m \leq k$ and $n \leq l$. Then the convolution operator $*$ is a mapping from $(\mathbb{C}^M \times \mathbb{C}^n) \times (\mathbb{C}^k \times \mathbb{C}^l) \mapsto \mathbb{C}^{(m-k)\times(n-l)}$. Then the convolution is defined as

$$(u * v)_{t,\tau} = \sum_{i=1}^{k}\sum_{j=1}^{l} u_{i,j} v_{i+t, j+\tau}.$$

2.1 Convolutional Layers

In this section we talk about 2D convolution component (pytorch link) inside of an ANNs. The convolution operations module contains more detailed parameters.

Definition 4 (2D Convolution Layers)**.** Assuming that we have a single sample. Let (C, H, W) be the shape of the input tensor. C is the number of channel, and H, W are the height and width. We use this because image tensors are usually in the shape of $(3, H, W)$. Define the component to be a function, mapping from (C', H', W'). Let (C, K, L) denotes the dimension of the kernel: $\mathcal{K}$. Then mathematically, the computation of the output tensor Y given input tensor X can be computed as

$$Y_{c',h',w'} = \mathrm{ReLU}\left(b_{c'} + \sum_{n=1}^{C} (\mathcal{K}_{c'} * X)_{h',w'}\right).$$

Remark 2.1.1. Observe that each of the output channel is the sum of C many kernels convluted with all inputs channels and summed up.

Definition 5 (2D Max Pooling Layers)**.**

A Appendix Section 1

This is the appendix section.