

# Modeling and Algorithms for Prof Shi's Project

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## Abstract

We propose some better algorithm for a problem in detecting structure of probability transition matrices from data.

## 1 Introduction

We describe an optimization problem introduced by Prof Shi and his student Yining. To start we define the following quantities for the optimization problem.

1.  $n \in \mathbb{N}$ . It denotes the number of states for a Markov Chain.
2.  $p \in \mathbb{R}^{n \times n}$  denotes the probability transition matrix. It's in small case because it's also the variable for the optimization problem. It supports 2 types of indexing,  $p_{ij}$  for  $i, j \in \{1, \dots, n\}$ , or  $p_j$  with  $j \in \{1, \dots, n^2\}$ . More on this later.
3.  $\eta_{ij} \geq 0$  for  $i, j \in \{1, \dots, n\}$  is a parameter of the problem.
4.  $\hat{p}$  is the empirically measured probability transition matrix. They are the maximal likelihood estimators for the transition probability in the transition probability matrix.
5.  $\lambda$  is the regularization parameter.

When  $p$  is referred to as a vector we may say  $p \in \mathbb{R}^{n^2}$ , if it's referred to as the matrix, we will use  $p \in \mathbb{R}^{n \times n}$ . When indexing  $p$  using a tuple, or a single number, it's possible to translate between the two type of indexing scheme using the following bijective map:

$$\begin{aligned}(i, j) &\mapsto k := i \times n + j \\ k &\mapsto (i, j) := (\lfloor k/n \rfloor, \text{mod}(k, n) + 1).\end{aligned}$$

We emphasize, in different programming languages and development environments, the convention of indexing a multi-array using different kind of tuples can be very different. For now we use the above indexing, which is a row major index convention (Like Python).

### 1.1 The Optimization Problem

## 2 Preliminaries

[This](#) is the preliminary (hyperref without text labeling).



(a) Cute Alto



(b) Minty and Alto

### 3 Blah Blah Bleeeh

#### A Bleeh Bleeh Bleeh I am not Listening

This is the Bleeh Bleeh Bleeh I am not Listening section.

#### B This section is in another .tex file

This is a new section.

##### B.1 Subsection

This is a subsection.

##### B.2 Cute Subsection

Check out this cute figure. In [fig 1a](#) is cute pink unicorn, and in [fig 1b](#), the green earth pony is minty, a cookie pone. Theyare cute together. Read source to understand the use of “subfloat” and “figure” together with “hyperref”.