A Discussion on The Nesterov Momentum and Variants of FISTA with TV Minmizations Application

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ToC

Paper Under Review

Nesterov's Method for Convex Optimization*

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Abstract. While Nesterov's algorithm for computing the minimum of a convex function is now over forty years old, it is rarely presented in texts for a first course in optimization. This is unfortunate since for many problems this algorithm is superior to the ubiquitous steepest descent algorithm, and it is equally simple to implement. This article presents an elementary analysis of Nesterov's algorithm that parallels that of steepest descent. It is envisioned that this presentation of Nesterov's algorithm could easily be covered in a few lectures following the introductory material on convex functions and steepest descent included in every course on optimization.

Key words. convex optimization, Nesterov's algorithm, steepest descent

MSC codes. 65K10, 60C46, 60C25

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Noel J. Walkington, SIAM REVIEW Jun 2023 Education, Volume 65 Number 2, pp. 539-562. [1]

Presentation Outline and Objective

1. Introducing the Application of TV Minimization for Signal Recovery.
2. Literature Review.
3. Nesterov lower bound complexity claim clarified.
4. Our proof for V-FISTA convergence under strong convexity inspired by Beck [2, 10.7.7].
5. Some exciting numerical results for our method, which we refer to as "The method of Spectral Momentum".

Total Variance Minimization Formulation

Total Variance Minimization (TV) problem recovers the digital signal from observations of a signal with noise. Let $u(t) : [0, 1] \mapsto \mathbb{R}$ be the signal and $\hat{u}$ be a noisy observation, then

Variational Formulation

$$f(u) = \int_0^1 \frac{1}{2}(u - \hat{u})^2 + \alpha |u'| dt.$$

- Minimizing $f(u)$ with pentalty term constant $\alpha > 0$ yield a recovered signal.
- Original signal u is assumed to be piecewise constant with bounded variation.
- Sparsity is imposed on u', making u' to be Dirac Delta function.

Discritizations

Implementations on modern computing platforms **necessitate discretization** of signal u to $\mathbb{R}^{N+1}$. With $s_i = u_i - \hat{u}_i$, $h_k = t_k - t_{k-1}, k \geq 1$ using the trapezoid rule and first-order forward difference yields:

$$\frac{1}{2}\int_0^1 (u-\hat{u})^2 dt + \alpha \int_0^1 |u'| dt \approx \frac{1}{2}\sum_{i=0}^{N}\left(\frac{s_i^2 + s_{i+1}^2}{2}\right) h_{i+1} + \alpha \sum_{i=1}^{N}\left|\frac{u_i - u_{i-1}}{h_{i+1}}\right|$$

- ▷ let $C \in \mathbb{R}^{N\times(N+1)}$ be upper bi-diagonal with $(1, -1)$

$$= \frac{1}{2}\left(\frac{s_0^2 h_1}{2} + \frac{s_N^2 h_N}{2} + \sum_{i=1}^{N-1} s_i^2 h_i\right) + \alpha \|Cu\|_1$$

- ▷ using $D \in \mathbb{R}^{N\times(N+1)}$,
- ▷ $D := \text{diag}(h_1/2, h_1, h_2, \cdots, h_N, h_N/2)$

$$= \frac{1}{2}\langle u - \hat{u}, D(u-\hat{u})\rangle + \alpha\|Cu\|_1.$$

Discretized Model

Recall D is diagonal, strictly positive entry, $C \in \mathbb{R}^{N \times N+1}$ is bidiagonal.

Discretized Formulation

$$f(u) = \frac{1}{2}\langle u - \hat{u}, D(u - \hat{u})\rangle + \alpha \|Cu\|_1.$$

If we were to use the Forward-Backward(FB) splitting, then we have unresolved implementation difficulties:

1. ADMM, Chambolle Pock, would apply; however, when using the FB Splitting, $\alpha\|Cu\|_1$ would be prox unfriendly.
2. Prox over $\alpha\|Cu\|_1$ is possible with D being bi-diagonal, but it would be a hassle if done for generic C.

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Remedy via Lagrangian Dual Reformulation

Let $p = Cu$, $C \in \mathbb{R}^{(N+1)\times N}$ with $D \in \mathbb{R}^{(N+1)\times(N+1)}$, we reformulate it into

$$\min_{u\in\mathbb{R}^{N+1}} \left\{ \underbrace{\frac{1}{2}\langle (u-\hat{u}), D(u-\hat{u})\rangle}_{f(u)} + \underbrace{\alpha\|p\|_1}_{h(p)} \middle| \, p = Cu \right\},$$

producing Lagrangian of the form

$$\mathcal{L}((u,p),\lambda) = f(u) + h(p) + \langle \lambda, p - Cu\rangle.$$

The Dual Problem is

$$
\begin{aligned}
-g(\lambda) &:= \inf_{(u,p)\in\mathbb{R}^{N+1}\times\mathbb{R}^N} \{\mathcal{L}((u,p),\lambda)\} \\
&= \inf_{(u,p)\in\mathbb{R}^{N+1}\times\mathbb{R}^N} \{f(u)+h(p)+\langle\lambda, p-Cu\rangle\} \\
&= \inf_{u\in\mathbb{R}^{N+1}} \left\{ f(u) - \langle\lambda, Cu\rangle + \inf_{p\in\mathbb{R}^N} \{h(p)+\langle\lambda,p\rangle\} \right\} \\
&\leq -f^\star(-C^T\lambda) - h^\star(p).
\end{aligned}
$$

So

$$
-g(\lambda) = -\frac{1}{2}\|C^T\lambda\|^2_{D^{-1}} - \langle\hat{u}, C^T\lambda\rangle - \delta_{[-\alpha,\alpha]^N}(p).
$$

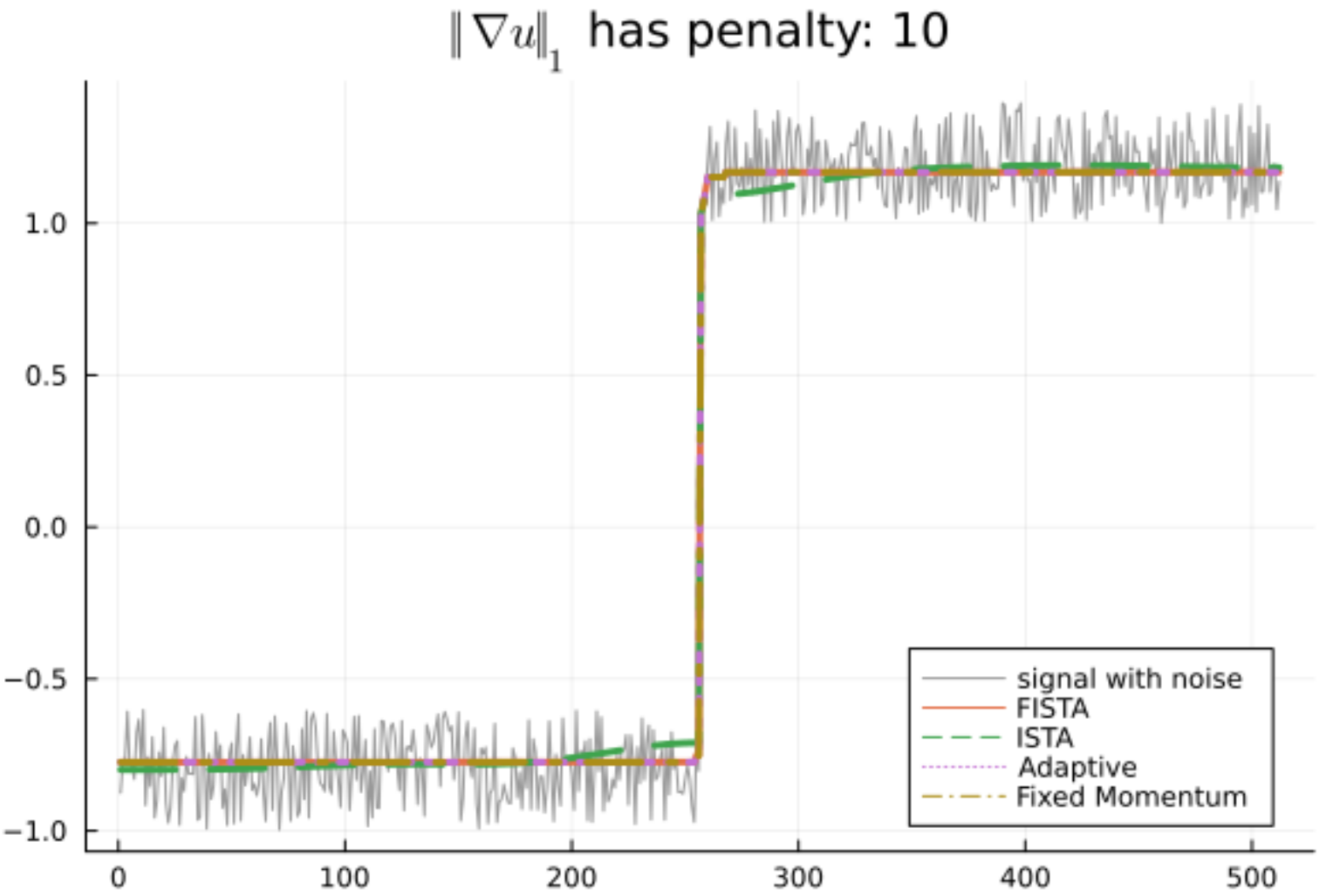
Dual

$$-g(\lambda) = -\frac{1}{2}\|C^T\lambda\|^2_{D^{-1}} - \langle \hat{u}, C^T\lambda \rangle - \delta_{[-\alpha,\alpha]^N}(p).$$

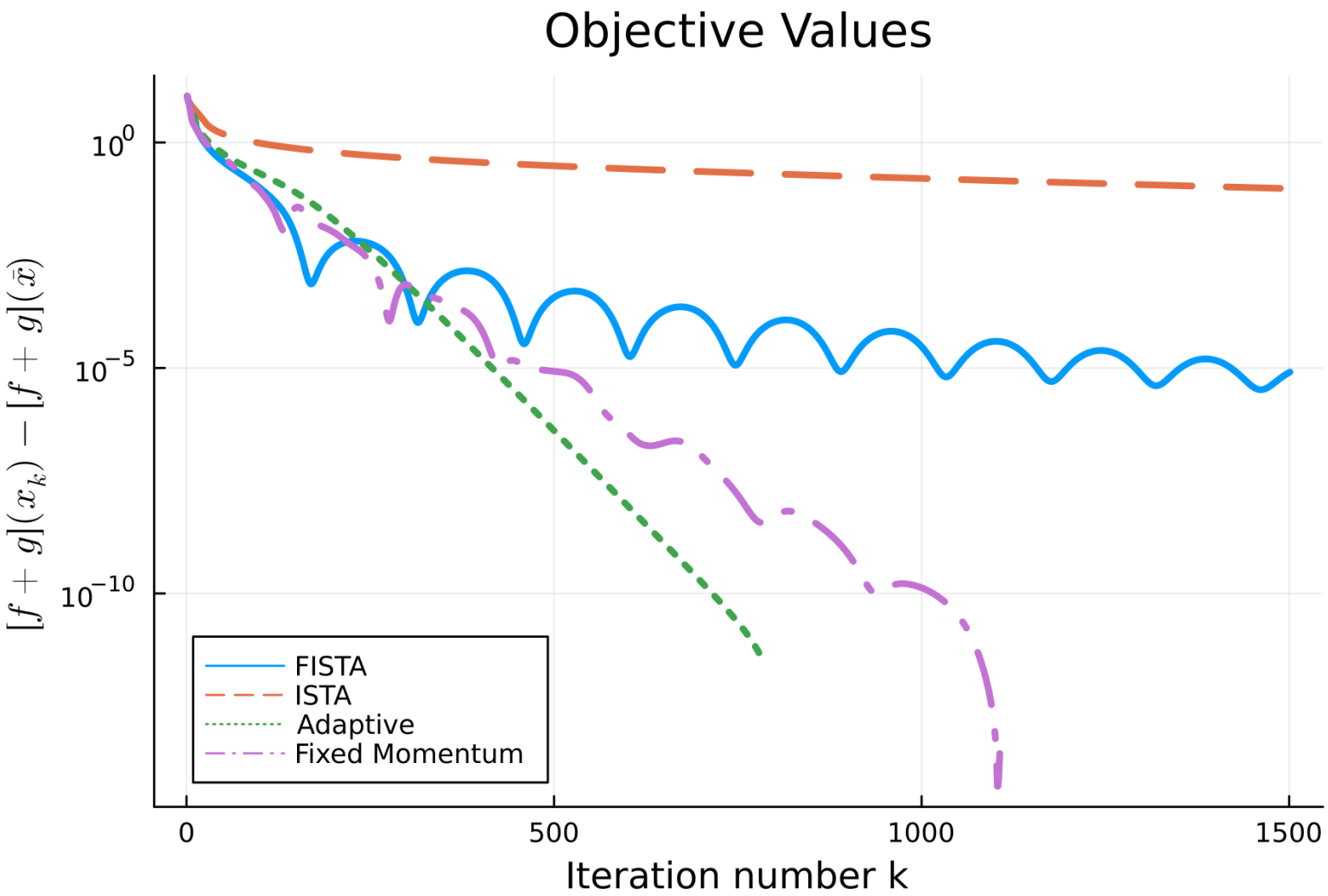
- Fact: $u = \hat{u} + D^{-1}C^T\lambda$, for the primal.
- D^{-1} is Positive Definite and diagonal, very easy to invert.
- $-g(\lambda)$ would be strongly convex.

Numerical Results

Implemented with Julia[3], with several variants of FISTA, we have



One Big Bummer



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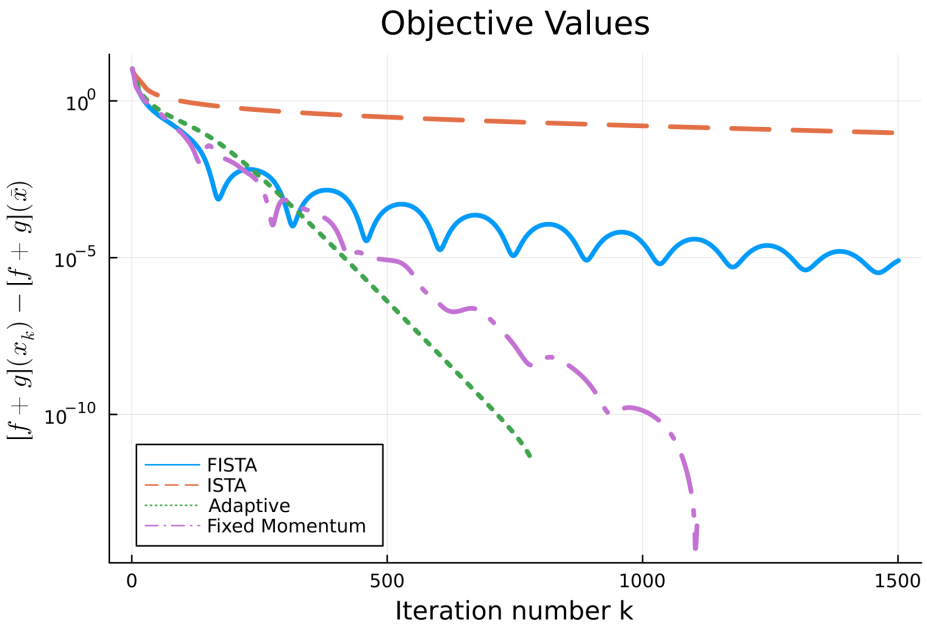

Main observations

1. FISTA is non-robust to strong convexity; it experiences the same $(1/k^2)$.
2. However, ISTA would be $\mathcal{O}(1-1/\kappa)^k)$ under strong convexity, with $\kappa = L/\sigma$, for L-Lipschitz smooth and σ strongly on the smooth part of the FB splitting objective.

Generic FISTA

We introduce the below algorithm 1 to expedite presentation. Consider Smooth, non-smooth Additive composite objective $F = g + h$.

A Good Template

Algorithm Generic FISTA

1: **Input:** $(g, h, x^{(0)})$
2: $y^{(0)} = x^{(0)}$, $\kappa = L/\sigma$
3: **for** $k = 0, 1, \cdots$ **do**
4: $\quad x^{(k+1)} = T_L y^{(k)}$
5: $\quad y^{(k+1)} = x^{(k+1)} + \theta_{k+1}(x^{(k+1)} - x^{(k)})$
6: $\quad$ Execute subroutine $\mathcal{S}$.
7: **end for**

Changing T_L, θ_{k+1}, and $\mathcal{S}$ yield different variants.

Variant (1.), FISTA Original

Algorithm Generic FISTA

Input: $(g, h, x^{(0)})$
$y^{(0)} = x^{(0)}$, $\kappa = L/\sigma$
for $k = 0, 1, \cdots$ **do**
    $x^{(k+1)} = T_L y^{(k)}$
    $y^{(k+1)} = x^{(k+1)} + \theta_{k+1}(x^{(k+1)} - x^{(k)})$
    Execute subroutine $\mathcal{S}$.
end for

Original FISTA proposed by Beck [4] has

- $\theta_{k+1} = (t_k - 1)/t_{k+1}$, $t_{k+1}(t_{k+1} - 1) = t_k^2$, $t_0 = 1$.
- It achieves $\mathcal{O}(1/k^2)$ on the objective value; it doesn't improve for strongly convex function g.
- No known proof for the convergence of the iterates.

Variant (2.), Chambolle, Dossal

Algorithm Generic FISTA

1: **Input:** $(g, h, x^{(0)})$
2: $y^{(0)} = x^{(0)}$, $\kappa = L/\sigma$
3: **for** $k = 0, 1, \cdots$ **do**
4: $\quad x^{(k+1)} = T_L y^{(k)}$
5: $\quad y^{(k+1)} = x^{(k+1)} + \theta_{k+1}(x^{(k+1)} - x^{(k)})$
6: $\quad$ Execute subroutine $\mathcal{S}$.
7: **end for**

From Chamebolle, Dossal [5],

- $\theta_{k+1} = (n + a - 1)/a$, for $a > 2$.
- Proved in Chambolle, Dossal [5], its iterates of this version of FISTA exhibit weak convergence.
- T_L is the same as (1.); it experiences the same convergence rate for the function objective.

Variant (3.) (Also known as V-FISTA), From Beck

Algorithm Generic FISTA

Input: $(g, h, x^{(0)})$
$y^{(0)} = x^{(0)}$, $\kappa = L/\sigma$
for $k = 0, 1, \cdots$ **do**
    $x^{(k+1)} = T_L y^{(k)}$
    $y^{(k+1)} = x^{(k+1)} + \theta_{k+1}(x^{(k+1)} - x^{(k)})$
    Execute subroutine $\mathcal{S}$.
end for

Proposed in Beck[2], and also Nesterov[6].

- has $\theta_{k+1} = (\sqrt{\kappa} - 1)/(\sqrt{\kappa} + 1)$.
- Only for strong convexity g, (Or the weaker condition such as Quadratic Growth).
- Has $\mathcal{O}((1 - 1/\sqrt{\kappa})^k)$, for both objective value and iterates.
- T_L is still the same FB splitting.

Motivates Variants (4.)

V-FISTA is simple but requires knowledge of σ, the strong convexity index; it is a troublemaker.

- Exact estimation would involve inverting the Hessian or a closed-form formula tailored for a specific problem.
- Over estimation of σ invalidates the linear convergence results.
- Under estimation of σ slows down the linear-convergence.

To overcome this, we propose a real-time overestimation of σ using

$$\sigma \leq \langle \nabla g(y^{(k+1)}) - \nabla g(y^{(k)}), y^{(k+1)} - y^{(k)} \rangle / \|y^{(k+1)} - y^{(k)}\|^2.$$

We call this **Spectral Momentum**. The same formula is used for spectral stepsizes, an adaptive stepsize scheme for gradient descent[7, 4.1]. Its efficacy was demonstrated at the start of the talk. We are not sure why it works so well.

MFISTA

Algorithm Generic FISTA

Input: $(g, h, x^{(0)})$
$y^{(0)} = x^{(0)}$, $\kappa = L/\sigma$
for $k = 0, 1, \cdots$ **do**
    $x^{(k+1)} = T_L y^{(k)}$
    $y^{(k+1)} = x^{(k+1)} + \theta_{k+1}(x^{(k+1)} - x^{(k)})$
    Execute subroutine $\mathcal{S}$.
end for

MFISTA in Beck[8] is produced by adding $\mathcal{S}$ to be the procedure

$$(y^{(k+1)}, t_{k+1}) = \begin{cases} (x^{(k+1)}, 1) & F(y^{(k+1)}) > F(x^{(k+1)}), \\ (y^{(k+1)}, t_{k+1}) & \text{else.} \end{cases}$$

MFISTA

MFISTA was an early attempt at improving FISTA.

1. Requires frequent computing of the objective, slowing it by a constant factor compared to FISTA.
2. There is no better convergence rate than $\mathcal{O}(1/k^2)$ from our research.

FISTA Restart Refuses to Die

However, the idea of restarting FISTA refuses to die.
In our opinion, the interests gather around restarting FISTA because spending too much computational effort would compete against the Proximal Quasi-Newton method, questioning the use of momentum in the first place. Hence, these recent developments:

- Asymptotic linear convergence of conditional restart by Alamo et al. [9][10, text] et al., and Fercoq [11], under strong convexity, or local quadratic growth.
- Parameter Free FISTA with automatical restart by Aujol et al.[12], fast linear convergence on quadratic growth with proofs, no parameters needed.

Theories of Nesterov Momentum

Frontier developments of theories for Nesterov Momentum are not dying either.

1. Su et al. [13] identified a second-order ODE that is the exact limit of the variant (1.).
2. In Attouch and Peypouquet [14], they showed a $o(1/k^2)$ convergence rate of variant (2.) based on the ODE understanding.
3. In Nesterov[6], he proposed a generic algorithm that can derive variants (1.), (3.), and more.
4. In Ahn and Sra[15] they derived a lot of variants of APGs using the Proximal Point method of Rockafellar and discussed the unified theme of a "similar triangle."

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The Most Recent and Hardcore Developement

The paper is titled "Computer-Assisted Design of Accelerated Composite Optimization Methods: OptISTA" by Jang et al. [16]. They updated the manual script On Nov 1st, 2023.

- Based on the Performance Estimation Problem, they phrase the search of the fastest possible first-order algorithm(more on this later) as a QCQP.
- They squeeze out a constant factor on the convergence of the original FISTA.
- They also proved a lower bound for their OptISTA, showing its exact optimality.
- They claim they had concluded the search for the fastest first-order method.

Moral of the Story

This is the general trajectory:

- Improve the algorithm's robustness and applications.
- Weaken the conditions and add more convergence proofs for a broader scope.
- Searching for a general framework of understanding for the class of the Nesterov acceleration method.
- Connects with various other ideas to improve theoretical understanding.
- Extremely hardcore algorithmic tricks and improvements to squeeze out that last bit of performance.

Working on FISTA would amount to competing against some of the brightest researchers.

First-Order Methods

What we can do is to understand at least Nesterov's claim on lower complexity bound and why we believe there is a mistake in Walkington[theorem 2.4][1].

Definition (First Order Method)

We are in $\mathbb{R}^n$ for now. Given $x^{(0)} \in \mathbb{R}^n$, an iterative algorithm generates sequence of $\left(x^{(n)}\right)_{n\in\mathbb{N}}$ in the space. All $\mathcal{A} \in \mathrm{GA}^{1\mathrm{st}}$ satisfy that

$$x^{(j+1)} := \mathcal{A}_f^{j+1} x^{(0)} \in \left\{x^{(0)}\right\} + \operatorname{span}\left\{\nabla f\left(x^{(i)}\right)\right\}_{i=1}^{j} \qquad \forall f, \forall 1 \leq j \leq k-1.$$

We adapted the above definition from Nesterov[6, 2.1.4]. We came up with two examples for the definition.

Fixed-Step Descent

Example (Fixed Step Descent)

The method of fixed-step gradient descent, $x^{(k+1)} = x^{(k)} - L^{-1}\nabla f(x^{(k)})$ is $\bar{\mathcal{A}} \in \mathrm{GA}^{1\mathrm{st}}$ achieves a maximal decrease in objective value for all $f \in \mathcal{F}_L^{1,1}$ given $x^{(k)}$, it can be understood as

$$\bar{\mathcal{A}} \in \underset{\mathcal{A}\in \mathrm{GA}^{1\mathrm{st}}}{\operatorname{argmin}} \max_{f\in\mathcal{F}_L^{1,1}} \left\{ f\left(\mathcal{A}_f x^{(k)}\right)\right\}.$$

This method is memoryless because it only matters what $x^{(k)}$, prior iterate $x^{(i)}, 1 \leq i \leq k-1$ plays no role.

Steepest Descent

Example (Steepest Descent)

Fix some $f, x^{(k)}$, the method of steepest descent would be $\bar{\mathcal{A}} \in \mathrm{GA}^{1\mathrm{st}}$ and it's

$$\bar{\mathcal{A}} \in \underset{\mathcal{A} \in \mathrm{GA}^{1\mathrm{st}}}{\operatorname{argmin}} \left\{ f\left(\mathcal{A}_f x^{(k)} \right) \right\}.$$

This method is also memoryless.

We emphasize that f is fixed.

Nesterov Lower Complexity Bound

The following is Nesterov [6, Thm 2.1.7].

Theorem (Nesterov's Claim of Lower Bound)

For any $1 \leq k \leq 1/2(n-1)$, for all $x^{(0)} \in \mathbb{R}^n$, there exists a Lipschitz smooth convex function in $\mathbb{R}^n$ such that for all algorithm from GA^{1st}, we have the lower bound for the optimality gap for the function values and its iterates:

$$f\left(x^{(k)}\right) - f^* \geq \frac{3L\|x - x^*\|^2}{32(k+1)^2}, \quad \|x^{(k)} - x^*\|^2 \geq \frac{1}{8}\|x^{(0)} - x^*\|^2.$$

Where x^* is the minimizer of f, so that $f(x^*) = \inf_x f(x)$.

We emphasize that it didn't fix the function f, but it fixes the iteration counter k.

Walkington's claim of Lower Bound

We now quote Walkington [1, theorem 2.4]

Theorem (Walkington's Claim of Lower Bound)

Let X be an infinite-dimensional Hilbert Space and set $x^{(0)} = \mathbf{0}$. There exists a convex function $f : X \mapsto \mathbb{R}$ with Lipschitz gradient and minimum $f(x_*) > -\infty$ such that for any sequence satisfying

$$x_{i+1} \in Span\left\{\nabla f(x^{(0)}), \nabla f(x^{(1)}), \cdots, \nabla f(x^{(i)})\right\}, \quad i = 0, 1, 2, \cdots,$$

there holds

$$\min_{1 \le i \le n} f(x_i) - f(x_*) \ge \frac{3L\|x_1 - x_*\|^2}{32(n+1)^2},$$

where L is the Lipschitz constant of the gradient.

- The former claims there exists a single function from $\mathcal{F}_L^{1,1}(\mathcal{H})$ introduces the lower bound for all values of k, and all algorithms from GA^{1st}, but the latter didn't claim that.
- The difference would remain in infinite dimension Hilbert space if we were to generalize thorem 2.
- No contradiction from Attouch and Peypouquet[14] on $o(1/k^2)$ because they fixed ϕ in the proof.

There is no proof after Walkington's claim; we can't know if he had his way of proving the latter claim. It makes us think it is likely a missed detail in his writing.

Statement of Convergence Results for V-FISTA
Convergence under Strong Convexity

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