Detailed UCED-CCGA Formulation

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1 Notational Discrepancies

- We use s to represent the slack variables (instead of v)
- we use μ, v, w to represent the auxiliary variables for bi-linear reformulation instead of ρ, ξ

2 Master Problem

Initially only consider first stage

The objective for the master problem is to maximize the size of the uncertainty set

$$\max \gamma^+ + \sum_b \sum_t \gamma_{bt}^- \tag{1}$$

for a given generator you can not turn it on *and* off in the same time period

$$x_n^t + z_n^t \leq 1, \forall n \in \mathcal{G}^{\mathrm{T}}, \forall t \tag{2}$$

This constraint properly changes the value of y if a generator is turned on or off

$$y_n^t - y_n^{t-1} = x_n^t - z_n^t, \forall n \in \mathcal{G}^{\mathrm{T}}, \forall t, \tag{3}$$

when a generator is turned on, in must respect the minimum up time

$$\sum_{\tau=t-T_n^{minu}+1}^{t} x_n^\tau \leq y_n^t,$$

$$\forall n \in \mathcal{G}^{\mathrm{T}}, \forall t \in \{T_n^{minu}, \cdots, \tau_{end}\} \tag{4}$$

$$\tag{5}$$

when a generator is turned off, in must respect the minimum down time

$$\sum_{\tau=t-T_n^{mind}+1}^{t} z_n^\tau \leq 1 - y_n^t,$$

$$\forall n \in \mathcal{G}^{\mathrm{T}}, \forall t \in \{T_n^{mind}, \cdots, \tau_{end}\} \tag{6}$$

$$\gamma^+ \leq M, \gamma_{bt}^- \leq \hat{d}_b^t \text{(To ensure bounded, realistic solutions)} \tag{7}$$

3 Relaxation Formulation

The point of the relaxation problem is to determine the feasibility of the master problem solution. From the master problem, we have fixed first stage variables. For the relaxation problem, we fix 2nd stage integer variables. The values of these 2nd stage variables are initialized at zero, and after the sub problem is solved, more variables and constraints are added with new 2nd stage variable values, to this relaxation problem.

3.1 Objective

We want to maximize the auxiliary variables which corresponds to the slack, to find the worst case demand

$$\max \eta$$

3.2 Constraints

3.2.1 Slack-Auxiliary constraint

Bound the auxiliary variable by the sum of all slack variables

$$\eta \leq s_{budget}^{-} + \sum_t (s_{balance}^{t+} + s_{balance}^{t-}) + \sum_t \sum_n s_{Pmin}^{nt+} + \sum_t \sum_m s'^{mt+}_{Pmin} + \sum_t \sum_l \left(s_{transUB}^{lt-} + s_{transLB}^{lt+} \right)$$

$$\sum_t (s_{RREGU}^{t+} + s_{RREGD}^{t+} + s_{RNSP}^{t+} + s_{rfrr}^{t+}) + \sum_t \sum_{sg} (s_{tffr1}^{t,sg+} + s_{tffr2}^{t,sg-} + s_{inx1}^{t,sg+} + s_{inx2}^{t,sg-}) \tag{8}$$

Primal Constraints

Fuel Constraints

The sum of fuel costs, up and down costs must not exceed our budget

$$(\lambda_{budget}) \sum_t \sum_n c_n^t + \sum_t \sum_n c'^t_m - s_{budget}^{-} \leq \phi - \sum_t \left(\sum_n SU_n x_n^t + \sum_n SD_n z_n^t + \sum_m SU_m x'^t_m + \sum_m SD_m z'^t_m \right) \tag{9}$$

Piecewise quadratic function for conventional generator fuel costs

$$(\lambda_{fuel}^{jnt}) c_n^t - \beta_{nj} p_n^t \geq \alpha_{nj} y_n^t \quad \forall j, n, t \tag{10}$$

Piecewise quadratic function for quick start generator fuel costs

$$(\lambda'^{jmt}_{fuel}) c'^t_m - \beta'_{mj} p'^t_m \geq \alpha'_{mj} y'^t_m \quad \forall j, m, t \tag{11}$$

Capacity Constraints

ramp up limiting for conventional generator

$$(\lambda_{RU}^{nt}) p_n^t - p_n^{t-1} \leq RU_n y_n^{t-1} + \overline{RU}_n x_n^t \quad \forall n, t = 2...T \tag{12}$$

ramp down limiting for conventional generator

$$(\lambda_{RD}^{nt}) p_n^{t-1} - p_n^t \leq RD_n y_n^t + \overline{RD}_n z_n^t \quad \forall n, t = 2...T \tag{13}$$

spinning reserve capacity for conventional generator

$$(\lambda_{sr}^{nt}) sr_n^t \leq 10 R_n^{MT} y_n^t \quad \forall n, t \tag{14}$$

overall capacity for conventional generator

$$(\lambda_{Pmax}^{nt}) p_n^t + sr_n^t + regu_n^t \leq P_n^{max} y_n^t \qquad \forall n, t \tag{15}$$

minimum requirement for conventional generator

$$(\lambda_{Pmin}^{nt}) p_n^t - regd_n^t + s_{Pmin}^{nt+} \geq P_n^{min} y_n^t \quad \forall n, t \tag{16}$$

non-spinning reserve capacity for conventional generator

$$(\lambda_{nsp}^{nt}) nsp_n^t \leq (1 - y_n^t) NSP_n \quad \forall n, t \tag{17}$$

regulation up capacity for conventional generators

$$(\lambda_{regu}^{nt}) regu_n^t \leq y_n^t REGU_n^t \quad \forall n, t \tag{18}$$

regulation down capacity for conventional generators

$$(\lambda_{regd}^{nt}) regd_n^t \leq y_n^t REGD_n^t \quad \forall n, t \tag{19}$$

fast frequency response capacity for conventional generators

$$(\lambda_{ffr}^{bt}) ffr_b^t \leq \overline{FFR}_b^t \quad \forall b, t \tag{20}$$

Quick Start Binary Constraints (don't directly need this constraints for

CCGA)

for a given generator you can not turn it on *and* off in the same time period

$$x_n^t + z_n^t \leq 1, \forall n \in \mathcal{G}^{\mathrm{T}}, \forall t \tag{21}$$

This constraint properly changes the value of y if a generator is turned on or off

$$y_n^t - y_n^{t-1} = x_n^t - z_n^t, \forall n \in \mathcal{G}^{\mathrm{T}}, \forall t, \tag{22}$$

when a generator is turned on, in must respect the minimum up time

$$\sum_{\tau=t-T_n^{minu}+1}^{t} x_n^\tau \leq y_n^t,$$

$$\forall n \in \mathcal{G}^{\mathrm{T}}, \forall t \in \{T_n^{minu}, \cdots, \tau_{end}\} \tag{23}$$

$$\tag{24}$$

when a generator is turned off, in must respect the minimum down time

$$\sum_{\tau=t-T_n^{mind}+1}^{t} z_n^\tau \leq 1 - y_n^t,$$

$$\forall n \in \mathcal{G}^{\mathrm{T}}, \forall t \in \{T_n^{mind}, \cdots, \tau_{end}\} \tag{25}$$

$$\tag{26}$$

Quick Start Capacity Constraints

ramp up capacity for quick start generator

$$(\lambda_{RU}^{\prime mt}) p_m^t - p_m^{t-1} \leq RU_n {y'}_m^{t-1} + \overline{RU}_m {x'}_m^t \quad \forall m, t = 2...T \tag{27}$$

ramp down capacity for quick start generator

$$(\lambda_{RD}^{\prime mt}) {p'}_m^{t-1} - {p'}_m^t \leq RD_m {y'}_m^t + \overline{RD}_m {z'}_m^t \quad \forall m, t = 2...T \tag{28}$$

spinning reserve capacity for quick start generator

$$(\lambda_{sr}^{\prime mt}) {sr'}_m^t \leq 10 R_m^{MT} {y'}_m^t \quad \forall m, t \tag{29}$$

overall capacity for quick start generator

$$(\lambda_{Pmax}^{\prime mt}) {p'}_m^t + {sr'}_m^t + {regu'}_m^t \leq P_m^{max} {y'}_m^t \qquad \forall m, t \tag{30}$$

minimum requirement for quick start generator

$$(\lambda_{Pmin}^{\prime mt}) {p'}_m^t - {regd'}_m^t + s_{Pmin}^{\prime mt+} \geq P_m^{min} {y'}_m^t \quad \forall m, t \tag{31}$$

non-spinning reserve capacity for quick start generator

$$(\lambda_{nsp}^{\prime mt}) {nsp'}_m^t \leq (1 - {y'}_m^t) NSP_m \quad \forall m, t \tag{32}$$

regulation up capacity for quick start generators

$$(\lambda_{regu}^{\prime mt}) {regu'}_m^t \leq {y'}_m^t REGU_m^t \quad \forall m, t \tag{33}$$

regulation down capacity for quick start generators

$$(\lambda_{regd}^{\prime mt}) regd_m^t \leq {y'}_m^t REGD_m^t \quad \forall m, t \tag{34}$$

$$\tag{35}$$

Demand Balance Constraints

total power must equal total demand

$$(\lambda^t_{balance})\sum_n p^t_n + \sum_m {p'}^t_m + s^{t+}_{balance} - s^{t-}_{balance} = \sum_b D^t_b \quad \forall t \tag{36}$$

transmission line upper bound

$$(\lambda^{lt}_{transUB})\sum_b SF^l_b\Big(\sum_{n\in G^b} p^t_n + \sum_{m\in {G'}^b} {p'}^t_m\Big) - s^{lt-}_{transUB} \le \sum_b SF^l_b D^t_b + F_l \quad \forall l,t$$

transmission line lower bound

$$(\lambda^{lt}_{transLB})\sum_b SF^l_b\Big(\sum_{n\in G^b} p^t_n + \sum_{m\in {G'}^b} {p'}^t_m\Big) + s^{lt+}_{transLB} \ge \sum_b SF^l_b D^t_b - F_l \quad \forall l,t \tag{37}$$

Requirement Constraints

regulation up requirement

$$(\lambda^t_{RREGUreq})\sum_n regu^t_n + \sum_m {regu'}^t_m + s^{t+}_{RREGU} \ge RREGU^T \quad \forall t \tag{38}$$

regulation down requirement

$$(\lambda^t_{RREGDreq})\sum_n regd^t_n + \sum_m {regd'}^t_m + s^{t+}_{RREGD} \ge RREGD^T \quad \forall t \tag{39}$$

non-spinning reserve requirement

$$(\lambda^t_{NSPreq})\sum_n nsp^t_n + \sum_m {nsp'}^t_m + s^{t+}_{RNSP} \ge RNSP^T \quad \forall t \tag{40}$$

total frequency response requirement

$$(\lambda^t_{rfrrreq}) frr^t - rfrr^t + s^{t+}_{rfrr} \ge 0 \quad \forall t \tag{41}$$

total frequency response is the total fast frequency response plus spinning reserve

$$(\lambda^t_{frr}) frr^t - tffr^t - \sum_n sr^t_n - \sum_m {sr'}^t_m = 0 \tag{42}$$

Inertia Constraints

total fast frequency response is equal to the sum of fast frequency response at each bus multiplied by the conversion factor for the correct inertia interval

$$(\lambda_{tffr1}^{t,sg}) tffr^t - K_{sg} \sum_b ffr_b^t + s_{tffr1}^{t,sg+} \geq -M_{sg}^t (1 - \delta_{sg}^t + \delta_{sg+1}^t) \quad \forall sg = 1, ...SG-1, t \tag{43}$$

total fast frequency response is equal to the sum of fast frequency response at each bus multiplied by the conversion factor for the correct inertia interval

$$(\lambda_{tffr2}^{t,sg}) tffr^t - K_{sg} \sum_b ffr_b^t - s_{tffr2}^{t,sg-} \leq M_{sg}^t (1 - \delta_{sg}^t + \delta_{sg+1}^t) \quad \forall sg = 1, ..., SG-1, t \tag{44}$$

calculation of required frequency response as a piecewise linear function with respect to inertia

$$(\lambda_{rfrr}^t) rfrr^t - \sum_{sg} IRinx_{sg}^t = RFRR_1 \quad \forall t \tag{45}$$

computation of inertia in each interval, which is determined by delta values

$$(\lambda_{inx1}^{t,sg}) inx_{sg}^t + s_{inx1}^{t,sg+} \geq \delta_{sg+1}^t (IN_{sg+1} - IN_{sg}) \quad sg = 1, ..., SG-1, t \tag{46}$$

computation of inertia in each interval, which is determined by delta values

$$(\lambda_{inx2}^{t,sg}) inx_{sg}^t - s_{inx2}^{t,sg-} \leq \delta_{sg}^t (IN_{sg+1} - IN_{sg}) \quad sg = 1, ...SG-1, t \tag{47}$$

$$(\delta_{sg+1}^t \leq \delta_{sg}^t \quad \forall sg, t \quad \text{only include this constraint for sub-problem}) \tag{48}$$

The total inertia isequal to the number of on generators multiplied by a conversion factor

$$(\lambda_{inertia}^t) \sum_{sg \in N} inx_{sg}^t = \sum_n H_n S_n y_n^t + \sum_m H'_m S'_m y'^t_m - IN_1 \quad \forall t \tag{49}$$

$c_n^t, c'^t_m, p_n^t, regu_n^t, regd_n^t, sr_n^t, nsp_n^t, p'^t_m, regu'^t_m, regd'^t_m, sr'^t_m, nsp'^t_m, ffr_b^t, tffr^t, frr^t, rfrr^t, inx_{sg}^t,$
$s_{budget}^-, s_{balance}^{t+}, s_{balance}^{t-}, s_{Pmin}^{nt+} s'^{mt+}_{Pmin}, s_{transUB}^{lt-}, s_{transLB}^{lt+}, s_{RREGU}^{t+}, s_{RREGD}^{t+}, s_{RNSP}^{t+}, s_{rfrr}^{t+}, s_{tffr1}^{t,sg+}, s_{tffr2}^{t,sg-},$
$s_{inx1}^{t,sg+}, s_{inx2}^{t,sg-} \geq 0$

4 Dual Objective

$$\max \Big(\phi - \sum_t \Big(\sum_n SUx_n^t + \sum_n SD_n z_n^t + \sum_m SU_m x'^t_m + \sum_m SD_m z'^t_m \Big) \Big) \lambda_{budget}$$
$$+ \sum_j \sum_t \Big[\sum_n \alpha_{nj} y_n^t \lambda_{fuel}^{jnt} + \sum_m \alpha'_{mj} y'^t_m \lambda'^{jmt}_{fuel} \Big]$$
$$+ \sum_{t=2}^T \sum_n \Big[(RU_n y_n^{t-1} + \overline{RU}_n x_n^t) \lambda_{RU}^{nt} + (RD_n y_n^t + \overline{RD}_n z_n^t) \lambda_{RD}^{nt} \Big]$$
$$+ \sum_t \sum_n \Big[10 R_n^{MT} y_n^t \lambda_{sr}^{nt} + P_n^{max} y_n^t \lambda_{Pmax}^{nt} + P_n^{min} y_n^t \lambda_{Pmin}^{nt}$$

$$
\begin{aligned}
&+(1-y_n^t)NSP_n\lambda_{nsp}^{nt}+y_n^tREGU_n^t\lambda_{regu}^{nt}+y_n^tREGD_n^t\lambda_{regd}^{nt}\Big] \\
&+\sum_t\sum_b\overline{FFR}_b^t\lambda_{ffr}^{bt} \\
&+\sum_{t=2}^{T}\sum_m\Big[(RU_n{y'}_m^{t-1}+\overline{RU}_m{x'}_m^t){\lambda'}_{RU}^{mt}+(RD_m{y'}_m^t+\overline{RD}_m{z'}_m^t){\lambda'}_{RD}^{mt}\Big] \\
&+\sum_t\sum_m\Big[10R_m^{MT}{y'}_m^t{\lambda'}_{sr}^{mt}+P_m^{max}{y'}_m^t{\lambda'}_{Pmax}^{mt}+P_n^{min}{y'}_m^t{\lambda'}_{Pmin}^{mt}+ \\
&(1-{y'}_m^t)NSP_m{\lambda'}_{nsp}^{mt}+{y'}_m^tREGU_m^t{\lambda'}_{regd}^{nt}+{y'}_m^tREGD_m^t{\lambda'}_{regd}^{mt}\Big] \\
&+\sum_t\Big[\sum_b D_b^t\lambda_{balance}^t\Big]+\sum_t\sum_l\Big[(\sum_b SF_b^lD_b^t+F_l)\lambda_{transUB}^{lt}+(\sum_b SF_b^lD_b^t-F_l)\lambda_{transLB}^{lt}\Big] \\
&+\sum_t\Big[RREGU^t\lambda_{RREGUreq}^t+RREGD^T\lambda_{RREGDreq}^t+NSP^T\lambda_{NSP}^t\Big] \\
&+\sum_t\sum_{sg}^{SG-1}\Big[-M_{sg}^t(1-\delta_{sg}^t+\delta_{sg+1}^t)\lambda_{tffr1}^{t,sg}+M_{sg}^t(1-\delta_{sg}^t+\delta_{sg+1}^t)\lambda_{tffr2}^{t,sg}\Big]+\sum_t RFRR_1\lambda_{rfrr}^t \\
&+\sum_t\sum_{sg}^{SG-1}\Big[\delta_{sg+1}^t(IN_{sg+1}-IN_{sg})\lambda_{inx1}^{t,sg}+\delta_{sg}^t(IN_{sg+1}-IN_{sg})\lambda_{inx2}^{t,sg}\Big] \\
&+\sum_t\Big[\Big(\sum_n H_nS_ny_n^t+\sum_m {H'}_m{S'}_m{y'}_m^t-IN_1\Big)\lambda_{inertia}^t\Big] \qquad (50)
\end{aligned}
$$

5 Dual Constraints

$$(c^t_n)\lambda_{budget} + \sum_j \lambda^{jnt}_{fuel} \leq 0 \quad \forall n,t \tag{51}$$

$$(c'^t_m)\lambda_{budget} + \sum_j \lambda'^{jmt}_{fuel} \leq 0 \quad \forall m,t \tag{52}$$

$$(p^1_n) - \sum_j \beta_{nj}\lambda^{jn1}_{fuel} + -\lambda^{n2}_{RU} + \lambda^{n2}_{RD} + \lambda^{n1}_{Pmax} + \lambda^{n1}_{Pmin} + \lambda^{T1}_{balance}$$
$$+ \sum_l SF^l_{b(n)}\lambda^{l1}_{transUB} + \sum_l SF^l_{b(n)}\lambda^{l1}_{transLB} \leq 0 \quad \text{where } b(n) := \{b : n \in G^b\}, \forall n \tag{53}$$

$$(p^t_n) - \sum_j \beta_{nj}\lambda^{jnt}_{fuel} + \lambda^{nt}_{RU} - \lambda^{nt+1}_{RU} - \lambda^{nt}_{RD} + \lambda^{nt+1}_{RD} + \lambda^{nt}_{Pmax} + \lambda^{nt}_{Pmin} + \lambda^{t}_{balance}$$
$$+ \sum_l SF^l_{b(n)}\lambda^{lt}_{transUB} + \sum_l SF^l_{b(n)}\lambda^{lt}_{transLB} \leq 0 \quad \text{where } b(n) := \{b : n \in G^b\}, \forall n, t = 2...T-1$$
$$(p^T_n) - \sum_j \beta_{nj}\lambda^{jnT}_{fuel} + \lambda^{nT}_{RU} - \lambda^{nT}_{RD} + \lambda^{nT}_{Pmax} + \lambda^{nT}_{Pmin} + \lambda^{T}_{balance}$$
$$+ \sum_l SF^l_{b(n)}\lambda^{lT}_{transUB} + \sum_l SF^l_{b(n)}\lambda^{lT}_{transLB} \leq 0 \quad \text{where } b(n) := \{b : n \in G^b\}, \forall n \tag{54}$$

$$(p'^1_m) - \sum_j \beta'_{mj}\lambda'^{jm1}_{fuel} - \lambda'^{m2}_{RU} + \lambda'^{m2}_{RD} + \lambda'^{m1}_{Pmax} + \lambda'^{m1}_{Pmin} + \lambda^{1}_{balance}$$
$$+ \sum_l SF^l_{b(m)}\lambda^{l1}_{transUB} + \sum_l SF^l_{b(m)}\lambda^{l1}_{transLB} \leq 0 \quad \text{where } b(m) := \{b : m \in G^b\}, \forall n \tag{55}$$

$$(p'^t_m) - \sum_j \beta'_{mj}\lambda'^{jmt}_{fuel} + \lambda'^{mt}_{RU} - \lambda'^{mt+1}_{RU} - \lambda'^{mt}_{RD} + \lambda'^{mt+1}_{RD} + \lambda'^{mt}_{Pmax} + \lambda'^{mt}_{Pmin} + \lambda^{t}_{balance}$$
$$+ \sum_l SF^l_{b(m)}\lambda^{lt}_{transUB} + \sum_l SF^l_{b(m)}\lambda^{lt}_{transLB} \leq 0 \quad \text{where } b(m) := \{b : m \in G^b\}, \forall n, t = 2...T-1$$
$$(p'^T_m) - \sum_j \beta'_{mj}\lambda'^{jmT}_{fuel} + \lambda'^{mT}_{RU} - \lambda'^{mT}_{RD} + \lambda'^{mT}_{Pmax} + \lambda'^{mT}_{Pmin} + \lambda^{T}_{balance}$$
$$+ \sum_l SF^l_{b(m)}\lambda^{lT}_{transUB} + \sum_l SF^l_{b(m)}\lambda^{lT}_{transLB} \leq 0 \quad \text{where } b(m) := \{b : m \in G^b\}, \forall n \tag{56}$$

$$(regu^t_n)\lambda^{nt}_{Pmax} + \lambda^{nt}_{regu} + \lambda^{t}_{RREGUreq} \leq 0 \quad \forall n,t \tag{57}$$

$$(regu'^t_m)\lambda'^{mt}_{Pmax} + \lambda'^{mt}_{regu} + \lambda^{t}_{RREGUreq} \leq 0 \quad \forall n,t \tag{58}$$

$$(regd^t_n) - \lambda^{nt}_{Pmin} + \lambda^{nt}_{regd} + \lambda^{t}_{RREGDreq} \leq 0 \quad \forall n,t \tag{59}$$

$$(regd'^t_m) - \lambda'^{mt}_{Pmin} + \lambda'^{mt}_{regd} + \lambda^{t}_{RREGDreq} \leq 0 \quad \forall n,t \tag{60}$$

$$(sr^t_n)\lambda^{nt}_{sr} + \lambda^{nt}_{Pmax} - \lambda^{t}_{frr} \leq 0 \quad \forall n,t \tag{61}$$

$$(sr'^t_m)\lambda'^{mt}_{sr} + \lambda'^{mt}_{Pmax} - \lambda^{t}_{frr} \leq 0 \quad \forall n,t \tag{62}$$

$$(nsp^t_n)\lambda^{nt}_{nsp} + \lambda^{t}_{NSPreq} \leq 0 \quad \forall n,t \tag{63}$$

$$(nsp'^t_m)\lambda'^{mt}_{nsp} + \lambda^{t}_{NSPreq} \leq 0 \quad \forall n,t \tag{64}$$

$$(ffr^t_b)\lambda^{bt}_{ffr} - \sum_{sg} K_{sg}\lambda^{t,sg}_{tffr1} - \sum_{sg} K_{sg}\lambda^{t,sg}_{tffr2} \leq 0 \quad \forall b,t \tag{65}$$

$$(tffr^t) - \lambda^{t}_{frr} + \sum_{sg} (\lambda^{sg,t}_{tffr1} + \lambda^{t,sg}_{tffr2}) \leq 0 \quad \forall t \tag{66}$$

(67)

$$(frr^t)\lambda^t_{rfrrreq} + \lambda^t_{frr} \leq 0 \quad \forall t \tag{68}$$

$$(rfrr^t) - \lambda^t_{rfrrreq} + \lambda^t_{rfrr} \leq 0 \quad \forall t \tag{69}$$

$$(inx^t_{sg})\lambda^{t,sg}_{inx1} + \lambda^{t,sg}_{inx2} - IR_{sg}\lambda^t_{rfrr} + \lambda^t_{inertia} \leq 0 \tag{70}$$

$$\tag{71}$$

Slack Dual Constraints

$$-\lambda_{budget} \leq 1 \tag{72}$$

$$\lambda^t_{balance} \leq 1 \quad \forall t \tag{73}$$

$$-\lambda^t_{balance} \leq 1 \quad \forall t \tag{74}$$

$$-\lambda^{lt}_{transUB} \leq 1 \quad \forall l, t \tag{75}$$

$$\lambda^{lt}_{transLB} \leq 1 \quad \forall L, t \tag{76}$$

$$\lambda^{nt-}_{Pmin} \leq 1 \tag{77}$$

$$\lambda'^{mt-}_{Pmin} \leq 1 \tag{78}$$

$$\lambda^t_{RREGUreq} \leq 1 \quad \forall t \tag{79}$$

$$\lambda^t_{RREGDreq} \leq 1 \quad \forall t \tag{80}$$

$$\lambda^t_{NSPreq} \leq 1 \quad \forall t \tag{81}$$

$$\lambda^t_{rfrrreq} \leq 1 \quad \forall t \tag{82}$$

$$\lambda^{t,sg}_{tffr1} \leq 1 \quad \forall t, sg \tag{83}$$

$$-\lambda^{t,sg}_{tffr2} \leq 1 \quad \forall t, sg \tag{84}$$

$$\lambda^{t,sg+}_{inx1} \leq 1 \quad \forall t, sg \tag{85}$$

$$-\lambda^{t,sg-}_{inx2} \leq 1 \quad \forall t, sg \tag{86}$$

$$\tag{87}$$

$$\lambda_{budget}, \lambda^{nt}_{RU}, \lambda'^{mt}_{RU}, \lambda^{nt}_{RD}, \lambda'^{mt}_{RD}, \lambda^{nt}_{sr}, \lambda'^{mt}_{sr}, \lambda^{nt}_{Pmax}, \lambda'^{mt}_{Pmax},$$

$$\lambda^{nt}_{nsp}, \lambda'^{mt}_{nsp}, \lambda^{nt}_{regu}, \lambda'^{mt}_{regu}, \lambda^{nt}_{regd}, \lambda'^{mt}_{regd}, \lambda^{bt}_{ffr}, \lambda^{lt}_{transUB}$$

$$\lambda^{t,sg}_{tffr2}, \lambda^{t,sg}_{inx2} \leq 0$$

$$\lambda^{nt}_{fuel}, \lambda'^{mt}_{fuel}, \lambda^{nt}_{Pmin}, \lambda'^{mt}_{Pmin}, \lambda^{lt}_{transLB}, \lambda^t_{RREGUreq}, \lambda^t_{RREGDreq}, \lambda^t_{NSPreq}, \lambda^t_{rfrrreq}, \lambda^{t,sg}_{tffr1}, \lambda^{t,sg}_{inx1} \geq 0$$

$$\lambda^t_{balance}, \lambda^t_{frr}, \lambda^t_{rfrr}, \lambda^t_{inertia} \quad unrestricted$$

6 Strong Duality Constraint

$$s^-_{budget} + \sum_t (s^{t+}_{balance} + s^{t-}_{balance}) + \sum_t \sum_n s^{nt+}_{Pmin} + \sum_t \sum_m s'^{mt+}_{Pmin} + \sum_t \sum_l \left(s^{lt-}_{transUB} + s^{lt+}_{transLB}\right)$$

$$= \left(\phi - \sum_t \left(\sum_n SUx^t_n + \sum_n SD_n z^t_n + \sum_m SU_m x'^t_m + \sum_m SD_m z'^t_m\right)\right)\lambda_{budget}$$

$$+\sum_j\sum_t\Big[\sum_n\alpha_{nj}y_n^t\lambda_{fuel}^{jnt}+\sum_m\alpha'_{mj}{y'}_m^t{\lambda'}_{fuel}^{jmt}\Big]$$

$$+\sum_{t=2}^T\sum_n\Big[(RU_ny_n^{t-1}+\overline{RU}_nx_n^t)\lambda_{RU}^{nt}+(RD_ny_n^t+\overline{RD}_nz_n^t)\lambda_{RD}^{nt}\Big]$$

$$+\sum_t\sum_n\Big[10R_n^{MT}y_n^t\lambda_{sr}^{nt}+P_n^{max}y_n^t\lambda_{Pmax}^{nt}+P_n^{min}y_n^t\lambda_{Pmin}^{nt}$$

$$+(1-y_n^t)NSP_n\lambda_{nsp}^{nt}+y_n^tREGU_n^t\lambda_{regu}^{nt}+y_n^tREGD_n^t\lambda_{regd}^{nt}\Big]$$

$$+\sum_t\sum_b\overline{FFR}_b^t\lambda_{ffr}^{bt}$$

$$+\sum_{t=2}^T\sum_m\Big[(RU_n{y'}_m^{t-1}+\overline{RU}_m{x'}_m^t){\lambda'}_{RU}^{mt}+(RD_m{y'}_m^t+\overline{RD}_m{z'}_m^t){\lambda'}_{RD}^{mt}\Big]$$

$$+\sum_t\sum_m\Big[10R_m^{MT}{y'}_m^t{\lambda'}_{sr}^{mt}+P_m^{max}{y'}_m^t{\lambda'}_{Pmax}^{mt}+P_n^{min}{y'}_m^t{\lambda'}_{Pmin}^{mt}+$$

$$(1-{y'}_m^t)NSP_m{\lambda'}_{nsp}^{mt}+{y'}_m^tREGU_m^t{\lambda'}_{regd}^{nt}+{y'}_m^tREGD_m^t{\lambda'}_{regd}^{mt}\Big]$$

$$+\sum_t\Big[\sum_b D_b^t\lambda_{balance}^t\Big]+\sum_t\sum_l\Big[(\sum_b SF_b^lD_b^t+F_l)\lambda_{transUB}^{lt}+(\sum_b SF_b^lD_b^t-F_l)\lambda_{transLB}^{lt}\Big]$$

$$+\sum_t\Big[RREGU^t\lambda_{RREGUreq}^t+RREGD^T\lambda_{RREGDreq}^t+NSP^T\lambda_{NSP}^t\Big]+\sum_t\sum_{sg}^{SG-1}\Big[-M_{sg}^t(1-\delta_{sg}^t+\delta_{sg+1}^t)\lambda_{tffr1}^{t,sg}$$

$$+M_{sg}^t(1-\delta_{sg}^t+\delta_{sg+1}^t)\lambda_{tffr2}^{t,sg}\Big]+\sum_t RFRR_1\lambda_{rfrr}^t$$

$$+\sum_t\sum_{sg}^{SG-1}\Big[\delta_{sg+1}^t(IN_{sg+1}-IN_{sg})\lambda_{inx1}^{t,sg}+\delta_{sg}^t(IN_{sg+1}-IN_{sg})\lambda_{inx2}^{t,sg}\Big]$$

$$+\sum_t\Big[\Big(\sum_n H_nS_ny_n^t+\sum_m H'_mS'_m{y'}_m^t-IN_1\Big)\lambda_{inertia}^t\Big] \tag{88}$$

$$\iff$$

$$s_{budget}^-+\sum_t(s_{balance}^{t+}+s_{balance}^{t-})+\sum_t\sum_n s_{Pmin}^{nt+}+\sum_t\sum_m {s'}_{Pmin}^{mt+}+\sum_t\sum_l\Big(s_{transUB}^{lt-}+s_{transLB}^{lt+}\Big)$$

$$=\Big(\phi-\sum_t\Big(\sum_n SUx_n^t+\sum_n SD_nz_n^t+\sum_m SU_m{x'}_m^t+\sum_m SD_m{z'}_m^t\Big)\Big)\lambda_{budget}$$

$$+\sum_j\sum_t\Big[\sum_n\alpha_{nj}y_n^t\lambda_{fuel}^{jnt}+\sum_m\alpha'_{mj}{y'}_m^t{\lambda'}_{fuel}^{jmt}\Big]$$

$$+\sum_{t=2}^{T}\sum_{n}\Big[(RU_n y_n^{t-1}+\overline{RU}_n x_n^t)\lambda_{RU}^{nt}+(RD_n y_n^t+\overline{RD}_n z_n^t)\lambda_{RD}^{nt}\Big]$$

$$+\sum_{t}\sum_{n}\Big[10R_n^{MT}y_n^t\lambda_{sr}^{nt}+P_n^{max}y_n^t\lambda_{Pmax}^{nt}+P_n^{min}y_n^t\lambda_{Pmin}^{nt}$$

$$+(1-y_n^t)NSP_n\lambda_{nsp}^{nt}+y_n^t REGU_n^t\lambda_{regu}^{nt}+y_n^t REGD_n^t\lambda_{regd}^{nt}\Big]$$

$$+\sum_{t}\sum_{b}\overline{FFR}_b^t\lambda_{ffr}^{bt}$$

$$+\sum_{t=2}^{T}\sum_{m}\Big[(RU_m y'^{t-1}_m+\overline{RU}_m x'^t_m)\lambda'^{mt}_{RU}+(RD_m y'^t_m+\overline{RD}_m z'^t_m)\lambda'^{mt}_{RD}\Big]$$

$$+\sum_{t}\sum_{m}\Big[10R_m^{MT}y'^t_m\lambda'^{mt}_{sr}+P_m^{max}y'^t_m\lambda'^{mt}_{Pmax}+P_n^{min}y'^t_m\lambda'^{mt}_{Pmin}+$$

$$(1-y'^t_m)NSP_m\lambda'^{mt}_{nsp}+y'^t_m REGU_m^t\lambda'^{nt}_{regd}+y'^t_m REGD_m^t\lambda'^{mt}_{regd}\Big]$$

$$+\sum_{t}\Big[\sum_{b}\hat{d}_b^t\lambda_{balance}^t+\gamma^+w_b^{t+}-\gamma_{bt}^-w_b^{t-}\Big]+\sum_{t}\sum_{l}\Big[\sum_{b}SF_b^l(\hat{d}_b^t\lambda_{transUB}^{lt}+\gamma^+v_{UB}^{blt+}-\gamma_{bt}^-v_{UB}^{blt-})+F_l\lambda_{transUB}^{lt}$$

$$\sum_{b}SF_b^l(\hat{d}_b^t\lambda_{transLB}^{lt}+\gamma^+v_{LB}^{blt+}-\gamma_{bt}^-v_{LB}^{blt-})-F_l\lambda_{transLB}^{lt}\Big]$$

$$+\sum_{t}\Big[RREGU^t\lambda_{RREGUreq}^t+RREGD^T\lambda_{RREGDreq}^t+NSP^T\lambda_{NSP}^t\Big]+\sum_{t}\sum_{sg}^{SG-1}\Big[-M_{sg}^t(1-\delta_{sg}^t+\delta_{sg+1}^t)\lambda_{tffr1}^{t,sg}$$

$$+M_{sg}^t(1-\delta_{sg}^t+\delta_{sg+1}^t)\lambda_{tffr2}^{t,sg}\Big]+\sum_{t}RFRR_1\lambda_{rfrr}^t$$

$$+\sum_{t}\sum_{sg}^{SG-1}\Big[\delta_{sg+1}^t(IN_{sg+1}-IN_{sg})\lambda_{inx1}^{t,sg}+\delta_{sg}^t(IN_{sg+1}-IN_{sg})\lambda_{inx2}^{t,sg}\Big]$$

$$+\sum_{t}\Big[\Big(\sum_{n}H_nS_ny_n^t+\sum_{m}H'_mS'_my'^t_m-IN_1\Big)\lambda_{inertia}^t\Big] \tag{89}$$

(for reference: $w_b^{t+}=\mu_b^{t+}\lambda_{balance}^t, v_{UB}^{blt+}=\mu_b^+\lambda_{transUB}^{lt}$)

Auxiliary Constraints

these auxiliary constraints linearize the bilinaer demand term in the strong duality constriant

$$\lambda_{balance}^t-(1-\mu_b^{t+})\le w_b^{t+}\le\lambda_{balance}^t+(1-\mu_b^{t+}) \tag{90}$$

$$-\mu_b^{t+}\le w_b^{t+}\le\mu_b^{t+} \tag{91}$$

$$\lambda_{balance}^t-(1-\mu_b^{t-})\le w_b^{t-}\le\lambda_{balance}^t+(1-\mu_b^{t-}) \tag{92}$$

$$-\mu_b^{t-}\le w_b^{t-}\le\mu_b^{t-} \tag{93}$$

$\forall b,\forall t$

$$\lambda^{lt}_{transUB} \leq v^{blt+}_{UB} \leq \lambda^{lt}_{transUB} + (1 - \mu^{t+}_b) \tag{94}$$
$$-\mu^{t+}_b \leq v^{blt+}_{UB} \leq 0 \tag{95}$$
$$\lambda^{lt}_{transUB} \leq v^{blt-}_{UB} \leq \lambda^{lt}_{transUB} + (1 - \mu^{t-}_b) \tag{96}$$
$$-\mu^{t-}_b \leq v^{blt-}_{UB} \leq 0 \tag{97}$$
$$\lambda^{lt}_{transLB} - (1 - \mu^{t+}_b) \leq v^{blt+}_{LB} \leq \lambda^{lt}_{transLB} \tag{98}$$
$$0 \leq v^{blt+}_{LB} \leq \mu^{t+}_b \tag{99}$$
$$\lambda^{lt}_{transLB} - (1 - \mu^{t-}_b) \leq v^{blt-}_{LB} \leq \lambda^{lt}_{transLB} \tag{100}$$
$$0 \leq v^{blt-}_{LB} \leq \mu^{t-}_b \tag{101}$$
$$\mu^{t+}_b + \mu^{t-}_b = 1 \tag{102}$$
$$\forall b, \forall t, \forall l$$