Flexible Robust Optimization for 2-stage UCED

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***Abstract*—we study economic dispatch**

***Index Terms*—Economic Dispatch, Linear Programming Formulation.**

I. Introduction

In this project, our goal is to find some solution of on/off decisions w, and generation decisions u, q for the unit commitment problem with frequency response that satisfies physical constraints and our pre-determined budget ϕ, while maximizing γ: the range of random demand D that w and u, q is feasible for.

The 1st stage problem considers binary variables, $w \in \{0,1\}$, and $\gamma \in \mathbb{R}$ and the 2nd stage problem contains continuous variables, $u \in \mathbb{R}^N$, and integer variables $q \in \mathbb{Z}^M$.

We first find a solution the 1st stage problem, $\bar{w}, \bar{\gamma}$ then check if it leads to a feasible 2nd stage solution for all possible demand realizations, D, in our uncertainty set: $[\hat{d} - \bar{\gamma}\mathbf{1}, \hat{d} + \bar{\gamma}\mathbf{1}]$, where $\hat{d}$ is the average demand, and $\mathbf{1} := (1, \ldots, 1)^T$ represents a vector of all ones. We use the above notation for brevity, but in reality, $d \in \mathbb{R}^{B \times T}$, where B is the number of buses and T is the time horizon. Therefore, the uncertainty set is a $B \times T$ dimensional hypercube.

We accomplish this feasibility check by using a *Feasibility Check Problem*. If it is not feasible, we generate a *feasibility cut*, and return to the 1st stage problem. If it is feasible, we solve the KKT conditions to find the maximum γ for this $\bar{w}$, which we denote $\underline{\gamma}$. And generate an *optimality cut* to ensure that $\gamma > \underline{\gamma}$ for future iterations. Then we return to the 1st stage problem.

The details of this procedure are explained below.

II. Nomenclature

A. Sets

$\mathcal{B}$	Set of buses.
$\mathcal{G}$	Set of Generators.
$\mathcal{G}'$	Set of quick start Generators.
$\mathcal{G}^b$	Set of Generators at bus b.
$\mathcal{G}'^b$	Set of quick start Generators at bus b.
$\mathcal{L}$	Set of transmission lines.
$\mathcal{S}$	Set of storage.

B. Parameters

T_{end}	Time Horizon
F_l	Transmission capacity of transmission line l (MW).
$\mathrm{P}_n^{\min}$	Minimum generation amount (MW).
$\mathrm{P}^{\max}$	Maximum generation amount (MW).
$REGU_n$	Maximum regulation up amount of generator n.
$REGD_n$	Maximum regulation down amount of generator n.
SR_n	Maximum spinning reserve amount of generator n.
σ_b^l	Power flow distribution factors of conventional unit and load at bus b for transmission line l.
μ_s^l	Power flow distribution factors of conventional unit and load at storage s for transmission line l.
SU_n	cost of starting up generator n
SD_n	cost of shutting down generator n
T_n^{minu}	minimum up time for generator n
T_n^{mind}	minimum down time for generator n
τ_{end}	last time period
RU_n	maximum allowable increase in generation amount for generator n if on in previous period
$\overline{RD}_n$	maximum allowable increase in generation amount for generator n if on in current period
RD_n	Maximum allowable decrease in generation amount for generator n if on in current period
$\overline{RU}_n$	Maximum allowable decrease in generation amount for generator n if turned off in current period
NSP_n	Maximum non-spinning reserve for generator n
RREGU^t	Required regulation up at time t
RREGD^t	Required regulation down at time t
R^{MT}	spinning reserve capacity
RNSP^t	Required non-spinning reserve at time t
$\hat{d}_b$	Nominal value for demand at bus b
$\bar{d}_b$	Unit deviation for demand at bus b
$\bar{H}_s$	ESS Storage Capacity of storage s.
$\bar{G}_s^+$	ESS sCharging Capacity of storage s.
$\bar{G}_s^-$	ESS Discharging Capacity of storage s.
ν_s^+	ESS Charging Efficiency of storage s.
ν_s^-	ESS Discharging Efficiency of storage s.
M_{sg}^t	Big number
α_n^k	y intercept for k^{th} piecewise component fuel cost calculation
β_n^k	slope for k^{th} piecewise component fuel cost calculation
α'^k_m, β'^k_m	the corresponding coefficients for calculating the quick start fuel cost
K	number of breakpoints for calculating the piece-wise approximation for quadratic fuel cost
ϕ	Allowable budget

C. Continuous Decision Variables

c_n^t, c'^t_m	Generation cost for generator n, and quick start generator m, respectively at time period t (\$).
p_n^t	generation amount of generator n (MW).
$regu_n^t$	regulation up amount of generator n (MW).
$regd_n^t$	regulation down amount of generator n (MW).
sr_n^t	Spinning reserve of generator n (MW).

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γ Size of uncertainty set for demand realizations.
h_s^t ESS storage level of storage s at time t
g_s^{t+} ESS charging level of storage s at time t
g_s^{t-} ESS discharging level of storage s at time t
nsp_n^t non-spinning reserve from generator n in period t
$p'^t_m, regu'^t_m, regd'^t_m, sr'^t_m, nsp'^t_m$ are corresponding variables for quick start generators
ξ^+, ξ^- auxiliary variable for linearizing strong duality term

D. Binary Decision Variables
$x_n^t = 1$ if we decided to turn on generator n in period t
$z_n^t = 1$ if we decide to shut down generator n in period t
$y_n^t = 1$ if generator n is on at period t
x'^t_m, y'^t_m, z'^t_m are corresponding decision variables for quick start generators
ρ^+, ρ^- binary variable for linearizing strong duality term

A. Random Variables
d_b uncertainty load on bus b (MW).

III. Problem Formulation

In our overall problem, which we denote (P), the objective is to maximize the size of our uncertainty set:

$$\max \gamma \tag{1}$$

subject to the following 1st and 2nd-stage constraints

A. 1st-Stage Constraints

The first stage problem consists of the binary on-off decisions for the generators.

$$x_n^t + z_n^t \leq 1, \forall n \in \mathcal{G}, \forall t, \tag{2}$$

$$y_n^t - y_n^{t-1} = x_n^t - z_n^t, \forall n \in \mathcal{G}, \forall t, \tag{3}$$

$$\sum_{\tau=t-T_n^{\mathrm{T,minu}}+1}^{t} x_n^\tau \leq y_n^t, \quad \forall n \in \mathcal{G}, \forall t \in \{T_n^{\mathrm{T,minu}}, \cdots, T_{end}\}, \tag{4}$$

$$\sum_{\tau=t-T_n^{\mathrm{T,mind}}+1}^{t} z_n^\tau \leq 1 - y_n^t, \quad \forall n \in \mathcal{G}, \forall t \in \{T_n^{\mathrm{T,mind}}, \cdots, T_{end}\} \tag{5}$$

(2) says that a generator cannot be turned on and turned off at the same time. (3) enforces the relationship between the on/off status and the decision to turn a generator on or off. (4) and (5) enforce the min up and min down time, respectively.

B. 2nd-Stage Constraints

For this formulation, we we will assume that binary variables are chosen in the first stage, the remaining 2nd stage problem is an LP. We will first try to find a feasible solution for the nominal demand, which is the average demand, $\hat{d}$. Below are the constraints.

1) Fuel Constraints:

$$\sum_{t\in\mathcal{T}} \Big(\sum_{n\in\mathcal{G}} (\mathrm{SU}_n x_n^t + \mathrm{SD}_n z_n^t + c_n^t) + \sum_{m\in\mathcal{G}'} (\mathrm{SU}_m x'^t_m + \mathrm{SD}_m z'^t_m + c'^t_m) \Big) \leq \phi \tag{6}$$

$$c_n^t \geq \alpha_n^k p_n^t + \beta_n^k y_n^t \quad \forall n \in \mathcal{G}, \forall k, \forall t \tag{7}$$

$$c'^t_m \geq \alpha'^k_m p'^t_m + \beta'^k_m y'^t_m \quad \forall m \in \mathcal{G}', \forall k, \forall t \tag{8}$$

(6) says that we must satisfy the pre-determined budget. (7) and (8) are constraints used to approximate the quadratic fuel cost as a piece-wise linear function, using the same method as in [2].

2) Quick Start Binary Constraints:

$$x'^t_m + z'^t_m \leq 1, \forall m \in \mathcal{G}', \forall t, \tag{9}$$

$$y'^t_m - y'^{t-1}_m = x'^t_m - z'^t_m, \forall m \in \mathcal{G}', \forall t, \tag{10}$$

$$\sum_{\tau=t-T_n^{\mathrm{T,minu}}+1}^{t} x'^\tau_m \leq y'^t_m \quad \forall m \in \mathcal{G}', \forall t \in \{T_n^{\mathrm{T,minu}}, \cdots, T_{end}\}, \tag{11}$$

$$\sum_{\tau=t-T_n^{\mathrm{T,mind}}+1}^{t} z'^\tau_m \leq 1 - y'^t_m \quad \forall m \in \mathcal{G}', \forall t \in \{T_n^{\mathrm{T,mind}}, \cdots, T_{end}\} \tag{12}$$

Constraints (9)-(12) follow the same logic as (2)-(5). The only difference is that they represent the on/off decision for quick start generators, and are therefore 2nd-stage constraints.

3) Capacity Constraints:

$$p_n^t - p_n^{t-1} \leq RU_n y_n^{t-1} + \overline{RU}_n x_n^t, \forall n \in \mathcal{G}, \forall t, \tag{13}$$

$$p_n^{t-1} - p_n^t \leq RD_n y_n^t + \overline{RD}_n z_n^t, \forall n \in \mathcal{G}, \forall t, \tag{14}$$

$$sr_n^t \leq R_n^{\mathrm{MT}} y_n^t, \forall n \in \mathcal{G}, \forall t, \tag{15}$$

$$p_n^t + sr_n^t + regu_n^t \leq P_n^{\max} y_n^t, \forall n \in \mathcal{G}, \forall t, \tag{16}$$

$$p_n^t - regd_n^t \geq P_n^{\min} y_n^t, \forall n \in \mathcal{G}, \forall t, \tag{17}$$

$$nsp_n^t \leq (1 - y_n^t)\mathrm{NSP}_n \forall n \in \mathcal{G}, \forall t, \tag{18}$$

$$regu_n^t \leq y_n^t REGU_n, \forall n \in \mathcal{G}, \tag{19}$$

$$regd_n^t \leq y_n^t REGD_n, \forall n \in \mathcal{G}, \tag{20}$$

Constraints (13) and (14) represent the ramp-up and ramp-down constraint, respectively. (15) is the spinning reserve upper bound. (16) and (17) describe upper and lower capacity limits, respectively. (18) is the non-spinning reserve limit. (19) and (20) and the regulation up and regulation down limits, respectively.

4) Quick Start Capacity Constraints:

$$p'^t_m - p'^{t-1}_m \leq RU_m y'^{t-1}_m + \overline{RU}_n x'^t_m, \forall m \in \mathcal{G}', \forall t \tag{21}$$

$$p'^{t-1}_m - p'^t_m \leq RD_m y'^t_m + \overline{RD}_m z'^t_m, \forall m \in \mathcal{G}', \forall t \tag{22}$$

$$sr'^t_m \leq R_m^{\mathrm{MT}} y'^t_m, \forall m \in \mathcal{G}', \forall t \tag{23}$$

$$p'^t_m + sr'^t_m + regu'^t_m \leq P_m^{\max} y'^t_n, \forall m \in \mathcal{G}', \forall t \tag{24}$$

$$p'^t_m - regd'^t_m \geq P_m^{\min} y'^t_m, \forall m \in \mathcal{G}'^T, \forall t \tag{25}$$

$$nsp'^t_m \leq (1 - y'^t_m)\mathrm{NSP}_m, \forall m \in \mathcal{G}', \forall t, \tag{26}$$

$$regu'^t_n \leq y'^t_m REGU_m, \forall m \in \mathcal{G}, \tag{27}$$

$$regd'^{t}_{n} \leq y'^{t}_{n}\text{R}EGD_n, \forall m \in \mathcal{G} \tag{28}$$
$$\tag{29}$$

Constraints (21)-(28) represent the same logic as Constraints (13)-(20) but for quick start generators.

5) Minimum Requirement Constraints:

$$\sum_{n\in\mathcal{G}} regu^t_n + \sum_{m\in\mathcal{G}'} regu'^t_m \geq \text{RREGU}^t \tag{30}$$
$$\sum_{n\in\mathcal{G}} regd^t_n + \sum_{m\in\mathcal{G}'} regd'^t_m \geq \text{RREGD}^t \tag{31}$$
$$\sum_{n\in\mathcal{G}} nsp^t_n + \sum_{m\in\mathcal{G}'} nsp'^t_m \geq \text{RNSP}^t \tag{32}$$
$$\tag{33}$$

(30), (31), and (32) describe lower limits for regulation up, regulation down, and non-spinning reserve, respectively.

6) Battery Constraints:

$$h^{t+1}_s = h^t_s + \nu^+_s g^{t+}_s - \nu^-_s g^{t-}_s, \quad \forall t = 1, ...T-1, \forall s \tag{34}$$
$$h^t_s \leq \bar{H}_s \quad \forall t \tag{35}$$
$$g^{t+}_s \leq \bar{G}^+_s \quad \forall t \tag{36}$$
$$g^{t-}_s \leq \bar{G}^-_s \quad \forall t \tag{37}$$
$$\tag{38}$$

Here, (34) represents the charge balance constraints, (35),(36), and (37) represents the storage, discharging, and charging capacities for the ESS, respectively.

7) Demand Balance Constraints:

$$\sum_{n\in\mathcal{G}} p^t_n + \sum_{m\in\mathcal{G}'} p'^t_m + \sum_{s\in\mathcal{S}} (g^{t-}_s - g^{t+}_s) = \sum_{b\in\mathcal{B}} d^t_b, \forall t \tag{39}$$
$$\sum_{b\in\mathcal{B}} \sigma^l_b (\sum_{n\in\mathcal{G}^b} p^t_n + \sum_{m\in\mathcal{G}'^b} p'^t_m - d^t_b) + \sum_s \mu^l_s (g^{t-}_s - g^{t+}_s) \leq F_l \quad \forall l \tag{40}$$
$$-F_l \leq \sum_{b\in\mathcal{B}} \sigma^l_b (\sum_{n\in\mathcal{G}^b} p^t_n + \sum_{m\in\mathcal{G}'^b} p'^t_m - d^t_b) + \sum_s \mu^l_s (g^{t-}_s - g^{t+}_s), \ \forall l \tag{41}$$
$$\forall d^t_b \in [\hat{d}_b - \gamma\mathbf{1}, \hat{d}_b + \gamma\mathbf{1}]. \tag{42}$$
$$p_n \geq 0, regu^t_n \geq 0, regd^t_n \geq 0, sr^t_n \geq 0, d^t_b \geq 0 \tag{43}$$
$$x^t_n, y^t_n, z^t_n, \in \{0,1\}, \forall n \in \mathcal{G}, \forall t, \tag{44}$$

(39) says that the sum of all generation must equal demand. (40) and (41) are the transmission line limits constraints.

(42) states that we consider all possible demands within the uncertainty set. (43) is non-negativity, and (44) is binary constraints.

We now formulate the overall problem. Let w be a vector of 1st stage binary decision variables x, y, and z. Let u and q be of 2nd stage continuous and integer decision variables, respectively. Let A and b be the the constraint matrix and right hand side for the 1st-stage binary decision variables, respectively. Let B be the constraint matrix for binary decision variables in the 2nd-stage. Let C and G be the constraint matrices of 2nd-stage continuous and integer decision variables, respectively. Let H be a constraint matrix for demand. Lastly, let h be the constants on the right hand side of the 2nd-stage constraints. Our overall objective is to find the largest uncertainty set where we can find a feasible w, u $\forall d$ in the uncertainty set. In other words, our main problem is:

$$\max \gamma \tag{45}$$

s.t.

$$Aw \leq b$$
$$Bw + Cu + Gq \leq Hd + h$$
$$\forall d \in [\hat{d} - \gamma\mathbf{1}, \hat{d} + \gamma\mathbf{1}]$$

Note that in reality, there are equality and inequality constraints, but we present all constraints as inequality constraints for brevity.

IV. Overall Flow

Before we introduce the details of how to solve this problem, we first outline the overall flow of the proposed method below.

1) Solve Master Problem for 1st stage variables $\bar{w}, \bar{\gamma}$
2) Solve FCP using the CCGA to check whether $\bar{w}, \bar{\gamma}$ leads to a feasible 2nd stage problem
   a) if not feasible add feasibility cuts and return to Step 1)
   b) if feasible, terminate the algorithm. $\bar{w}, \bar{\gamma}$ is optimal

V. Master Problem (MP)

In order to solve our overall problem, we must first solve the following Master problem to find an initial 1st-stage solution:

$$\max \gamma \tag{46}$$

s.t.

$$Aw \leq b$$
$$\gamma \leq M$$

for $\bar{\gamma}$ and $\bar{w}$. And we initialize the index $i = 0$ where i represents the i^{th} feasibility cut added to the problem. Initially, there are no feasibility cuts.

VI. Feasibility Check Problem

Next, we must check the feasibility of for $\bar{\gamma}, \bar{w}$, which were obtained from solving the Master Problem. We do this by solving the Feasibility Check Problem (FCP):

$$\psi(\bar{w}, \bar{\gamma}) := \max_{d\in\Gamma_{\bar{\gamma}}} \min_{q,u\geq 0, v\geq 0} \sum_{j=1}^{J} v_j \tag{47}$$

s.t.

$$Cu + Gq - v \leq Hd + h - B\bar{w} \tag{48}$$

where $\Gamma_{\bar{\gamma}} = [\hat{d} - \bar{\gamma}\mathbf{1}, \hat{d} + \bar{\gamma}\mathbf{1}]$ and J is the number of second stage constraints. Here, v is a J dimensional slack variable, where the j^{th} component represents the slack of the j^{th} inequality constraint. As stated before, the true system includes equality constraints, and two slack variables should be used for equality constraints, but we present the case of only inequality constraints for brevity

Note that ψ represents the worst-case violation of Constraint (48) for all d in the uncertainty set. Therefore if $\psi > 0$ then the choice of $\bar{w}, \bar{\gamma}$ from the master problem is infeasible.

We can reformulate FCP as a 3-stage problem by separating the binary and continuous variables.

$$\max_{d} \min_{q \in Q} \min_{u \geq 0, v \geq 0} \sum_{j=1}^{J} v_j \tag{49}$$

s.t.

$$\hat{d} - \bar{\gamma}\mathbf{1} \leq d \leq \hat{d} + \bar{\gamma}\mathbf{1}$$

$$Cu - v \leq Hd + h - B\bar{w} - Gq$$

where Q is the set of all values for possible discrete variables, q. (Note that not all $q \in Q$ need to be feasible to the overall problem, they are guaranteed to be feasible to the FCP because of the slack variables in FCP).

We define the "inner-problem" as θ in the following way:

$$\theta(q, d) := \min_{u \geq 0, v \geq 0} \sum_{j=1}^{J} v_j \tag{50}$$

s.t.

$$Cu - v \leq Hd + h - B\bar{w} - Gq$$

Therefore, the FCP problem can be represented as

$$\max_{D} \min_{q} \theta(q, d) \tag{51}$$

s.t.

$$\hat{d} - \bar{\gamma}\mathbf{1} \leq d \leq \hat{d} + \bar{\gamma}\mathbf{1}$$

Furthermore, we can reformulate the FCP as:

$$\max \eta \tag{52}$$

s.t.

$$\eta \leq \theta(q, d) \qquad \forall q \in Q$$

$$\hat{d} - \bar{\gamma}\mathbf{1} \leq d \leq \hat{d} + \bar{\gamma}\mathbf{1}$$

Using duality, we can show that the value of $\theta(q, d)$ is the same as the feasible solution of the following KKT conditions:

$$Cu - v \leq Hd + h - B\bar{w} - Gq \tag{53}$$

$$\lambda^T C \leq 0 \tag{54}$$

$$\lambda \geq -1 \tag{55}$$

$$\lambda^T (Hd + h - B\bar{w} - Gq) = \sum_{j=1}^{J} v_j \tag{56}$$

$$\lambda \leq 0$$

Here, (53) are primal feasibility constraints, (54) and (55) are dual feasibility constraints, and (56) is a strong duality constraint. Note that (56) has a bilinear term $\lambda^T Hd$, which can be linearized with the following procedure. We first introduce new decision variables $\xi^+, \xi^- \in \mathbb{R}^{|\mathcal{B}|}$ and $\rho^+, \rho^- \in \{0,1\}^{|\mathcal{B}|}$, and we substitute $\lambda^T Hd$ as follows, using the fact that $d = \hat{d} + \bar{\gamma}\mathbf{1}$ or $d = \hat{d} - \bar{\gamma}\mathbf{1}$ at optimality:

$$\lambda^T (Hd) = \lambda^T (H\hat{d}) + \bar{\gamma} H \xi^+ - \bar{\gamma} H \xi^- \tag{57}$$

in addition to the following constraints;

$$\lambda - (1 - \rho^+) \leq \xi^+ \leq \lambda + (1 - \rho^+)$$

$$-\rho^+ \leq \xi^+ \leq \rho^+$$

$$\lambda - (1 - \rho^-) \leq \xi^- \leq \lambda + (1 - \rho^-)$$

$$-\rho^- \leq \xi^- \leq \rho^-$$

$$\rho^+ + \rho^- = 1$$

$$\rho^+, \rho^- \in \{0,1\}^{|\mathcal{B}|}$$

This method takes advantage of the fact that at optimality, d will be equal to $\hat{d} + \bar{\gamma}\mathbf{1}$ or $\hat{d} - \bar{\gamma}\mathbf{1}$.

Using these KKT conditions, we are able to provide a new equivalent formulation of the FCP:

$$\psi(\bar{w}, \bar{\gamma}) = \max \eta \tag{58}$$

s.t.

$$\eta \leq \sum_{j=1}^{J} v_j^r \quad r = 1, ..., |Q|$$

$$Cu^r - v^r \leq Hd + h - B\bar{w} - Gq^r \quad r = 1, ..., |Q|$$

$$(\lambda^r)^T C \leq 0, \quad r = 1, ..., |Q|$$

$$\lambda^r \geq -1 \quad r = 1, ..., |Q|$$

$$(\lambda^r)^T (Hd + h - B\bar{w} - Gq^r) = \sum_{j=1}^{J} v_j^r, \quad r = 1, ..., |Q|$$

$$\hat{d} - \bar{\gamma}\mathbf{1} \leq d \leq \hat{d} + \bar{\gamma}\mathbf{1}$$

$$\lambda^r \leq 0$$

VII. Column and Constraint Generation Algorithm

We now present a Column and Constraint Generation Algorithm (CCGA) to solve (58)

Step 1: Initialization

Set iteration $k \leftarrow 1$.

Arbitrarily choose some $q^1 \in Q$ (note that q^1 can be chosen arbitrarily because the FMP presented in Step 2 is guaranteed to have a feasible solution $\forall q \in Q$)

ϵ = a pre-determined tolerance level for termination.

$LB \leftarrow -\infty$

Step 2: Solve FMP

Solve the following *Feasibility Master Problem (FMP)*:

$$\psi(\bar{w}, \bar{\gamma}) = \max \eta \tag{59}$$

s.t.

$$\eta \leq \sum_{j=1}^{J} v_j^r \quad r = 1, ..., k$$

$$Cu^r - v^r \leq Hd + h - B\bar{w} - Gq^r \quad r = 1, ..., k$$

$$(\lambda^r)^T C \leq 0, \quad r = 1, ..., k$$

$$\lambda^r \geq -1 \quad r = 1, ..., k$$

$$(\lambda^r)^T (Hd + h - B\bar{w} - Gq^r) = \sum_{j=1}^{J} v_j^r, \quad r = 1, ..., k$$

$$\hat{d} - \bar{\gamma}\mathbf{1} \leq d \leq \hat{d} + \bar{\gamma}\mathbf{1}$$

Obtain an optimal demand value, which we denote d^{k+1*}. Note that this is the same as our reformulation (66), but we have k of each constraints instead of $|Q|$ constraints, where $k \leq |Q|$.
$UB \leftarrow \psi_{FMP}(\bar{w}, \bar{\gamma})$

Step 3: Update LB

Next, solve the following *Feasibility Sub Problem (FSP)*

$$\psi_{FSP}^{k+1}(\bar{w}, \bar{\gamma}) := \min_{u \geq 0, v \geq 0, q} \sum_{j=1}^{J} v_j \tag{60}$$

$$Cu - v \leq H(d^{k+1})^* + h - B\bar{w} - Gq$$

and obtain the optimal solution $(u^{k+1})^*, (v^{k+1})^*, (q^{k+1})^*$
Update $LB \leftarrow \max\{LB, \psi_{FSP}^{k+1}\}$
If $UB - LB \leq \epsilon$, **terminate the CCGA.**,

Step 4:Constraint and Column Generation

Create variables $(u^{k+1}, v^{k+1}, \lambda^{k+1})$, and add the following constraints to (FMP).

$$\eta \leq \sum_{j=1}^{J} v_j^{k+1}$$

$$Cu^r - v^{k+1} \leq Hd + h - B\bar{w} - G(q^{k+1})^*$$

$$(\lambda^{k+1})^T C \leq 0,$$

$$\lambda^{k+1} \geq -1$$

$$(\lambda^{k+1})^T (Hd + h - B\bar{w} - G(q^{k+1})^*) = \sum_{j=1}^{J} v_j^{k+1},$$

$k \leftarrow k + 1$ and return to *Step 2*

VIII. Feasibility Cut

Let us assume that we terminate the CCGA in step 3 at iteration k. Upon the termination of the CCGA, we are able to determine if our first stage decision $\bar{w}, \bar{\gamma}$ is feasible or not.

If $\psi_{FMP}(\bar{w}, \bar{\gamma}) = 0$ we conclude that $\bar{w}, \bar{\gamma}$ is feasible and we can terminate the entire algorithm, and report $\bar{\gamma}, \bar{w}, (u^{k+1})^*$ and $(q^{k+1})*$ as an *optimal solution*. If $\psi_{FMP}(\bar{w}, \bar{\gamma}) > 0$, we conclude that $\bar{w}, \bar{\gamma}$ is *not* feasible. and we add the following feasibility cut:

$$\sum_{j=1}^{J} v_j \leq 0 \tag{61}$$

$$Cu^i + Gq^i - v^i \leq H(\hat{d} + (\rho^+)^{k+1}\gamma\mathbf{1} - (\rho^-)^{k+1}\gamma\mathbf{1}) + h - Bw \tag{62}$$

$$v^i, u^i \geq 0, q^i \in \{0, 1\}$$

Recall that the uncertainty set is an $|\mathcal{B}| \times T$ dimensional hypercube. Therefore, the CCGA algorithm chooses a different extreme point of this hypercube at each iteration. $(\rho^+)^{k+1}$ and $(\rho^+)^{k+1}$ are indicator vectors that indicate whether D has achieved the upper or lower bound of the interval for each dimension, respectively, at the termination of CCGA.

At conclusion of the CCGA, we know that (62) can only be satisfied with $\bar{\gamma}$ and $\bar{w}$ with positive slack. Therefore, the system (61)-(62) will have no solution if $\bar{\gamma}$ and $\bar{w}$ are chosen in the 1st stage. Adding these cuts to the master problem ensures that $\bar{\gamma}$ and $\bar{w}$ will not be chosen again. Furthermore, in the $(i+1)^{st}$ iteration, we will find a value of γ that is less that $\bar{\gamma}$. (Recall that i is the iteration counter for the master problem)

The last step of the overall procedure is to add these constraints to the master problem, and update $i \leftarrow i + 1$.

IX. References

1) Liu, C. Participation of Load Resources in Day-Ahead Market to Provide Primary-Frequency Response Reserve
2) Zhao, C. Unified Stochastic and Robust Unit Commitment