

May 8, 2008

Interest Rate Strategy

## Long Vol: The New Carry Trade

### Optimizing Global Volatility Rolldown & Carry

As realized volatility has picked up globally, short expiry volatility has rallied significantly over the past year. Long dated volatility however has tended to lag in most markets. As such, expiry curves remain inverted in global volatility markets, with longer dated volatility well below levels reached the last time short dated volatility was so high.

Investors who share our view that global interest rate volatility is likely continue to rally, or at a minimum fail to decline in a significant way, can therefore establish positions in global volatility that carry very positively – the gains from delta hedging high realized volatility outweigh the theta decay and the structures roll up the expiry curve.

In this report, we attempt to provide a metric to search global interest rate volatility for trades with attractive rolldown and carry characteristics. While volatility rolldown is easy to quantify, carry (the profitability from delta hedging near term expectations of higher volatility) is more difficult. We use an analogy from interest rate swaps to come up with a measure that allows an investor to express the expected carry in normalized bp of vega. We also compare this carry and rolldown to the variation in the underlying implied volatility to risk-adjust our metric, which we call a carry quotient.

Using this new volatility carry quotient metric, along with other considerations – such as relative levels vs history and correlation with global markets, we suggest the following long volatility trades:

**Long 4y30y UK Swaption Volatility**

**Long 5y5y AUD Straddles**

**Long 6m Fwd 6m5y US Volatility**

**Long 1y5y EUR Swaption Volatility**

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May 8, 2008

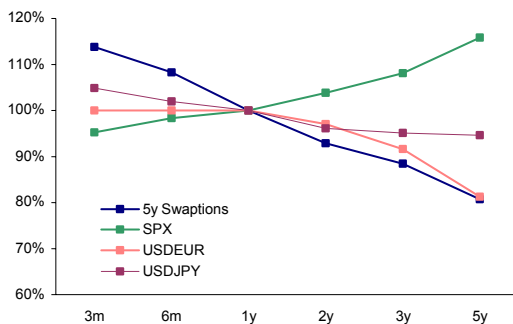
Long Vol: The New Carry Trade

Optimizing Global Volatility Rolldown &amp; Carry

## US Interest Rate Strategy

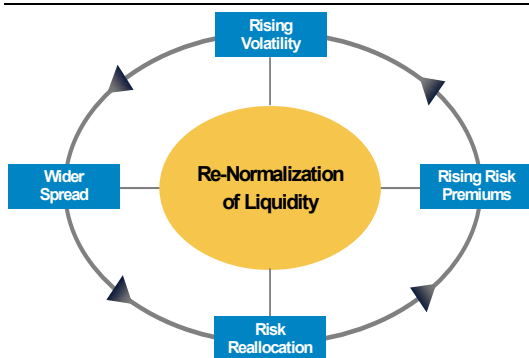
### Quantifying Rolldown & Carry for Volatility

Exhibit 1  
**Interest Rate Volatility Term Structure  
Steepest Across US Asset Classes**



Source: Morgan Stanley

Exhibit 2  
**Renormalization of Liquidity**



Source: Morgan Stanley

### Expiry Curve Is Extreme Globally – Creates Significant Positive Rolldown and Carry to be Long Volatility

The credit crunch over the past year has caused a dramatic pick-up in realized interest rate volatility globally with shorter expiries leading and longer expiries lagging. The inversion in the expiry curve is now at extremes. Thus, it is positive carry to own longer expiries as it rolls positive to shorter expiries. While expectations of mean reversion in volatility could partially explain this lag, in our view it has more to do with an overhang of volatility supply than it does market expectations. For instance, the argument of mean reversion could apply across multiple asset classes, as volatility is high in most asset classes. However, the inversion of the term structure in interest rate volatility has tended to be more extreme than that of other asset classes. As an example, Exhibit 1 graphs the term structure of US interest rate, FX and equity volatility expressed as a percentage of 1y implied volatility for each asset class. As one can see, the shape of the interest rate expiry curve is flatter than the rest.

In our view, the shape of the term structure of interest rate volatility is more a function of an overhang of volatility supply than it is a function of volatility expectations. This is simply a remnant of our ReNormalization theme (Exhibit 2). Over the past several years, low yields, tight spreads and flat curves sparked a global search for carry. One popular way to earn carry was to sell interest volatility through callable bonds or structured notes, adding significant supply of long-dated volatility to the interest rate markets. While short-expiry options have seen a significant rise, due to their sensitivity to realized volatility, this overhang of volatility supply has depressed the rise in long-dated interest rate volatility globally, inverting expiry curves. As the ReNormalization theme continues, wider spreads and steeper curves should depress demand for yield-enhancing strategies. This should cause longer-dated volatility to rise globally.

However, the creation of excess liquidity was a multi-year process, and while the renormalization of this liquidity has started very quickly, it may take time to play out. As result, despite our views above, finding cheap interest rate volatility involves more than identifying an attractive level, as it may take time for long-dated volatility to mean revert. In order to satisfy potential return hurdles, attractive investments must also include volatility that offers significant potential returns from carry and rolldown as well.

In this piece, we introduce a measure to quantify carry and rolldown in the volatility markets similar to the way it is done for swaps and bonds. We will then show how to use the metric to search for volatility trades that offer attractive potential risk-adjusted returns in both short-expiry and long-expiry volatility markets.

May 8, 2008  
 Long Vol: The New Carry Trade  
 Optimizing Global Volatility Rolldown & Carry

### Carry: The Interest Rate Swap Paradigm

The carry for a bond or swap is the difference between the fixed coupon on a bond or rate on a swap and the floating financing costs of the position (repo for a bond, or the floating rate of a swap). Investors who want to analyze the potential profitability of a fixed income position must take into consideration both the current price and the carry. The steeper the shape of the yield curve generally, the larger the carry becomes and therefore it becomes an increasingly important consideration. Carry can be expressed in two ways that are mathematically equivalent:

1. **Difference between Fixed and Floating** - For swaps and bonds, this is the difference between the fixed coupon received and the financing costs (repo for a bond or floating LIBOR for a standard swap) to the horizon of the trade. We say that this is the carry in **bps upfront**. This is typically used in analyzing notional neutral positions
2. **Difference between Forward and Spot** - Alternatively, using absence of arbitrage arguments, the carry on any asset to a given horizon is the difference between its spot price and its forward price at that horizon. This can be also be expressed in **bps running**, the difference between the forward rate and the spot rate.

These two formulations give us different ways of thinking about carry and, for swaps, they are linked by the standard identity

$$P_T r_T = P_t r_t + P_{t,T} r_{t,T}, \quad (1)$$

where  $r_T$ ,  $r_t$  and  $r_{t,T}$  are the swap rate to time T, the swap rate to horizon t, and the forward rate from the horizon; and the  $P$  s are their respective PV01s. If we rewrite this identity, we find that

$$P_{t,T} (r_{t,T} - r_T) = P_t (r_T - r_t),$$

we see the equality of these two views of carry.

1. The right hand side shows that the carry is simply the difference between what is paid ( $r_t$ ) and what is received ( $r_T$ ) until time  $t$ . This is the carry in **bps upfront**.
2. The left hand side shows is the difference between the spot price and forward price (recall the spot price is 0) and shows that, in **bps running**, the carry is the difference between the forward rate and the spot rate. Note that the bps running refers to the forward PV01.
3. There is a third way to view carry, as the PNL from the security if the value of is the same at horizon as it is today. In this case, since the position is "paid" carry up until horizon, the total PNL is this carry. For swaps, this would say that both the initial PV and the PV at horizon is zero. That is, at horizon, the swap rate, for the remaining time, is still  $r_T$ . The PNL is then the difference between the price of the original swap and the swap at horizon. That is

$$(P_t r_t + P_{t,T} r_{t,T}) - (P_t r_t + P_T r_T) = P_{t,T} (r_{t,T} - r_T),$$

May 8, 2008

Long Vol: The New Carry Trade

Optimizing Global Volatility Rolldown &amp; Carry

which equals the forward price. We can view this as the effective rate of the swap; i.e., what rate  $r'$ , over the life of the swap, would give this value. That is,

$$P_T(r_T - r') = P_{t,T}(r_{t,T} - r_T).$$

This third viewpoint will be helpful for understanding vol carry.

If an investor expects the curve to stay steep, **rolldown** is also an important consideration, as any long bond or swap position will experience capital appreciation over time as the yield rolls down the curve.

### Carry for Volatility

We believe it is useful to look at similar metrics in the volatility markets. Below we walk through our definitions of rolldown and carry for volatility. In the trade idea section we walk through an example calculation for 5y5y AUD straddles.

The PNL of a delta-hedged option can be described as the difference between 'realized' volatility of rates and the volatility implied by the price. More precisely, the PNL is given by

$$\text{Delta Hedging PNL} \sim \frac{1}{2} \sum \gamma(i)(\sigma_R^2(i) - \sigma_I^2(i)),$$

where  $\gamma(i)$  is the gamma of the option at time  $i$ ,  $\sigma_R^2(i) = (r_i - r_{i-1})^2$  is the realized variance, and  $\sigma_I^2(i)$  is the implied variance. In particular, a delta-hedged option can be viewed as a swap (pay fixed variance and receive floating variance) whose cash flows are weighted by gamma, which may vary with time and rates. Therefore, when trading volatility, whether through variance swaps or delta-hedged options, we can express carry exactly as we do for interest rate swaps.

In general, we express carry (for rates and volatility) in two manners:

1. **Difference between Floating and Fixed:** Following our swap analogy, the fixed side is the volatility paid during each period; it's helpful to view this as the theta decay, typically a function of the fixed implied volatility over the life of the option. The floating side of a delta-hedged option is the realized volatility/variance. To a fixed horizon, the expected value – or price – of the realized volatility is derived from the implied volatility of a shorter expiry option, specifically a midcurve (discussed below). We say that this is the carry in **bp upfront**.
2. **Difference between Spot and Forward:** Alternatively, carry can be expressed as the difference between the forward volatility and spot volatility. We will refer to this as **normalized bp running**. Note that the 'bp running' refers to the forward expected vega.

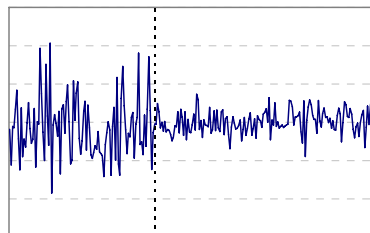
Since the gamma of a vanilla swaption is dependent on time and the path of interest rates, calculating carry is a bit delicate. However, subject to certain assumptions, gamma will, on average, be constant over the life of an initially ATM option. Using an assumption of constant gamma, the calculation of carry becomes exactly the

May 8, 2008  
 Long Vol: The New Carry Trade  
 Optimizing Global Volatility Rolldown & Carry

Exhibit 3

## Total Variance Is a Combination of Midcurve Variance and the Forward Variance of the Underlying Swap

Midcurve Variance	Swap
Forward Variance	Swap
Underlying Variance	Swap



Source: Morgan Stanley

same as that of a variance swap, the difference between the fixed (implied) variance and floating (realized) variance to our horizon. Therefore, we can use the analogy of a swap to calculate the carry as the *expected value* of the difference between floating and fixed; actual results will depend on the path of interest rates and realized volatility.

### Building Blocks of Volatility Carry: Forward and Midcurve Variance

As with interest rate swaps, the building blocks for the carry calculations come from the forward prices. We will assume we have forward prices at our horizon dates. As forward volatility agreements gain popularity, these forward prices are becoming more available. Alternatively, we discuss in Appendix A how to estimate the forwards.

In general, the variance until the expiry of a volatility trade can be represented as the average variance for different periods of the trade. If we know the variance of an underlying to the expiry of an option and to our horizon date, we can calculate the forward variance at our horizon date, which is the level of implied volatility needed in order to breakeven on our carry.

That is, if the normalized volatility of a 5y10y swaption is  $\sigma_5$ , then the total variance for the 5 years should be  $5\sigma_5^2$ ; this can be viewed as the “price” of variance. On the other hand, if the 2y forward volatility of the same underlying swap is  $\sigma_{2,5}$ , then the total variance obtained from years 3-5 is implied to be  $3\sigma_{2,5}^2$ . In order for the total variance over the five years to be  $5\sigma_5^2$ , the implied total variance for the first two years must be

$$2\sigma_2^2 = 5\sigma_5^2 - 3\sigma_{2,5}^2.$$

**Midcurve volatility:** The annualized volatility  $\sigma_2$  for the first two years is the *midcurve* volatility; that is, a midcurve is an option on a forward rate. In this case, it is the 2y option on the 3y10y forward swap rate. See Exhibit 3 for a visual example. Details on how to estimate midcurve volatility appear in Appendix A.

### Definition of Volatility Carry

If we are a bit more precise and account for the time value of the cash flows we note that this is the **exact same formula as (1)**. That is, using the same notation conventions as we used for swaps,

$$V_T \sigma_T^2 = V_t \sigma_t^2 + V_{t,T} \sigma_{t,T}^2.$$

Here, we denote by  $V_T$  the “variance vega” of the variance swaps. While this is simply the PV01 for a variance swap, by using variance vega  $V_T$ , we obtain a more general formula for all volatility products. Just as with swaps, this formula will illuminate our calculation; specifically, we will build formulations 1 and 2 for carry by rewriting as

$$V_{t,T} (\sigma_{t,T}^2 - \sigma_T^2) = V_t (\sigma_t^2 - \sigma_T^2).$$

May 8, 2008

Long Vol: The New Carry Trade

Optimizing Global Volatility Rolldown &amp; Carry

1. Until the horizon  $t$ , an investor long variance pays the fixed rate  $\sigma_T^2$  and receives the floating, realized variance, whose expected value is  $\sigma_t^2$ . This gives the right hand side of the equation. As before, this is the carry in dollar terms.
2. Alternatively, the left hand side shows the difference between the spot price and the forward price. Taking out the PV01, we get the carry in *normalized variance points running*.
3. For the 3<sup>rd</sup> way of looking at carry, we are looking for the “effective” variance  $\bar{\sigma}_T^2$  received if, at horizon  $t$ , the remaining variance is the same as the spot variance. That is, exactly as we had for swaps

$$V_T(\bar{\sigma}_T^2 - \sigma_T^2) = V_t(\sigma_t^2 - \sigma_T^2).$$

We will appeal to all three characterizations of carry.

**The first characterization tells us how much (realized) volatility is priced in for the period until the horizon.** Since our horizon is often shorter than the life of the option, it is useful to understand how much “realized” volatility we need to achieve in order to realize the carry.

Just as with swaps, **the second characterization tells us how much the market price (specifically, the forward implied vol) needs to move to lose the gains from carry** – or make it up in the case of negative carry.

The first two characterizations allow us to view relative value based on 1) the level of the forwards or 2) the near-term realized volatility priced in.

**The third characterization, as with rates, allows us to estimate an “effective volatility” over the life of the option if the forwards are not met.** That is if the implied volatility of the straddle is unchanged at the horizon, how much is the carry worth?

### Carry Across Vol Products: Variance Swaps and Swaptions

Historically, investors have traded interest rate volatility through delta-hedged swaption straddles. The development of more sophisticated vol products (e.g., volatility/variance swaps and forward volatility) has allowed investors to take pure views on volatility without the maintenance or rate exposure inherent in delta hedging. But the fact is most investors still trade volatility through delta-hedged swaptions.

Mathematically, the P&L of a delta-hedged option is roughly

$$\text{Delta Hedging PNL} \sim \frac{1}{2} \int e^{-rt} \gamma(t, r) (\sigma_r^2(t, r) - \sigma_t^2(t, r)) dt$$

where  $\sigma_R^2$  and  $\sigma_I^2$  are the realized and the implied volatilities, respectively, and  $\gamma$  is the gamma of the option. In practice, this means that, to capture the difference between realized and implied volatility, a delta-hedged straddle must have high gamma (i.e., must be close to the strike).

On the other hand, a variance swap allows perfect exposure to the squared changes in rates. That is, a variance swap would have the same PNL as delta

May 8, 2008

Long Vol: The New Carry Trade

Optimizing Global Volatility Rolldown &amp; Carry

hedged swaption if the gamma were *constant*. For more on variance swaps, refer to our publication *Trading Interest Rate Volatility: Volatility and Variance Swaps*, (6 November 2006).

Carry should be the difference between what is expected to be paid and what is expected to be received until horizon, the formula for the PNL of a delta-hedged swaption suggest that

$$\text{Carry} = E \left[ \frac{1}{2} \sum_{i=1}^T e^{-rt} \gamma(i) (\sigma_R^2(i) - \sigma_i^2(i)) \right].$$

Since gamma is non-constant for a swaption, the dollar value of carry on a swaption is not independent of the path of rates. If we assume volatility is independent of rates, then the formula simplifies to

$$\text{Carry} = \frac{1}{2} \sum_{i=1}^T e^{-rt} E[\gamma(i)] (\sigma_R^2(i) - \sigma_i^2(i)).$$

In Appendix B, we show that – assuming a normal model for interest rates – the expected value of gamma of an ATM option is constant over time and that

$$V_{t_1, t_2} = E \left[ \frac{1}{2} \int_{t_1}^{t_2} e^{-rt} \gamma(t, r) dt \right] = \lambda(t_2 - t_1)$$

where  $\lambda$  is a fixed constant. In particular, the carry of the delta-hedged swaption is

$$\text{Carry} = V_t(\sigma_t^2 - \sigma_T^2).$$

More generally, using the same analysis, we can rewrite our fundamental identity

$$V_T \sigma_T^2 = V_t \sigma_t^2 + V_{t,T} \sigma_{t,T}^2,$$

giving rise to all three views of carry. Given the our results that the expected gamma is constant over the life of an option this simplifies to the following for an ATM swaption.

$$T \sigma_T^2 = t \sigma_t^2 + (T - t) \sigma_{t,T}^2$$

### Implied to Realized Premium

Investors at times talk of the implied / realized differential when referring to carry for volatility. Technically, since this does not impact forward volatility prices we are hesitant to call this carry. But with implied volatilities trading at a significant discount to recent realized volatilities it is important to consider. The **implied to realized premium** is the difference between historical realized volatility and *the midcurve volatility* priced in to the horizon; the **adjusted carry**, then, is the carry plus this premium. In normal markets the implied to realized premium is typically a negative adjustment to carry for volatility longs. In today's environment if trailing realized volatilities are used the implied / realized premium produces a positive adjustment to carry.

Note as said above we will be looking at the difference between realized volatility and the midcurve. The first characterization of carry tells us the market is pricing –

May 8, 2008  
 Long Vol: The New Carry Trade  
 Optimizing Global Volatility Rolldown & Carry

through the midcurve – a certain amount of realized volatility at the beginning of the life of the option. In order to achieve this carry, the realized volatility must exceed the midcurve, not simply the implied volatility of the option.

## Calculating Rolldown

Rolldown for a swap is the expected capital gain on the swap if rates roll down the yield curve. For a horizon  $t$  the rolldown in **bps running** is equal to the difference in the longer maturity rate and the shorter maturity rate. The rolldown in **bps upfront** is the difference in rates multiplied by the PV01 of the swap at horizon.

$$P_{T-t}(r_T - r_{T-t})$$

For variance the same analogy applies. For a horizon  $t$  the rolldown in **normalized variance points running** is equal to the difference in the longer expiry implied variance and the shorter expiry variance. The rolldown in **bps upfront** is the difference in rates multiplied by the vega of the swap at horizon.

$$V_{T-t}(\sigma_T^2 - \sigma_{T-t}^2)$$

Again, for pure volatility products like variance swaps (and with some modification vol swaps and forward volatility agreements) this analogy is perfect as these trades have constant volatility exposure. For traditional swaptions this is correct in the expected case.

## Carry as a percent of volatility

Since rate volatility is best understood in normalized absolute basis points, we divide the rate carry, in bps running, by the realized, normalized volatility. Vol carry, on the other hand, typically trades more lognormally. That is, the higher the volatility, the higher the vol of vol (Exhibit 5).

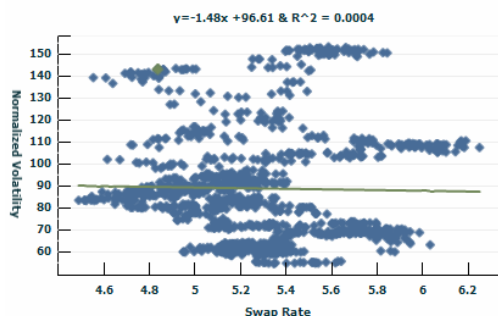
In particular, the absolute level of carry should be viewed relative to the level of volatility; a high value for carry is more attractive if volatility is low than if volatility is high. Therefore, it is useful to measure the percent carry as the **percent of the level of volatility/variance**. Using the notation from our third characterization of carry, the percent carry for a variance swap can be written  $\ln(\bar{\sigma}_T^2 / \sigma_T^2)$ . For a volatility swap or a straddle, the percent carry is  $\ln(\bar{\sigma}_T / \sigma_T)$ . These are related by

$$\text{Var Swap Carry} = \ln(\bar{\sigma}_T^2 / \sigma_T^2) = 2 \ln(\bar{\sigma}_T / \sigma_T) = 2 \times \text{Straddle Carry}.$$

## Carry Quotient

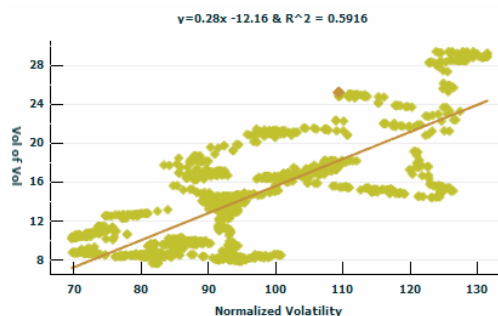
When looking at carry trades on swaps, we use the Morgan Stanley carry quotient to seek out relative value across the curve. The carry quotient is simply the carry divided by the realized volatility of the rate. We “annualize” the volatility to the horizon of the carry. For volatility trades, we use the same metric, dividing the percent carry by the “lognormal” realized volatility, that is, the volatility of the daily percent returns. Just as the percent carry is doubled for variance swaps, so is its volatility. In particular, by dividing by the “vol of vol”, **the carry quotient is the same for all volatility products**.

Exhibit 4  
**Rate Volatility Is Uncorrelated with the Level of Rates**



Source: Morgan Stanley

Exhibit 5  
**Vol of Vol Rises as Implied Volatility Rises**



Source: Morgan Stanley



May 8, 2008  
Long Vol: The New Carry Trade  
Optimizing Global Volatility Rolldown & Carry

### Using Carry and Rolldown to Find Attractive Volatility Trades

The inversion of expiry curves globally implies that forward volatilities are cheaper than spot, and therefore, imply volatility will decline from current levels. For investors that expect volatility to stay high, entering trades with significant rolldown and carry offers an opportunity to profit should volatility not meet the forwards. The risk investors are taking when entering a long volatility trade is that the capital losses from any fall in implied volatility will be larger than the rolldown and carry from the position. That is, in searching for attractive trades, we want to look at two metrics; one measuring the carry and a second measuring the level of volatility relative to history.

- 1) **Carry quotient-volatility adjusted carry and rolldown:** Any capital losses on a volatility position will be dependent on changes in the implied volatility of the option. As such, in order to identify attractive trades we need to find trades with a high level of rolldown and carry as compared to the volatility of implied volatility of the option (vol of vol). We will define **carry quotient** as the ratio of rolldown and/or carry to the volatility of volatility.
- 2) **Current versus historical level of volatility:** Entering a long volatility position with positive rolldown and carry is a bet that the volatility will not fall as much as the forward volatilities imply. We would prefer to make that bet where levels of implied volatility are low compared to historical levels. We will measure this using a 5y percentile of the current level of implied volatility.

#### Results:

We ran our analysis on G4 markets. We also include the AUD swaption market as the inversion in the expiry curve there is quite severe. Note our calculations of carry are based on ATM volatilities and assumptions of correlation based on 6m trailing realized correlation. We did not use market prices for forward volatility or midcurves nor did we incorporate skew into the results as market prices are available only in certain markets and parts of the grid. As such, these calculations should serve as a guide to find positive carry trades across multiple markets. Pricing for forward volatility agreements for example may not be consistent with the carry we calculate.

May 8, 2008

Long Vol: The New Carry Trade

Optimizing Global Volatility Rolldown &amp; Carry

Based on our analysis we suggest the following trades:

- **Buy AUD 5y5y:** For options with an expiry of more than one year, AUD 5y5y scores highest on our carry quotient metric, offering 23.4% 1y rolldown and carry, scoring a carry quotient of 1.0.
- **Buy 4y30y UK Vol:** 4y30y UK vol is the cheapest vol in our search relative to history that also offers a carry quotient > 0.5.

Exhibit 6

**Global Rolldown & Carry Screen – Vega (Expiries ≥ 2y, 1y Horizon)**

Market	Swaption	Implied Vol	Carry	Rolldown	RD&C	Carry Quotient	5y Z-Score
AUD	5y5y	71.0	14.9%	8.5%	23.4%	1.01	0.57
AUD	5y7y	68.3	14.1%	8.4%	22.4%	0.83	0.46
GBP	2y1y	91.1	5.4%	9.6%	15.0%	0.77	0.68
AUD	5y10y	66.5	12.9%	6.0%	18.9%	0.67	0.39
AUD	7y10y	62.0	9.9%	2.5%	12.4%	0.63	0.31
GBP	2y2y	86.6	2.4%	9.3%	11.8%	0.62	0.62
AUD	10y3y	68.2	8.2%	3.3%	11.5%	0.61	0.56
AUD	3y10y	78.0	9.7%	6.6%	16.4%	0.61	0.23
GBP	3y30y	48.0	5.5%	4.0%	9.5%	0.59	-1.23
AUD	4y7y	77.9	10.2%	5.0%	15.2%	0.58	2.47
GBP	2y3y	80.2	1.9%	8.9%	10.8%	0.58	0.44
AUD	4y10y	73.0	11.0%	4.1%	15.1%	0.58	2.32
AUD	5y2y	83.6	13.4%	3.4%	16.8%	0.58	1.02
GBP	3y1y	79.9	3.5%	8.5%	12.1%	0.58	0.07
GBP	4y30y	45.4	5.6%	3.7%	9.2%	0.56	-1.47

Source: Morgan Stanley Research

- **Buy 1y5y EUR Vol:** 1y10y UK vol and 1y5y EUR vol score highest on our carry quotient metric for options < 1 year with scores of 1.1 and 0.8, respectively, for 3 months of carry and rolldown. Given we have already suggested a UK trade above, we recommend longs in 1y5y EUR straddles.

Exhibit 7

**Global Rolldown & Carry Screen – Gamma (Expiries ≤ 1y, 3m Horizon)**

Market	Swaption	Implied Vol	Carry	Rolldown	RD&C	Carry Quotient	Z-Scores
GBP	1y10y	67.1	4.4%	3.9%	8.3%	1.14	0.15
GBP	1y7y	73.2	4.4%	4.1%	8.5%	1.03	0.45
GBP	1y20y	59.7	3.6%	3.5%	7.1%	0.95	-0.21
GBP	1y15y	62.5	4.0%	3.5%	7.5%	0.94	-0.13
GBP	1y30y	55.7	3.6%	3.5%	7.1%	0.92	-0.40
EUR	6m5y	72.9	5.1%	4.2%	9.3%	0.90	0.40
GBP	1y5y	81.8	3.6%	3.8%	7.4%	0.85	0.92
EUR	1y5y	67.2	4.4%	2.9%	7.3%	0.81	-0.05
GBP	6m7y	82.1	3.5%	3.0%	6.6%	0.79	1.16
EUR	6m4y	75.4	5.2%	3.9%	9.2%	0.79	0.57
EUR	6m7y	68.0	3.8%	3.3%	7.1%	0.79	0.20
EUR	1y4y	69.3	4.9%	2.9%	7.8%	0.76	0.09
GBP	6m10y	74.9	3.2%	2.7%	5.8%	0.74	0.94
EUR	6m3y	77.5	5.8%	4.2%	10.0%	0.74	0.71
GBP	1y4y	87.7	3.3%	3.8%	7.0%	0.71	1.16

Source: Morgan Stanley Research

May 8, 2008

Long Vol: The New Carry Trade

Optimizing Global Volatility Rolldown &amp; Carry

- **Buy 6m Fwd 6m5y USD Volatility:** In the US, active markets in forward volatility have developed. As opposed to the trades suggested above which are based on expectations of carry, investors can lock in the carry in the US through a forward volatility agreement. 6m forward 6m5y volatility is our favorite place in the US to buy forward volatility, scoring a carry quotient of 0.35.

Exhibit 8

**USD Rolldown & Carry Screen – Gamma (Expiries ≤ 1y, 3m Horizon)**

Market	Swaption	Implied Vol	Carry	Rolldown	RDC	Carry Quotient	Z-Scores
USD	1y4y	120.5	4.2%	3.8%	8.0%	0.36	0.64
USD	1y5y	120.6	3.6%	3.9%	7.5%	0.35	0.63
USD	1y3y	120.7	4.0%	3.6%	7.6%	0.33	0.65
USD	1y7y	116.3	3.2%	3.7%	6.9%	0.33	0.59
USD	1y30y	96.7	3.4%	3.6%	6.9%	0.32	0.91
USD	1y20y	99.9	3.1%	3.4%	6.5%	0.31	0.76
USD	1y15y	102.8	2.8%	3.2%	6.1%	0.28	0.59
USD	1y10y	110.3	2.5%	3.0%	5.5%	0.27	0.54
USD	1y2y	120.5	3.5%	3.2%	6.6%	0.27	0.67
USD	1y1y	117.8	-0.5%	-0.5%	-1.1%	-0.04	0.81

Source: Morgan Stanley Research

Since each of these trades is long volatility, they are at risk to longer-dated volatility continuing to fall. Moreover, as hedge funds tend to be long cheap vega, continued deleveraging could put downward pressure on the longer-dated volatilities. However, the attractive carry profile should help to offset these risks.

May 8, 2008  
Long Vol: The New Carry Trade  
Optimizing Global Volatility Rolldown & Carry

Exhibit 9

## 5y, 10y and 5y5y 6m Trailing Realized Volatility



Source: Morgan Stanley

Exhibit 10

## 5y and 10y 6m Realized Correlation



Source: Morgan Stanley

## Buy 5y5y AUD Swaption Straddles

The trade with the highest carry quotient globally is AUD 5y5y swaption volatility. We see the expected rolldown and carry on the 5y5y options as being worth 23.4% of the original premium of the option, producing a carry quotient of 1.0 (Exhibit 14).

The high level of carry is driven by both the inversion of the expiry curve and the relatively high volatility of forward rates in AUD. The inversion of the expiry curve the AUD market is much more severe than other global markets. For instance, 1y5y volatility is 140% of 5y5y volatility, equivalent to the same ratio in the UK and higher than similar ratios in USD and EUR. Furthermore, the realized volatility of forward rates in AUD have been much higher than spot as the yield curve has shown little directionality with the level of rates (Exhibit 9). This lack of correlation within the yield curve has created huge differential between implied and actual volatilities for 5y5y options (implied to trailing 6m realized ratio is 50%!). Despite having much less gamma, the much bigger differential between near-term expectations of volatility and longer-term expectations of volatility points to more potential delta hedging profits per unit of vega risk compared to short-expiry options.

Below is a table of the 1y rolldown and carry values for the AUD 5y5y swaption. We will walk through these calculations as an example.

**1) Midcurve Volatility** - The first step is to compute what the market expects the midcurve volatility of the underlying forward rate to be over the next year (a 1y option on a 4y5y rate). In order to calculate this we need the 1y4y implied volatility (100 bp normalized), 1y9y implied volatility (95 bp normalized) and an assumption of correlation (6m trailing realized correlation has been 87%).

Exhibit 11

## Midcurve Calculation

	Expiry	Tail	Vol	PV01		Expiry	Tail	Vol	PV01
Option1	1y	4y	100.4	3.16	Option1	1y	4y	100.4	3.16
Option 2	1y	9y	95.5	6.05	Option 2	1y	9y	95.5	6.05
Correlation			87%		Correlation			95%	
Midcurve	1y	4y5y	117.4	2.89	Midcurve	1y	4y5y	101.6	2.89

Source: Morgan Stanley

Using these values we get a midcurve volatility of 117 bp normalized. Initially this may seem high given the 1y4y and 1y9y volatilities above, but it is consistent with the fact that realized volatility of the 5y5y rate has been systematically higher than the realized volatility of its underlying 5y and 10y rates. In fact the average realized volatility of the 4y5y and 5y5y rate has been close to 140 bp normalized over the past 6 months (see Exhibit 9). If we assume a correlation closer to the historical average of 95% this provides us with a midcurve volatility of 102 norm bp.

**2) Carry** - Given a 1y volatility of 117 bp normalized, the 1y forward 4y5y volatility would need to be 53 bp normalized in one year's time to justify the 71 bp normalized implied volatility over the life of the 5y5y option. We therefore calculate the carry in norm bp running to be 17 bp (this uses our second characterization of carry).

The 1y midcurve volatility of 113 bp normalized results in an effective volatility of 82 bp normalized if the implied volatility of the option is still 71 norm bp in 1y time. The carry is therefore worth 15% in lognormal terms using our 3<sup>rd</sup> characterization of carry.

Exhibit 12

## 5y5y AUD Swaption Implied Volatility



Source: Morgan Stanley

May 8, 2008

Long Vol: The New Carry Trade

Optimizing Global Volatility Rolldown &amp; Carry

If we use the more conservative 101.6 midcurve volatility, this carry reduces to 10 bp or 9.5% in lognormal terms.

Exhibit 13

**Midcurve Calculation**

	Expiry	Tail	Vol		Expiry	Tail	Vol
Midcurve	1y	4y5y	117.4	Midcurve	1y	4y5y	101.6
Option 3	5y	5y	71.0	Option 3	5y	5y	71.0
<b>Fwd Vol</b>	<b>1y</b>	<b>4y5y</b>	<b>53.4</b>	<b>Fwd Vol</b>	<b>1y</b>	<b>4y5y</b>	<b>60.9</b>
Carry norm bp			17.6	Carry norm bp			10.0
Effective Vol			82.4	Effective Vol			78.1
Carry %			14.9%	Carry %			9.5%

Source: Morgan Stanley

**3) Implied / Realized Premium** - If realized volatility equals the 140 bp normalized seen over the past 6 months for the rest of our horizon, our breakeven on implied vol in 1 years time pushes down another 17 bp normalized to 36 bp normalized. The effective vol pushes up to 89.6 norm bp or 8.5%.

**4) Rolldown** - 4y5y implied volatility is trading 10 normalized higher than 5y5y volatility creating 10 bp normalized of rolldown. That would push the effective volatility up another 8.7%.

Exhibit 14

**Volatility Rolldown and Carry for 5y5y AUD Swaptions**

	Norm bp	%
Carry	17.6	14.9%
Imp / Real Premium	17.4	8.4%
<b>Adjusted Carry</b>	<b>35.0</b>	<b>23.3%</b>
Rolldown	10.3	8.5%
<b>Rolldown &amp; Carry</b>	<b>27.9</b>	<b>23.4%</b>
<b>Adjusted RD&amp;C</b>	<b>45.3</b>	<b>31.8%</b>

Adjusted values include implied / realized premium

Assumes 1y midcurve vol on 4y5y rate of 117.4 norm bp

computed from 1y4y Vol = 100.4, 1y9y Vol = 95.5, Corr = 0.87

The most conservative assumption of 100% correlation

would reduce carry by 7.6 norm bp or 5.4%

Trailing realized vol is 141.4 or 120.4% of the implied midcurve vol

Source: Morgan Stanley

Rolldown & Carry	23.4%
Vol of Vol	22.7%
<b>Carry Quotient</b>	<b>1.0</b>

As shown above our rolldown and carry calculation is sensitive to our assumption of correlation. Using a longer-term historical correlation of 95% (Exhibit 10) reduces the carry by 6%, putting total rolldown and carry at 16%. Note if realized volatility exceeds what our midcurves imply rolldown & carry could be much larger than our 23.4% value (it could be as high as 31.8% if realized vol over the next year matches what we have seen in the past 6 months).

Given the recent retracement and the high levels of rolldown and carry, we think now is an attractive time to look at AUD volatility longs. We think economic uncertainties in the AUD market are quite high and expect the RBA to revert to cutting rates in the next year (see Vincenzo Guzzo's *Antipodean Steepeners*, April 24, 2008). A retracement of the curve steepening has also caused a significant decline in implied AUD volatility (Exhibit 12), creating attractive entry levels relative to history, in our view. When compared to history, the absolute level of 5y5y implied volatility is cheaper than other parts of the AUD volatility surface, providing less downside risk from a decline in implied volatility. The 5y5y point optimizes the carry in both absolute and risk-adjusted measures.

May 8, 2008  
 Long Vol: The New Carry Trade  
 Optimizing Global Volatility Rolldown & Carry

## Buy 4y30y UK Swaption Straddles

When compared to history, longer-dated UK volatility is very cheap with various points on the grid, 1-2 standard deviations below 5y averages. In *UK Volatility Curve Too Inverted*, April 15, 2008, we suggested that even allowing for long-standing technical downward pressure on UK long-tail volatility, recent selling (apparently driven by risk reduction in parts of the leveraged community) has pushed some areas of the UK volatility surface to levels which look unsustainably low (Exhibit 15).

Our new rolldown and carry metric highlights why the 4y30y point is an attractive place to put this trade as it maximizes the rolldown and carry of the position on the swaption surface. The 1y rolldown and carry of 8.4% (5.6bp normalized) is high compared to the low annualized lognormal vol of vol, producing a carry quotient of 0.5 (Exhibit 16). If the volatility of the forward matches what they realized over the past six months, the 1y carry and rolldown including the implied/realized premium could be even larger – as high as 14.5% (10.4 bp normalized - see Exhibit 16)

Exhibit 15

### 4y30y UK Swaption Implied Volatility



Source: Morgan Stanley

Exhibit 16

### Volatility Rolldown and Carry for 4y30y UK Swaptions

	Norm bp	%
Carry	3.1	4.7%
Imp / Real Premium	4.8	6.0%
<b>Adjusted Carry</b>	<b>7.9</b>	<b>10.7%</b>
Rolldown	2.5	3.7%
<b>Rolldown &amp; Carry</b>	<b>5.6</b>	<b>8.4%</b>
<b>Adjusted RD&amp;C</b>	<b>10.4</b>	<b>14.5%</b>

Rolldown & Carry	8.4%
Vol of Vol	16.1%
<b>Carry Quotient</b>	<b>0.5</b>

Adjusted values include implied / realized premium  
 Assumes 1y midcurve vol on 3y30y rate of 53.6 norm bp  
 computed from 1y3y Vol = 95.1, 1y33y Vol = 54.4, Corr = 0.70  
 The most conservative assumption of 100% correlation  
 would reduce carry by 2.8 norm bp or 4.3%  
 Trailing realized vol is 63.5 or 118.5% of the implied midcurve vol

Source: Morgan Stanley Research

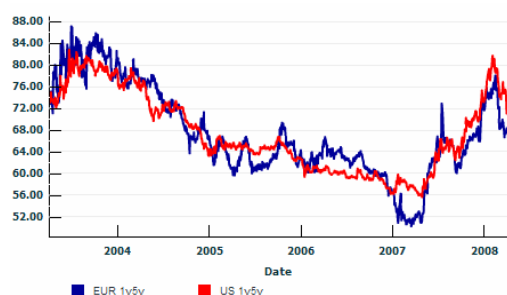
Granted UK volatility has remained cheap for some time now and is likely to stay cheap in the near term even if it normalizes from recent lows. However, given the potential gains from rolldown and carry alone and the lack of volatility of the underlying implied volatility, we think 4y30y straddles offer attractive risk-adjusted returns in an unchanged volatility environment for investors willing to buy the straddles and delta hedge them over time. Any rally in volatility in that part of the surface would only be upside.

May 8, 2008  
**Long Vol: The New Carry Trade**  
**Optimizing Global Volatility Rolldown & Carry**

## Buy 1y5y EUR Vol

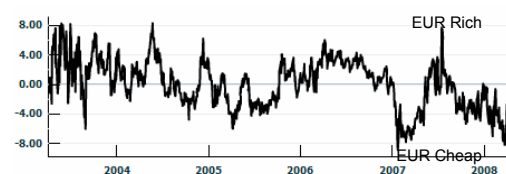
A search of shorter expiry options with a 6m horizon highlights 1y10y UK volatility and 1y5y EUR volatility as having the highest carry quotients of 1.1 and 0.8, respectively. As we have already suggested a UK trade, we will focus on the suggestion in EUR. The calculation of rolldown and carry is below, with the total worth 7.1 norm bp or 7.3% (Exhibit 18).

Exhibit 17  
**1y5y EUR and USD Volatility**



Source: Morgan Stanley

Exhibit 19  
**USD vs. EUR Volatility Regression Residual**



Source: Morgan Stanley

Exhibit 18  
**3m Volatility Rolldown and Carry for 1y5y EUR Swaption**

	Norm bp	%
Carry	4.3	4.4%
Imp / Real Premium	0.6	0.6%
<b>Adjusted Carry</b>	<b>4.8</b>	<b>5.0%</b>
Rolldown	2.8	2.9%
<b>Rolldown &amp; Carry</b>	<b>7.1</b>	<b>7.3%</b>
<b>Adjusted RD&amp;C</b>	<b>7.7</b>	<b>7.8%</b>

Adjusted values include implied / realized premium  
 Assumes 0.25y midcurve vol on 0.75y5y rate of 78.6 norm bp  
 computed from 0.25y0.75y Vol = 86.8, 0.25y5.75y Vol = 77.1, Corr = 0.82  
 The most conservative assumption of 100% correlation  
 would reduce carry by 1.3 norm bp or 1.2%  
 Trailing realized vol is 80.0 or 101.8% of the implied midcurve vol

Rolldown & Carry 7.3%  
 Vol of Vol 8.8%  
**Carry Quotient 0.8**

Source: Morgan Stanley

Aside from having the highest carry among its global counterparts, 1y5y EUR volatility also looks cheap versus other markets, especially the US (Exhibit 18 and Exhibit 19).

The key risk to owning volatility in EUR is the potential for a sustained period of the ECB remaining on hold and the curve remaining flat. However, recent data have highlighted the risks to the Euro economy (see Eric Chaney's *Turning Point; Risk of Manufacturing Recession*, April 30, 2008) and our economists still forecast that the ECB will be considering rate cuts by the end of the year. In our view, the relative uncertainty as to the economic outlook, coupled with the positive carry and rolldown, argues risk reward to owning 1y5y volatility in EUR is attractive.

## Buy 6m Fwd 6m USD Vol

All the trade suggestions above were based on current levels of rolldown and carry. But locking in the carry on those positions is relatively difficult. To do so, investors must sell a portfolio aimed at replicating the midcurve volatility against the underlying option they are buying. Such a portfolio of options would be subject to strike risk and correlation risk.

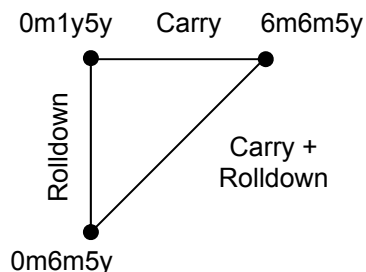
In the US, active markets in forward volatility have developed which allow investors to take lock in attractive rolldown and carry easily. In addition, forward volatility positions have gained appeal, given that they require no delta hedging and are therefore relatively easy to manage.

We can use our carry quotient metric to find the optimal place to buy forward volatility in the US – which happens to be the 1y5y point scoring a carry quotient of 0.35 for 6m of carry worth 19 bp normalized (Exhibit 21). Current prices for 6m forward 6m5y volatility are about 19bp below spot 6m5y volatility (112bp normalized versus 131bp normalized as of May 7, 2008 close).

May 8, 2008  
Long Vol: The New Carry Trade  
Optimizing Global Volatility Rolldown & Carry

Exhibit 20

## Carry and Rolldown For Volatility



Source: Morgan Stanley

Note that the difference between the 1y5y point and the 6m fwd 6m5y point is the carry on the 1y5y trade. The difference between the 1y5y point and the 6m5y vol is the rolldown on the position. Combining those two indicates that the difference between the 6m fwd 6m5y vol and spot 6m5y vol is equivalent to the rolldown and carry on a 1y5y option (see Exhibit 20).

Exhibit 21

## Volatility Rolldown and Carry for 1y5y Swaptions

	Norm bp	%
Carry	9.4	3.6%
Rolldown	10.0	3.9%
<b>Rolldown &amp; Carry</b>	<b>19.4</b>	<b>7.5%</b>

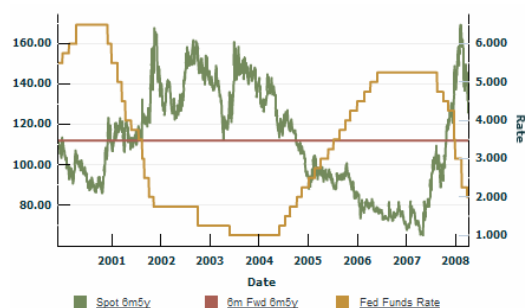
Rolldown & Carry	7.5%
Vol of Vol	21.5%
<b>Carry Quotient</b>	<b>0.3</b>

Source: Morgan Stanley Research

The key risk to owning 6m forward 6m5y volatility is the fact that even with the large spread to spot, the level of 6m fwd 6m5y volatility is above the long-term average of spot 6m5y volatility (Exhibit 22). However, in our view the last 10 years is not necessarily the best comparison for what to expect over the next year. In fact, we believe the most comparable period to what we expect in the next year is the 2002-04 period when the Fed was on hold at low interest rates and the market was characterized by a high degree of inflation uncertainty, a steep yield curve and high level of negative convexity in the mortgage market. In this period, the lowest 6m5y vol was 118bp normalized, above current pricing for 6m forward 6m5y volatility. We think all the factors that kept volatility elevated in 2002-04 will be present in the year to come. As such, even though it looks high relative to history, the rolldown versus spot and cheapness relative to the 2002-04 period make 6m forward 6m5y volatility attractive, in our view.

Exhibit 22

## Spot 6m5y vs. 6m Forward 6m5y



Source: Morgan Stanley



## Appendix A: Calculating Forward and Midcurve Volatility

As shown above, for swaptions, the fundamental identity linking forward, midcurve and spot volatilities reduces to

$$T\sigma_T^2 = t\sigma_t^2 + (T-t)\sigma_{t,T}^2$$

As long as we know the midcurve volatility  $\sigma_{t,T}$ , we can determine the forwards as

$$\sigma_{t,T} = \sqrt{\frac{T\sigma_T^2 - t\sigma_t^2}{(T-t)}}$$

Since midcurve swaptions are customized and typically not readily observable from typical option pricing sources, it is necessary, in most cases, to estimate them from the prices of vanilla swaption. To demonstrate this, we appeal to the standard identity for swaps and solve for the forward rate

$$r_{t,T} = \frac{P_T r_T - P_t r_t}{P_{t,T}}$$

That is, we can express a forward rate as a linear combination of spot rates. For example, a 2yr forward 10yr rate is a linear combination of the 12yr rate and the 2yr rate (weighted by PV01s). For the midcurve calculation, we will add a twist. We will **write a forward rate as a linear combination of two forward rates**. For example, the 5yr forward 10yr rate is a linear combination of the 2yr forward 13yr rate and the 2yr forward 3yr rate. Write  $r_{t_1,t_2,T}$  for the  $t_1$ -forward  $t_2$ -forward  $T$ -year rate (so our example would be  $r_{2,3,10}$ ). Note that this is just the  $(t_1 + t_2)$ -forward  $T$ -year rate. Then, our relation is

$$r_{t_1,t_2,T} = \frac{P_{t_1,t_2+T} r_{t_1,t_2+T} - P_{t_1,t_2} r_{t_1,t_2}}{P_{t_1,t_2,T}} = q_1 r_{t_1,t_2+T} - q_2 r_{t_1,t_2}$$

where  $q_i$  is defined by the equation. For the midcurve, we need the implied volatility of the underlying rate for the next  $t_1$  years. However, we know the volatilities of  $r_{t_1,t_2}$  and  $r_{t_1,t_2+T}$  for the next  $t_1$  years; these are standard swaption volatilities  $\sigma_{t_1,t_2}$  and  $\sigma_{t_1,t_2+T}$ . Therefore, the variance of  $r_{t_1,t_2,T}$  is

$$\sigma_{t_1,t_2,T}^2 = q_1^2 \sigma_{t_1,t_2}^2 + q_2^2 \sigma_{t_1,t_2+T}^2 - 2\rho q_1 q_2 \sigma_{t_1,t_2} \sigma_{t_1,t_2+T}$$

where  $\rho$  is the correlation of  $r_{t_1,t_2}$  and  $r_{t_1,t_2+T}$ . Therefore, we can determine implied volatility of the midcurve if we know this “implied” correlation.

The relevant volatilities may not be liquid points on the surface, but at least in principle, they are the volatilities of vanilla swaptions. On the other hand, the implied correlation cannot be determined from the prices of vanilla instruments. As such, **the implied correlation will generally be an assumption that can be approximated using historical realized correlations**.

In general for computing the carry of long volatility positions a high number will bias the results downward and a low number will bias the results upward (high

May 8, 2008

Long Vol: The New Carry Trade

Optimizing Global Volatility Rolldown &amp; Carry

correlation reduces the midcurve volatility, increasing the computed forward volatility and vice versa). In the current environment where we are looking for positive carry long volatility positions an assumption of 100% correlation is the most conservative choice. Using this value can be useful as a conservative measure to reduce the chance that we are overstating the carry of the position. Using trailing realized correlations can give a more aggressive view of potential profits from carry.

## Appendix B: The Normal Model for Swaptions

We defined

$$V_{t_1, t_2} = E \left[ \frac{1}{2} \int_{t_1}^{t_2} \gamma(t, x) dt \right]$$

If rates are independent of volatility, then

$$V_{t_1, t_2} = \frac{1}{2} \int_{t_1}^{t_2} e^{-rt} E[\gamma(t, x)] dt .$$

Now, while gamma is more uncertain as we increase time to expiry, its expected value is essentially constant in time. To make this precise, assume time dependent volatility  $\sigma(t)$  and a rate process  $dx = \sigma(t)dW$ , where  $x$  is the “moneyness” of the rate. Write

$$S(t_1, t_2) = \int_{t_1}^{t_2} \sigma^2(s) ds .$$

(Note that, if  $\sigma^2(t)$  is piecewise constant, then this is just  $\sum T_i \sigma_i^2$  for the relevant periods.) Using the standard calculations of options prices and greeks, gamma is

$$\gamma(t, x) = \exp(-r(T-t)) \frac{1}{\sqrt{2\pi S(t, T)}} \exp\left(-\frac{x^2}{2S(t, T)}\right) .$$

Now, at time  $t$ ,  $\gamma(t, x) \sim N(0, S(0, t))$ ; i.e., gamma is normally distributed with variance  $S(0, t)$ . Then, taking expectations for an ATM straddle,

$$\begin{aligned} E[\gamma(t, x)] &= \exp(-r(T-t)) \frac{1}{2\pi\sqrt{S(0, t)S(t, T)}} \int \exp\left(-\frac{x^2}{2S(t, T)}\right) \exp\left(-\frac{x^2}{2S(0, t)}\right) dx \\ &= \exp(-r(T-t)) \frac{1}{2\pi\sqrt{S(0, t)S(t, T)}} \int \exp\left(-\frac{x^2}{2} \left(\frac{S(0, t)}{S(0, t)S(t, T)}\right)\right) dx \\ &= \frac{\exp(-r(T-t))}{\sqrt{2\pi S(0, T)}} \end{aligned}$$

May 8, 2008

Long Vol: The New Carry Trade

Optimizing Global Volatility Rolldown &amp; Carry

In particular, with the exception of a discount factor, gamma depends only on the total volatility  $\sigma\sqrt{T} = \sqrt{S(0,T)}$  over the life of the option. Therefore,

$$V_{t_1, t_2} = \frac{e^{-rT}}{2\sqrt{2\pi S(0,T)}}(t_2 - t_1)$$

### Appendix C: Relationship Between Gamma and Vega

In our notation we refer to the expected gamma and vega of an option as simply as  $V$ . This is not by coincidence. The variance vega of an option is a function of the expected gamma of an option over its life.

For instance using the expected gamma function above until expiry of the option produces the following formula

$$V_T = \frac{e^{-rT}}{2\sqrt{2\pi S(0,T)}}T$$

We know that  $S(0,T) = \sigma^2 T$  substituting into the formula above

$$V_T = \frac{e^{-rT}}{2\sigma\sqrt{2\pi}}\sqrt{T}$$

We know the volatility vega of an ATM straddle for a normal distribution is given as follows:

$$\frac{dP}{d\sigma} = \frac{e^{-rT}}{\sqrt{2\pi}}\sqrt{T}$$

Using the chain rule we can relate the variance vega to the volatility vega as follows:

$$\frac{dP}{d\sigma^2} = \frac{dP}{d\sigma} 2\sigma$$

Combining the two formulas above shows that in fact the expected gamma over the life of the option is equal to the variance vega of the option.

$$V_T = \frac{dP}{d\sigma^2}$$

May 8, 2008  
Long Vol: The New Carry Trade  
Optimizing Global Volatility Rolldown & Carry

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