Advice Methodology for IWFE 4.0

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- Update on Nov. 11, 2006: (1) Employer match in Human Capital Calculation Section. (2) Default Saving Sequence in Appendix 1
- Update on Nov. 13, 2006: (1) Add a method on how human capital determines the asset allocation of an advisable asset in Section "Model Portfolio for W_t."
- Update on Nov. 14, 2006: Delete "All future cash flows should be treated as outside assets. I will clarify this part later." in Section "Model Portfolio for W_t."
- Update on Nov. 15, 2006: Add the asset allocation switch algorithm.
- Update on Nov. 21, 2006: Use actual asset allocation instead of model portfolios for the advisable asset in Section "Model Portfolio for W_t." I also made some changes on notations to make them more consistent. Constrained Model Portfolio Advice Update in WFE has been added into the document too.
- Update on Nov. 25, 2006: The location method is updated in Section "Model Portfolio for W_t." The file, Asset Location Algorithm Test Nov. 23 2006.xls, is a test program for the new location method.
- Update on Nov. 27, 2006: Add joint case into Human Capital calculation.
- Update on Feb. 1, 2007: Modify joint case in Section "Human Capital at time t" to make it clear based on Eric concerns.
- Update on Feb. 1, 2007: Add the annuitization formula in Section "Lifetime Income Converter" to address Eric concerns.
- Update on March 23, 2007: Add a section "A Practical Issue on the Determination of Model Portfolios" in Section "Advice Problem Overview" to address the issue where an investor may have n different accounts and each account has its own set of model portfolios.

Objective

During the past 10 years, a lot of new features have been added into WFE from time to time due to new developments in academic studies, regulation evolvements, and clients' requirements. The objective of this document is to take into account all these features to standardize WFE methodologies and to make the methodologies in WFE consistent.

Issue list:

- Solve the saving amount in dollar amount or in percentage: using dollar amount
- Saving rates interpretation: having one page pdf to explain outputs
- Adding accounts: relying on application levels. We only create a taxable account. Any other account types will be determined by applications. When we create a taxable account, we should provide a message to say something like: we have created a taxable account for you with a saving amount \$. You may be better off to create a tax-advantage account such as ....

1 Advice Problem Overview

1.1 Investors' problem:

Suppose that investors are at age x. (Note, in forecast, x is always the first year in the Sim data.)

1. How much labor incomes do the investors need to save to support their retirement lives?

   SR_x - Saving Rate at Age x

2. What are their current investment strategies?

   MP_x - Model Portfolio at Age x

3. What ages can they retire at comfortably?

   T - Retirement Age

4. What chance do they have to meet their desired retirement incomes?

   α - Confidence Level (Chance)

   DRI - Desired Retirement Income (We use the replacement ratio to determine DRI)

1.2 High Level Mathematical Description

For a given confidence level α and a given desired retirement income DRI, we solve for saving rate SR_x, Model Portfolio MP_x, and retirement age T in the equation below,

$$\Pr(RI_T \geq DRI) = \alpha \tag{1}$$

where,

$$RI_T = f\left(Wealth_x, LI_x, SR_x, MP_x, T, Others\right) \tag{2}$$

and LI_x is Labor Income at Age x, $Wealth_x$ is Financial Wealth at Age x, RI_T is Retirement Income. $f(\cdot)$ is an "system" that gives the income at age T based on the inputs at age x. The system consists of strategies, restrictions, and assumptions.

1.3 A Practical Issue on the Determination of Model Portfolios

There is possibility that an investor may have n different accounts. Each account is implemented with our 7 Model Portfolios (MP). However, the same "indexed" Model Portfolio (1 to 7) may have different asset allocations. For example, the asset allocation assigned to Model Portfolio 1 (MP_1) in Account 1 may be slightly different from that assigned to MP_1 in Account 2. This causes a problem, i.e. which asset allocations we should use for a Model Portfolio, say MP_1.

1.3.1 Solution: Representative Portfolios

Suppose an investor have n account at current age x. Denote,

- $MP_{ij,x}$ - model portfolio i associated with account j at age x, where, $i = 1, ..., 7$ and $j = 1, ..., n$
- $BMP_{ij,x}$ - bond proportion in model portfolio i associated with account j at age x, where, $i = 1, ..., 7$ and $j = 1, ..., n$
- $BL_{j,x}$ - balance associated with account j at age x, where $j = 1, ..., n$
- $RMP_{i,x}$ - representative model portfolio for Model Portfolio i at age x, $i = 1, ..., 7$

- $BRMP_{i,x}$ - bond proportion in representative model portfolio i at age x, $i = 1, ..., 7$

  $RMP_{i,x}$ is determined by,

$$BRMP_{i,x} = \frac{1}{\left(\sum_{j=1}^{n} BL_{j,x}\right)} \sum_{j=1}^{n} \left(BMP_{ij,x}\right)\left(BL_{j,x}\right) \tag{3}$$

  That is, use a value-weighted bond proportion as the bond proportion of a representative Model Portfolio. The representative model portfolios are used as the model portfolios in all subsequent sections. In other words, the model portfolios used in following sections are these representative model portfolios.

2 Labor Income and Savings

2.1 Labor Income Process

Denote LI_t labor income at time t, where, $t = x, ..., T$. Assume the labor income grows with the long-term inflation rate, i.e.,

$$LI_{t+1} = LI_t\left(1 + Inf\right) \tag{4}$$

Then, we have

$$LI_t = LI_x\left(1 + Inf\right)^{t-x}. \tag{5}$$

2.2 Saving Process

- SR_t - saving rate with respect to labor income at time t.

2.2.1 Employer Matching Sequence (Order) (Jim Delay)

See Appendix 1 for details on accounts.

1. Contribute first to accounts with matching; favor accounts with higher match percentages

2. After all matching has been fulfilled, favor Pre-Tax/Roth, then Post-Tax deferred, then Taxable

3. Within Item 2. above, favor accounts that are advisable vs. those that are not.

2.2.2 Saving Sequence (Order)

The following order is used as saving order to distribute savings to all accounts in the consideration sequentially.

1. Employer Matching Sequence
2. Default Saving Sequence

   If there is more savings that needs to be allocated to accounts, follow the default saving sequence that is provided in Appendix 1.

2.2.3 Saving Limits on Accounts

Suppose that an investor have N accounts.

- $SL_{n,t}$ - Saving limit scheme for account n at time t where $t = x...T$ and $n = 1, ..., N$. These limits are determined by *SAVING SEQUENCE*.
- $SL_{n,t}$, $n = 1, ..., N$, are determined through following process

1. (a) Distribute savings to the first account in the saving sequence up to the limit that is the smallest one among the account's contribution limit (402g), plan limit, and 415 limit to get maximum feasible saving amount.

   (b) Reduce corresponding limits based on the saving amount indicated in (a).

   (c) Distribute savings to next account in the saving sequence and repeat (a), (b), and (c) until all accounts are filled.

   All $SL_{n,t}$ will be obtained through the above 3 steps.

2.2.4 Overall Saving Rate

- $MINSR_{t,n}$ - Pre-determined *MINIMUM SAVING RATE* scheme for account n at time t where $t = x...T$ and $n = 1, ..., N$.

- $MAXSR_{t,n}$ - Pre-determined *MAXIMUM SAVING RATE* scheme for account n at time t where $t = x...T$ and $n = 1, ..., N$. We have,

$$MINSR_{t,n} \leq MAXSR_{t,n} \tag{6}$$

- $FMINSR_{t,n}$ - *Feasible MINIMUM SAVING RATE* scheme for account n at time t where $t = x...T$ and $n = 1, ..., N$. It is determined by,

$$FMINSR_{t,n} = \min\{MINSR_{t,n}, SL_{n,t}\} \tag{7}$$

- $FMAXSR_{t,n}$ - *Feasible MAXIMUM SAVING RATE* scheme for account n at time t where $t = x...T$ and $n = 1, ..., N$. It is determined by,

$$FMAXSR_{t,n} = \min\{MAXSR_{t,n}, SL_{n,t}\} \tag{8}$$

Let

$$FMINSR_t = \sum_{n=1}^{N} FMINSR_{n,t} \tag{9}$$

the sum of the feasible minimum saving rates across accounts at time t, and

$$FMAXSR_t = \sum_{n=1}^{N} FMAXSR_{n,t} \tag{10}$$

the sum of the feasible maximum saving rates across accounts at time t. The overall saving rate at time t where $t = x...T$ is determined by:

$$SR_t = \min\{\max\{SR_x, FMINSR_t\}, FMAXSR_t\} \tag{11}$$

.

- Note:
  - If an investor has a pre-determined *MINIMUM SAVING AMOUNT* scheme $MINSA_{n,t}$ (or *MAXIMUM SAVING AMOUNT* scheme $MAXSA_{n,t}$) for account n at time t where $t = x...T$ and $n = 1, ..., N$, we translate it to *MINIMUM SAVING RATE* scheme $MINSR_{n,t}$ (or *MAXIMUM SAVING AMOUNT* scheme $MAXSA_{n,t}$) by,

$$MINSR_{n,t} = \frac{MINSA_{n,t}}{LI_t}. \tag{12}$$

or

$$MAXSR_{n,t} = \frac{MAXSA_{n,t}}{LI_t}. \tag{13}$$

This translated scheme is unchanged in forecast.

- The frontloading is a particular case of the predetermined *MINIMUM SAVING RATE* scheme.
- The lock-in feature, e.g., account n is locked, can be implemented by setting

$$MINSR_{t,n} = MAXSR_{t,n} = 0. \tag{14}$$

- SMART advice can be implemented by setting appropriate $FMINSR_t$ and $FMAXSR_t$.

- Denote SV_t overall savings at time t, $t = x...T$. And then,

$$SV_t = (SR_t)(LI_t) \tag{15}$$

2.3 Saving Strategy (Default, User can override saving sequence)

Assume that an investor is at x and plans to retire at age T. $t = x, ..., T$. The overall saving rate follows the saving process discussed before. SV_t, overall savings, is distributed to accounts at time t, $t = x...T$ through *SAVING SEQUENCE* sequentially through the following process.

1. Employer Matching Sequence

   (a) Distribute savings to the first account in the saving sequence up to the limit that is the smaller one between the account's saving limit and the limit that has a corresponding employer matching amount less than or equal to the employer matching limit.

   (b) Reduce the corresponding limits (employer matching limit, plan limit, contribution limit (402g) and 415 limit) based on the saving amount indicated in (a). Note that employer matching limit is not reduced by the savings and is reduced according to employer matching rules.

   (c) Distribute savings to next account in the saving sequence and repeat (a), (b), and (c) until all accounts that are with employer matching are filled.

2. Default Saving Sequence

   If there is more savings that needs to be allocated to accounts, follow the default saving sequence in Appendix 1.

3 Human Capital at time t

Human Capital (HC) is defined as present value of job related future "cash inflows" weighted by mortality.

Following items are taken into account in HC calculation.

- Savings from labor income (before retirement)
- Social Security Payments (before or after retirement, inflation adjusted)
- Pension Payments (before or after retirement, inflation adjusted)
- Pension Payments (before or after retirement, not-inflation adjusted)
- Employer Matching (before retirement)

3.1 Human Capital Calculation - Preliminary

1. Savings from labor income

   SV_t - the saving amount at time t; $t = x, ..., T$.

   $$SV_t = (SR_t)(LI_x)(1 + Inf)^{t-x} \tag{16}$$

   Note

   $$SR_t = \min\{\max\{SR_x, FMINSR_t\}, FMAXSR_t\} \tag{17}$$

   where $FMINSR_t, FMAXSR_t$, LI_x and Inf are known. SV_t is independent of the simulation runs.

2. Social Security Payments (before or after retirement, inflation adjusted)

   SP_t - Social Security Payment at time t, $t = x, ..., m$. Where m is the maximum age in a related mortality table.

The labor income is calculated as follows.

- When $i = x + 1, ..., t$, LI_i, is calculated by,

$$LI_i = LI_x \left(1 + Inf\right)^{i-x}. \tag{18}$$

- When $i < x$, LI_i, use historical labor income, or a "back forecast" labor income using historical inflation rate.

- Apply the Social Security Calculator to get SP_t.

  Note that let y be the year starting the social security payment, and then,

$$if \;\; t < y, \;\; SP_t = 0 \;\; and \;\; if \;\; t \geq y, \;\; SP_t = SP_t. \tag{19}$$

3. Pension Payments (before or after retirement, inflation adjusted)

   $PPIA_t$ - inflation-adjusted pension payment at time t. Users' inputs.

4. Pension Payments (before or after retirement, not inflation adjusted)

   $PPNIA_t$ - not inflation-adjusted pension payment at time t. Users' inputs.

5. EM_t - Employer Matching (before retirement)

3.2 Human Capital Definition at Time t

3.2.1 Single Case

- At any time t (corresponding to an age), HC_t is calculated (defined) by:

$$HC_t = \sum_{s=1}^{m-t} \frac{Y_{t+s}\left({}_sP_t\right)}{\left(1 + Inf\right)^s \left(1 + r_{HC}\right)^s} \tag{20}$$

  Where,

$$Y_{t+s} = SV_{t+s} + SP_t \left(1 + Inf\right)^s + PPIA_t \left(1 + Inf\right)^s + PPNIA_{t+s} + E_t \left[EM_{t+s}\right] \tag{21}$$

  and Y_{t+s} is the value at time $t + s$ with $t = x, ..., T$ and $s = 0, ..., m - t$.

- Notes on Human Capital calculation

- For a given age t, we need to calculate Y_{t+s} for $s = 0, ..., m - t$, i.e., Y_t, Y_{t+1}, ..., Y_m
- For each Y_{t+s} for $s = 0, ..., m - t$, we need to calculate its components, i.e., SV_{t+s}, $SP_t (1 + Inf)^s$, $PPIA_t (1 + Inf)^s$, $PPNIA_{t+s}$, and $E_t [EM_{t+s}]$.

3.2.2 Joint Case

- Denote $G1$ and $G2$ are two people in a joint case.
- Denote $SP_{t,G1}$ and $SP_{t,G2}$ social securities they receive respectively.
- Denote t_{G1} as the eligible year for $G1$ starting to receive Social Security payments and t_{G2} as the eligible year for $G2$ starting to receive Social Security payments. Not to loss of generality, we assume $t_{G1} \leq t_{G2}$. i.e. $G1$ receives Social Security payments before $G2$.

At any time t, HC_t is calculated (defined) as follows.

$$HC_t = \sum_{s=1}^{m-t} \left[\frac{Y_{t+s,1}}{(1 + Inf)^s (1 + r_{HC})^s} + \frac{Y_{t+s,2}}{(1 + r_{HC})^s} \right] \tag{22}$$

Where,

$$Y_{t+s,1} = \{SV_{t+s} + PPIA_t (1 + Inf)^s + PPNIA_{t+s} + E_t [EM_{t+s}]\} [({}_sP_{t,G1}) + ({}_sP_{t,G2})] \tag{23}$$

and

- 1. If $t \geq t_{G2}$ (both receive Social Security payments), then

$$\begin{aligned} Y_{t+s,2} &= (SP_{t,G1} + SP_{t,G2}) ({}_sP_{t,G1}) ({}_sP_{t,G2}) \\ &\quad + \max (SP_{t,G1}, SP_{t,G2}) (1 -_s P_{t,G1}) ({}_sP_{t,G2}) + \max (SP_{t,G1}, SP_{t,G2}) ({}_sP_{t,G1}) (1 -_s P_{t,G2}) \end{aligned} \tag{24}$$

  2. If $t_{G1} \leq t < t_{G2}$ (only $G1$ receives Social Security payments), then

$$Y_{t+s,2} = (SP_{t,G1}) ({}_sP_{t,G1}) \tag{25}$$

  3. If $t < t_{G1}$ (no one receives Social Security payments), then

$$Y_{t+s,2} = 0 \tag{26}$$

and $Y_{t+s,1}$ (for items except for social security payments) and $Y_{t+s,2}$ (for social security payment only) are the values at time $t+s$ with $t = x, ..., T$ and $s = 0, ..., m-t$.

- Notes on joint case
  - We separate the social security calculation from other items in joint case.

3.3 Employer Match in Human Capital Calculation

Note that in both Equations (20) and (21), SV_t, $PPIA_t$, $PPNIA_t$, and SP_t do NOT depend on the simulation runs. SP_t (Social Security Payment) needs to be evaluated at each age due to salary update.

The key is to evaluate employ matching amounts in the simulation runs since the employer match relates to contribution limits that are set by IRS. According to current rules, the contribution limits are changing to reflect inflation rates. As a consequence, the contribution limit at time t should be calculated using inflation rates in a particular run and the contribution limits at time above t should be calculated using the long-term inflation rate, e.g., Inf. In other words, Human capital will depend on the simulation runs through contribution limits.

3.3.1 Evaluating Employer Matching Amount in Human Capital Calculation

1. Use the long-term forecast inflation in Human Capital calculation. This assumption makes the calculation of employer matching amounts independent of the simulation runs. The assumption is mainly based on the concern on WFE application performance and the stability of Human capital estimation, e.g., the human capital should not be too sensitive to the changes of contribution limits.

2. At age t, $t = x+1, ..., T$, use *SAVING SEQUENCE* to determine overall employer matching amount., e.g., EM_t.

3. Equation (21) in Human Capital calculation becomes,

$$Y_{t+s} = SV_{t+s} + SP_t\,(1+Inf)^s + PPIA_t\,(1+Inf)^s + PPNIA_{t+s} + EM_{t+s} \tag{27}$$

and(23) becomes,

$$Y_{t+s,1} = \{SV_{t+s} + PPIA_t\,(1+Inf)^s + PPNIA_{t+s} + EM_{t+s}\}\,[({}_sP_{t,G1}) + ({}_sP_{t,G2})] \tag{28}$$

As a result, Human Capital is independent of the runs.

4 Determine Model Portfolios at Time t

The asset location rule states that the most tax inefficient investment should be located in tax-advantage accounts and the most tax efficient investment should be located in no-tax-advantage accounts. As an application, in WFE, Model portfolio 1 tends to be located in tax-advantage accounts and Model portfolio 7 tents to be located in no-tax-advantage accounts.

4.1 Tax-Advantage and No-Tax-Advantage Account Categories

All accounts are classified into two categories: with tax advantage or without tax advantage

- Accounts with tax advantage
  - Pre-tax money and tax deferred
  - After-tax money and earnings deferred
  - After-tax money and tax -free
  - Some specific tax-advantage accounts
- Accounts without tax advantage
  - Taxable account
- All accounts in one category have the same investment option, i.e., same model portfolio.

See Appendix 1 for details on account categories.

4.2 Notation

- HC_t - Human Capital time t
- AA_{HC} - Asset allocation for Human Capital
- OAA_x - Overall asset allocation at age x (Market Portfolio, Currently, it is independents of age x)
- $MITR_t$ - Marginal income tax rate at time t
- LCT - Long term capital gain tax rate
- W_t - Combined advisable wealth from all accounts at time t in pre-tax basis
- $W_{1,t}$ - Combined advisable wealth from all accounts in the *tax-advantage category* at time t in pre-tax basis. Use the $MITR_t$ to convert after-tax money into pre-tax money.
- $W_{2,t}$ - Combined advisable wealth from all accounts in the *no-tax-advantage category* at time t in pre-tax basis. Use $ATR = \frac{MITR_t + LCT}{2}$ to convert after-tax money into pre-tax money.
- $W_{out,t}$ - Outside asset
- $AA_{out,t}$ - Outside asset allocation at time t
- $AA_{adv,t}$ - Asset allocation for W_t. It is determined by HC method.
- $MP_{1,t}$ - Model portfolio for $W_{1,t}$. It is one of 7 model portfolios.
- $MP_{2,t}$ - Model portfolio for $W_{2,t}$. It is one of 7 model portfolios.

We have

$$W_t = W_{1,t} + W_{2,t} \tag{29}$$

4.3 Model Portfolio for W_t

Human Capital Method

$$AA_{adv,t} = f_{MP}\left(HC_t,\ W_t,\ OAA_x,\ AA_{HC},\ W_{out,t},\ AA_{out,t}\right) \tag{30}$$

where, $t = x, ..., T-1$, and $f_{MP}(\cdot)$ is a general function relationship that determines the model portfolio for W_t at time t. The $f_{MP}(\cdot)$ is determined as follows.

4.3.1 Notation

- z - Bond proportion of W_t, the unknown variable
- $BOAA_x$ - Bond proportion in OAA_x, known
- $BAA_{out,t}$ - Bond proportion in $AA_{out,t}$, known
- BAA_{HC} - Bond proportion in AA_{HC}, known

4.3.2 Solve for z

Since

$$BOAA_x\left(HC_t + W_t + W_{out,t}\right) = BAA_{HC}\left(HC_t\right) + zW_t + BAA_{out,t}\left(W_{out,t}\right) \tag{31}$$

we have

$$z = \frac{BOAA_x\left(HC_t + W_t + W_{out,t}\right) - BAA_{HC}\left(HC_t\right) - BAA_{out,t}\left(W_{out,t}\right)}{W_t} \tag{32}$$

where if

$$BOAA_x\left(HC_t + W_t + W_{out,t}\right) - BAA_{HC}\left(HC_t\right) - BAA_{out,t}\left(W_{out,t}\right) \leq 0, \tag{33}$$

then

$$z = 0; \tag{34}$$

and if

$$z = \frac{BOAA_x\left(HC_t + W_t + W_{out,t}\right) - BAA_{HC}\left(HC_t\right) - BAA_{out,t}\left(W_{out,t}\right)}{W_t} > 1, \tag{35}$$

then

$$z = 100\%. \tag{36}$$

Note:

1. Do not map z to one of 7 model portfolios at this stage.
2. When $W_t/W_{out,t}$ is small, z is very sensitive to changes on the value of Human Capital and OAA_x. We may need an algorithm to stable z after testing real cases.

4.4 Model Portfolios for $MP_{1,t}$ and $MP_{2,t}$

4.4.1 Notation

- MP_i - model portfolio i, $i = 1, ..., 7$.
- BMP_i - bond proportion in model portfolio i, $i = 1, ..., 7$.
- $BAA_{adv,t}$ - bond proportion in $AA_{adv,t}$. It is equal to z in Equation (32).
- $BMP_{1,t}$ - bond proportion in $MP_{1,t}$.
- $BMP_{2,t}$ - bond proportion in $MP_{2,t}$.

4.4.2 Conditions

Let $MP^*_{1,t}$ and $MP^*_{2,t}$ the solutions for $MP_{1,t}$ and $MP_{1,t}$ respectively. Denote $BMP^*_{1,t}$ and $BMP^*_{2,t}$ are bond proportions in $MP^*_{1,t}$ and $MP^*_{2,t}$ respectively. $MP^*_{1,t}$ and $MP^*_{2,t}$ satisfy following conditions.

1. The first condition is:

$$\left|\left(BAA_{adv,t}\right) W_t - \left[\left(BMP^*_{1,t}\right) W_{1,t} + \left(BMP^*_{2,t}\right) W_{2,t}\right]\right| \leq Threshold \tag{37}$$

   where, $Threshold$ is a threshold that will be defined in next section. The condition says that the bond proportion difference between the combined portfolio and $AA_{adv,t}$, is less than or equal to a predetermined threshold.

2. The second condition is:

$$BMP_{1,t}^{*} \geq BMP_{2,t}^{*} \tag{38}$$

That is, the bond proportion in a model portfolio for tax-advantage assets is larger than or equal to that in a model portfolio for no-tax-advantage assets.

3. Denote Ψ a set that includes all possible combinations of 7 model portfolios, $\{MP_{1,t}, MP_{2,t}\}$, that satisfy condition 1 and 2. The final condition is that $MP_{1,t}^{*}$ and $MP_{2,t}^{*}$ satisfy:

$$\left|BMP_{1,t}^{*} - BMP_{2,t}^{*}\right| = \max_{\{MP_{1,t},\, MP_{2,t}\} \in \Psi} \{|BMP_{1,t} - BMP_{2,t}|\} \tag{39}$$

4.4.3 Simple Algorithm for Finding $MP_{1,t}^{*}$ and $MP_{2,t}^{*}$

- Step 1: The threshold is defined by:

$$Threshold = \max\left\{\min_{k=1,\ldots,7} \{|BAA_{adv,t} - BMP_k|\},\ 2.5\%\right\} \tag{40}$$

where, BMP_k, $k = 1, ..., 7$, are bond proportions of 7 model portfolios. Note that 2.5% is a subjective number that measures trade-off between having asset location and having the closest allocation to the target allocation (the allocation of advisable assets).

- Step 2: Find all possible combinations of 7 model portfolios. e.g.,

$$(MP_1, MP_1), ..., (MP_1, MP_7),$$

$$(MP_2, MP_1), ..., (MP_2, MP_7),$$

...,

$$(MP_7, MP_1), ..., (MP_7, MP_7).$$

- Step 3: Eliminate the combinations that do not meet condition 1 and 2, e.g., find $\dot{\Psi}$.

- Step 4: In $\dot{\Psi}$, find the combination that has the biggest difference between bond proportions of 2 model portfolios in the combination, e.g., condition (39).

5 Profit Sharing Process

5.1 Overall Profit Sharing Modeling

Profit sharing depends on the performance of company businesses. Since we are not going to model the performance of individual companies, we will model overall profit sharing as a percentage of labor income.

Let

$$PSR_x = \frac{PS_x}{LI_x}, \tag{41}$$

where, PS_x is profit sharing amount at age x. PSR_x is employer match to labor income ratio at time x. Then the overall profit sharing, PS_t, at time t is modeled as

$$PS_t = (PSR_x)\, LI_t, \tag{42}$$

where $t = x + 1, ...T$.

Note that PSR_x may be supplied by users directly.

6 Wealth Process (Yearly Sequence)

6.1 Determine Wealth at Time t

$$W_{t+1} = f_2\left(SR_x, W_t, MP_t, R_{t+1}, Other\ Factors\right) \tag{43}$$

where, $t = x, ..., T - 1$, and $f_2(\cdot)$ is a general function relationship that determines the wealth at time t from the inputs $SR_x, W_t, MP_t, R_{t+1}, Other\ Factors$.

6.2 Determine Income at Retirement

Wealth Income Conversion Method

$$RI_T = f_3\left(Wealth_T\right) \tag{44}$$

$f_3(\cdot)$ is a general function relationship that determines the income at time T.

7 Advice Problem

We solve for the saving rate SR_x and the retirement age T in the equation below,

$$\Pr(RI_T \geq TRI) = \alpha \tag{45}$$

subject:

$$\text{All Process Discussed} \tag{46}$$

$$HC_t = \sum_{k=1}^{m-t} \frac{Y_{t+k}\left({}_kP_t\right)}{(1+Inf)^k\,(1+r_{HC})^k} \tag{47}$$

$$MP_t = f_2\left(HC_t, Wealth_t\right) \tag{48}$$

$$W_{t+1} = f_1\left(SR_x, W_t, MP_t, R_{t+1}, Other\ Factors\right) \tag{49}$$

for $t = x, ..., T-1$, and

$$RI_T = f_3\left(Wealth_T\right) \tag{50}$$

$$SR_x \leq MSRL \tag{51}$$

and

$$T \leq MRAL \tag{52}$$

where, $MSRL$ is the maximum saving limit allowed (business rule), and $MRAL$ is the maximum retirement age allowed (business rule).

8 Advice Methods

8.1 Lifetime Income Converter

Objective: Convert a wealth to a lifetime payment stream through an fixed payout annuity contract.

8.1.1 Variables

- n - an account, $n = 1, ..., N$.
- $W_{n,0}$ - Wealth at time 0, i.e., at the beginning of year 1, in account n.
- Type A cash flows

- Type A cash flows are cash flows that are contingent on mortality, i.e., the cash flows only have starting dates and NO ending dates. The present value of a Type A cash flow is obtained by using mortality-weighted present value calculation formula.
- $ab_{n,t}$ - cash flows (in or out flows) for account n at the beginning of year t and measured in post-tax.
- $ae_{n,t}$ - cash flows (in or out flows) for account n at the end of year t and measured in post-tax.

- Type B cash flows
  - Type B cash flows are cash flows that are NOT contingent on mortality, i.e., the cash flows have both starting dates and ending dates. The present value of a Type B cash flow is obtained by using the general present value calculation formula.
  - $bb_{n,t}$ - cash flows (in or out flows) for account n at the beginning of year t and measured in post-tax.
  - $be_{n,t}$ - cash flows (in or out flows) for account n at the end of year t and measured in post-tax.

Note: see the document "RIS - Annuitization Level and SS DB Existing Immediate Annuity Adjustment Updated August 4 2006.doc" for how to calculate the present values of Type A and Type B cash flows.

8.1.2 Calculation Steps

- Convert $W_{n,0}$ to post-tax amount $W^*_{n,0}$ by assuming that account n is liquidated and pays all taxes, $n = 1, ..., N$.
- Get present values of $ab_{n,t}$, $ae_{n,t}$, $bb_{n,t}$, and $be_{n,t}$. Denote these values as $PVab_{n,0}$, $PVae_{n,0}$, $PVbb_{n,0}$, and $PVbe_{n,0}$ respectively.
- Get total wealth, denoted as W_0, that is available for annuitization by,

$$W_0 = \sum_{n=1}^{N} \left(W^*_{n,0} + PVab_{n,0} + PVae_{n,0} + PVbb_{n,0} + PVbe_{n,0} \right) \tag{53}$$

- Apply a standard inflation-adjusted immediate fixed payout annuity formula to W_0 to get the income. The formula is as follows.

$$Income = \frac{W_0}{\sum_{t=0}^{m-t} \left(\frac{1+g}{1+r}\right)^t ({}_tp_x)} \tag{54}$$

Where,

1. x is retirement age, i.e. $t = 0$.
2. g is the long-term inflation rate and r is the long term government yield. Both are supplied by Research Department in Ibbotson/Morningstar.
3. m is the maximum age in a related mortality table.
4. ${}_tp_x$ is the survival probability that one will live t years conditional survival at age x.

8.2 Spend-Down Income Estimator

Objective: Convert a wealth to an income assuming a systematic withdrawal over a pre-determined period and the returns on related asset classes are known during the period.

8.2.1 Variables

- T - Investment horizon and measured in years
- n - an account, $n = 1, ..., N$.
- $W_{n,0}$ - Wealth at year 0, i.e., at the *beginning* of year 1, in account n.
- $W_{B,T}$ - Wealth amount (post-tax) left at year T.
- $R_{n,t}$ - Total return on account n during time period t. Note that when asset allocation for each account is determined, the return of the account is determined too because the returns of underlying asset classes are known. It may or may not be deducted by tax at year t depending on the account (i.e., such as pre-tax 401(k) and a taxable account) and its rules on how tax should be paid according to Yearly Sequence document.
- $TH_{n,t}$ - tax withholding payment for account n at year t. It is generated at year $t-1$, but pays at the *beginning* of year t.

- $c_{n,t}$ - cash flows (in or out flows) for account n at the *beginning* of year t and measured in post-tax.

- $d_{n,t}$ - cash flows (in or out flows) for account n at the *ending* of year t and measured in post-tax.

- $W_{n,t}$ - Wealth in account n at the end of year t.

- X - the overall withdrawal amount measured in post-tax and happens at the *beginning* of a year.

8.2.2 Withdrawal Sequence (James Daley)

The withdrawal starts from the accounts in Group 1, then Group 2, and so on until required withdrawal amount is fulfilled. The accounts in the same group follow the order indicated by a, b, and c below.

1. Group 1
   (a) Taxable (except for Muni Bonds)
   (b) Municipal Bonds Held in Taxable Accounts
2. Group 2
   (a) Non-Deductible IRA
   (b) Non-Qualified Tax Deferred
   (c) Deferred Variable Annuity
3. Group 3
   (a) Post-Tax 401(k)
   (b) Post-Tax 403(b)
4. Group 4

    (a) Deductible IRA
    (b) Qualified Tax Deferred

5. Group 5
    (a) Pre-Tax 401(k)
    (b) Pre-Tax 403(b)
    (c) 457

6. Group 6
    (a) Roth IRA

7. Group 7
    (a) Roth 401(k)
    (b) Roth 403(b)

8.2.3 Rebalance

- At any year t, the overall wealth portfolio is rebalanced back to the asset allocation at year 0, or any pre-determined asset allocation.
- The timing of rebalancing follows the rules in Yearly Sequence document.
- Model portfolios for each account at t is determined by the same way in Section "Determine Model Portfolios at Time t".

8.2.4 The Problem

- Wealth dynamics associated with account n is given by,

$$W_{n,t} = \left[W_{n,t-1} - x_{n,t} - c_{n,t} - TH_{n,t}\right] R_{n,t} + d_{n,t} \tag{55}$$

and

$$TH_{n,t} = \tau_{n,t-1} \left(1 - BP_{n,t-1}\right) x_{n,t-1} \tag{56}$$

subject to

$$\sum_{n=1}^{N} x_{n,t} = X \tag{57}$$

where $t = 1, ..., T$ and $n = 1, ..., N$ and,

- $x_{n,t}$ is withdrawal amount from account n at year t (assume that the withdrawal happens at the beginning of a year) and determined by the withdrawal sequence.
- $TH_{n,1} = 0$, i.e., there is no withholding tax that needs to be paid at the first year.
- $\tau_{n,t}$ is the tax rate
- $BP_{n,t}$ is the proportion of the basis in account n at year t. Note, for a pre-tax 401(k), $\tau_{n,t}$ is the income tax rate and $BP_{n,t} = 0$.

- Find X such that

$$\sum_{n=1}^{N} W^*_{n,T} = W_{B,T} \tag{58}$$

where $W^*_{n,T}$ is post-tax wealth corresponding to $W_{n,T}$ in account n, $n = 1, ..., N$.

8.2.5 Solution and Consideration

A binary search plus some modification may be sufficient for solving the problem. Specifically, since income tax rate is linked to the withdrawal amount, and the capital gain tax rates are linked to income tax rate, an increase in withdrawal amount may results in a jump in capital gain tax rate, which results in discontinuous in terminal wealth.

8.3 SMART

8.3.1 Variables

- SR_S - Planned Starting Savings Rate. It is defaulted as the participant's current savings rate
- SR_E - Desired Ending Savings Rate (how much the participant is willing to "save up to" in the future)
- Δ - Planned Increase Period measured in years (how often the participant is willing to increase the savings rate)

- Δ_{SR} - Planned Increase Saving Rate measured in 1% increment (how much the participant is willing to increase the savings rate at each Increase Period)

- T - Retirement Age

- k_T - a positive integer for a given retirement age T.

8.3.2 SMART Advice Problem

We solve for the retirement age T (Not saving rate) and a minimum k_T in the equation below,

$$\Pr(RI_T \geq TRI) = \alpha \tag{59}$$

subject:

$$\text{All Process Discussed} \tag{60}$$

$$HC_t = \sum_{k=1}^{m-t} \frac{Y_{t+k}\left({}_kP_t\right)}{(1+Inf)^k\,(1+r_{HC})^k} \tag{61}$$

$$MP_t = f_2\left(HC_t, Wealth_t\right) \tag{62}$$

$$W_{t+1} = f_1\left(SR_x, W_t, MP_t, R_{t+1}, Other\ Factors\right) \tag{63}$$

for $t = x, ..., T-1$, and

$$RI_T = f_3\left(Wealth_T\right) \tag{64}$$

$$SR_x \leq MSRL \tag{65}$$

and

$$T \leq MRAL \tag{66}$$

where, $MSRL$ is the maximum saving limit allowed (the business rule), and $MRAL$ is the maximum retirement age allowed (the business rule).

(Constraints for SMART)

If $t < x + k_T$ then

$$SR_{t+\Delta} = SR_t + \Delta_{SR} \tag{67}$$

and if $t \geq x + k_T$ then

$$SR_{t+\Delta} = SR_t \tag{68}$$

In SMART,

- In the saving process section, we have defined $FMINSR_{t,n}$ - *Feasible MINIMUM SAVING RATE* scheme for the account n at time t where $t = x...T$ and $n = 1, ..., N$. It is determined by,

$$FMINSR_{t,n} = \min\{MINSR_{t,n}, SL_{n,t}\} \tag{69}$$

  Let

$$FMINSR_t = \sum_{n=1}^{N} FMINSR_{n,t}$$

  If

$$SR_t < FMINSR_t \tag{70}$$

  then

$$SR_t = SR_t \tag{71}$$

  e.g., we use SR_t and ignore $FMINSR_t$ requirement. In this case, the sum of feasible minimum saving rates across all accounts is larger than the saving rate specified in the SMART saving plan. As a consequence, not all feasible minimum saving rates across the accounts can be implemented. In this case, the accounts are filled by savings according to their priorities to receive savings. In other words, the account with the lowest priority does not receive savings. If

$$SR_t \geq FMINSR_t \tag{72}$$

  then all feasible minimum saving rates across accounts can be implemented. In this case, the accounts are filled by savings according to their priorities to receive savings.

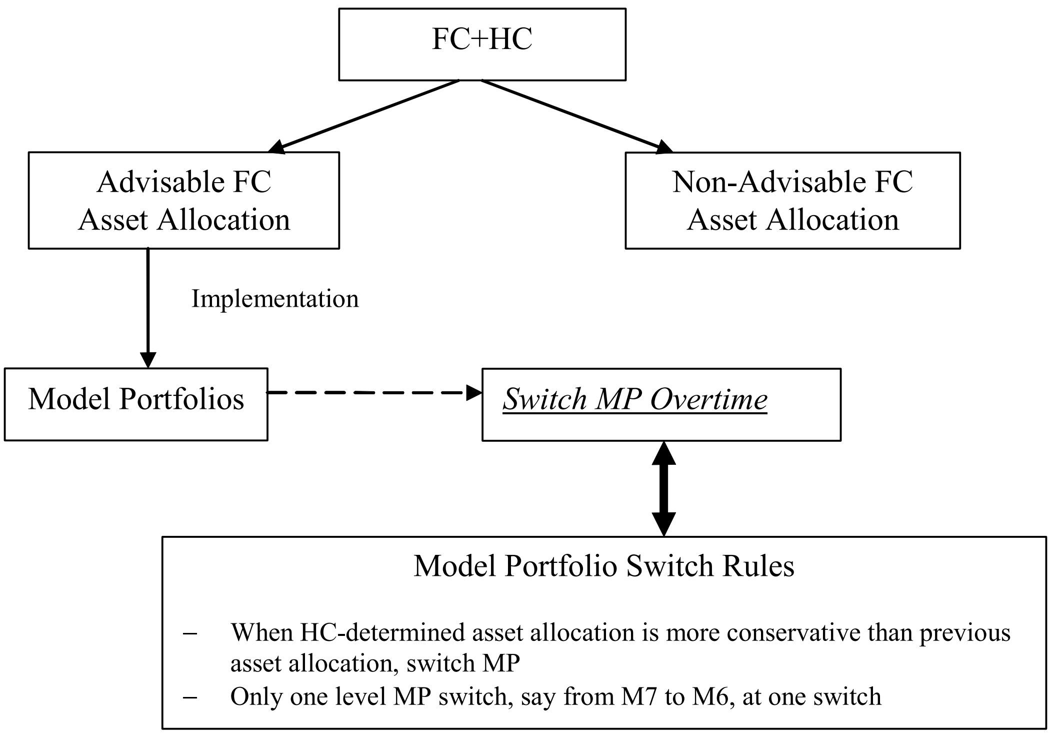
- *SAVING SEQUENCE* Rule is applied.

8.4 Portfolio Switch Update in WFE (November 15, 2006)

Incorporating the asset location method into WFE brings the issue on the existing portfolio switching algorithm in WFE. The existing switching algorithm applies to model portfolios, which now is hard to be implemented when we use 2 model portfolios to implement tax-advantage and no-tax-advantage assets. The proposed algorithm applies to asset allocation and NOT model portfolios, which is reasonable because investment strategies and implementing investment strategies should be separated. The proposed algorithm switches investment strategies.

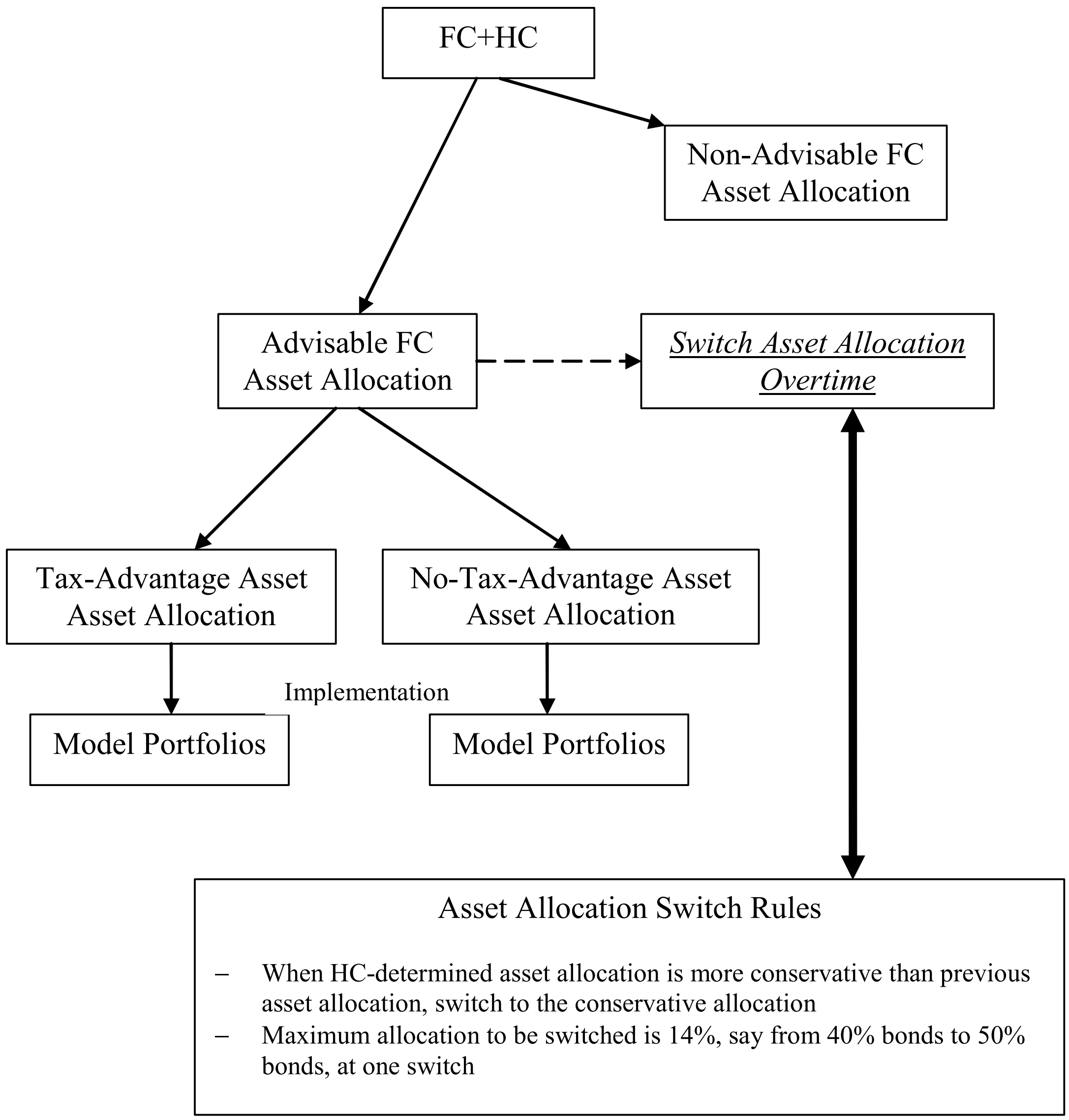
8.4.1 Existing Algorithm

Existing Portfolio Switch Process



8.4.2 New Algorithm

New Portfolio Switch Process



8.4.3 Note

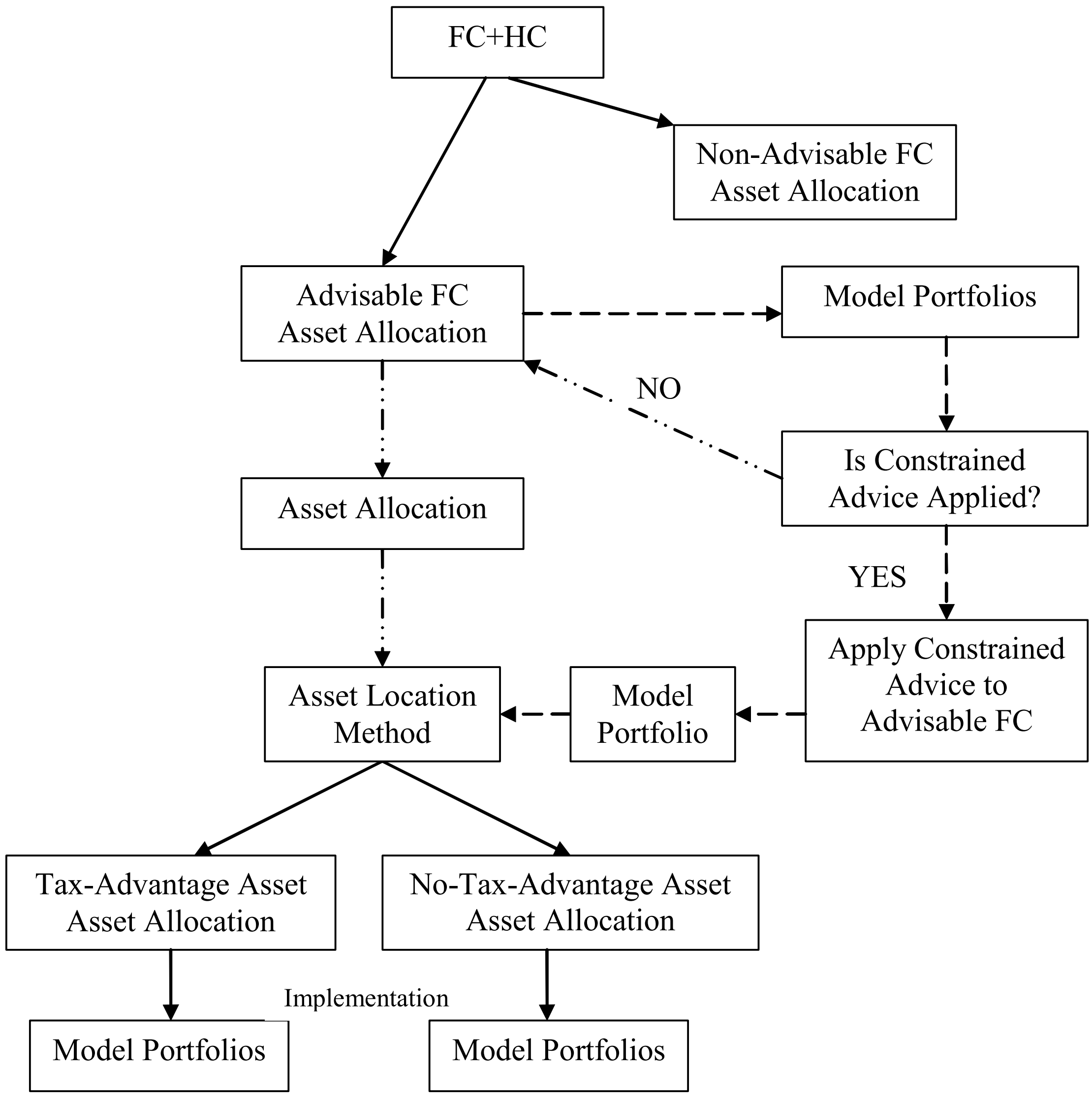
The frequency of asset allocation switch should be flexible, e.g., users can set asset allocation switch frequency.

8.5 Constrained Model Portfolio Advice Update in WFE (November 16, 2006)

1. There is no change on the existing constraint model portfolio advice method.

2. The flow to implement the method in WFE 4.0 is provided in the graph below. There are 2 paths: a) if the constrained model portfolio advice is not applied, go back to the regular advice process; b) if the constrained model portfolio advice is applied, use the method to find the appropriate model portfolio, and then use the location method to implement this portfolio for both tax-advantaged and no-tax-advantaged assets.

3. Incorporate the original constrained model portfolio advice (WFE 3.94) into WFE 4.0



9 Appendix 1: Accounts and Default Saving Sequence in WFE 4.0 (James Daley)

9.1 Account Types and Account Classification

- Pre-Tax Contributions and Tax-Deferred Earnings:
  - With Matching
    - Pre-Tax 401(k)

    * Pre-Tax 403(b)
    * 457
  - Without Matching
    * Deductible IRA
    * Qualified Tax Deferred
    * Deferred Compensation Fixed
    * Deferred Compensation Variable
- Post-Tax Contributions and Tax-Deferred Earnings:
  - With Matching
    * Post-Tax 401(k)
    * Post-Tax 403(b)
  - Without Matching
    * Non-Deductible IRA
    * Non-Qualified Tax Deferred
    * Deferred Variable Annuity
- After-Tax Contributions and Tax-Free Earnings:
  - With Matching
    * Roth 401(k) **
    * Roth 403(b) **
  - Without Matching
    * Roth IRA **
    * Municipal Bonds Held in Taxable Accounts
- Non Tax-Advantaged
  - Without Matching

∗ Taxable (except for Muni Bonds)

** Please note that certain rules have to be met by Roth 401(k), Roth 403(b), and Roth IRA in order for the contributions and earnings to get tax-free treatment.

9.2 Default Saving Sequence

9.2.1 A 4-level sort according to the following hierarchical rules:

Sort 1: Tier-1 Match Percentage

Sort 2: Tax Grouping

Sort 3: Account Status - Advised vs. Considered

Sort 4: Order of Accounts Passed to IWFE

Note: "Considered" means that accounts are for saving rate advice but not for portfolio advice.

9.2.2 Details

1. Sort 1 looks at each account's match rules as passed in by a User or pulled from a RK system.

2. Sort 2 Define the Tax Grouping Order as follows

Group	Contributions	Earnings	Taxation of Account	Examples
Group A:	Pre-Tax	Pre-Tax	Contributions and Earnings taxed when withdrawn	Pre-Tax 401(k),Deductible IRA
Group B:	Post-Tax	Post-Tax	Contributions and Earningsare tax-free	Roth 401(k), Roth IRA
Group C:	Post-Tax	Pre-Tax	Earnings taxed when withdrawn	Post-Tax 401(k), Non-Deductible IRA
Group D:	Post-Tax	Post-Tax	Earnings and Realized Capital Gains taxed in current year. Withdrawals may trigger additional Realized Capital Gains.	Taxable

3. Sort 3 Define the Account Status as Follows:

   Account Status A: Asset Class Model Portfolio will be Recommended for the Account

   Account Status C: Asset Class Model Portfolio will NOT be Recommended for the Account; Existing Account Allocation will be Considered

4. Sort 4 Define the Order Passed as Follows

   This is the physical order that the accounts are passed in the XML input.

10 Appendix: Future Reference

10.1 Optimal Saving Combination across Accounts with Employer Matching at Time t

The follows are only applied to the accounts with employer matching. The resulting saving strategy plus the saving sequence for the accounts that are without matching provides a complete saving strategy for all accounts.

10.1.1 Assumptions, Notations, Shared Local Limit Modeling

- Assumptions
  - There are N accounts that are with employer matching.
  - The minimum saving rate for an account is 0% and the maximum saving rate for a account is 25% measured in pre-tax. (Business rule).
  - Incremental saving rate - 1%. This assumption may result in a solution that is not global optimal. (We can improve this next version.)
  - Each account has 2 limits: one is a local limit (e.g. 402(g) limit) that could be shared with some accounts in the consideration and the other is a global limit (i.e., 415 limit) that is shared with all accounts in the consideration. The assumption implies that the number of the shared limits is less than or equal to the number of accounts.
- Notations
  - AC_n: Account n, $n = 1, ..., N$
  - DCL: A global limit that is shared by all account in the consideration (Total DC Limit, Ibbotson Wealth Forecasting Engine Specifications Document Version 4.0, page 27).

If there is profit sharing, this limit should be reduced accordingly to accommodate the profit sharing. In other words, the profit sharing has the highest priority and has to be added into investors' portfolio first.

- LCL: An overall limit (Limit on compensation limits, Ibbotson Wealth Forecasting Engine Specifications Document Version 4.0, page 28). I do not see that this limit is applied to the employer matching case. I put it here for discussion.
- $x_{i,j,n}$: Employer matching amount for account n at ith 1% saving increment associated with total j percent saving rate
- $SR_{n,t}$: Saving rate for account n at time t and it can take 0% to 25%.
- LI_t: Labor income at time t
- SR_t: Overall saving rate at time t

- Shared Local Limit Modeling
  - Φ_m: A set that consists of the accounts that share the same local limit and $m = 1, ..., M$.
  - SCL_m: The local limit that is shared by the accounts in set Φ_m, $m = 1, ..., M$.
  - Since each account has only one local limit, we have,

    1) $M \leq N$

    2) If $i \neq j$, then

    $$\Phi_i \bigcap \Phi_j = \phi \tag{73}$$

    where ϕ represents the empty set. The condition says that there are no accounts that belong to both set Φ_i and set Φ_j if two sets are different.

    3)

    $$\bigcup_{m=1}^{M} \Phi_m = \Omega \tag{74}$$

    where Ω is the set that consists of all the accounts in the consideration. In this case, Ω consists of all accounts that have employer matching.

10.1.2 Mathematical Description of the Problem

For account n, the possible saving combinations are given in the following matrix,

$$
\begin{array}{cc}
AC_n & : \text{ Possible Saving Combinations} \\
\begin{array}{c} 0\% \\ 1\% \\ 2\% \\ 3\% \\ \vdots \\ 25\% \end{array} & \begin{pmatrix} 0\% & 0\% & 0\% & ... & 0\% \\ 1\% & 0\% & 0\% & ... & 0\% \\ 1\% & 1\% & 0\% & ... & 0\% \\ 1\% & 1\% & 1\% & ... & 0\% \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1\% & 1\% & 1\% & ... & 1\% \end{pmatrix}
\end{array}
$$

where the columns show the 1% incremental in savings and the rows show the total saving rates. The corresponding employer matching amount for each saving combination is given by,

$$
\begin{array}{cc}
AC_n & : \text{ Corresponding Employer Matching Amounts} \\
\begin{array}{c} 0 \\ x_{1,1,n} \\ \sum_{k=1}^{2} x_{k,2,n} \\ \sum_{k=1}^{3} x_{k,3,n} \\ \vdots \\ \sum_{k=1}^{25} x_{k,25,n} \end{array} & \begin{pmatrix} 0 & 0 & 0 & ... & 0 \\ x_{1,1,n} & 0 & 0 & ... & 0 \\ x_{1,2,n} & x_{2,2,n} & 0 & ... & 0 \\ x_{1,3,n} & x_{2,3,n} & x_{3,3,n} & ... & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{1,25,n} & x_{2,25,n} & x_{3,25,n} & ... & x_{25,25,n} \end{pmatrix}
\end{array}
$$

Note that when a particular employer matching rule is specified for an account, we should be able to fill in the above matrix for the account. Employ matching limits and plan limits should be incorporated in this stage. For instance, if an additional 1% incremental savings to a account makes the total matching above the employer matching limit or plan limits on this account, the matching associated with this 1% incremental savings is 0.

For a given overall saving rate SR_t, we look for a saving combination $SR^*_{n,t}$, $n = 1, ..., N$ such that

$$\sum_{n=1}^{N} SR^*_{n,t} = SR_t \tag{75}$$

and the overall employer matching

$$\sum_{n=1}^{N} \left(\sum_{k=1}^{j^*_n} x_{k,j^*_n,n} \right) \tag{76}$$

is maximized. Where j_n^* is the saving rate for account n and $SR_{n,t}^*$ is the saving rate that is corresponding to the employer matching,

$$\sum_{k=1}^{j^*} x_{k,j^*,n} \tag{77}$$

The following constraints are satisfied as well.

1) Saving amount limits for each account

$$LI_t \left(SR_{n,t}^* \right) \leq CL_n \tag{78}$$

where $n = 1, ..., N$. That is, the saving amount can not exceed the contribution limit.

2) Shared local limit constraints

$$LI_t \left(\sum_{AC_k \in \Phi_m} SR_{k,t}^* \right) \leq SCL_m \tag{79}$$

where $m = 1, ..., M$ and $\sum_{AC_k \in \Phi_m} SR_{k,t}^*$ is the sum of the saving rates of all accounts that belong to Φ_m, i.e., the accounts that share the same local limit SCL_m.

3) Total DC Limit (global limit)

$$(LI_t) \sum_{n=1}^{N} SR_{n,t}^* + \sum_{n=1}^{N} \left(\sum_{k=1}^{j^*} x_{k,j^*,n} \right) \leq DCL_t \tag{80}$$

and

4) General Constraints

$$SR_{n,t}^* \geq 0 \tag{81}$$

for $n = 1, ..., N$.

10.1.3 Discussion

If all employer matching schemes in the consideration are tiered with saving amounts (piecewise function of saving amounts), the solution of the problem should provide an optimal saving sequence that can be used to be a is linear function of saving amounts or saving rates, that is,

$$\sum_{n=1}^{N} \left(\sum_{k=1}^{j_n^*} x_{k,j_n^*,n} \right) \tag{82}$$

$$x_{k,j_n^*,n} = (c_{n,t}) \left(SR_{n,t} \right) \tag{83}$$

10.1.4 Algorithm for Solving the Problem

When the number of account is small, an enumeration method may be used to solve the problem.