Gravitational Collapse in a Superfluid Vacuum I: Emergent Spacetime and the Acoustic Horizon

Il Woong Choi[1, *]

[1] *Independent Researcher, Oxford, United Kingdom*

(Dated: December 10, 2025)

The presence of singularities in the Schwarzschild and Kerr solutions of General Relativity indicates a breakdown of the classical geometric description of spacetime at high curvatures. In this work, we propose a regularization mechanism based on the hydrodynamics of a superfluid vacuum. We treat the physical vacuum as a Bose-Einstein Condensate (BEC) governed by the Gross-Pitaevskii equation. We demonstrate that the metric tensor of General Relativity emerges as the effective acoustic metric for phonon fluctuations propagating on this condensate background. Within this framework, the Event Horizon is identified as a sonic horizon where the radial inflow velocity of the vacuum exceeds the local sound speed ($v_r > c_s$). We derive the effective line element for a spherically symmetric inflow and show that it reproduces the Schwarzschild geometry in the hydrodynamic limit, while allowing for distinct ultraviolet corrections that preserve unitarity.

I. INTRODUCTION

Classical General Relativity (GR) describes gravity as the curvature of a pseudo-Riemannian manifold. While successful at macroscopic scales, GR predicts its own demise in the form of singularities—regions where geodesic incompleteness implies the loss of predictive power. Furthermore, the Information Paradox associated with the Hawking evaporation of black holes suggests a fundamental conflict between the equivalence principle and the unitary evolution of quantum mechanics.

We address these issues by adopting the framework of *Analog Gravity* or *Emergent Gravity*, specifically identifying the vacuum sector with a Superfluid Bose-Einstein Condensate (BEC) [4]. In this picture, "spacetime" is not a fundamental entity but a hydrodynamic approximation of the collective excitations (Goldstone modes) of the condensate.

In this first section, we derive the effective spacetime metric from first principles using the scalar field action and identify the hydrodynamic conditions required to form a trapping horizon.

II. FIELD THEORETIC FORMALISM

A. The Gross-Pitaevskii Action

We model the vacuum as a complex scalar field $\Psi(\mathbf{r}, t)$ obeying a $U(1)$ global symmetry. The dynamics are governed by the Gross-Pitaevskii (GP) action:

$$S = \int d^4x \sqrt{-g}\left[g^{\mu\nu}\partial_\mu \Psi^* \partial_\nu \Psi - V(|\Psi|)\right] \tag{1}$$

where the potential $V(|\Psi|)$ includes the chemical potential μ and the self-interaction term λ:

$$V(|\Psi|) = -\mu|\Psi|^2 + \frac{\lambda}{2}|\Psi|^4 \tag{2}$$

B. Hydrodynamic Representation

To extract the fluid variables, we employ the Madelung transformation, decomposing the field into a density ρ and a phase S:

$$\Psi = \sqrt{\rho}e^{iS/\hbar} \tag{3}$$

Substituting this ansatz into the Euler-Lagrange equations derived from Eq. (1) yields the coupled hydrodynamic equations for the vacuum fluid:

1. **Continuity Equation:**

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{4}$$

2. **Quantum Euler Equation:**

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\left(\Phi_{grav} + h(\rho) + Q\right) \tag{5}$$

Here, $\mathbf{v} \equiv \frac{\hbar}{m}\nabla S$ is the irrotational superfluid velocity, $h(\rho)$ is the specific enthalpy, and Q is the Quantum Potential, which contains the higher-derivative corrections:

$$Q = -\frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \tag{6}$$

III. THE EMERGENT METRIC

A. Linearized Fluctuations

Gravity and matter fields correspond to the low-energy excitations (phonons) propagating on top of this background condensate flow. We linearize the field variables:

$$\rho = \rho_0 + \rho_1, \quad S = S_0 + S_1 \tag{7}$$

* iwchoikr@gmail.com

where (ρ_0, S_0) describes the bulk vacuum flow (the "geometry") and (ρ_1, S_1) describes the fluctuations (the "particles").

Following the derivation by Unruh [1], the equation of motion for the massless scalar fluctuation field $\phi \equiv S_1$ can be written as a d'Alembertian wave equation in a curved spacetime:

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) = 0 \tag{8}$$

B. The Acoustic Metric Tensor

The effective metric $g_{\mu\nu}$ experienced by these fluctuations is determined entirely by the background flow variables ρ_0, $\mathbf{v}_0$, and the local sound speed c_s:

$$g_{\mu\nu} = \frac{\rho_0}{c_s}\begin{pmatrix} -(c_s^2 - v_0^2) & -v_{0j} \\ -v_{0i} & \delta_{ij} \end{pmatrix} \tag{9}$$

This metric depends on the vacuum density ρ_0 and flow velocity $\mathbf{v}_0$.

IV. THE BLACK HOLE AS A TRANSONIC FLOW

A. Spherically Symmetric Inflow

Consider a static, spherically symmetric inflow of the vacuum induced by a central mass concentration. In spherical coordinates (t, r, θ, ϕ), the radial velocity is $v_r(r) < 0$ (inflow). The line element becomes:

$$ds^2 = \frac{\rho_0}{c_s}\left[-(c_s^2 - v_r^2)dt^2 - 2v_r dr dt + dr^2 + r^2 d\Omega^2\right] \tag{10}$$

B. The Event Horizon

The metric component g_{00} vanishes when the coefficient of dt^2 becomes zero. This defines the **Sonic Horizon** (or Event Horizon) at radius r_H:

$$c_s^2(r_H) - v_r^2(r_H) = 0 \implies |v_r(r_H)| = c_s \tag{11}$$

Inside this radius ($r < r_H$), the vacuum flows inward faster than the speed of sound ($|v_r| > c_s$). Since all physical signals (phonons/photons) propagate at c_s relative to the fluid, no signal can propagate outward to infinity. This reproduces the causal structure of a Schwarzschild black hole, identifying the event horizon not as a boundary in space, but as a boundary in flow velocity.

To recover the Schwarzschild metric exactly, we transform to the Schwarzschild time coordinate T:

$$dT = dt + \frac{v_r}{c_s^2 - v_r^2}dr \tag{12}$$

which yields the diagonalized metric:

$$ds^2 = -\left(1 - \frac{v_r^2}{c_s^2}\right)c^2 dT^2 + \left(1 - \frac{v_r^2}{c_s^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{13}$$

Setting $v_r^2/c_s^2 = 2GM/c^2 r$ recovers the standard Schwarzschild solution. Thus, classical gravity is recovered as the low-momentum limit of the vacuum hydrodynamics.

V. RESOLUTION OF THE SINGULARITY

Classical General Relativity assumes the validity of the Einstein Field Equations down to arbitrarily small length scales. This assumption leads inevitably to the prediction of a singularity at $r = 0$ in the Schwarzschild solution. However, in the superfluid vacuum framework, the hydrodynamic approximation breaks down at the scale of the coherence length (or healing length) of the condensate, ξ. Below this scale, the Quantum Potential term Q in the Euler equation (Eq. 5) becomes dominant, providing a repulsive force that prevents infinite density accumulation.

A. The Healing Length Scale

The superfluid vacuum is characterized by a fundamental length scale ξ, defined by the competition between the kinetic energy and the interaction energy of the condensate:

$$\xi = \frac{\hbar}{\sqrt{2m\rho_0 g}} \tag{14}$$

Physically, this represents the minimum size of a vortex core or the thickness of a domain wall. For a vacuum boson mass of $m \sim 10^{-22}$ eV (typical for Fuzzy Dark Matter models), ξ is macroscopic (on the order of parsecs), but for a Planck-scale constituent, ξ would be the Planck length l_P. We posit that ξ acts as the UV cutoff for the effective field theory of gravity.

B. Quantum Pressure Repulsion

Consider the vacuum flow as $r \to 0$. The conservation of mass requires the density ρ or the velocity v to diverge. In the classical Euler equation, nothing prevents this divergence. However, including the quantum potential Q:

$$Q = -\frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \tag{15}$$

We examine the force density arising from this term:

$$F_Q = -\rho\nabla Q = \frac{\hbar^2\rho}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right) \tag{16}$$

Assuming a trial density profile for the collapsing core similar to a dark soliton or vortex core:

$$\rho(r) = \rho_\infty \tanh^2\left(\frac{r}{\xi}\right) \tag{17}$$

At small radii ($r \ll \xi$), the density scales as $\rho \sim r^2$. Substituting this into the Quantum Potential expression:

$$\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \approx \frac{\nabla^2 r}{r} \sim \frac{1}{r^2} \tag{18}$$

The resulting quantum force scales as:

$$F_Q \sim \frac{\hbar^2}{m^2 r^3} \tag{19}$$

This force is **strictly repulsive** (directed outward) and diverges as $1/r^3$. Gravity, by contrast, scales as $F_G \sim 1/r^2$. Therefore, at sufficiently small radii ($r \sim \xi$), the repulsive quantum pressure **always** overcomes the attractive gravitational force.

C. Formation of the Gravastar Core

The competition between gravitational attraction and quantum repulsion leads to a stable equilibrium state. The collapse does not proceed to a singularity but halts at a finite radius $R_{core} \sim \xi$. This object creates a "hole" in the vacuum condensate—a region where the superfluid order parameter Ψ is suppressed but non-zero. This structure is physically isomorphic to the **Gravastar** (Gravitational Vacuum Star) proposed by Mazur and Mottola [7].

- **Exterior ($r > r_H$):** Schwarzschild vacuum (Inflow ¡ c_s).
- **Interior ($R_{core} < r < r_H$):** Supersonic inflow region.
- **Core ($r < R_{core}$):** A de Sitter-like region of constant negative pressure maintained by the quantum potential.

D. Thermodynamic Regularity

The energy density ϵ inside the core remains finite. The singularity is "smeared out" over the volume of the soliton core. This resolution preserves unitarity. Information that falls into the hole is not destroyed at a singularity; it is stored in the phase correlations of the condensate within the core and can be recovered via the long-range coherence of the superfluid wavefunction.

VI. THERMODYNAMICS OF THE ACOUSTIC HORIZON

The identification of the black hole with a transonic fluid flow allows for a direct derivation of Hawking Radiation via the Unruh effect in acoustic systems. In this framework, Hawking radiation is not a mysterious conversion of vacuum energy, but the emission of thermal phonons from the sonic horizon.

A. Acoustic Surface Gravity

In classical black hole thermodynamics, the temperature of the horizon is determined by the surface gravity κ. For the acoustic metric derived in Eq. (9), the effective surface gravity is determined by the gradient of the flow velocity at the sonic point. The surface gravity κ is given by:

$$\kappa = \frac{1}{2}\left|\frac{\partial(c_s^2 - v_r^2)}{\partial r}\right|_{r=r_H} \tag{20}$$

Assuming a constant sound speed c_s for simplicity, this reduces to:

$$\kappa = c_s\left|\frac{\partial v_r}{\partial r}\right|_{r=r_H} \tag{21}$$

This quantity represents the rate at which the fluid accelerates through the sound barrier.

B. Hawking Temperature

We quantize the linear fluctuation field ϕ (phonons) on this background metric. Following the standard Bogoliubov transformation technique for particle creation in curved spacetime, we find that an observer at infinity detects a thermal flux of phonons emitted from the horizon. The temperature of this radiation is the **Hawking Temperature**:

$$T_H = \frac{\hbar\kappa}{2\pi k_B c_s} \tag{22}$$

Substituting the expression for κ:

$$T_H = \frac{\hbar}{2\pi k_B}\left|\frac{\partial v_r}{\partial r}\right|_{r=r_H} \tag{23}$$

This result confirms that any system with a sonic horizon—whether astrophysical or a laboratory Bose-Einstein Condensate—will emit thermal radiation. The "Black Hole" is simply a specific case of this universal hydrodynamic phenomenon.

C. Entropy and Information Storage

The Bekenstein-Hawking entropy formula states that $S_{BH} = A/4l_P^2$. In our model, we reinterpret this entropy as the number of available microstates of the condensate phase at the horizon boundary. Since the vacuum is a superfluid, it possesses long-range off-diagonal long-range order (ODLRO). This implies that the quantum state inside the horizon is phase-locked with the state outside.

$$\langle \Psi(r_{in}) \Psi^\dagger(r_{out}) \rangle \neq 0 \tag{24}$$

This non-vanishing correlation function means that information is **not** lost. The horizon is a "one-way membrane" for classical matter (phonons), but it is transparent to the quantum phase S. The "Information Paradox" is resolved because the vacuum state remains a pure state throughout the collapse and evaporation process. The information is encoded in the phase winding numbers and vortex loop configurations of the condensate, which are topologically protected.

D. The Area Law from Entanglement

The area law scaling of entropy ($S \propto A$) arises naturally in this system as the entanglement entropy between the interior and exterior regions of the BEC. Calculating the Von Neumann entropy $S_{ent} = -\mathrm{Tr}(\rho_{red} \ln \rho_{red})$ for a spherical region in a ground-state BEC yields:

$$S_{ent} \sim \frac{R^2}{\xi^2} \tag{25}$$

Identifying the horizon radius R with r_H and the UV cutoff ξ with the Planck length l_P, we recover the Bekenstein scaling $S \sim A/l_P^2$. Thus, Black Hole entropy is effectively the "Entanglement Entropy of the Vacuum."

VII. CONCLUSION

We have presented a hydrodynamic resolution to the problem of gravitational collapse. By modeling the physical vacuum as a Superfluid Bose-Einstein Condensate, we have replaced the geometric singularities of General Relativity with the regular, unitary physics of quantum fluids.

Our analysis yields three key results:

1. **The Emergent Metric:** The Lorentzian metric of spacetime is derived as the acoustic metric for phonon fluctuations on the condensate background. The "Event Horizon" is identified as the sonic horizon where vacuum inflow becomes supersonic ($v_r > c_s$).

2. **Singularity Avoidance:** The repulsive Quantum Pressure force ($F_Q \sim r^{-3}$), which arises from the stiffness of the condensate wavefunction, prevents the formation of a singularity. The collapse stabilizes into a finite-size "Dark Soliton" or Gravastar core of radius $R \sim \xi$.

3. **Thermodynamic Consistency:** Hawking radiation is recovered as the thermal emission of phonons from the sonic horizon. The entropy area law is identified with the entanglement entropy across the horizon boundary, preserving information via phase coherence.

This framework suggests that "Black Holes" are not punctures in the fabric of reality, but rather macroscopic quantum vortices—tornados in the superfluid ether. This perspective unifies the physics of the Planck scale with the hydrodynamics of the macroscopic universe, offering a promising pathway to a full theory of Quantum Gravity.

[1] W. G. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981).
[2] M. Visser, *Class. Quantum Grav.* **15**, 1767 (1998).
[3] C. Barceló, S. Liberati, and M. Visser, *Living Rev. Relativ.* **8**, 12 (2005).
[4] G. E. Volovik, *The Universe in a Helium Droplet*, Oxford Univ. Press (2003).
[5] W. Hu, R. Barkana, and A. Gruzinov, *Phys. Rev. Lett.* **85**, 1158 (2000).
[6] P.-H. Chavanis, *Phys. Rev. D* **84**, 043531 (2011).
[7] P. O. Mazur and E. Mottola, *Proc. Nat. Acad. Sci.* **101**, 9545 (2004).
[8] G. Chapline, *Int. J. Mod. Phys. A* **20**, 4984 (2005).
[9] M. Visser and D. L. Wiltshire, *Class. Quantum Grav.* **21**, 1135 (2004).
[10] S. W. Hawking, *Nature* **248**, 30 (1974).
[11] T. Jacobson, *Phys. Rev. D* **44**, 1731 (1991).
[12] J. Steinhauer, *Nature Phys.* **12**, 959 (2016).
[13] W. G. Unruh, *Phys. Rev. Lett.* **113**, 231301 (2014).
[14] G. Dvali and C. Gomez, *Fortsch. Phys.* **61**, 742 (2013).
[15] S. Liberati, M. Visser, and S. Weinfurtner, *Class. Quantum Grav.* **23**, 3129 (2006).