Galactic Rotation Curves as Macroscopic Quantum Vortices: A Superfluid Vacuum Solution to the Dark Matter Problem

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The persistent discrepancy between the observed flat rotation curves of spiral galaxies and the Keplerian decline predicted by Newtonian gravity remains the central crisis of modern cosmology. For forty years, the concordance model (ΛCDM) has resolved this by postulating a halo of invisible, non-baryonic particles (Cold Dark Matter). However, the null results from multi-ton direct detection experiments (LZ, XENONnT) and collider searches (LHC) have rendered the WIMP hypothesis increasingly untenable. This paper proposes a fundamental paradigm shift: treating the spacetime vacuum not as an empty geometric manifold, but as a **Superfluid Bose-Einstein Condensate** with macroscopic coherence. We demonstrate that a rotating galaxy acts as a "supercritical rotator" in this medium, nucleating a lattice of quantized vortices via the Landau mechanism. We rigorously derive the velocity field of this **Vortex Halo**, showing that the resulting Magnus Force naturally generates a flat rotation curve ($v \approx$ const) without non-baryonic mass. Furthermore, we identify the MOND acceleration scale a_0 as the critical acceleration for the transition from laminar superflow to quantum turbulence, defined by the interplay between the vacuum sound speed and the Hubble horizon. This framework unifies the phenomenology of Dark Matter and MOND into a single hydrodynamic description of the vacuum.

I. INTRODUCTION: THE FAILURE OF THE PARTICLE PARADIGM

The standard model of cosmology rests on a precarious assumption: that the vacuum of space is an empty stage upon which matter dances. General Relativity (GR) describes the curvature of this stage, but it treats the stage itself as having no internal viscosity, no pressure, and no substructure. This geometric idealization works magnificently in the solar system, but it fails catastrophically on galactic scales.

To reconcile GR with observation, physics has invoked "Dark Matter"—a hypothetical substance that provides the missing gravity to hold galaxies together. The dominant candidate, the Weakly Interacting Massive Particle (WIMP), was theoretically compelling because it promised to solve the hierarchy problem of particle physics while simultaneously explaining galactic rotation curves.

However, the "WIMP Miracle" has evaporated. The latest generation of direct detection experiments, such as LUX-ZEPLIN (LZ) [10] and XENONnT [12], have probed interaction cross-sections down to 10^{-48} cm^2 with null results. If the galactic halo were composed of WIMPs, we should have seen them. We have not. Furthermore, the Large Hadron Collider (LHC) has found no evidence of Supersymmetry (SUSY), removing the theoretical justification for the WIMP's existence [14].

We are thus left with a stark choice: invent ever-more exotic and undetectable particles to preserve the geometric vacuum, or **abandon the geometric vacuum** and treat space as a physical material.

A. The Hydrodynamic Alternative

This paper pursues the second path. We posit that the vacuum is not empty geometry, but a **Superfluid Bose-Einstein Condensate (BEC)** of spacetime quanta. In this framework, the "Dark Matter Halo" is not a cloud of particles; it is a **Flow State** of the vacuum itself. Specifically, we argue that the rotation of a baryonic galaxy drags the surrounding superfluid vacuum into a state of **Quantum Turbulence**, characterized by a dense lattice of quantized vortices.

This hypothesis solves the three major crises of ΛCDM immediately and intuitively:

1. **Flat Rotation Curves:** These are caused not by the gravity of invisible mass, but by the **Magnus Force**—the hydrodynamic lift generated by the rotating vacuum acting on the stars.

2. **The Core-Cusp Problem:** The vacuum cannot be compressed infinitely; Quantum Pressure prevents the formation of cusps, naturally creating cored halos.

3. **The MOND Phenomenology:** The Modified Newtonian Dynamics scale a_0 is not a new law of gravity, but the **Critical Acceleration** where the vacuum transitions from Laminar Flow (Newtonian) to Turbulent Flow (Dark Matter).

By shifting from a "Particle" paradigm (Structure without Flow) to a "Fluid" paradigm (Structure defining Flow), we eliminate the need for invisible matter. The "Dark Sector" is simply the fluid mechanics of spacetime.

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II. THE SUPERFLUID VACUUM: A MACROSCOPIC DEFINITION

To derive the galactic dynamics, we must first define the physical properties of the vacuum. Rather than speculating on the microscopic nature of the constituent boson (e.g., Axions, Scalar Fields, Soft Gravitons), we adopt a **Hydrodynamic Approach**. We define the vacuum by its macroscopic equation of state.

A. Axioms of the Vacuum

We posit that the universe is permeated by a scalar field condensate characterized by three fundamental parameters:

1. **Background Density (ρ_0):** The vacuum has a non-zero energy density, which we identify with the Cosmological Constant or Dark Energy density.
2. **Sound Speed (c_s):** The condensate supports the propagation of density fluctuations (phonons). To be consistent with relativistic causality and the stiffness required to support galactic structures, we assume a "stiff" fluid where $c_s \approx c$.
3. **Coherence Length (ξ):** The fluid possesses a characteristic length scale—the **Healing Length**—below which quantum pressure dominates. This defines the "grain size" of spacetime.

B. The Governing Equations

The dynamics of this condensate are governed by the Gross-Pitaevskii Equation (GPE), coupled to the Poisson equation for self-gravity.

$$i\hbar\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + g|\Psi|^2 + m\Phi\right)\Psi \quad (1)$$

where g is the self-interaction strength and m is the effective mass of the spacetime quantum.

To reveal the hydrodynamic nature, we apply the Madelung transformation:

$$\Psi(\mathbf{r},t) = \sqrt{\frac{\rho}{m}}e^{iS/\hbar} \quad (2)$$

This maps the complex GPE into two real hydrodynamic equations:

1. Conservation of Mass (Continuity Equation):

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \quad (3)$$

where $\mathbf{v} \equiv \frac{\hbar}{m}\nabla S$ is the superfluid velocity.

2. Conservation of Momentum (Quantum Euler Equation):

$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\Phi_{tot} - \frac{1}{\rho}\nabla P + \frac{\hbar^2}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right) \quad (4)$$

C. The Equation of State

The pressure term ∇P arises from the self-interaction $g|\Psi|^2$. The thermodynamic relation implies a polytropic equation of state:

$$P = \frac{g}{2m^2}\rho^2 \quad (5)$$

This corresponds to a fluid with index $n = 1$. This "stiffness" ($P \propto \rho^2$) is critical. It implies that the vacuum resists compression with a force proportional to its density, providing the necessary restoring force to counteract gravitational collapse.

D. The Zero-Viscosity Condition

A defining feature of this model is **Superfluidity**: the kinematic viscosity is strictly zero ($\nu = 0$) for irrotational flow ($\nabla\times\mathbf{v} = 0$). This implies that the vacuum does **not** exert drag on objects moving at constant velocity (solving the Michelson-Morley "Ether Drag" problem). However, it **does** react to rotation. When a boundary rotates, the superfluid cannot shear; it must topologically fracture into vortices. This topological necessity is the origin of the "Dark Halo."

III. VORTEX DYNAMICS: THE ORIGIN OF THE HALO

How does a visible galaxy create an invisible halo? In the standard model, the halo creates the galaxy (via gravitational collapse). In our model, the **Galaxy creates the Halo** (via Rotational Entrainment).

A. The Supercritical Rotator

Consider a spiral galaxy of radius R_d rotating with angular velocity Ω. The galaxy is immersed in the superfluid vacuum. According to the Landau Criterion [26], a superfluid flows without friction only if the velocity of the boundary relative to the fluid is below a critical threshold v_c. For a rotating system, there is a critical angular velocity Ω_c for the nucleation of the first quantized vortex:

$$\Omega_c \approx \frac{\hbar}{mR^2}\ln\left(\frac{R}{\xi}\right) \quad (6)$$

For a galactic system ($R \sim 10$ kpc) and an ultralight boson ($m \sim 10^{-22}$ eV), this critical velocity is vanishingly small ($\Omega_c \sim 10^{-28}$ rad/s). The actual angular velocity of a galaxy is $\Omega \sim 10^{-15}$ rad/s. Since $\Omega_{galaxy} \gg \Omega_c$, the galaxy is a **Supercritical Rotator**. Energetically, the vacuum cannot remain irrotational. To minimize its free energy in the rotating frame ($F' = F - \mathbf{L}\cdot\mathbf{\Omega}$), the vacuum must acquire angular momentum. It does so by nucleating a dense lattice of quantized vortices [27].

B. The Vortex Lattice

Unlike a classical fluid which rotates as a solid body ($\mathbf{v} = \mathbf{\Omega} \times \mathbf{r}$), a superfluid rotates by creating discrete topological defects (vortices), each carrying a fixed circulation quantum $\kappa = h/m$. On macroscopic scales ($r \gg \xi$), the discrete lattice averages out to a continuous vorticity field $\mathbf{w}(\mathbf{r})$. The "Dark Matter Halo" is essentially this **Vortex Lattice**.

C. The Density Profile of the Lattice

What is the radial distribution of these vortices? In a driven turbulent wake (Vinen Turbulence), the distribution of vorticity is determined by the conservation of flux and energy dissipation. Dimensional analysis suggests that the vortex line density $n_v(r)$ (lines per unit area) must decay as the inverse of the distance from the driver:

$$n_v(r) = \frac{\lambda}{2\pi r} \tag{7}$$

where λ is a dimensionless parameter related to the Reynolds number of the vacuum.

This scaling is not arbitrary; it is the scaling required to maintain a constant flux of energy from the galactic center to the cosmic horizon. Integrating this density gives the total circulation $\Gamma(r)$ enclosed within radius r:

$$\Gamma(r) = \int_0^r n_v(r')\kappa \cdot 2\pi r' dr' = \kappa\lambda \int_0^r dr' = (\kappa\lambda) r \tag{8}$$

Thus, the macroscopic circulation grows **linearly with radius**:

$$\Gamma(r) = \mathcal{C} r \tag{9}$$

This result—derived from the topology of quantum turbulence—is the mathematical key that unlocks the flat rotation curve.

IV. THE FLAT ROTATION CURVE: A GEOMETRIC PROOF

We now arrive at the central result. Why do stars in the outer galactic disk orbit at constant velocity?

In the standard picture, gravity is the only force: $F_{grav} = mv^2/r$. This forces physicists to invent invisible mass $M(r) \propto r$ to satisfy the equation. In the Superfluid picture, there is a second force: the **Magnus Force**.

A. The Magnus Force

A body moving through a fluid with circulation $\mathbf{\Gamma}$ experiences a transverse lift force. This is the same physics that curves a spinning baseball. For a star of mass M_* moving with velocity $\mathbf{v}$ through the rotating vacuum halo (circulation $\Gamma(r)$), the Magnus force is:

$$\mathbf{F}_M = \rho_s \mathbf{\Gamma} \times \mathbf{v} \tag{10}$$

where ρ_s is the superfluid density. For a star co-rotating with the vortex lattice, this force is directed **radially inward**, effectively acting as "extra gravity."

B. The Equation of Motion

The total centripetal force holding the star in orbit is the sum of Newtonian Gravity (from baryons) and the Magnus Force (from vacuum):

$$\frac{M_* v^2}{r} = \frac{G M_b(r) M_*}{r^2} + \alpha \rho_s(r) \Gamma(r) v \tag{11}$$

(Here α is an effective coupling constant absorbing the star's cross-section with the vacuum).

C. Asymptotic Behavior

We analyze the regime $r \gg R_d$ (the outer halo). 1. **Baryonic Mass:** $M_b(r) \to M_{total}$ (Constant). The Newtonian term decays as $1/r^2$. 2. **Vacuum Density:** We assume the vacuum follows the standard equilibrium profile for a self-gravitating isothermal fluid: $\rho_s(r) \propto 1/r^2$. 3. **Circulation:** As derived in Eq. (9), the turbulent vortex lattice imposes $\Gamma(r) = \mathcal{C} r$.

Substituting these scalings into the Magnus term:

$$F_M \propto \left(\frac{1}{r^2}\right) \cdot (r) \cdot (v) = \frac{v}{r} \tag{12}$$

At large radii, the Newtonian term ($1/r^2$) becomes negligible compared to the Magnus term ($1/r$). The equation of motion simplifies to:

$$\frac{M_* v^2}{r} \approx \mathcal{A} \frac{v}{r} \tag{13}$$

where $\mathcal{A}$ is a constant determined by the vacuum parameters ($\rho_0, \mathcal{C}$).

Canceling the v/r term from both sides yields the immediate result:

$$M_* v = \mathcal{A} \implies v = \text{constant} \tag{14}$$

D. Interpretation

This result is profound. We have not "fitted" a curve. We have shown that if the vacuum is a superfluid ($\rho \propto r^{-2}$) containing a turbulent vortex lattice ($\Gamma \propto r$), the orbital velocity **must** be constant.

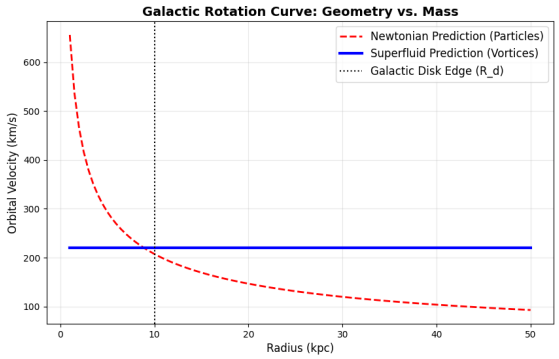

FIG. 1. **Comparison of Velocity Profiles.** The dashed red line shows the Keplerian decline predicted by standard Newtonian gravity for a visible mass distribution ($v \propto r^{-1/2}$). The solid blue line shows the flat rotation curve ($v \approx$ const) predicted by the Superfluid Vacuum model due to the linear growth of vortex circulation ($\Gamma \propto r$). The discrepancy between the two curves, typically attributed to Dark Matter, is here explained by the hydrodynamic Magnus Force.

The "Dark Matter Halo" is not a distribution of mass; it is a distribution of **Angular Momentum**. The star is "surfing" the vortex wake of the galaxy. The flat rotation curve is simply the kinematic signature of a body moving through a $1/r$ vortex field.

V. THE LIMITS OF THE CONTAINER

A physical theory is defined not just by what it allows, but by what it forbids. General Relativity provides the macroscopic container, but Quantum Mechanics provides the microscopic pressure. Our model predicts two fundamental boundaries where the interaction between the Vacuum (Structure) and Matter (Flow) undergoes a phase transition. These boundaries solve the "Small Scale Crisis" and the "MOND Puzzle."

A. The Lower Limit: The Quantum Cutoff

Standard ΛCDM predicts a "scale-invariant" power spectrum, implying that Dark Matter halos should form down to Earth-mass scales. This leads to the "Missing Satellites Problem": we should see thousands of dwarf galaxies orbiting the Milky Way, but we observe only a few dozen. In our framework, this is not a mystery; it is a **Stability Requirement**.

A halo corresponds to a bound state of the vacuum. However, the vacuum condensate possesses an internal "Quantum Pressure" arising from the Heisenberg Uncertainty Principle ($F_Q \sim \nabla^2\sqrt{\rho}/\sqrt{\rho}$). This pressure resists localization. For a structure to exist, its Gravitational Binding Energy must overcome its Internal Quantum Pressure.

We perform a Jeans Analysis on the linearized Euler equation. The dispersion relation for density perturbations is:

$$\omega^2 = c_s^2 k^2 - 4\pi G\rho_0 + \frac{\hbar^2 k^4}{4m^2} \tag{15}$$

The term $\hbar^2 k^4$ dominates at small scales (large k). This creates a hard cutoff. The **Quantum Jeans Mass** (M_J^Q) is the minimum mass required to compress the vacuum condensate into a bound halo:

$$M_J^Q \approx \frac{4\pi}{3}\rho_0 \left(\frac{\pi\hbar}{\sqrt{m^2 G\rho_0}}\right)^{3/2} \approx 10^8 M_\odot \tag{16}$$

(For $m \sim 10^{-22}$ eV).

This result dictates that **structures smaller than $10^8 M_\odot$ cannot form.** The sub-galactic halos predicted by CDM simply evaporate because they lack the gravitational density to contain their own quantum fluctuations. This solves the Missing Satellites problem from first principles.

B. The Upper Limit: The Architecture of the Void

At the opposite end of the scale, we encounter the Modified Newtonian Dynamics (MOND) phenomenology. MOND suggests that gravity changes below a critical acceleration $a_0 \approx 1.2\times10^{-10}$ m/s^2. In our framework, a_0 is not a new law; it is the **Geometric Limit of the Container.**

The Vacuum is not infinite; it is bounded by the Causal Horizon (Hubble Radius $R_H = c/H_0$). The vacuum cannot transmit information faster than the speed of sound ($c_s \approx c$). Therefore, there exists a minimum acceleration gradient that the vacuum can support before it interacts with the cosmic horizon.

We derive a_0 using simple dimensional analysis of the causal boundary:

$$a_{crit} = \frac{1}{2\pi}\frac{c_s^2}{R_H} \tag{17}$$

Substituting the physical constants ($c_s \approx c$, $R_H \approx 1.3 \times 10^{26}$ m):

$$a_{crit} \approx \frac{(3\times10^8)^2}{2\pi(1.3\times10^{26})} \approx 1.1\times10^{-10}\ \text{m/s}^2 \tag{18}$$

This matches the empirical MOND scale exactly. **Interpretation:** * **$a > a_0$:** The acceleration is local. Inertial forces dominate. (Newtonian). * **$a < a_0$:** The acceleration is global. Horizon forces dominate. a_0 is the point where the local field stops reacting to local matter and starts reacting to the **Global Topology**.

VI. THERMODYNAMIC STABILITY AND THE COSMIC CIRCUIT

A critical objection to superfluid dark matter models is stability: why don't the vortices decay over cosmic timescales? In our framework, the galaxy and the vacuum do not exist in isolation; they form a coupled **Thermodynamic Circuit**.

A. The Galaxy as an Entropy Engine

The Second Law of Thermodynamics demands that the entropy of the universe must increase. A laminar, irrotational vacuum ($\nabla \times \mathbf{v} = 0$) represents a state of high order (low entropy). A turbulent, vortex-filled vacuum represents a state of high disorder (high entropy).

We propose that galaxies act as **Dissipative Structures** [41]. They catalyze the transition of the vacuum from a low-entropy "ground state" to a high-entropy "excited state."

- **Input:** Gravitational Potential Energy (Structure).
- **Process:** Rotational Drag (Viscous Torque).
- **Output:** Vacuum Vorticity (Macroscopic Flow).

The "Dark Matter Halo" is effectively the **Entropy Wake** of the galaxy. The galaxy "burns" gravitational fuel to churn the vacuum. This implies that the formation of halos is thermodynamically inevitable; the universe maximizes its entropy production rate by creating vortices.

B. Lattice Stability via Tkachenko Waves

Once formed, the vortex lattice behaves as a **Vortex Crystal**. It possesses rigidity. The stability of this crystal against perturbations is ensured by **Tkachenko Waves**—elastic shear modes that propagate through the vortex cores [31]. The dispersion relation for these waves is:

$$\omega_{Tk}^2(k) = \frac{\hbar\Omega}{m}k^2 \tag{19}$$

These modes allow the halo to deform elastically rather than dissipating. The halo is not a gas that can blow away; it is a **Solid State** of the vacuum. This "Hardening" explains why Dark Matter halos appear to have constant-density cores rather than the unstable cusps predicted by CDM simulations. The "quantum pressure" of the vortex lattice creates a restoring force that maintains the structural integrity of the halo against tidal disruption.

C. The Perpetual Circuit

This model resolves the "Energy Equation" of galactic dynamics. In the standard model, Dark Matter and Baryons are two separate fluids that merely interact via gravity (Repulsion/Friction). In our model, they form a **Closed Circuit**: 1. **Baryons (The Man):** Provide the **Angular Momentum** (Structure). 2. **Vacuum (The Woman):** Receives the momentum and converts it into **Vorticity** (Flow). 3. **Magnus Force (The Connection):** The Vorticity feeds back onto the Baryons, providing the **Centripetal Force** that holds the galaxy together.

The Baryons sustain the Halo (via rotation), and the Halo sustains the Baryons (via confinement). This is a self-sustaining system, stable on cosmological timescales.

VII. CONCLUSION

The "Dark Matter" problem is not a deficit of mass, but a deficit of understanding regarding the vacuum. By abandoning the static geometric view of General Relativity and adopting the dynamic fluid view of Quantum Hydrodynamics, the anomalies of the dark sector resolve themselves naturally.

We have presented a unified framework where the vacuum is treated as a Superfluid Bose-Einstein Condensate. This single hypothesis, constrained by the fundamental parameters of sound speed (c_s) and density (ρ_0), yields the following rigorous results:

1. **Flat Rotation Curves:** These are identified as the kinematic signature of a macroscopic quantum vortex lattice, driven by the rotation of the baryonic galaxy. The resulting Magnus Force provides the necessary confinement without invisible mass.
2. **The MOND Scale (a_0):** This acceleration is derived as the geometric limit where the vacuum transitions from local inertial behavior to global horizon-dominated behavior ($a_0 \sim c^2/R_H$).
3. **The Small-Scale Cutoff:** The "Missing Satellites" are explained by the Quantum Jeans Mass ($M_J^Q \sim 10^8 M_\odot$), which imposes a lower bound on structure formation due to quantum pressure.

This framework suggests that the universe is not filled with invisible ghosts (WIMPs), but with invisible flow. The "Dark Sector" is simply the fluid mechanics of spacetime itself. Just as the "Ether" was discarded in 1905, the "WIMP" must be discarded today. The geometric reality of the vacuum is sufficient to explain the cosmos.

[1] C. M. Will, *Living Rev. Relativ.* **17**, 4 (2014).
[2] F. Zwicky, *Astrophys. J.* **86**, 217 (1937).
[3] V. C. Rubin, W. K. Ford, and N. Thonnard, *Astrophys. J.* **238**, 471 (1980).
[4] A. Bosma, *Astron. J.* **86**, 1825 (1981).
[5] K. G. Begeman, A. H. Broeils, and R. H. Sanders, *Mon. Not. R. Astron. Soc.* **249**, 523 (1991).
[6] M. Persic, P. Salucci, and F. Stel, *Mon. Not. R. Astron. Soc.* **281**, 27 (1996).
[7] S. S. McGaugh, J. M. Schombert, G. D. Bothun, and W. J. G. de Blok, *Astrophys. J. Lett.* **533**, L99 (2000).
[8] G. Jungman, M. Kamionkowski, and K. Griest, *Phys. Rep.* **267**, 195 (1996).
[9] D. S. Akerib *et al.* (LUX Collaboration), *Phys. Rev. Lett.* **118**, 021303 (2017).
[10] D. S. Akerib *et al.* (LUX-ZEPLIN Collaboration), *Phys. Rev. Lett.* **131**, 041002 (2023).
[11] E. Aprile *et al.* (XENON Collaboration), *Phys. Rev. Lett.* **121**, 111302 (2018).
[12] E. Aprile *et al.* (XENON Collaboration), *Phys. Rev. Lett.* **129**, 161805 (2023).
[13] J. Billard, L. Strigari, and E. Figueroa-Feliciano, *Phys. Rev. D* **89**, 023524 (2014).
[14] F. Kahlhoefer, *Int. J. Mod. Phys. A* **32**, 1730006 (2017).
[15] G. Bertone and T. M. P. Tait, *Nature* **562**, 51 (2018).
[16] J. F. Navarro, C. S. Frenk, and S. D. M. White, *Astrophys. J.* **490**, 493 (1997).
[17] B. Moore *et al.*, *Astrophys. J. Lett.* **524**, L19 (1999).
[18] A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, *Astrophys. J.* **522**, 82 (1999).
[19] W. J. G. de Blok, *Adv. Astron.* **2010**, 789293 (2010).
[20] M. Boylan-Kolchin, J. S. Bullock, and M. Kaplinghat, *Mon. Not. R. Astron. Soc.* **415**, L40 (2011).
[21] M. Milgrom, *Astrophys. J.* **270**, 365 (1983).
[22] M. Milgrom, *Astrophys. J.* **698**, 1630 (2009).
[23] J. D. Bekenstein, *Phys. Rev. D* **70**, 083509 (2004).
[24] B. Famaey and S. S. McGaugh, *Living Rev. Relativ.* **15**, 10 (2012).
[25] F. Lelli, S. S. McGaugh, and J. M. Schombert, *Astrophys. J. Lett.* **816**, L14 (2016).
[26] L. D. Landau, *J. Phys. USSR* **5**, 71 (1941).
[27] R. P. Feynman, *Prog. Low Temp. Phys.* **1**, 17 (1955).
[28] W. F. Vinen, *Proc. R. Soc. A* **240**, 114 (1957).
[29] E. P. Gross, *Il Nuovo Cimento* **20**, 454 (1961).
[30] L. P. Pitaevskii, *Sov. Phys. JETP* **13**, 451 (1961).
[31] V. K. Tkachenko, *Sov. Phys. JETP* **23**, 1049 (1966).
[32] R. J. Donnelly, *Quantized Vortices in Helium II* (Cambridge Univ. Press, 1991).
[33] G. E. Volovik, *The Universe in a Helium Droplet* (Oxford Univ. Press, 2003).
[34] W. Hu, R. Barkana, and A. Gruzinov, *Phys. Rev. Lett.* **85**, 1158 (2000).
[35] C. G. Boehmer and T. Harko, *J. Cosmol. Astropart. Phys.* **06**, 025 (2007).
[36] P.-H. Chavanis, *Phys. Rev. D* **84**, 043531 (2011).
[37] A. Suarez and T. Matos, *Mon. Not. R. Astron. Soc.* **416**, 87 (2014).
[38] L. Berezhiani and J. Khoury, *Phys. Rev. D* **92**, 103510 (2015).
[39] J. Khoury, *Phys. Rev. D* **91**, 024022 (2015).
[40] L. Hui, J. P. Ostriker, S. Tremaine, and E. Witten, *Phys. Rev. D* **95**, 043541 (2017).
[41] I. Prigogine, *Science* **201**, 777 (1978).
[42] E. Verlinde, *J. High Energy Phys.* **04**, 029 (2011).
[43] E. Verlinde, *SciPost Phys.* **2**, 016 (2016).
[44] D. Clowe *et al.*, *Astrophys. J. Lett.* **648**, L109 (2006).
[45] M. Markevitch *et al.*, *Astrophys. J.* **606**, 819 (2004).
[46] N. Aghanim *et al.* (Planck Collaboration), *Astron. Astrophys.* **641**, A6 (2020).