Untitled

February 18, 2021

[121]:

```python
"""Hyperboloid manifold."""

import torch

from manifolds.base import Manifold
from utils.math_utils import arcosh, cosh, sinh


class Hyperboloid():
    """
    Hyperboloid manifold class.

    We use the following convention: -x0^2 + x1^2 + ... + xd^2 = -K

    c = 1 / K is the hyperbolic curvature.
    """

    def __init__(self):
        super(Hyperboloid, self).__init__()
        self.name = 'Hyperboloid'
        self.eps = {torch.float32: 1e-7, torch.float64: 1e-15}
        self.min_norm = 1e-15
        self.max_norm = 1e6

    def minkowski_dot(self, x, y, keepdim=True):
        #print(x.shape, y.shape)
        res = torch.sum(x * y, dim=-1) - 2 * x[..., 0] * y[..., 0]
        if keepdim:
            res = res.view(res.shape + (1,))
        return res

    def inner(self, p, c, x, y=None, keepdim=True):
        #print(x.shape, y.shape)
        if y == None:
            y = x
        res = torch.sum(x * y, dim=-1) - 2 * x[..., 0] * y[..., 0]
        if keepdim:
```

```
            res = res.view(res.shape + (1,))
        return res

    def minkowski_norm(self, u, keepdim=True):
        dot = self.minkowski_dot(u, u, keepdim=keepdim)
        return torch.sqrt(torch.clamp(dot, min=self.eps[u.dtype]))

    def sqdist(self, x, y, c):
        K = 1. / c
        prod = self.minkowski_dot(x, y)
        theta = torch.clamp(-prod / K, min=1.0 + self.eps[x.dtype])
        sqdist = K * arcosh(theta) ** 2
        # clamp distance to avoid nans in Fermi-Dirac decoder
        return torch.clamp(sqdist, max=50.0)

    def proj(self, x, c):
        K = 1. / c
        d = x.size(-1) - 1
        y = x.narrow(-1, 1, d)

        y_sqnorm = torch.norm(y, p=2, dim=1, keepdim=True) ** 2
        mask = torch.ones_like(x)
        mask[:, 0] = 0
        vals = torch.zeros_like(x)
        vals[:, 0:1] = torch.sqrt(torch.clamp(K + y_sqnorm, min=self.eps[x.
↪dtype]))
        return vals + mask * x

    def proj_tan(self, u, x, c):
        K = 1. / c
        d = x.size(1) - 1
        ux = torch.sum(x.narrow(-1, 1, d) * u.narrow(-1, 1, d), dim=1,␣
↪keepdim=True)
        mask = torch.ones_like(u)
        mask[:, 0] = 0
        vals = torch.zeros_like(u)
        vals[:, 0:1] = ux / torch.clamp(x[:, 0:1], min=self.eps[x.dtype])
        return vals + mask * u

    def proj_tan0(self, u, c):
        narrowed = u.narrow(-1, 0, 1)
        vals = torch.zeros_like(u)
        vals[:, 0:1] = narrowed
        return u - vals

    def expmap(self, u, x, c, verbose=False):
        if verbose:
```

```
            print(f'####### EXPMAP #######')
            print(f'Input: {torch.isnan(u).sum().item()} nans')

        K = 1. / c
        sqrtK = K ** 0.5
        normu = self.minkowski_norm(u)
        normu = torch.clamp(normu, max=self.max_norm)
        theta = normu / sqrtK
        theta = torch.clamp(theta, min=self.min_norm)
        result = cosh(theta) * x + sinh(theta) * u / theta
        if verbose:
            print(f'{(theta == 0).sum().item()} zeros in denominator')
        if verbose:
            print(f'theta: {torch.isnan(theta).sum().item()}, \
                  cosh: {torch.isnan(cosh(theta)).sum().item()}, \
                  sinh: {torch.isnan(sinh(theta)).sum().item()}')

            print(f'Before proj: {torch.isnan(result).sum().item()} nans')
        out = self.proj(result, c)
        if verbose:
            print(f'Output: {torch.isnan(out).sum().item()} nans')
            print(f'$$$$$$ EXPMAP END $$$$$$')
        return out

    def logmap(self, x, y, c, verbose=False):
        if verbose:
            print(f'####### LOGMAP #######')
            print(f'Input: {torch.isnan(x).sum().item()} nans')
        K = 1. / c
        xy = torch.clamp(self.minkowski_dot(x, y) + K, max=-self.eps[x.dtype]) - K
        u = y + xy * x * c
        normu = self.minkowski_norm(u)
        normu = torch.clamp(normu, min=self.min_norm)
        dist = self.sqdist(x, y, c) ** 0.5
        result = dist * u / normu
        if verbose:
            print(f'Before proj: {torch.isnan(result).sum().item()} nans')
        out = self.proj_tan(result, x, c)
        if verbose:
            print(f'Output: {torch.isnan(out).sum().item()} nans')
            print(f'$$$$$$ LOGMAP END $$$$$$')

        return out

    def expmap0(self, u, c, verbose=False):
        if verbose:
```

```
            print(f'####### EXPMAP0 #######')
            print(f'Input: {torch.isnan(u).sum().item()} nans')
        K = 1. / c
        sqrtK = K ** 0.5
        d = u.size(-1) - 1
        x = u.narrow(-1, 1, d).view(-1, d)
        x_norm = torch.norm(x, p=2, dim=1, keepdim=True)
        x_norm = torch.clamp(x_norm, min=self.min_norm)
        theta = x_norm / sqrtK
        res = torch.ones_like(u)
        res[:, 0:1] = sqrtK * cosh(theta)
        res[:, 1:] = sqrtK * sinh(theta) * x / x_norm
        if verbose:
            print(f'max theta is: {torch.max(theta)}, sinh: {torch.
↪max(sinh(theta))}, cosh: {torch.max(cosh(theta))}')
            print(f'res[0:1]: {torch.isnan(res[:, 0:1]).sum().item()} nans')
            print(f'res[1:]: {torch.isnan(res[:, 1:]).sum().item()} nans')
            print(f'x: {torch.isnan(x).sum().item()}, x/x_norm: {torch.isnan(x /
↪ x_norm).sum().item()}')
        out = self.proj(res, c)

        if verbose:
            print(f'zeros in denominator {(x_norm == 0).sum().item()}')
            print(f'theta: {torch.isnan(theta).sum().item()}, cosh: {torch.
↪isnan(cosh(theta)).sum().item()}, sinh: {torch.isnan(sinh(theta)).sum().
↪item()}')
            print(f'Before proj: {torch.isnan(res).sum().item()} nans')

        if verbose:
            print(f'Output: {torch.isnan(out).sum().item()} nans')
            print(f'$$$$$$ EXPMAP0 END $$$$$$')

        return out

    def logmap0(self, x, c, desc=None, verbose=False):
        if verbose:
            print(f'####### LOGMAP0 #######')
            print(f'Input: {torch.isnan(x).sum().item()} nans')

        K = 1. / c
        sqrtK = K ** 0.5
        d = x.size(-1) - 1
        y = x.narrow(-1, 1, d).view(-1, d)
        y_norm = torch.norm(y, p=2, dim=1, keepdim=True)
        y_norm = torch.clamp(y_norm, min=self.min_norm)
        res = torch.zeros_like(x)
        if len(x.shape) > 1:
```

```
            theta = torch.clamp(x[:, 0:1] / sqrtK, min=1.0 + self.eps[x.dtype])
            res[:, 1:] = sqrtK * arcosh(theta) * y / y_norm
        else:
            theta = torch.clamp(x[0:1] / sqrtK, min=1.0 + self.eps[x.dtype])
            res[1:] = sqrtK * arcosh(theta) * y / y_norm
        if verbose:
            print(f'zeros in denominator {(y_norm == 0).sum().item()}')
            print(f'theta: {torch.isnan(theta).sum().item()}, \
                  arcosh: {torch.isnan(arcosh(theta)).sum().item()}')
            print(f'Output: {torch.isnan(res).sum().item()} nans')
            print(f'$$$$$$ LOGMAP0 END $$$$$$')


        return res

    def mobius_add(self, x, y, c):
        u = self.logmap0(y, c)
        v = self.ptransp0(x, u, c)
        return self.expmap(v, x, c)

    def mobius_matvec(self, m, x, c, desc=None, verbose=False):
        if verbose:
            print(f'################ MOBIUS MATVEC #################')
            print(f'Input: {torch.isnan(x).sum().item()} nans')
        u = self.logmap0(x, c)
        if verbose:
            print(f'After logmap0: {torch.isnan(u).sum().item()} nans')
            print(f'u shape: {u.shape}, w^T shape: {m.transpose(-1, -2).shape}')

        mu = u @ m.transpose(-1, -2)
        if verbose:
            print(f'After mu: {torch.isnan(mu).sum().item()} nans')
        out = self.expmap0(mu, c)
        if verbose:
            print(f'After expmap0: {torch.isnan(out).sum().item()} nans')
            print(f'$$$$$$$$$$$$$$$$ MOBIUS MATVEC END $$$$$$$$$$$$$$$$$$')
        return out

    def ptransp(self, x, y, u, c):
        logxy = self.logmap(x, y, c)
        logyx = self.logmap(y, x, c)
        sqdist = torch.clamp(self.sqdist(x, y, c), min=self.min_norm)
        alpha = self.minkowski_dot(logxy, u) / sqdist
        res = u - alpha * (logxy + logyx)
        return self.proj_tan(res, y, c)

    def ptransp0(self, x, u, c):
```

```
        K = 1. / c
        sqrtK = K ** 0.5
        x0 = x.narrow(-1, 0, 1)
        d = x.size(-1) - 1
        y = x.narrow(-1, 1, d)
        y_norm = torch.clamp(torch.norm(y, p=2, dim=1, keepdim=True), min=self.
↪min_norm)
        y_normalized = y / y_norm
        v = torch.ones_like(x)
        v[:, 0:1] = - y_norm
        v[:, 1:] = (sqrtK - x0) * y_normalized
        if len(u.shape) > 1:
            alpha = torch.sum(y_normalized * u[:, 1:], dim=1, keepdim=True) /␣
↪sqrtK
        else:
            alpha = torch.sum(y_normalized * u[1:], dim=1, keepdim=True) / sqrtK
        res = u - alpha * v
        return self.proj_tan(res, x, c)

    def to_poincare(self, x, c, verbose=False):
        if verbose:
            print(f'####### TO POINCARE #######')
            print(f'Input: {torch.isnan(x).sum().item()} nans')
        K = 1. / c
        sqrtK = K ** 0.5
        d = x.size(-1) - 1
        out = sqrtK * x.narrow(-1, 1, d) / (x[:, 0:1] + sqrtK)
        if verbose:
            print((x[:, 0:1] + sqrtK == 0).sum().item())
            print(f'Input: {torch.isnan(out).sum().item()} nans')
            print(f'$$$$$$$ TO POINCARE END $$$$$$$')
        return out

    def to_hyperboloid(self, x, c, verbose=False):
        if verbose:
            print(f'####### TO HYPERBOLOID #######')
            print(f'Input: {torch.isnan(x).sum().item()} nans')
        K = 1./ c
        sqrtK = K ** 0.5
        sqnorm = torch.norm(x, p=2, dim=1, keepdim=True) ** 2
        out = sqrtK * torch.cat([K + sqnorm, 2 * sqrtK * x], dim=1) / (K -␣
↪sqnorm)

        if verbose:
            print(f'Zeros in denominator: {(K - sqnorm == 0).sum().item()}')
            print(f'Output: {torch.isnan(out).sum().item()} nans')
            print(f'$$$$$$$ TO HYPERBOLOID END $$$$$$$')
```

```
        return out

    def egrad2rgrad(self, p, dp, c):
        K = 1. / c
        sqrtK = K ** 0.5
        u = dp
        x = p
        u.narrow(-1, 0, 1).mul_(-1 / sqrtK)
        u.addcmul_(self.minkowski_dot(x, u, keepdim=True).expand_as(x), x)

        return u
```

[117]:
```
a = torch.tensor([[1, 2, 3], [10, 10, 10]]).float()
hyp = Hyperboloid()
```

[118]:
```
a_exp = hyp.expmap0(a, 1)
a_exp
```

[118]:
```
tensor([[1.8415e+01, 1.0199e+01, 1.5299e+01],
        [6.9314e+05, 4.9012e+05, 4.9012e+05]])
```

[119]:
```
hyp.minkowski_dot(a_exp, a_exp)
```

[119]:
```
tensor([[-0.9999],
        [ 0.0000]])
```

[61]:
```
hyp.minkowski_norm(hyp.proj(a_exp, 2))
```

[61]:
```
tensor([[0.0003],
        [0.0003]])
```

[63]:
```
a_proj = hyp.proj_tan0(a, 2)
a_exp = hyp.expmap0(a_proj, 2)
a_exp
```

[63]:
```
tensor([[5.7936e+01, 3.2135e+01, 4.8202e+01],
        [1.6254e+05, 1.0578e+05, 1.2341e+05]])
```

[67]:
```
hyp.minkowski_dot(a_exp, a_exp)
```

[67]:
```
tensor([[-0.4995],
        [ 0.0000]])
```

[147]:
```
def minkowski_dot(x, y, keepdim=True):
    #print(x.shape, y.shape)
    res = torch.sum(x * y, dim=-1) - 2 * x[..., 0] * y[..., 0]
```

```
    if keepdim:
        res = res.view(res.shape + (1,))
    return res
```

[154]:
```
a = torch.ones(100).float().unsqueeze(0)
a_exp = hyp.expmap0(hyp.proj_tan0(a, c=1), c=1)
a_exp = hyp.proj(a_exp, c=1)
hyp.minkowski_dot(a_exp, a_exp).item()
```

[154]:
```
-16.0
```

[156]:
```
%pylab inline
```

```
Populating the interactive namespace from numpy and matplotlib
```

```
/opt/conda/lib/python3.7/site-packages/IPython/core/magics/pylab.py:160:
UserWarning: pylab import has clobbered these variables: ['cosh', 'sinh']
`%matplotlib` prevents importing * from pylab and numpy
  "\n`%matplotlib` prevents importing * from pylab and numpy"
```

[201]:
```
scales = np.arange(start=0, stop=0.2, step=0.01)
vec = torch.ones(100).float().unsqueeze(0)
dots = np.zeros_like(scales)

hyp = Hyperboloid()

for i, scale in enumerate(scales):
    vec_scaled = vec * scale
    exp = hyp.expmap0(vec_scaled, c=10)
    dots[i] = hyp.minkowski_dot(exp, exp)
```

[202]:
```
scales
```

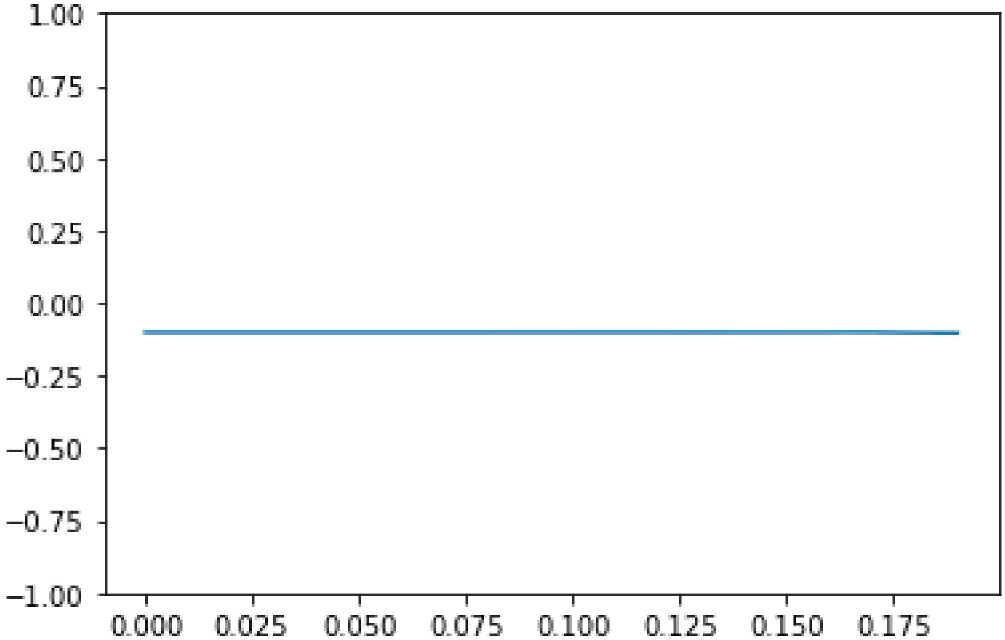
[202]:
```
array([0.  , 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1 ,
       0.11, 0.12, 0.13, 0.14, 0.15, 0.16, 0.17, 0.18, 0.19])
```

[203]:
```
dots
```

[203]:
```
array([-0.1       , -0.09999999, -0.10000001, -0.1000002 , -0.09999973,
       -0.10000014, -0.10000062, -0.10000086, -0.09999657, -0.0999918 ,
       -0.10000038, -0.09996796, -0.09998322, -0.10009766, -0.09991455,
       -0.09991455, -0.09985352, -0.09960938, -0.1015625 , -0.10302734])
```

[204]:
```
plt.plot(scales, dots)
plt.ylim(-1, 1)
```

[204]:
```
(-1.0, 1.0)
```


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175

```
[ ]:
```