$$\frac{\bar{X}-\mu}{\sqrt{\frac{\hat{\sigma}^2}{n}}} \sim t(n-1)$$

1) χ^2_n

2) t_n

3)

$X_1, \dots, X_n \sim iid\ N(\mu, \sigma^2)$

$X_i \sim N(\mu, \sigma^2)$

$$\sum_{i=1}^{n}\left(\frac{X_i-\mu}{\sigma}\right)^2 = \qquad \sum_{i=1}^{n}\left(\frac{X_i-\overset{\approx \mu}{\bar{X}}}{\sigma}\right)^2 =$$

$$\sum_{i=1}^{n}\left[N(0,1)\right]^2 \qquad \sum_{i=1}^{n}\left[N(0,1)\right]^2$$

$$\sum_{i=1}^{n}\left[Z_i\right]^2 \sim \chi^2_n \qquad \sum_{i=1}^{n}\left[Z_i\right]^2 \sim \chi^2_{n-1}$$

$$\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{X_i-\bar{X}}{\sigma}\right)^2 =$$

$$\frac{1}{\sigma^2}\cdot\underbrace{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2}_{\hat{\sigma}^2} =$$

$$\sigma^2$$

$$= \frac{\hat{\sigma}^2}{\sigma^2}$$

$$(n-1)\frac{\hat{\sigma}^2}{\sigma^2} \sim \chi^2_{n-1}$$