$$\mathbb{E}[X] = -1 \qquad var[X] = 9 \qquad corr(X,Y) = 1$$
$$\mathbb{E}[Y] = -1 \qquad var[Y] = 4$$

$$\mathbb{E}[\,Y-2X-3\,] = \mathbb{E}\underbrace{[Y]}_{-1} - 2\,\mathbb{E}\underbrace{[X]}_{-1} - 3 = -1+2-3 = -2$$

$$var[\,Y-2x-3\,] = var[1.Y-2X] =$$

$$= \underbrace{var[Y]}_{4} + \underbrace{(-2)^2}_{4}\underbrace{var[X]}_{9} + \underbrace{2\cdot 1\cdot(-2)}_{-4}\underbrace{cov(X,Y)}_{6} =$$

$$= 4+4\cdot 9 - 4\cdot 6 = 4 + 36 - 24 = 16$$

$$var[\,aX+bY\,] = a^2\, var[X] + b^2\, var[Y] + 2\cdot a\cdot b\cdot cov(X,Y)$$

$$corr(X,Y) = 1 = \frac{cov(X,Y)}{\sqrt{var(X)var(Y)}} = \frac{cov(X,Y)}{\sqrt{9\cdot 4}}$$

$$\frac{cov(X,Y)}{6} = 1 \Rightarrow cov(X,Y) = 6$$

$$\mathbb{E}[\,2Y\cdot(X-1)\,] = \mathbb{E}[\,2XY - 2Y\,] =$$

$$= 2\,\mathbb{E}[XY] - 2\,\mathbb{E}[Y] = \overset{14}{2\cdot 7} - \overset{+2}{2\cdot(-1)} = 16$$

$$cov[X,Y] = \mathbb{E}[X\cdot Y] - \mathbb{E}[X]\mathbb{E}[Y] =$$
$$6 = \mathbb{E}[X\cdot Y] - (-1)\cdot(-1)$$
$$\mathbb{E}[X\cdot Y] = \boxed{7}$$

$$\mathrm{corr}(Y-2X-3,\ X) = \frac{\overbrace{\mathrm{cov}(Y-2X-3,\ X)}^{-12}}{\sqrt{\underbrace{\mathrm{Var}(Y-2X-3)}_{=16}\cdot \underbrace{\mathrm{Var}(X)}_{=9}}} = \frac{-12}{4\cdot 3} = -1$$

$$\mathrm{cov}(Y-2X-3,\ X) = \mathrm{cov}(Y-2X,\ X) =$$

$$\mathrm{cov}(Y+a,\ X) = \mathbb{E}[(Y+a)X] - \mathbb{E}[Y+a]\cdot\mathbb{E}[X] =$$

$$= \mathbb{E}[YX] + \cancel{a\mathbb{E}[X]} - \mathbb{E}[Y]\mathbb{E}[X] - \cancel{a\mathbb{E}[X]}$$

$$= \mathrm{cov}(Y,\ X) + \mathrm{cov}(-2X,\ X) =$$

$$= \mathrm{cov}(Y,\ X) - 2\,\mathrm{cov}(X,\ X) =$$

$$= 6 - 2\cdot 9 = 6 - 18 = \boxed{-12}$$

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$\mathbb{E}[X] = \mathbb{E}[Y] = 0$$

$$var[X] = var[Y] = 1$$

$$cov(X, Y) = 0$$

1) $$var(X-Y) = \underbrace{var(X)}_{=1} + \underbrace{var(Y)}_{=1} - 2\underbrace{cov(X,Y)}_{=0}$$

$$var(X-Y) < var(X) + var(Y)$$

$$1 + 1 < 1 + 1$$

$$2 < 2$$

2) $$\underbrace{cov(X,Y)}_{=0} = \underbrace{corr(X,Y)}_{=0} = \underbrace{\underbrace{var(X)}_{=1} - \underbrace{var(Y)}_{=1}}_{1-1=0}$$

3) $$corr(X,Y) = \frac{cov(X,Y)}{\sqrt{var(X)\, var(Y)}} = \frac{0}{\sqrt{1 \cdot 1}} = 0$$

4) $$\mathbb{E}[X \cdot Y] = \mathbb{E}[X]\, \mathbb{E}[Y]$$

$$cov[X,Y] = \mathbb{E}[X \cdot Y] - \mathbb{E}[X]\, \mathbb{E}[Y] = 0$$

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim \begin{pmatrix} \text{рост} \\ \text{вес} \end{pmatrix} \sim N\left(\begin{pmatrix} 180 \\ 90 \end{pmatrix}, \begin{pmatrix} 100 & 35 \\ 35 & 25 \end{pmatrix} \right)$$

$$U = X - Y$$

$$\mathbb{E}[U] = \mathbb{E}[X] - \mathbb{E}[Y] = 180 - 90 = 90$$

$$\operatorname{var}(U) = \operatorname{var}(X) + \operatorname{var}(Y) - 2\operatorname{cov}(X,Y) =$$

$$= 100 + 25 - 2 \cdot 35 = 125 - 70 = 55$$

$$\mathbb{P}(U \leq 80) =$$

$$= \mathbb{P}\left(\underbrace{\frac{U-90}{\sqrt{55}}}_{\sim N(0,1)} \leq \frac{80-90}{\sqrt{55}} \right) = \mathbb{P}\left(N(0,1) \leq \frac{-10}{\sqrt{55}} \right) =$$

$$= F_{N(0,1)}\left(\frac{-10}{\sqrt{55}} \right) = CDF_{N(0,1)}\left(\frac{-10}{\sqrt{55}} \right) \approx 0.09$$