1) Integrable Singularities

A) Convergence for a non-singular integrand

For this part of the problem I used the specified parameters for integration, run my code, and recorded the results. I then used the results to plot the following graph.

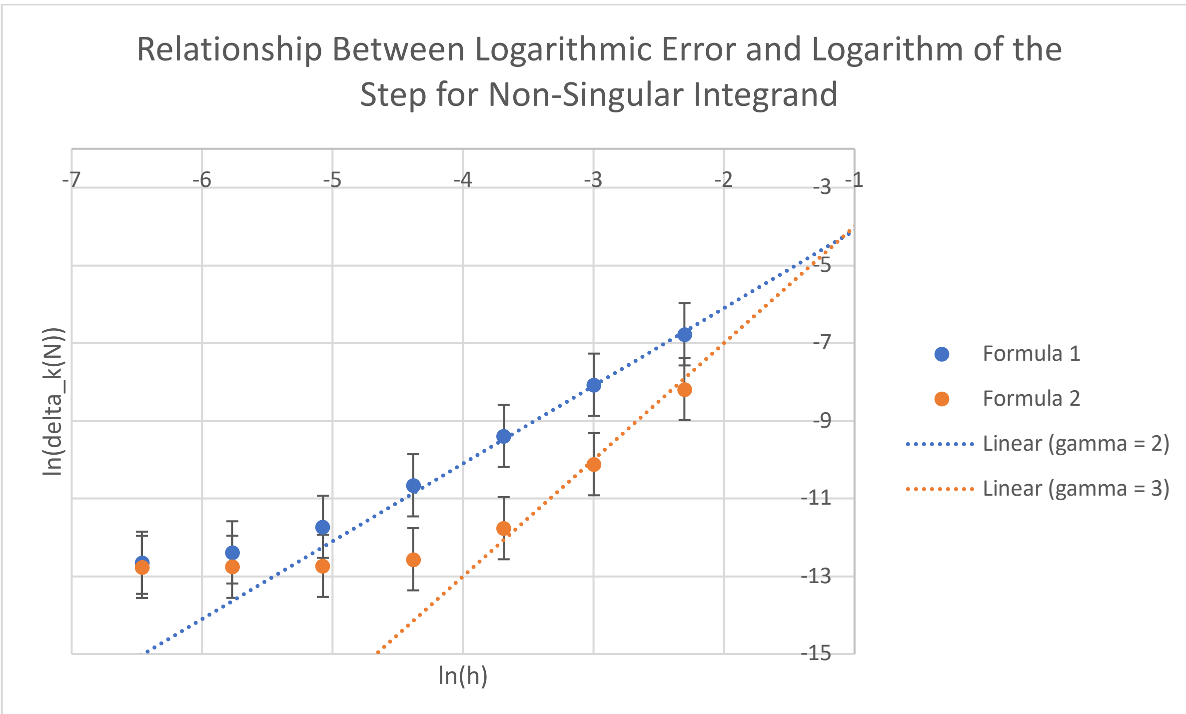

Figure 1: Relationship Between Logarithmic Error and Logarithm of the Step for Non-Singular Integrand

We can see that the results produced by two different integral approximation formulas produce the results that seem to be close to the expected values with slopes being close to the values of 2 and 3.

B) Convergence when the integrand is singular

Here again I used the parameters specified by the instructions, recorded the results of the run, and used them to obtain the following graphs.

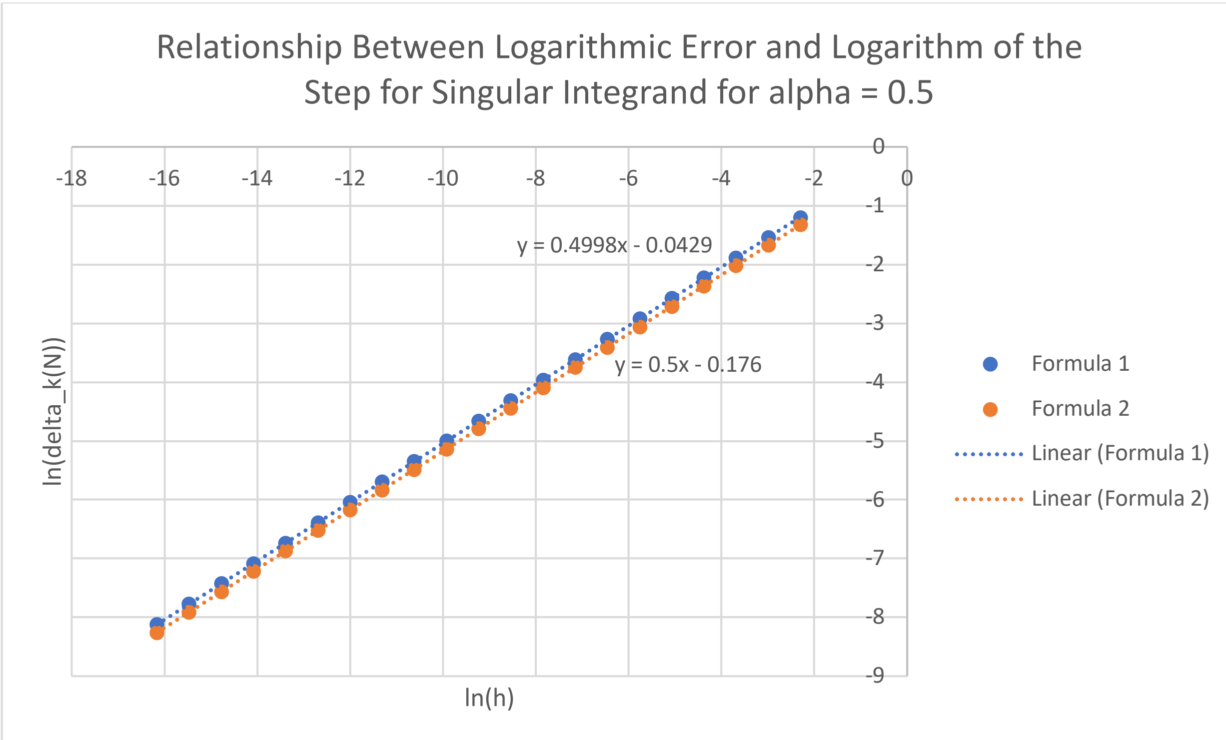

Figure 2: Relationship Between Logarithmic Error and Logarithm of the Step for Singular Integrand for alpha = 0.5

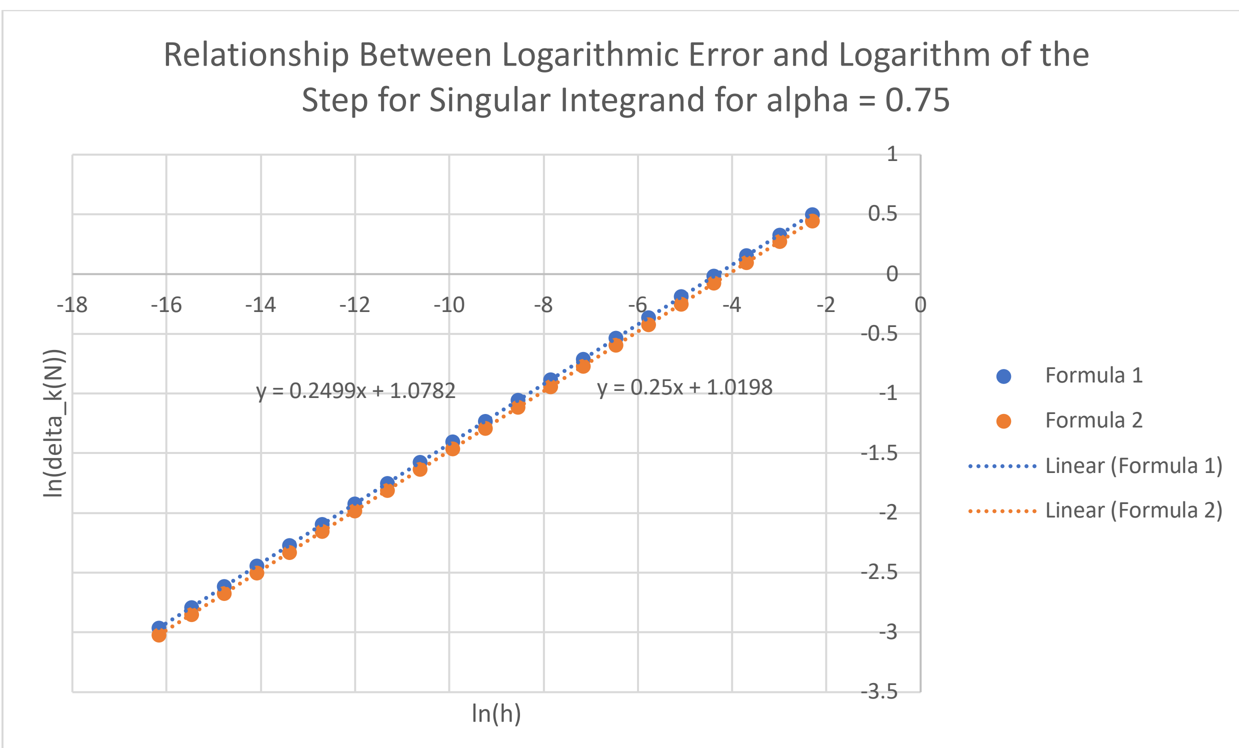

Figure 3: Relationship Between Logarithmic Error and Logarithm of the Step for Singular Integrand for alpha = 0.75

As can be seen from the graphs, I plotted the lines of best fit for each set of calculations and obtained the following slopes (exponents).

$$\alpha = \frac{1}{2} \quad \gamma_1 \approx \gamma_2 \approx \frac{1}{2}$$

$$\alpha = \frac{3}{4} \quad \gamma_1 \approx \gamma_2 \approx \frac{1}{4}$$

This result suggests that the stronger singularity, the better the estimation formulas perform, causing the exponent to decrease with increasing alpha. In addition, equal values of exponents for two different formulas yield that the main source of error is not the error in estimation anymore, rather the presence of singularity.

C) Convergence when the integrand is almost singular

I repeated the procedure from previous parts with new parameters, collected data, and used it to produce the following graph.

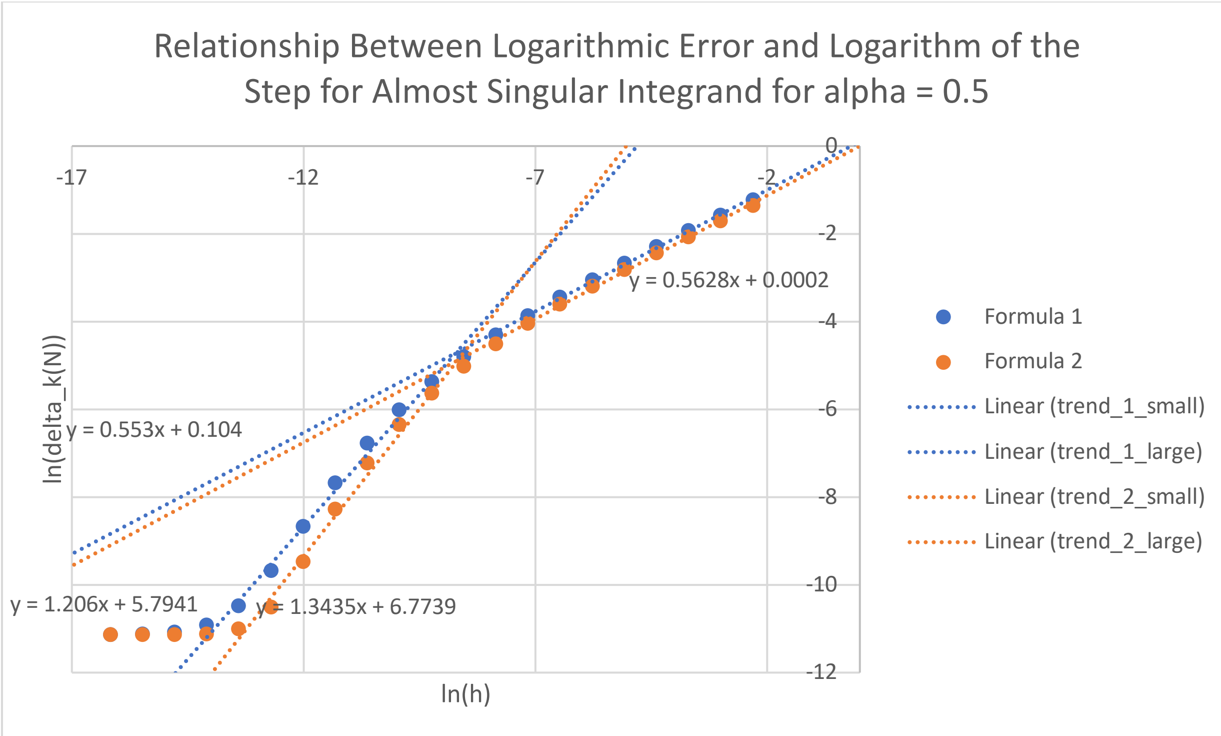

Figure 4: Relationship Between Logarithmic Error and Logarithm of the Step for Almost Singular Integrand for alpha = 0.5

From this graph we notice that for almost singular integrands the behavior of the logarithmic error is different from the one observed in previous parts. More specifically, there is now a visible "twist" in the curve, causing the slope change. Below are the exponent values I obtained from the graph.

$$small\ n: \ \ \gamma_1 \approx 0.553; \quad \gamma_2 \approx 0.563$$

$$large\ n: \ \ \gamma_1 \approx 1.206; \quad \gamma_2 \approx 1.344$$

We can notice how moving from large n to small n the exponent decreases. This behavior of the function is most likely due to the effect on the discretization on the total error. As we start increasing the step, the error initially grows at a bigger rate due to fact that main source of error in this region is actual error of the integral estimation. However, the larger the step becomes, the closer the slope to alpha, meaning that the main source of error becomes the actual singularity.

When the change of main source of error happens the graph has cross-over region. In fact, the graph seems to be a combination of graphs from parts (A) and (B). For larger values of n the graph behaves similar to the one in part (A) with different exponent values, as if there is no singularity, meaning that the main source of error here (just like in part (A)) is error in estimation and it is different for two formulas, which is confirmed by different slopes of trendlines for larger values of n. As the value of n becomes smaller, we notice that the slopes of trendlines become closer to each other and also to alpha, making this part of the graph look like graphs from part (B). This means that in this region singularity became the main source of error. Therefore, cross-over region appears where one source of error starts affecting the result more than the other.

2) Monte Carlo Integration

Running the code for this problem for 3 different values of nbi I obtained the following results.

nbi value	I_z average, $kg * m^2$	I_z standard deviation, $kg * m^2$	I_x average, $kg * m^2$	I_x standard deviation, $kg * m^2$
50	0.004692	$2.822509 * 10^{-6}$	0.004947	$3.971773 * 10^{-6}$
500	0.004691	$3.121148 * 10^{-6}$	0.004947	$3.787780 * 10^{-6}$
5000	0.004691	$3.033898 * 10^{-6}$	0.004947	$3.611122 * 10^{-6}$

In addition, using the collected bin data I plotted the histograms of the bin averages of the two moments of inertia that are presented below.

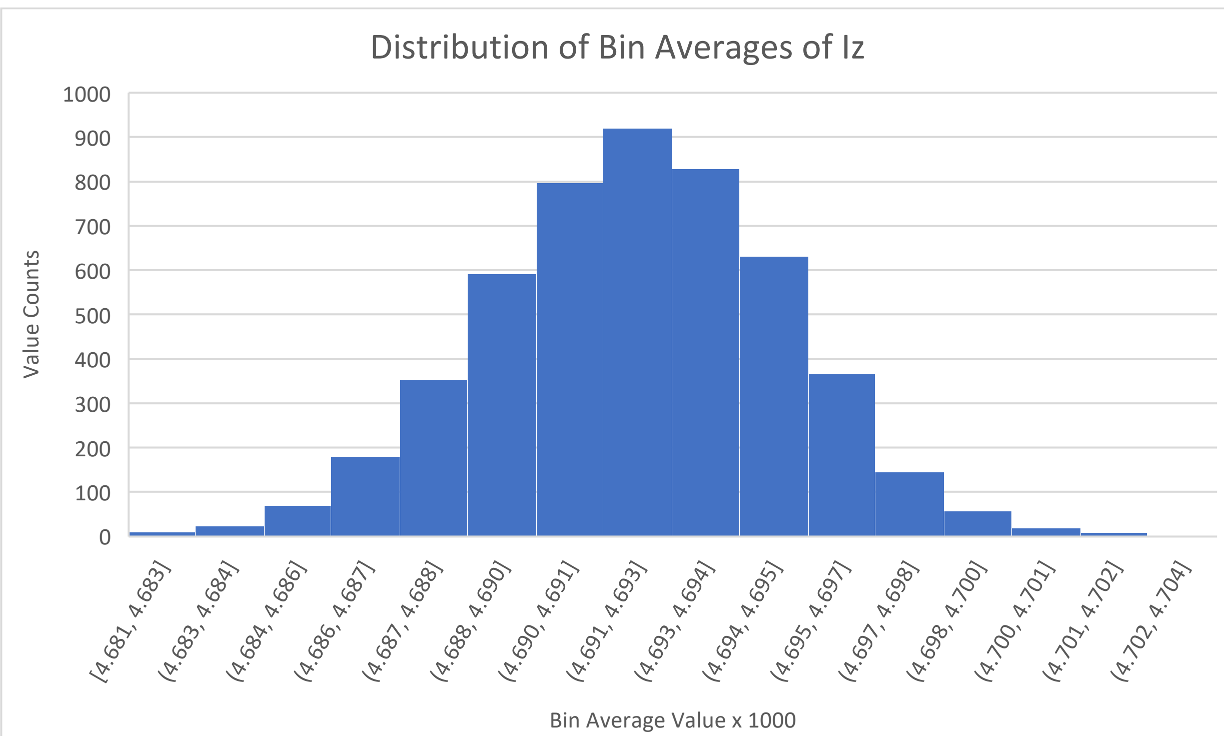

Figure 5: Distribution of Bin Averages of Iz

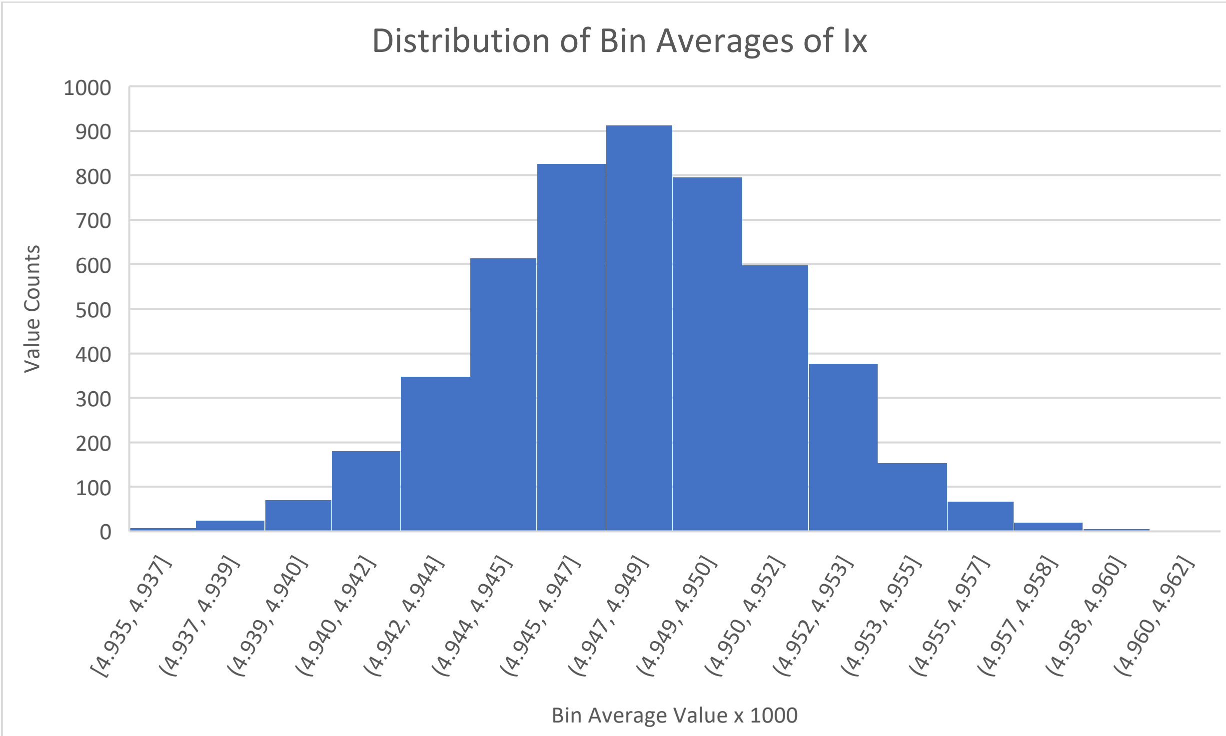

Figure 6: Distribution of Bin Averages of Ix