2.1

(a). $P(B=1 \mid J=1)$

$$= \frac{P(B=1, J=1)}{P(J=1)} = \frac{\sum_A P(B=1, J=1 \mid A)\, P(A)}{P(J=1)}$$

$\because P(A=1) = \left(P(A=1 \mid Z=1, B=0) \cdot P(B=0) + P(A=1 \mid Z=1, B=1) P(B=1)\right)$
$+ \left(P(A=1 \mid Z=0, B=0) \cdot P(B=0) + P(A=1 \mid Z=0, B=1) P(B=1)\right)$

$\therefore P(A=1 \mid Z=0) = 1.939 \times 10^{-3}$

$P(A=1 \mid Z=1) = 0.29006$

$\therefore P(A=1) = 2.5164 \times 10^{-3}$

$P(J=1) = P(J=1 \mid A=1) \cdot P(A=1) + P(J=1 \mid A=0) P(A=0)$

$= 0.05213$

$\sum_A P(B=1, J=1 \mid A) = \sum_A P(B=1 \mid A) P(J=1 \mid A)$

$\therefore P(B=1 \mid J=1) = 0.01628$

(b) $P(B=1 \mid J=1, Z=1) = \frac{P(J=1, E=1, B=1)}{P(J=1, E=1)}$

$\because P(J=1, Z=1, B=1) = \sum_A P(J=1, Z=1, B=1 \mid A)$

$= \sum P(J=1 \mid A) \cdot P(Z, B \mid A) \cdot P(A)$

$P(J=1, Z=1) = \sum_A P(J=1, Z=1 \mid A)$

$\therefore P(B=1 \mid J=1, Z=1) = 0.002887$

(c). $P(A=1 \mid J=1) = \frac{P(J=1 \mid A=1) \cdot P(A=1)}{P(J=1)} = 0.0434$

(d). $P(A=1 \mid J=1, M=0) = \dfrac{P(A=1, J=1, M=0)}{P(J=1, M=0)}$

$= \dfrac{P(J=1, M=0 \mid A=1) \cdot P(A=1)}{P(J=1, M=0)}$ ⇒ $= \sum P(J=1, M=0 \mid A)\, P(A)$

$\because \sum_A P(J=1, M=0 \mid A)\, P(A)$

$= P(J=1 \mid A=1) \cdot P(M=0 \mid A=1)\, P(A=1)$

$+ P(J=1 \mid A=0) \cdot P(M=0 \mid A=0)\, P(A=0)$

$= 0.0135.$

(e) $P(A=0 \mid M=1) = \dfrac{P(A=0, M=1)}{P(M=1)} = \dfrac{P(M=1 \mid A=0)\, P(A=0)}{P(M=1)}$

$P(M=1) = P(M=1 \mid A=0) \cdot P(A=0) + P(M=1 \mid A=1) \cdot P(A=1)$

$= 0.8499$

(f) $P(A=0 \mid M=1, Z=0) = \dfrac{P(M=1, Z=0 \mid A=0) \cdot P(A=0)}{\sum P(M=1, Z=0, A)}$

$= \dfrac{P(M=1 \mid A=0) \cdot P(Z=0 \mid A=0) \cdot P(A=0)}{\sum P(M=1, Z=0 \mid A) \cdot P(A)}$

$= 0.8803.$

2.2.

(a). $\because P(S_1=1 \mid D=0) = \dfrac{f(k-1)}{f(k)}$, where $f(k) = 2^k + (-1)^k$

$P(S_k=1 \mid D=1) = \frac{1}{2}$

$\therefore P(D=1 \mid S_1 \ldots S_k=1) = \dfrac{P(S_1 \ldots S_k=1 \mid D=1)\, P(D=1)}{P(S_1 \ldots S_k)}$

$= \dfrac{P(S_1=1 \mid D=1) \ldots P(S_k=1 \mid D=1) \cdot P(D)}{P(S_1 \ldots S_k=1)} = \dfrac{(\frac{1}{2})^k P(D=1)}{P(S_1 \ldots S_k=1)}$

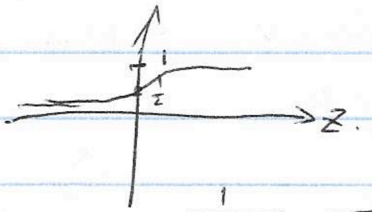
where $P(D=0 \mid S_1 \ldots S_k=1 = (2^k+(-1)^k) \cdot P(D=0)$

$\therefore$ original equation $= \dfrac{2^k+(-1)^k}{2^k} = 1+(-\frac{1}{2})^k.$

$\therefore$ when $k=$ odd, $r_k<1$; $k=$ even, $r_k>1$.

(b) if $k \to \infty$, $r_k \to 1$, then it becomes more ambiguous, which means less certainty.

2.3



(a). $\sigma(z)\sigma(-z) = \frac{1}{1+e^{-z}} \cdot \frac{1}{1+e^{z}}$

where $\sigma'(z) = -e^{-z} \cdot \frac{-1}{(1+e^{z})^2} = \frac{e^{-z}}{(1+e^{-z})^2} = \frac{e^{-z}}{1+e^{-z}} + \frac{1}{1+e^{-z}}$

(b). $\sigma(-z) + \sigma(z)$

$= \frac{1}{1+e^{z}} + \frac{1}{1+e^{-z}} = \frac{1}{1+e^{z}} + \frac{e^{z}}{1+e^{z}} = 1$

(c). $L(\sigma(z)) = \log\left(\frac{\sigma(z)}{1-\sigma(z)}\right) = \log\left(\frac{\frac{1}{1+e^{-z}}}{1-\frac{1}{1+e^{-z}}}\right)$

$= \log\left(\frac{\frac{1}{1+e^{-z}}}{\frac{1+e^{-z}-1}{1+e^{-z}}}\right) = \log(e^{z}) = z$

(d). $P_i = P(Y=1 \mid x_i = 1, x_j = 0 \text{ for all other}) = \sigma(w_i x_i)$

$\therefore P_i = \sigma(w_i x_i)$

$L(P_i) = L(\sigma(w_i \cdot x_i)) = w_i x_i = w_i \cdot 1 = w_i$

(e). We can see:

for Noisy-OR: $P(Y=1 \mid x_i=1, x_{j\neq i}=0) = P_i$

for sig: $P(Y=1 \mid x_i=1, x_{j\neq i}) = \sigma(w_i) = P_i$

$\therefore$ for $P(Y=1 \mid x_i=1, x_j=0)$, either Noisy-OR / sig are same.

However, other case could be different.

e.g. $P(Y=1 \mid x_1=1, x_2=1, x_3=1) = 1-(1-P_1)(1-P_2)(1-P_3)$

$\neq$ Sigmoid $(P(Y=1 \mid x_1=1, x_2=1, x_3=1))$.

2.4.

~~X Y Z~~

M: month, S: sprinkler. R: rain.
P: Puddle, A: Accident.

X	Y	Z
M	P	{S, R}, {S, R, A}
M	A	{S, R} {P} {S, P} {R, P} {S, R, P}
S	R	{M}
S	A	{P} {M, P} {R, P} {M, R, P}
R	A	{P} {M, P} {S, P} {M, S, P}

2.5

According to d-separation rule 1, we could get

① : $1 \in Pa(A_1)$, $A_1 \in Pa(X)$, choose A_1

③. $X \in Pa(A_1)$, $A_1 \in Pa(3)$, choose A_1.

⑤. $5 \in Pa(A_1)$, $A_1 \in Pa(A_2)$, $X \in Pa(A_2)$, choose A_1

According to d-separation rule 2, we could get

② : $A_1 \in Pa(X)$, $A_1 \in Pa(2)$, choose A_1

④ $A_1 \in Pa(4)$, $A_1 \in Pa(A_2)$, $X \in Pa(A_2)$ choose A_2.

2.6.

1> $P(A|Z) = P(A)$ T

2> $P(A,Z|D) = P(A|D)\,P(Z|D)$ F

3> $P(A,B,C,F,G,H)$

$= P(A,B,C)\,P(F,G,H)$ T

4> $P(D,Z|B) = P(D|B)\,P(Z|B)$ F

5> $P(D,Z|F,H) = P(D|F)\,P(Z|H)$ F

6> $P(A|B,C,D) = P(A|B,D)$ T

7> $P(B,C,G,H|Z)$

$= P(B,C|Z)\,P(G,H|Z)$ F

8> $P(B,C,G,H) = P(B,H)\,P(C,G)$ T

2.7.

1> $P(A|C)$ $S = \{C, Z, B, D\}$

2> $P(A|F)$ $S = \{B, F\}$

3> $P(C)$ $S = \{B, D\}$

4> $P(C|Z)$ $S = \{D, Z\}$

5> $P(C|A,B,Z)$ $S = \{A, B, D, Z, F\}$

6> $P(Z)$ $S = \{D\}$

7> $P(Z|B)$ $S = \{B, D\}$

8> $P(Z|C)$ $S = \{A, C\}$

2.8.

(a) $>$ (e) $<$

(b) $>$ (f) $>$

(c) $>$ (g) $<$

(d) $=$

2.9.

(a) $P(C|A,B,D)$

$$= \frac{P(D|A,B,C) \cdot \cancel{P(C|A,B)}}{P(D|AB)}$$

$$= \frac{P(D|B,C) \cdot P(C|A)}{\sum_{C} P(D|BC) \cdot P(C|A)}$$

(b) $P(E|A,B,D) = \sum_{C} P(C,E|A,B,D)$

$$= \sum P(E|A,B,C,D) \cdot P(C|A,B,D)$$

$$= \sum P(E|C) \cdot P(C|AB,D)$$

(c) $P(G|A,B,D) = \sum P(G,E|A,B,D)$

$$= \sum P(G|A,B,D,E) \cdot P(E|A,B,D)$$

$$= \sum P(G|E) \cdot P(E|ABD)$$

(d) $P(F|A,B,D,G) = \dfrac{P(F,G|A,B,D)}{P(G|ABD)}$

$$= \frac{P(FG)}{P(G|A,BD)} = \frac{P(F) \cdot P(G|F)}{P(G|A,B,D)}$$