3.2

(a). $P(Y_1|x_1) = \sum_{x_0} P(Y_1, x_0|x_1)$

$= \sum_{x_0} P(Y_1|x_1, x_0) \cdot P(x_0|x_1)$

$= \sum_{x_0} P(Y_1|x_1, x_0) \cdot P(x_0)$

from CPT, we know $P(Y_1|x_0, x_1)$, $P(x_0)$

$\therefore$ we get $P(Y_1|x_1)$

(b) $P(Y_1) = \sum_{x_1}\sum_{x_0} P(Y_1, x_0, x_1)$

$= \sum_{x_1}\sum_{x_0} \left(P(Y_1|x_0, x_1) \cdot P(x_0|x_1) \cdot P(x_1) \right)$

$= \sum_{x_1}\sum_{x_0} \left(P(Y_1|x_0, x_1) \cdot P(x_0) \cdot P(x_1) \right)$

$\therefore$ we get $P(Y_1)$

(c). $P(X_n|Y_1, \dots Y_{n-1})$

$= P(X_n)$

(d) $P(Y_n|X_n, Y_1, \dots Y_{n-1}) = \sum_{X_{n-1}} P(Y_n, X_{n-1}|X_n, \dots Y_{n-1})$

$$= \sum_{X_{n-1}} \frac{P(Y_n, X_n, X_{n-1}|Y_1 \dots Y_{n-1})}{P(X_n|Y_1 \dots Y_{n-1})}$$

$$= \sum_{X_{n-1}} \frac{P(X_{n-1}|Y_1 \dots Y_{n-1})\, P(X_n|\cancel{Y_1 \dots Y_{n-1} X_{n-1}})\, P(Y_n|Y_1 \dots Y_{n-1}, X_n, X_{n-1})}{P(X_n|\cancel{Y_1 \dots Y_{n-1}})}$$

(independe from (c))

$= \sum_{X_{n-1}} P(Y_n|Y_1 \dots Y_{n-1}, X_n, X_{n-1})\, P(X_{n-1}|Y_1 \dots Y_{n-1})$

$= \sum_{X_{n-1}} P(X_{n-1}|Y_1 \dots Y_{n-1})\, P(Y_n|X_n, X_{n-1})$

(e) $P(Y_n \mid Y_1, \ldots Y_{n-1})$

$$= \sum_{Y_{n-1}} P(Y_n, X_{n-1} \mid Y_1, \ldots Y_{n-1})$$

$$= \sum_{X_{n-1}} P(X_{n-1} \mid Y_1 \ldots Y_n) \cdot P(Y_n \mid X_{n-1}, \ldots Y_1, Y_n)$$

$$= \sum_{X_{n-1}} P(X_{n-1} \mid Y_1 \ldots Y_n) \, P(Y_n \mid X_{n-1})$$

$$= \sum_{X_{n-1}} P(X_{n-1} \mid Y_1 \ldots Y_n) \cdot \sum_{X_n} P(Y_n \mid X_n, X_{n-1}) \cdot P(X_n \mid X_{n-1})$$

$$= \sum_{X_{n-1}} P(X_{n-1} \mid Y_1 \ldots Y_n) \cdot \sum_{X_n} P(Y_n \mid X_n, X_{n-1}) \cdot P(X_n)$$