(c)

$P(X_1 = x_1, X_2 = x_2 \ldots X_n = x_n)$

$= \prod_{i=1}^{n} P(X_i = x_i \mid Pa(x_i) = a)$

from G_1, where $Pa(x_i) = X_{i-1}$

$\because$ $Pa(x_1) = x_0$, which doesn't exist

$\therefore = P(X_1 = x_1) \prod_{i=2}^{n} P(X_i = x_i \mid X_{i-1} = x_{i-1})$

$= P(X_1 = x_1) \prod_{i=1}^{n-1} P(X_{i+1} = x_{i+1} \mid X_i = x_i)$

1 $= \dfrac{COUNT(x_1)}{data} \prod_{i=1}^{n-1} \dfrac{COUNT(X_i, X_{i+1})}{COUNT(X_i)}$

from G_2, where $Pa(X_i) = X_{i+1}$, doesn't exist

$\therefore = P(X_n = x_n) \prod_{i=1}^{n-1} P(X_i = x_i \mid X_{i+1} = x_{i+1})$

2 $= \dfrac{count(x_n)}{data} \prod_{i=1}^{n-1} \dfrac{COUNT(X_i, X_{i+1})}{COUNT(X_{i+1})}$

for 1

$= \dfrac{\prod_{i=1}^{n-1} COUNT(X_i, X_{i+1})}{data \prod_{i=2}^{n-1} COUNT(X_i)}$

for 2

$= \dfrac{count(x_n)}{data} \dfrac{\prod_{i=1}^{n-1} COUNT(X_i, X_{i+1})}{\prod_{i=2}^{n} COUNT(X_i)}$

$= \dfrac{\prod_{i=1}^{n-1} COUNT(X_i, X_{i+1})}{data \cdot \prod_{i=2}^{n-1} COUNT(X_i)}$

$\therefore$ 1 = 2

(d) Yes,

e.g for

$$\begin{tikzcd} & x_2 \arrow[dl] \arrow[dr] & \\ x_1 & & x_3 \end{tikzcd}$$

$P_{ML}(x_i \mid Pa)$

$\therefore P(x_1, x_2, x_3) = P(x_1 \mid x_2) \cdot P(x_2) \cdot P(x_3 \mid x_2)$

which share the similar pattern as G_1 / G_2.