

(c)

$$P(X_1=x_1, X_2=x_2, \dots, X_n=x_n) \\ = \prod_{i=1}^n P(X_i=x_i \mid P_a(x_i)=a)$$

from  $G_1$ , where  $P_a(x_i) = x_{i-1}$

$\therefore P_a(x_1) = x_0$ , which doesn't exist

$$\therefore = P(X_1=x_1) \prod_{i=2}^n P(X_i=x_i \mid x_{i-1}=x_{i-1})$$

$$= P(X_1=x_1) \prod_{i=2}^n P(X_{i+1}=x_{i+1} \mid x_i=x_i)$$

$$\textcircled{1} = \frac{\text{count}(x_1)}{\text{data}} \prod_{i=1}^{n-1} \frac{\text{count}(x_i, x_{i+1})}{\text{count}(x_i)}$$

from  $G_2$ , where  $P_a(x_i) = x_{i+1}$ , doesn't exist

$$\therefore = P(X_n=x_n) \prod_{i=1}^{n-1} P(X_i=x_i \mid x_{i+1}=x_{i+1})$$

$$\textcircled{2} = \frac{\text{count}(x_n)}{\text{data}} \prod_{i=1}^{n-1} \frac{\text{count}(x_i, x_{i+1})}{\text{count}(x_{i+1})}$$

for  $\textcircled{1}$

$$= \frac{\prod_{i=1}^{n-1} \text{count}(x_i, x_{i+1})}{\text{data} \prod_{i=2}^{n-1} \text{count}(x_i)}$$

for  $\textcircled{2}$

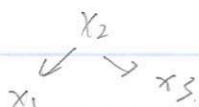
$$= \frac{\text{count}(x_n)}{\text{data}} \cdot \frac{\prod_{i=1}^{n-1} \text{count}(x_i, x_{i+1})}{\prod_{i=2}^n \text{count}(x_i)}$$

$$= \frac{\prod_{i=1}^{n-1} \text{count}(x_i, x_{i+1})}{\text{data} \cdot \prod_{i=2}^{n-1} \text{count}(x_i)}$$

$$\therefore \textcircled{1} = \textcircled{2}$$

(d) Yes,

e.g. for



$$P_{M2}(x_i | p_a)$$

$$\therefore P(x_1, x_2, x_3) = P(x_1 | x_2) \cdot P(x_2) \cdot P(x_3 | x_2)$$

which share the similar pattern as  $G_1/G_2$ .