0.7

(a) Show that the conditional distribution for binary to decimal conversion is normalized; namely, that $\sum_z P(Z=z|B_1, B_2, \ldots, B_n) = 1$, where the sum is over all integers $z \in [-\infty, +\infty]$.

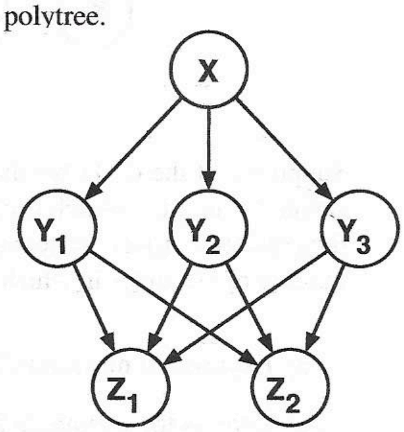
(b) Use the method of *likelihood weighting* to estimate the probability $P(B_8=1|Z=128)$ for a network with $n=10$ bits and noise level $\alpha=0.25$.

(c) Plot your estimate in part (b) *as a function of the number of samples*. You should be confident from the plot that your estimate has converged to a good degree of precision (say, at least two significant digits).

(d) Submit your source code (electronically). You may program in the language of your choice, and you may use any program at your disposal to plot the results.

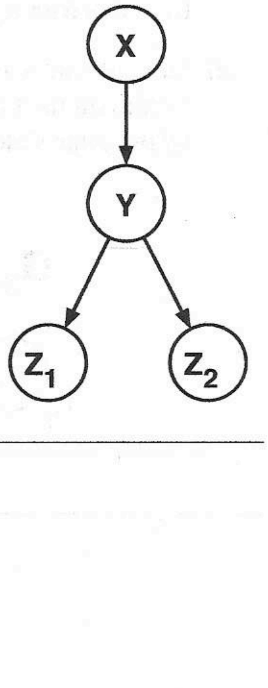
3.4 Node clustering

Consider the belief network shown below over binary variables X, Y_1, Y_2, Y_3, Z_1, and Z_2. The network can be transformed into a polytree by clustering the nodes Y_1, Y_2, and Y_3 into a single node Y. From the CPTs in the original belief network, fill in the missing elements of the CPTs for the polytree.



X	$P(Y_1=1\|X)$	$P(Y_2=1\|X)$	$P(Y_3=1\|X)$
0	0.1	0.3	0.5
1	0.8	0.6	0.4

Y_1	Y_2	Y_3	$P(Z_1=1\|Y_1,Y_2,Y_3)$	$P(Z_2=1\|Y_1,Y_2,Y_3)$
0	0	0	0.1	0.9
1	0	0	0.2	0.8
0	1	0	0.3	0.7
0	0	1	0.4	0.6
1	1	0	0.5	0.5
1	0	1	0.6	0.4
0	1	1	0.7	0.3
1	1	1	0.8	0.2



Y_1	Y_2	Y_3	Y	$P(Y\|X=0)$	$P(Y\|X=1)$	$P(Z_1=1\|Y)$	$P(Z_2=1\|Y)$
0	0	0	1	0.315	0.12	0.1	0.9
1	0	0	2	0.035	0.192	0.2	0.8
0	1	0	3	0.135	0.072	0.3	0.7
0	0	1	4	0.315	0.032	0.4	0.6
1	1	0	5	0.015	0.288	0.5	0.5
1	0	1	6	0.035	0.128	0.6	0.4
0	1	1	7	0.135	0.048	0.7	0.3
1	1	1	8	0.015	0.192	0.8	0.2
