1.1.

(a). $P(X, Y|E)$

$$= \frac{P(X, Y, Z)}{P(Z)} = \frac{P(X|Y, E)\,P(Y|E)\,P(E)}{P(E)} = P(X|Y, E)\,P(Y|E)$$

(b). $\because P(X|Y, E) \cdot P(Y|E) = P(X, Y|E)$

$$P(Y|X, Z) \cdot P(X|E) = P(X, Y|E)$$

$$\therefore P(X|Y, Z) = \frac{P(Y|X, Z)\,P(X|E)}{P(Y|E)}$$

1.2.

$\because$ we could see if (1) → (2) → (3) → (1), then everything could be proved.

$\therefore$ Assume (1) is correct, we get (1) → (3)

$\because P(X|Y, E) = P(X|E)$.

$\therefore P(X|Y, Z) \cdot P(Y|Z) = P(X, Y|E)$.

$$P(X|Z) \cdot P(Y|E) = P(X, Y|E)$$

(3) → (2).

$\because \frac{P(X, Y|E)}{P(X|E)} = \frac{P(X|E)\,P(Y|E)}{P(X|E)} \Rightarrow P(Y|X, E) = P(Y|E)$

(2) → (1)

$\because P(Y|X, E) = P(Y|E)$

$\therefore \frac{P(X, Y, E)}{P(X, E)} = \frac{P(Y, E)}{P(E)} \Rightarrow \frac{P(X, Y, E)}{P(Y, E)} = \frac{P(X, E)}{P(E)}$

$\therefore P(X|Y, E) = P(X|E)$

$\therefore$ all proved.

1.5.

(a) $P(Z=1) < P(Z=1 \mid X=1) < P(Z=1 \mid X=1, Y=1)$

∴ Z: Peter takes umbrella

X: John (Peter's roommate) takes umbrella.

Y: weather podcast says raining today.

(b) $P(X=1 \mid Y=1) > P(X=1)$,

$P(X=1 \mid Y=1, Z=1) < P(X=1 \mid Y=1)$

∴ X: as (a)'s Z, takes umbrella

Y: as (a)'s y, weather report.

Z: Peter's roommate John doesn't take umbrella.

(c) we know our coin is fair.

Y: first coin flip is head.

Z: second coin flip is head.

X: your first two flips were same

(d) we don't know either biased or fair.

Y: first flip is head.

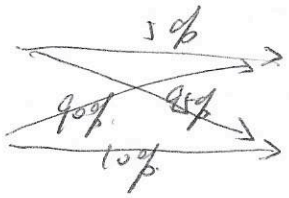
Z: second flip is head.

X: I gave you a biased coin.

1.4.

(a) $D=1,\ P=1\%$

$D=0,\ P=99\%$



	D
1	1%
0	99%

(D) → (T)

D	P(T=1|D)	P(T=0|D)
0	10%	90%
1	95%	5%

(b) $P(D=1|T=1) = \dfrac{P(T=1|D=1)\cdot P(D=1)}{P(T=1)} = \dfrac{0.95\times 0.01}{0.95\times 0.01 + 0.99\times 0.1}$

$= 8.76\%$

(c) $P(D=0|T=0) = \dfrac{P(T=0|D=0)\cdot P(D=0)}{P(T=0)} = 99.94\%$

1.5.

(a) $f = -\sum_{i=1}^{n} P_i \log P_i - \lambda\left(\sum_i P_i - 1\right)$

$\because \dfrac{\partial f}{\partial \lambda} = 0 \Rightarrow -\left(\sum_i P_i - 1\right) = 0 \ ; \ \dfrac{\partial f}{\partial P_i} = 0 \Rightarrow -\ln P_i - 1 - \lambda = 0$

$\therefore P_i = e^{-1-\lambda} \quad \because \sum P_i = 1 \quad \therefore \sum (e^{-1-\lambda}) = 1$

$\therefore e^{-1-\lambda} = \dfrac{1}{n} = P_i$

(b) $P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i) \quad \therefore \log(P(x_1 \ldots x_n)) = \log\left(\prod_{i=1}^{n} P(x_i)\right)$

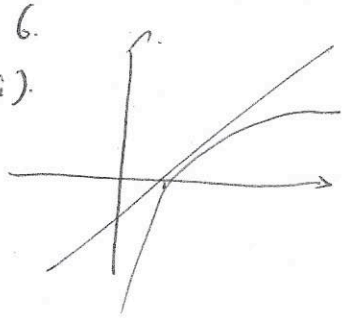
$\therefore \sum_{x_1} \cdots \sum_{x_n} (P(x_1) \cdots P(x_n)) \cdot (\log P(x_1) + \cdots + \log P(x_n)) = \sum_{i=1}^{n} \log(P x_i)$

$= \sum_{x_1} \cdots \sum_{x_{n-1}} (P(x_1) \cdots P(x_{n-1}) \cdot \left(P(x_n) \cdot (\log P(x_1) + \cdots + \log P(x_n)\right)$

$= \sum_{x_1} \cdots \sum_{x_{n-1}} (P(x_1) \cdots P(x_{n-1}) \cdot \sum P(x_i) \log P(x_i) = \text{same} = \sum_i H(x_i)$

1.6.

(a).



$\therefore \log(x) - x + 1 = f$

$\therefore f' = \frac{1}{x} - 1$, when $f' = 0$, $x = 1$

$\therefore$ the maximum of f is 0, when $x = 1$

$\therefore \log(x) \leq x - 1$, when $x = 1$, $\log(x) - x + 1 = 0$

(b). $KL(p,q) = \sum p_i \log\left(\frac{p_i}{q_i}\right) = -\sum p_i \log\left(\frac{q_i}{p_i}\right)$

$\therefore \geq -\log\left(\sum p_i \frac{q_i}{p_i}\right) = -\log 1 = 0$. $\therefore KL(p,q) \geq 0$

$\therefore -\sum p_i \log\left(\frac{q_i}{p_i}\right) \geq -\sum p_i \left(\frac{q_i}{p_i} - 1\right) = -\sum (q_i - p_i) = \sum (p_i - q_i)$.

$\therefore$ when $q_i = p_i$, it is 0.

(c) $KL(p,q) = \sum p_i \cdot 2 \log \sqrt{\frac{q_i}{p_i}} \geq -2 \sum p \left(\sqrt{\frac{q_i}{p_i}} - 1\right)$

$= -2 \sum (\sqrt{p_i q_i} - q_i) = 2 \sum p_i - 2 \sum \sqrt{p_i q_i} =$ ~~$2 \sum q_i - 2 \sum \sqrt{}$~~

$= \sum p_i + \sum q_i - 2 \sum \sqrt{p_i q_i} = \sum (\sqrt{p_i} - \sqrt{q_i})^2$

$\therefore KL(p,q) \geq \sum (\sqrt{p_i} - \sqrt{q_i})^2$ iff $\sqrt{\frac{q_i}{p_i}} = 1$.

(d). assume. $P(p=1) = P(p=0) = 0.5$.

$P(q=1) = 0.2 \neq P(q=0) = 0.8$.

$\therefore KL(p,q) = 0.2231$

$KL(q,p) = 0.1927$

$\therefore KL(p,q) \neq KL(q,p)$.

1.7.

(a). $I(X,Y) = -\sum_x \sum_y P(x,y) \log\left(\frac{P(x)P(y)}{P(x,y)}\right)$

$\geq -\log\left(\sum\sum P(x,y) \cdot \frac{P(x)P(y)}{P(x,y)}\right)$

$= -\log\left(\sum\sum P(x)\cdot P(y)\right) = 0.$

$\therefore I(X,Y) \geq 0.$

(b). if $P(x)$, $P(y)$ independent.

$P(x)P(y) = P(x,y)$

$\therefore I(x,y) = \sum_x \sum_y P(x,y) \cdot 0 = 0.$

if $I(X,Y) = 0.$

$\therefore H(Y) - H(Y|X) = 0.$

$\therefore H(Y) = H(Y|X)$ $\therefore$ x, y independ.

1.8.

(a). yes. #(2) $P(z|y)$ cannot find in #(1)

(b). no

(c). yes. #3's conditional dependent relationship are different from #2's.

(a)
The most frequency ten words(without ordering):
FIFTY, 0.013942726
AFTER, 0.014364521
ABOUT, 0.020541545
THEIR, 0.018974131
WOULD, 0.020858184
WHICH, 0.01854516
THREE, 0.03562715
EIGHT, 0.021626497
SEVEN, 0.023332724
FIRST, 0.014345604

the least frequency ten words(without ordering):
SERNA, 9.1325904E-7
NIAID, 9.1325904E-7
TOCOR, 9.1325904E-7
BOSAK, 7.8279345E-7
CLEFT, 9.1325904E-7
OTTIS, 7.8279345E-7
TROUP, 7.8279345E-7
CAIXA, 7.8279345E-7
MAPCO, 7.8279345E-7
FOAMY, 9.1325904E-7

(b) the answer corresponding to the row in table
1. E, 0.5394
2. O, 0.5340
3. E, 0.7715
4. E, 0.7127
5. R,0.7454

(c) source code:

```
import java.util.*;
import java.io.*;


public class guess{

        public static void main(String[] args) {
                String filePath = "file5.txt";
                int sum = 0;

   ArrayList<String> word = new ArrayList<>();
```

```
  ArrayList<Integer> count = new ArrayList<>();
  ArrayList<Float> prob = new ArrayList<>();

                try {
        FileReader file = new FileReader(filePath);
        BufferedReader buffer = new BufferedReader(file);
        String line;

        while((line = buffer.readLine()) != null){
           String[] part = line.split(" ");
           word.add(part[0]);
           int count_temp = Integer.parseInt(part[1]);
           sum += count_temp;
           count.add(count_temp);

        }

        System.out.println("sum: "+sum);

        for(int i = 0; i<word.size(); i++){
          float probility = (float)count.get(i)/sum;
          prob.add(probility);
        }

     }
    catch(FileNotFoundException ex) {
        System.out.println( "Unable to open file");
    }
    catch(IOException ex) {
        System.out.println("Error reading file");
    }

    HashSet<Character> p1 = new HashSet<>();
    String s1 = "D**I*";
    //p1.add('A');
    //p1.add('E');
    //p1.add('M');
    //p1.add('N');
    //p1.add('T');
    bestGuess(s1,p1,word,prob);

    System.out.println("the letter "+ ch+", the probility is "+f);
}
```

```
  public static void bestGuess(String correctGuess, HashSet<Character> wrongGuess,
ArrayList<String> word, ArrayList<Float> prob){
    float[] lguess = new float[26];
    ArrayList<Integer> tag = new ArrayList<>();
    float sumProb = 0;

    for(int i = 0; i< word.size();i++){
      String w = word.get(i);
      boolean check = true;

      for(int j = 0; j<5 ;j++){
        if((correctGuess.charAt(j) =='*' && contains(correctGuess, w.charAt(j))) ||
(correctGuess.charAt(j)!='*' && correctGuess.charAt(j)!= w.charAt(j))){
          check = false;
          break;
        }else if(wrongGuess.contains(w.charAt(j))){
          check = false;
          break;
        }

      }

      if(check){
        tag.add(1);
        sumProb += prob.get(i);

      }else{
        tag.add(0);
      }
    }

    System.out.println("sum of prob = "+sumProb);

    for(int i = 0; i<word.size();i++){
      if(tag.get(i)==0) continue;

      float p = (prob.get(i))/sumProb;
      for(int l = 0; l <26; l++){
        char temp = (char)('A'+l);
        if(!contains(correctGuess,temp) && !wrongGuess.contains(temp) &&
contains(word.get(i),temp)){
          lguess[l] += p;
        }
      }
```

```
    }

    float maxP = 0;
    char bestL = '\n';

    for(int i = 0; i<26; i++){
      if(maxP < lguess[i]){
        maxP = lguess[i];
        bestL = (char)('A'+i);
      }
    }

    ch = bestL;
    f = maxP;

  }

  private static char ch = '\n';
  private static float f = 0;

   private static boolean contains(String s, char c){
     for(int i = 0; i< s.length();i++){
       if(s.charAt(i) ==c) return true;
     }
     return false;
   }


}
```