DATA 605 Week 6 Homework

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QUICK ANWERS

Full explanations are below.

1. $P(red\ or\ blue) \approx 0.9348$
2. $P(red) = 0.25$
3. $P(not\ male\ OR\ do\ not\ live\ with\ parents) \approx 0.8463$
4. Events are DEPENDENT.
5. $211,680$
6. Evenet are INDEPENDENT.
7. $121,080,960$
8. $P(0\ red,\ 1\ orange,\ 3\ green) \approx 0.0459$
9. $7,920$
10. 33% of subscribers to a fitness magazine are 34 years old or younger
11. $E(X) = 1.75$. After 559 times, I would expect to win $978.25.
12. $E(X) = -1.5$. After 994 times, I would expect to loose $1,491.00.
13.

a) $P(Liar|Detect^{-}) \approx 0.596$
b) $P(Truth\ Teller|Detect^{+}) \approx 0.8978$
c) $P(Liar \cup Detect^{-}) = P(Liar) + P(Detect^{-}) - P(Liar) \times P(Detect^{-}|Liar) = 0.28$

Problem 1

A box contains 54 red marbles, 9 white marbles, and 75 blue marbles. If a marble is randomly selected from the box, what is the probability that it is red or blue? Express your answer as a fraction or a decimal number rounded to four decimal places.

ANSWER 1

$P(red\ or\ blue) = \frac{54+75}{54+9+75} = \frac{129}{138} \approx 0.9348$

Problem 2

You are going to play mini golf. A ball machine that contains 19 green golf balls, 20 red golf balls, 24 blue golf balls, and 17 yellow golf balls, randomly gives you your ball. What is the probability that you end up with a red golf ball? Express your answer as a simplified fraction or a decimal rounded to four decimal places.

ANSWER 2

$P(red) = \frac{20}{19+20+24+17} = \frac{20}{80} = \frac{1}{4} = 0.25$

Problem 3

A pizza delivery company classifies its customers by gender and location of residence. The research department has gathered data from a random sample of 1,399 customers. The data is summarized in the table below.

```
prob3 <- data.frame(male=c(81,116,215,130,129),
                    female=c(228,79,252,97,72),
                    row.names = c("apartment","dorm","with parents","greek house","other"))
prob3
```

```
##              male female
## apartment      81    228
## dorm          116     79
## with parents  215    252
## greek house   130     97
## other         129     72
```

What is the probability that a customer is not male or does not live with parents? Write your answer as a fraction or a decimal number rounded to four decimal places.

Solution 3

Number of customers:

```
cust_total <- sum(prob3)
cust_total
```

```
## [1] 1399
```

The complement of customers who are not male or do not live with parents is customers who are male and live with parents, which is 215 customers.

$P(male\ AND\ live\ with\ parents) = \frac{215}{1399}$

ANSWER 3

$P(not\ male\ OR\ do\ not\ live\ with\ parents) = 1 - P(male\ AND\ live\ with\ parents) = 1 - \frac{215}{1399} = \frac{1184}{1399} \approx 0.8463$

Problem 4

Determine if the following events are independent. Going to the gym. Losing weight.

Solution 4

Two events are independent if probability of one event is not affected by another event. In other words, if A and B are two events, then they are independent if $P(A) = P(A|B)$.

ANSWER 4

It is a realistic assumption that $P(losing\ weight)$ will not be the same as $P(losing\ weight|going\ to\ gym)$, so events are likely **DEPENDENT**. Of course, without additional information we cannot be sure.

Problem 5

A veggie wrap at City Subs is composed of 3 different vegetables and 3 different condiments wrapped up in a tortilla. If there are 8 vegetables, 7 condiments, and 3 types of tortilla available, how many different veggie wraps can be made?

ANSWER 5

Since repetition is not allowed (all vegetables and condiments must be different per stated problem), number of veggie wraps equals $8 \times 7 \times 6 \times 7 \times 6 \times 5 \times 3 = 211680$.

Problem 6

Determine if the following events are independent. Jeff runs out of gas on the way to work. Liz watches the evening news.

ANSWER 6

Similarly to problem 4, it is a realistic assumption that Jeff running out of gas has no influence on Liz watching the news. $P(Liz\ watches\ the\ evening\ news) = P(Liz\ watches\ the\ evening\ news|Jeff\ runs\ out\ of\ gas\ on\ the\ way\ to\ work)$, so events are likely **INDEPENDENT**. Of course, without additional information we cannot be sure (for instance, Liz and Jeff can be related and Jeff's morning commute may influence Liz's evening schedule).

Problem 7

The newly elected president needs to decide the remaining 8 spots available in the cabinet he/she is appointing. If there are 14 eligible candidates for these positions (where rank matters), how many different ways can the members of the cabinet be appointed?

ANSWER 7

Since rank (order) matters, then number of ways equals $14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 = 121080960$.

Problem 8

A bag contains 9 red, 4 orange, and 9 green jellybeans. What is the probability of reaching into the bag and randomly withdrawing 4 jellybeans such that the number of red ones is 0, the number of orange ones is 1, and the number of green ones is 3? Write your answer as a fraction or a decimal number rounded to four decimal places.

Solution 8

The bag contains 22 jellybeans. Since the order of selected jellybeans does not matter the number of possible combinations is $\binom{22}{4} = 7315$ (using `choose(22,4)`). The number of possible combinations of 3 out of 9 green jellybeans is $\binom{9}{3} = 84$ (using `choose(9,3)`), so the number of possible combinatios of 3 out of 9 green jellybeans and 1 out of 4 orange jellybeans is $84 \times 4 = 336$.

ANSWER 8

$P(0\ red,\ 1\ orange,\ 3\ green) = \frac{336}{7315} \approx 0.0459$

Problem 9

Evaluate the following expression. $\frac{11!}{7!}$

ANSWER 9

```
factorial(11)/factorial(7)
```

```
## [1] 7920
```

Problem 10

Describe the complement of the given event. 67% of subscribers to a fitness magazine are over the age of 34.

ANSWER 10

The complement is **33% of subscribers to a fitness magazine are 34 years old or younger**.

Problem 11

If you throw exactly three heads in four tosses of a coin you win \$97. If not, you pay me \$30. *Step 1.* Find the expected value of the proposition. Round your answer to two decimal places. *Step 2.* If you played this game 559 times how much would you expect to win or lose? (Losses must be entered as negative.)

Solution 11

A win is 3 heads in 4 tosses of a coin.

$P(win) = \frac{4}{2^4} = \frac{4}{16} = \frac{1}{4} = 0.25.$

$P(loss) = 1 - P(win) = 0.75.$

The return on a win is \$97. The return on a loss is -\$30.

ANSWER 11

Step 1. Expected value is $97 \times 0.25 - 30 \times 0.75 = 1.75$.

Step 2. After 559 times, the expectation will be $559 \times 1.75 = 978.25$. I would expect to win \$978.25.

Problem 12

Flip a coin 9 times. If you get 4 tails or less, I will pay you \$23. Otherwise you pay me \$26. *Step 1.* Find the expected value of the proposition. Round your answer to two decimal places. *Step 2.* If you played this game 994 times how much would you expect to win or lose? (Losses must be entered as negative.)

Solution 12

A win is 4 or less tails out of 9 tosses of a coin.

Number of no tails is 1. Number of 1 tail is 9. Number of 2 tails is $\binom{9}{2} = 36$. Number of 3 tails is $\binom{9}{3} = 84$. Number of 4 tails is $\binom{9}{4} = 126$

$P(win) = \frac{1+9+36+84+126}{2^9} = \frac{256}{512} = \frac{1}{2} = 0.5.$

$P(loss) = 1 - P(win) = 0.5.$

The return on a win is \$23. The return on a loss is -\$26.

ANSWER 12

Step 1. Expected value is $23 \times 0.5 - 26 \times 0.5 = -1.50$.

Step 2. After 994 times, the expectation will be $994 \times (-1.5) = -1491$. I would expect to loose \$1,491.00.

Problem 13

The sensitivity and specificity of the polygraph has been a subject of study and debate for years. A 2001 study of the use of polygraph for screening purposes suggested that the probability of detecting a liar was .59 (sensitivity) and that the probability of detecting a "truth teller" was .90 (specificity). We estimate that about 20% of individuals selected for the screening polygraph will lie.

a. What is the probability that an individual is actually a liar given that the polygraph detected him/her as such? (Show me the table or the formulaic solution or both.)
b. What is the probability that an individual is actually a truth-teller given that the polygraph detected him/her as such? (Show me the table or the formulaic solution or both.)
c. What is the probability that a randomly selected individual is either a liar or was identified as a liar by the polygraph? Be sure to write the probability statement.

Solution 13

$P(Liar) = 0.2$ and $P(Truth\ Teller) = 0.8$

$P(Detect^{-}|Liar) = 0.59$, so $P(Detect^{+}|Liar) = 0.41$

$P(Detect^{+}|Truth\ Teller) = 0.9$, so $P(Detect^{-}|Truth\ Teller) = 0.1$

ANSWER 13(a)

Per Bayes' Theorem,

$$\begin{aligned}
P(Liar|Detect^{-}) &= \frac{P(Detect^{-}|Liar) \times P(Liar)}{P(Detect^{-})} \\
&= \frac{P(Detect^{-}|Liar) \times P(Liar)}{P(Liar) \times P(Detect^{-}|Liar) + P(Truth\ Teller) \times P(Detect^{-}|Truth\ Teller)} \\
&= \frac{0.59 \times 0.2}{0.2 \times 0.59 + 0.8 \times 0.1} \\
&= \frac{0.118}{0.198} \approx 0.596
\end{aligned}$$

ANSWER 13(b)

Per Bayes' Theorem,

$$
\begin{aligned}
P(Truth\ Teller|Detect^{+}) &= \frac{P(Detect^{+}|Truth\ Teller) \times P(Truth\ Teller)}{P(Detect^{+})} \\
&= \frac{P(Detect^{+}|Truth\ Teller) \times P(Truth\ Teller)}{P(Liar) \times P(Detect^{+}|Liar) + P(Truth\ Teller) \times P(Detect^{+}|Truth\ Teller)} \\
&= \frac{0.9 \times 0.8}{0.2 \times 0.41 + 0.8 \times 0.9} \\
&= \frac{0.72}{0.802} \approx 0.8978
\end{aligned}
$$

ANSWER 13(c)

$$
\begin{aligned}
P(Liar \cup Detect^{-}) &= P(Liar) + P(Detect^{-}) - P(Liar \cap Detect^{-}) \\
&= P(Liar) + P(Detect^{-}) - P(Liar) \times P(Detect^{-}|Liar) \\
&= 0.2 + 0.198 - 0.2 \times 0.59 \\
&= 0.28
\end{aligned}
$$