DATA 605 Week 12 Homework

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Task

Using the `cars` dataset in R, build a linear model for stopping distance as a function of speed and replicate the analysis of your textbook chapter 3 (visualization, quality evaluation of the model, and residual analysis).

Data Import

```
# Import data
who <- read.csv('https://raw.githubusercontent.com/ilyakats/CUNY-DATA605/master/who.csv')

knitr::kable(head(who[,c(1,2,6,8,9,10)]))
```

Country	LifeExp	PropMD	PersExp	GovtExp	TotExp
Afghanistan	42	0.0002288	20	92	112
Albania	71	0.0011431	169	3128	3297
Algeria	71	0.0010605	108	5184	5292
Andorra	82	0.0032973	2589	169725	172314
Angola	41	0.0000704	36	1620	1656
Antigua and Barbuda	73	0.0001429	503	12543	13046

Data is real-world World Health Organization data from 2008. It includes 190 observations for 10 variables. Data dictionary:

- `Country`: name of the country
- `LifeExp`: average life expectancy for the country in years
- `InfantSurvival`: proportion of those surviving to one year or more
- `Under5Survival`: proportion of those surviving to five years or more
- `TBFree`: proportion of the population without TB
- `PropMD`: proportion of the population who are MDs
- `PropRN`: proportion of the population who are RNs
- `PersExp`: mean personal expenditures on healthcare in US dollars at average exchange rate
- `GovtExp`: mean government expenditures per capita on healthcare, US dollars at average exchange rate
- `TotExp`: sum of personal and government expenditures

Data Exploration

```
summary(who)
```

```
##                Country       LifeExp      InfantSurvival
##  Afghanistan       :  1   Min.   :40.00   Min.   :0.8350
##  Albania           :  1   1st Qu.:61.25   1st Qu.:0.9433
##  Algeria           :  1   Median :70.00   Median :0.9785
##  Andorra           :  1   Mean   :67.38   Mean   :0.9624
##  Angola            :  1   3rd Qu.:75.00   3rd Qu.:0.9910
```

```
##  Antigua and Barbuda:  1   Max.   :83.00   Max.   :0.9980
##  (Other)            :184
##  Under5Survival       TBFree           PropMD             PropRN
##  Min.   :0.7310   Min.   :0.9870   Min.   :0.0000196   Min.   :0.0000883
##  1st Qu.:0.9253   1st Qu.:0.9969   1st Qu.:0.0002444   1st Qu.:0.0008455
##  Median :0.9745   Median :0.9992   Median :0.0010474   Median :0.0027584
##  Mean   :0.9459   Mean   :0.9980   Mean   :0.0017954   Mean   :0.0041336
##  3rd Qu.:0.9900   3rd Qu.:0.9998   3rd Qu.:0.0024584   3rd Qu.:0.0057164
##  Max.   :0.9970   Max.   :1.0000   Max.   :0.0351290   Max.   :0.0708387
##
##     PersExp           GovtExp             TotExp
##  Min.   :   3.00   Min.   :    10.0   Min.   :    13
##  1st Qu.:  36.25   1st Qu.:   559.5   1st Qu.:   584
##  Median : 199.50   Median :  5385.0   Median :  5541
##  Mean   : 742.00   Mean   : 40953.5   Mean   : 41696
##  3rd Qu.: 515.25   3rd Qu.: 25680.2   3rd Qu.: 26331
##  Max.   :6350.00   Max.   :476420.0   Max.   :482750
##
```

Looking at the range of personal and government expenditures (13 to 482,750), I thought it was interesting to see top and bottom countries.

Table 2: Bottom 5 Countries by Total Expenditures

Country	LifeExp	PropMD	PersExp	GovtExp	TotExp
Burundi	49	0.0000245	3	10	13
Ethiopia	56	0.0000239	6	64	70
Democratic Republic of the Congo	47	0.0000961	5	66	71
Nepal	62	0.0001948	16	64	80
Bangladesh	63	0.0002749	12	75	87

Table 3: Top 5 Countries by Total Expenditures

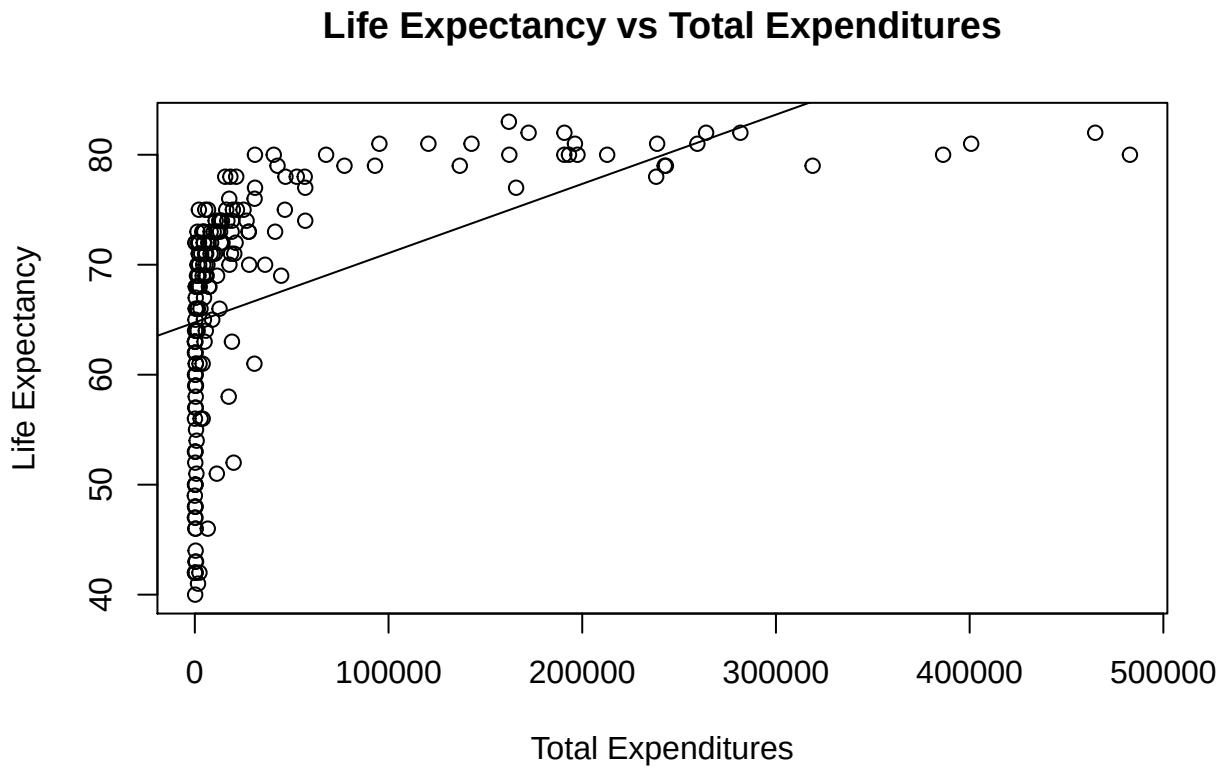
Country	LifeExp	PropMD	PersExp	GovtExp	TotExp
Denmark	79	0.0035519	4350	314588	318938
Norway	80	0.0037531	5910	380380	386290
Iceland	81	0.0037584	5154	395622	400776
Monaco	82	0.0056364	6128	458700	464828
Luxembourg	80	0.0027223	6330	476420	482750

Question 1

Let us build a linear regression model for predicting life expectancy by total expenditures. Below scatterplot shows the relationship along with the linear regression line.

```
# Linear regression model build
life_exp_lm <- lm(LifeExp ~ TotExp, data=who)

# Scatterplot of dependent and independent variables
plot(LifeExp~TotExp, data=who,
     xlab="Total Expenditures", ylab="Life Expectancy",
```

```
     main="Life Expectancy vs Total Expenditures")
abline(life_exp_lm)
```

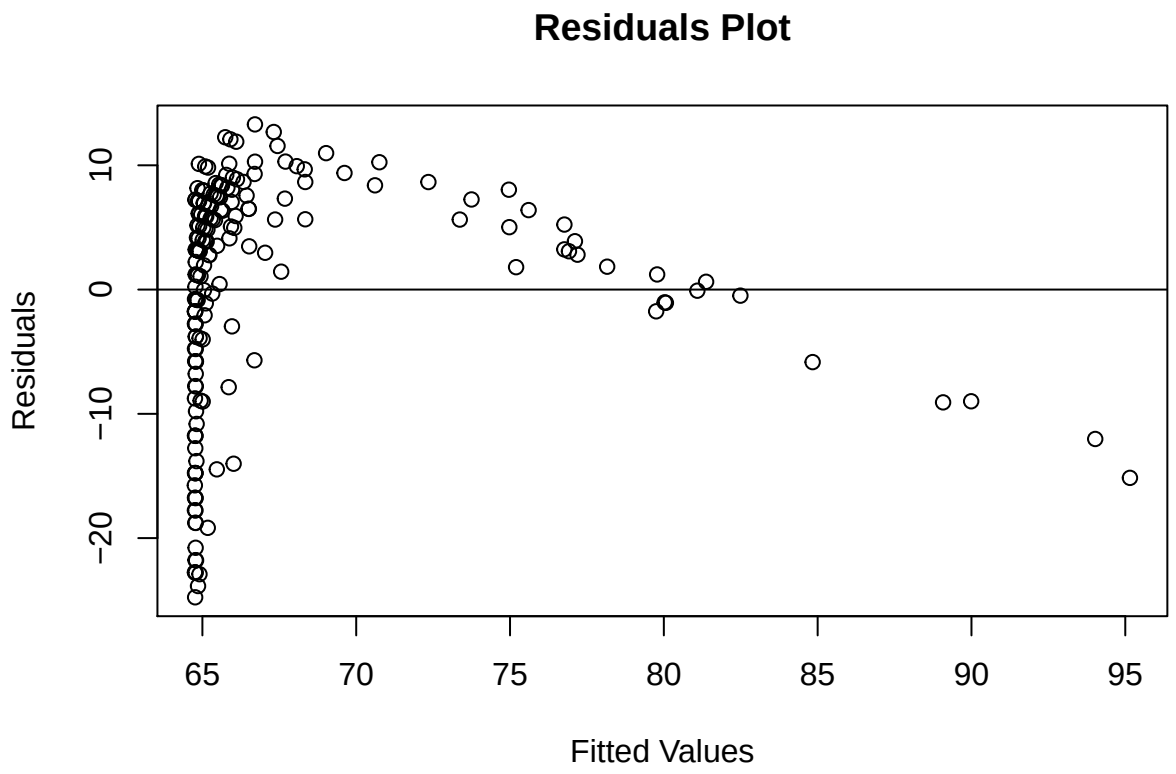



```
# Linear regression model summary
summary(life_exp_lm)
```

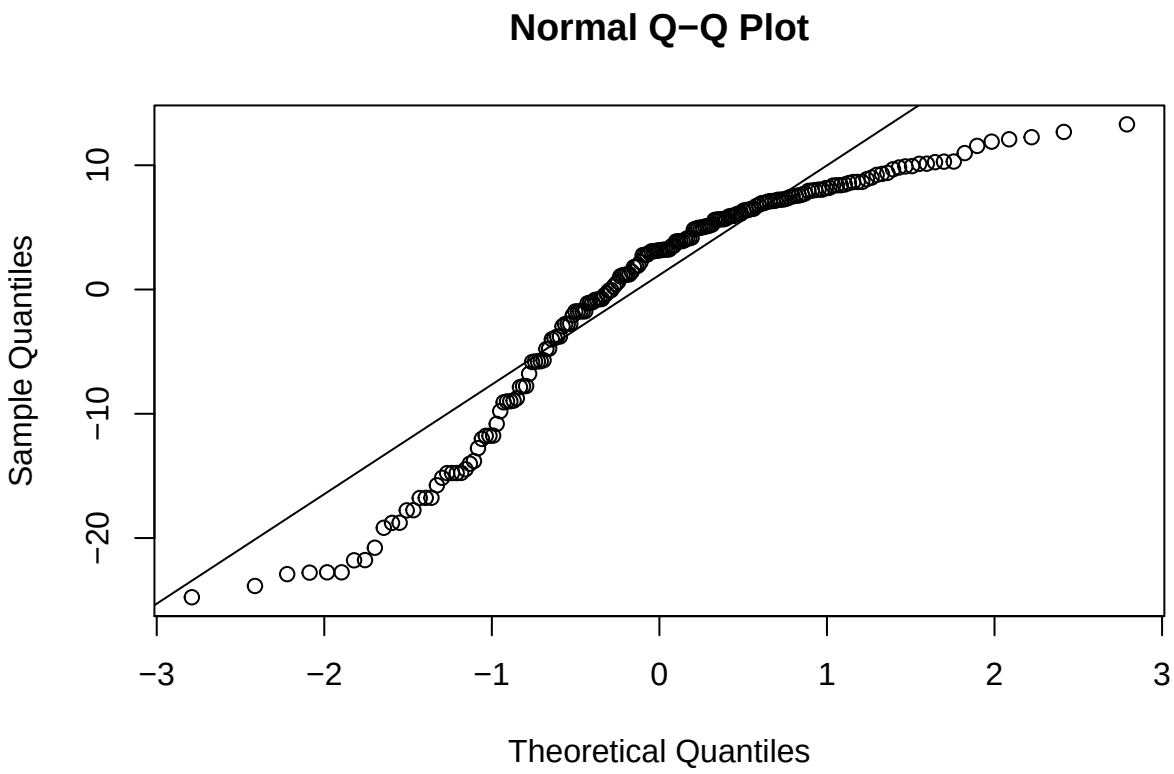
```
##
## Call:
## lm(formula = LifeExp ~ TotExp, data = who)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -24.764  -4.778   3.154   7.116  13.292
##
## Coefficients:
##                Estimate   Std. Error t value           Pr(>|t|)
## (Intercept) 64.753374534  0.753536611  85.933           < 2e-16 ***
## TotExp       0.000062970  0.000007795   8.079 0.0000000000000771 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.371 on 188 degrees of freedom
## Multiple R-squared:  0.2577, Adjusted R-squared:  0.2537
## F-statistic: 65.26 on 1 and 188 DF,  p-value: 0.00000000000007714
```

```
# Residuals variability plot
plot(life_exp_lm$fitted.values, life_exp_lm$residuals,
```

```
     xlab="Fitted Values", ylab="Residuals",
     main="Residuals Plot")
abline(h=0)
```




```
# Residuals Q-Q plot
qqnorm(life_exp_lm$residuals)
qqline(life_exp_lm$residuals)
```



Results

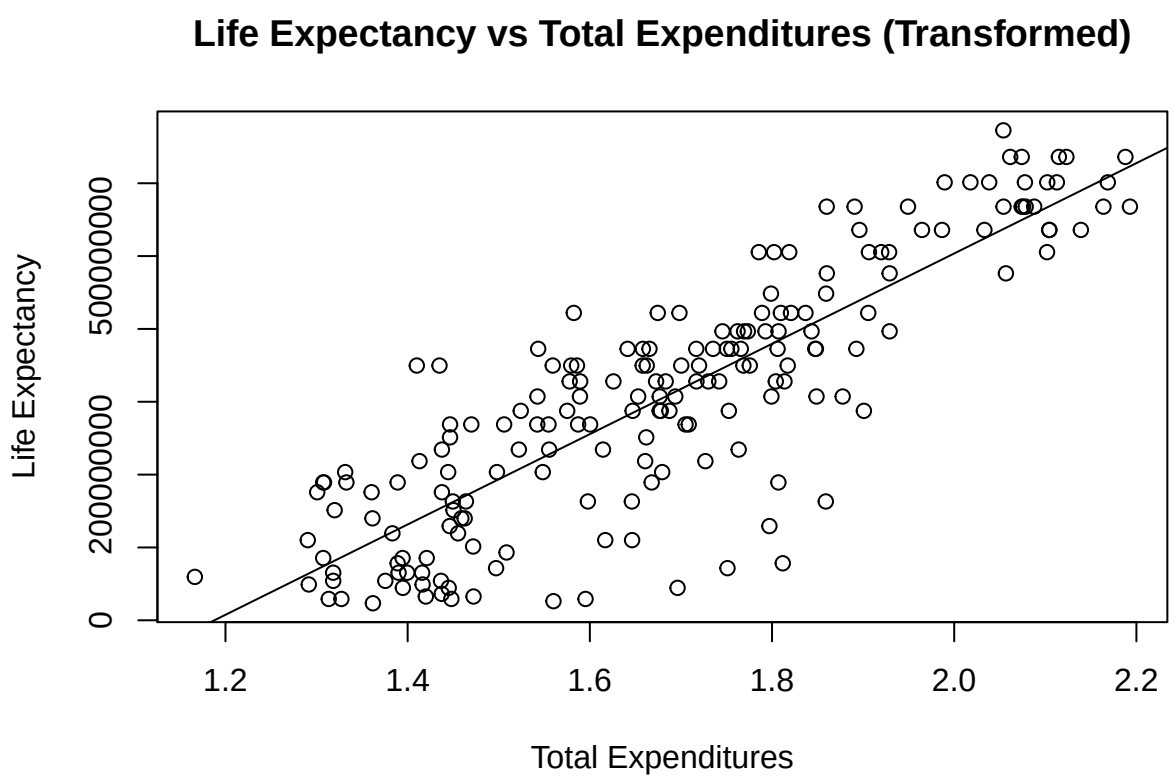
Residual standard error is 9.371 and **F-statistic** is 65.26. Considering that average life expectancy is 67.38, the SE is not terrible and F-statistics is high. However, R^2 is only 0.2577 (so the model explains only 25.77% of variability). **P-value** is nearly 0, so the relationship is not due to random variation.

Looking at residuals plots it is clear that there is no constant variability and that residuals are not normally distributed. This is **not a good model** to describe the relationship. It is clear from the scatterplot that the relationship is not linear.

Question 2

Let us transform variables and re-run the simple linear regression model - $LifeExp^{4.6}$ and $TotExp^{0.06}$.

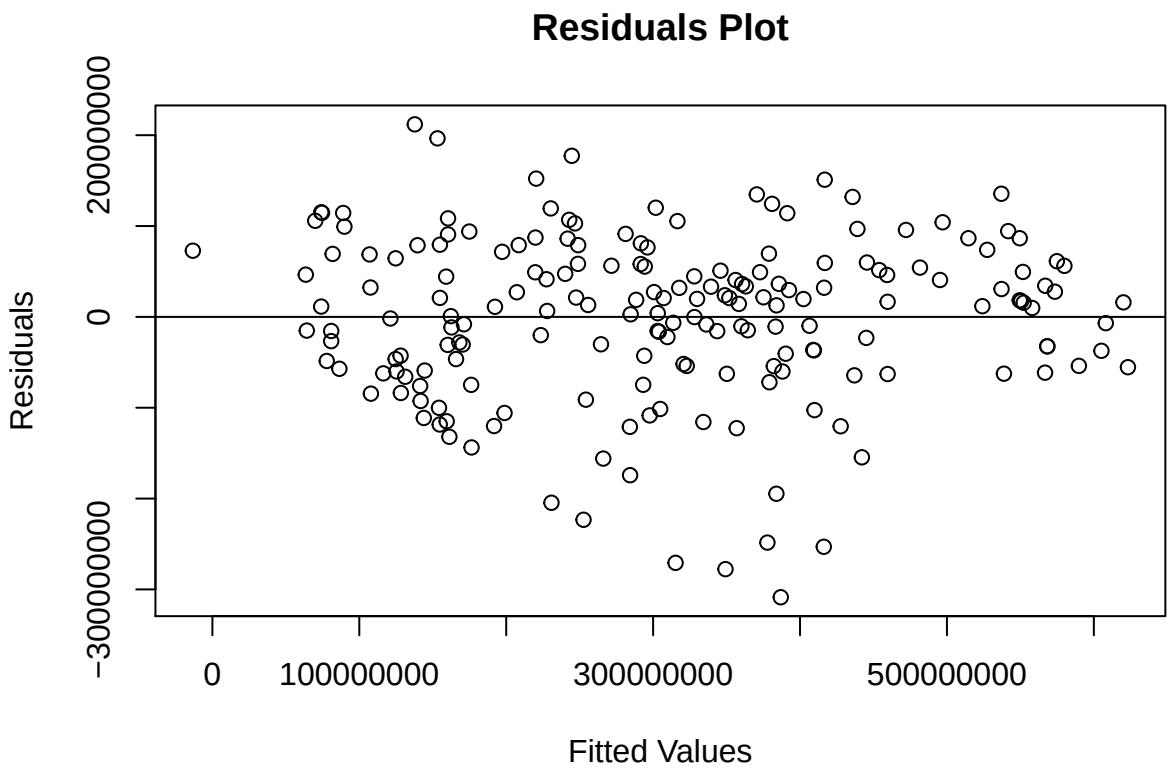
```
# Transformation
LifeExp4.6 <- who$LifeExp^4.6
TotExp0.06 <- who$TotExp^0.06

# Linear regression model build
life_exp_lm <- lm(LifeExp4.6 ~ TotExp0.06)

# Scatterplot of dependent and independent variables
plot(LifeExp4.6~TotExp0.06,
     xlab="Total Expenditures", ylab="Life Expectancy",
     main="Life Expectancy vs Total Expenditures (Transformed)")
abline(life_exp_lm)
```



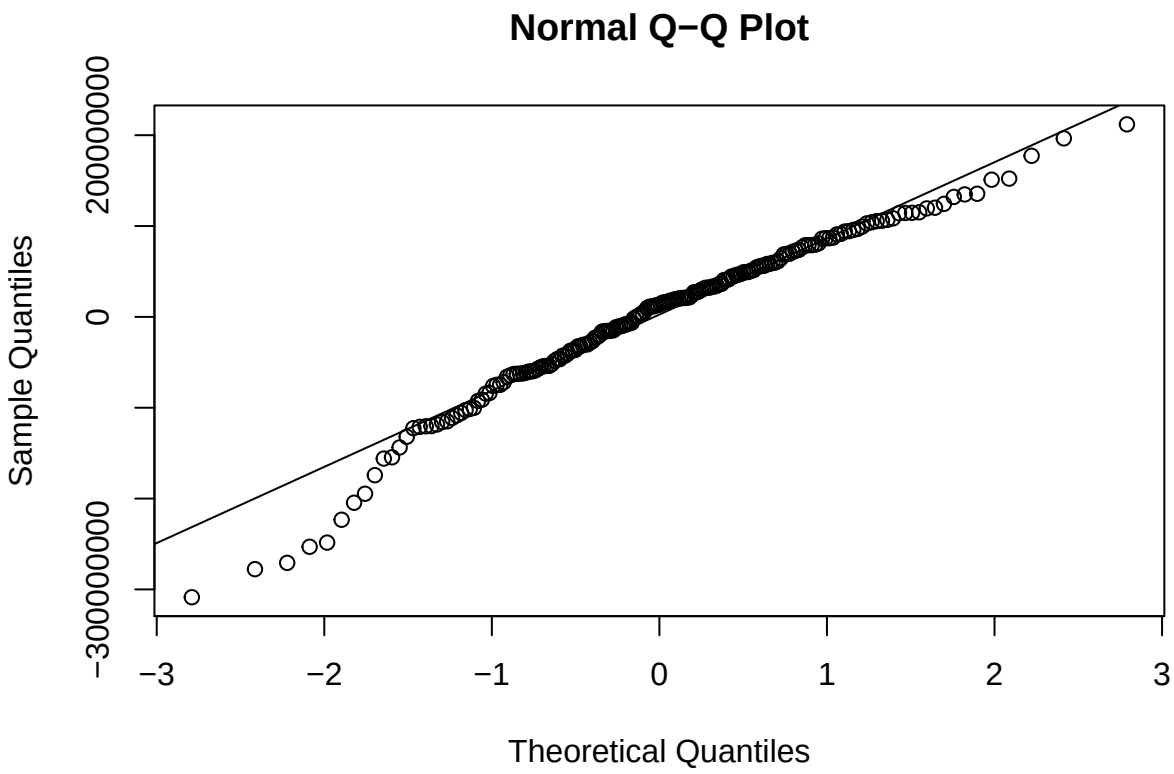

```r
# Linear regression model summary
summary(life_exp_lm)
```

```
##
## Call:
## lm(formula = LifeExp4.6 ~ TotExp0.06)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -308616089  -53978977   13697187   59139231  211951764
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -736527910   46817945  -15.73   <2e-16 ***
## TotExp0.06   620060216   27518940   22.53   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 90490000 on 188 degrees of freedom
## Multiple R-squared:  0.7298, Adjusted R-squared:  0.7283
## F-statistic: 507.7 on 1 and 188 DF,  p-value: < 2.2e-16
```

```r
# Residuals variability plot
plot(life_exp_lm$fitted.values, life_exp_lm$residuals,
     xlab="Fitted Values", ylab="Residuals",
     main="Residuals Plot")
abline(h=0)
```




```r
# Residuals Q-Q plot
qqnorm(life_exp_lm$residuals)
qqline(life_exp_lm$residuals)
```



Results

Residual standard error is 90,490,000 and **F-statistic** is 507.7. The F-statistic is good, but the SE is a bit high considering that it corresponds to 53.67 years if we reverse the transformation). R^2 is 0.7298, which is considerably better than in the first model (the model explains 72.98% of variability). **P-value** is again nearly 0, so the relationship is not due to random variation.

Looking at residuals plots, variability is fairly constant with a few outliers and distribution of residuals is nearly normal with some deviation at the tails. This is **a fairly good model** to describe the relationship and it is significantly better than the first model. The linear relationship between transformed variables is clear from the scatterplot.

Question 3

```
newdata <- data.frame(TotExp0.06=c(1.5,2.5))
predict(life_exp_lm, newdata,interval="predict")^(1/4.6)
```

```
##        fit      lwr      upr
## 1 63.31153 35.93545 73.00793
## 2 86.50645 81.80643 90.43414
```

Based on the second model, prediction for total expeditures of \$860.705 ($TotExp^{0.06} = 1.5$) is 63.31 years with a 95% confidence interval between 35.94 and 73.01.

Prediction for total expeditures of \$4,288,777 ($TotExp^{0.06} = 2.5$) is 86.51 years with a 95% confidence interval between 81.81 and 90.43.

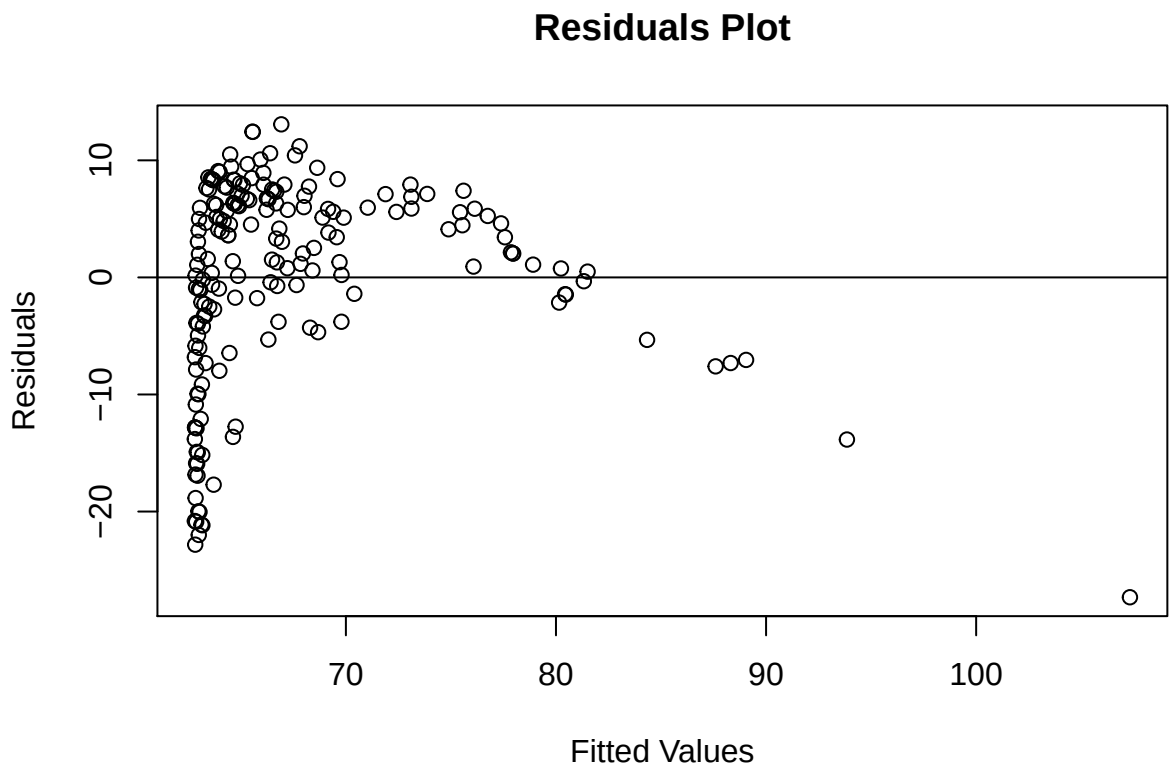
Question 4

Let us build the following model: $LifeExp = \beta_0 + \beta_1 \times PropMD + \beta_2 \times TotExp + \beta_3 \times PropMD \times TotExp$.

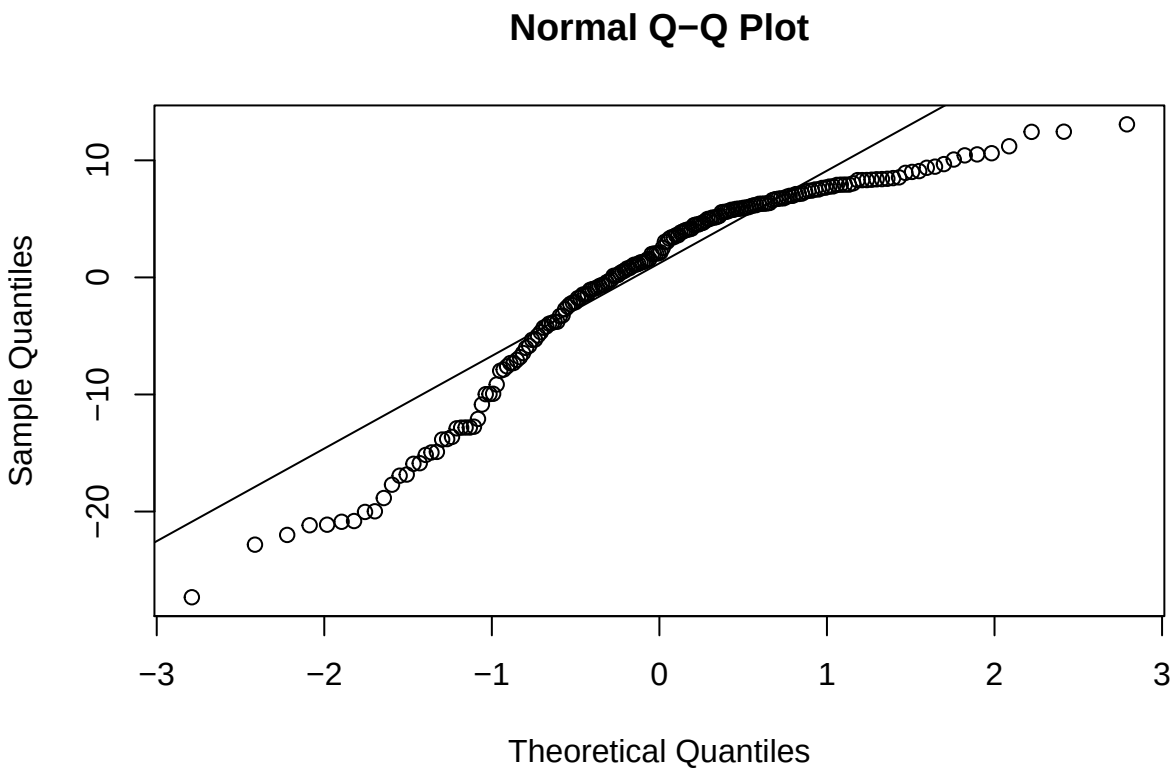
```
# Multiple linear regression model build
life_exp_lm <- lm(LifeExp ~ PropMD + TotExp + TotExp:PropMD, data=who)

# Linear regression model summary
summary(life_exp_lm)
```

```
##
## Call:
## lm(formula = LifeExp ~ PropMD + TotExp + TotExp:PropMD, data = who)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -27.320  -4.132   2.098   6.540  13.074
##
## Coefficients:
##                     Estimate     Std. Error t value           Pr(>|t|)
## (Intercept)     62.772703255    0.795605238  78.899            < 2e-16 ***
## PropMD        1497.493952519  278.816879652   5.371 0.0000002320602774 ***
## TotExp           0.000072333    0.000008982   8.053 0.0000000000000939 ***
## PropMD:TotExp   -0.006025686    0.001472357  -4.093 0.0000635273294941 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.765 on 186 degrees of freedom
## Multiple R-squared:  0.3574, Adjusted R-squared:  0.3471
## F-statistic: 34.49 on 3 and 186 DF,  p-value: < 2.2e-16
```

```
# Residuals variability plot
plot(life_exp_lm$fitted.values, life_exp_lm$residuals,
     xlab="Fitted Values", ylab="Residuals",
     main="Residuals Plot")
abline(h=0)
```




```
# Residuals Q-Q plot
qqnorm(life_exp_lm$residuals)
qqline(life_exp_lm$residuals)
```



Results

Residual standard error is 8.765 and **F-statistic** is 34.49. Considering that average life expectancy is 67.38, the SE is not terrible and F-statistics is fairly high (but lower than in the first model). R^2 is only 0.3574, so the model explains only 35.74% of variability, which is not high. **P-value** is nearly 0, so the relationship is not due to random variation.

Looking at residuals plots it is clear that there is no constant variability and that residuals are not normally distributed. This is **not a good model** to describe the relationship. Kind of similar to the first model.

Question 5

Consider forecast based on the last model with $PropMD = 0.03$ and $TotExp = 14$.

```
newdata <- data.frame(PropMD=0.03, TotExp=14)
predict(life_exp_lm, newdata,interval="predict")
```

```
##       fit      lwr      upr
## 1 107.696 84.24791 131.1441
```

The prediction is 107.70 years with 95% confidence interval between 84.25 and 131.14. The prediction is completely unrealistic. We do have individuals livings into their 100s; however, consider that the total expenditures of $14 is just a tad higher than the minimum value of $13 for Burundi and the life expectancy there is 49 years. The highest life expectancy in the data is 83 years. There is nothing in our data to support this prediction and it goes against common sense. As stated under question 4, this is not a good model.