DATA 605 Week 15 Homework

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QUICK ANSWERS

1. $y = -14.8 + 4.257x$
2. $(4, -2, 64), (-4, 2, -64)$: Both are saddle points.
3. $R(x, y) = 81x + 40y + 28xy - 21x^2 - 23y^2$ and $R(2.3, 4.1) = 116.62$
4. $x = 75$, $y = 21$
5. $\frac{1}{24}(e^{44} - e^{38} - e^{28} + e^{22})$

Problem 1

Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary.

$(5.6, 8.8), (6.3, 12.4), (7, 14.8), (7.7, 18.2), (8.4, 20.8)$

SOLUTION 1

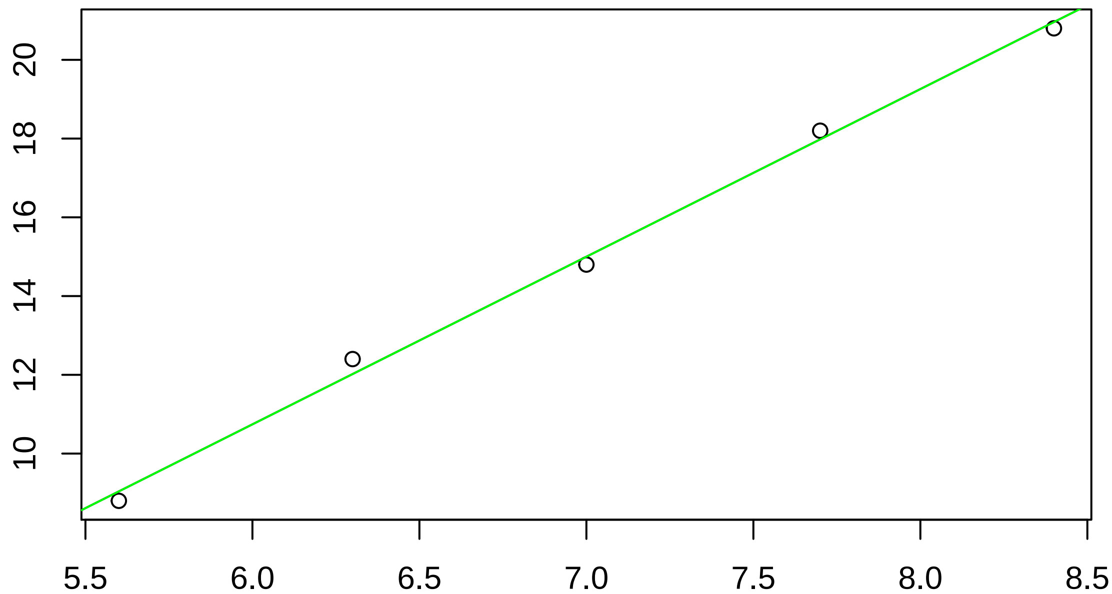
```
x <- c(5.6, 6.3, 7, 7.7, 8.4)
y <- c(8.8, 12.4, 14.8, 18.2, 20.8)

r <- lm(y~x)
r
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Coefficients:
## (Intercept)            x
##     -14.800        4.257
```

Based on the linear regression model, the regression line is $y = -14.8 + 4.257x$.

Check with plot.

```
plot(x,y, xlab="", ylab="")
abline(r)
lines(c(5,9), -14.8+4.257*c(5,9), col="green")
```



Problem 2

Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma.

$f(x, y) = 24x - 6xy^2 - 8y^3$

SOLUTION 2

Partial derivatives:

$f_x(x, y) = 24 - 6y^2$

$f_y(x, y) = -12xy - 24y^2$

f_x and f_y are never underfined.

If $24 - 6y^2 = 0$, then $y^2 = 4$ and $y = \pm 2$.

If $y = 2$ and $-12xy - 24y^2 = 0$, then $-24x = 24 \times 4$ and $x = -4$.

If $y = -2$ and $-12xy - 24y^2 = 0$, then $24x = 24 \times 4$ and $x = 4$.

Calculate $f(x, y)$.

$f(4, -2) = 24 \times 4 - 6 \times 4 \times (-2)^2 - 8 \times (-2)^3 = 64$

$f(-4, 2) = 24 \times (-4) - 6 \times (-4) \times 2^2 - 8 \times 2^3 = -64$

Two critical points: $(4, -2, 64)$ and $(-4, 2, -64)$.

Use Second Derivative test to determine if points are minimum, maximum or saddle.

Second partial derivatives:

$f_{xx} = 0$

$f_{yy} = -12x - 48y$

$f_{xy} = -12y$

Then $D(x, y) = f_{xx}f_{yy} - f_{xy}^2 = -(-12y)^2 = -144y^2$.

$D(x, y) < 0$ for all (x, y), so per Second Derivative Test, any critical point is a saddle point.

Problem 3

A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

Step 1. Find the revenue function $R(x, y)$. Step 2. What is the revenue if she sells the "house" brand for \$2.30 and the "name" brand for \$4.10?

SOLUTION 3

$$
\begin{aligned}
R(x, y) &= (81 - 21x + 17y)x + (40 + 11x - 23y)y \\
&= 81x - 21x^2 + 17xy + 40y + 11xy - 23y^2 \\
&= 81x + 40y + 28xy - 21x^2 - 23y^2
\end{aligned}
$$

$R(2.3, 4.1) = 81 \times 2.3 + 40 \times 4.1 + 28 \times 2.3 \times 4.1 - 21 \times (2.3)^2 - 23 \times (4.1)^2 = 116.62$

Problem 4

A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

SOLUTION 4

Consider $x + y = 96$, then $x = 96 - y$.

$$
\begin{aligned}
C(x, y) = C(96 - y, y) &= \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700 \\
&= \frac{1}{6}(96 - y)^2 + \frac{1}{6}y^2 + 7 \times (96 - y) + 25y + 700 \\
&= \frac{1}{6}(y^2 - 192y + 9216) + \frac{1}{6}y^2 + 672 - 7y + 25y + 700 \\
&= \frac{1}{6}y^2 - 32y + 1536 + \frac{1}{6}y^2 + 18y + 1372 \\
&= \frac{1}{3}y^2 - 14y + 2908 \\
&= C_1(y)
\end{aligned}
$$

$C_1'(y) = \frac{2}{3}y - 14$

To find the minimal value consider $C_1'(y) = \frac{2}{3}y - 14 = 0$, then $y = 21$. Then $x = 96 - y = 75$.

There should be 75 units produced in Los Angeles and 21 units produced in Denver.

Problem 5

Evaluate the double integral on the given region.

$$\int\int_R (e^{8x+3y})dA, R : 2 \leq x \leq 4 \ and \ 2 \leq y \leq 4$$

SOLUTION 5

$$\begin{aligned}
\int_2^4 \int_2^4 (e^{8x+3y}) \ dy \ dx &= \int_2^4 (\frac{1}{3}e^{8x+3y})|_2^4 \ dx \\
&= \int_2^4 ((\frac{1}{3}e^{8x+12}) - (\frac{1}{3}e^{8x+6})) \ dx \\
&= \int_2^4 \frac{1}{3}e^{8x+6}(e^6 - 1) \ dx \\
&= \frac{1}{24}e^{8x+6}(e^6 - 1)|_2^4 \\
&= \frac{1}{24}e^{32+6}(e^6 - 1) - \frac{1}{24}e^{16+6}(e^6 - 1) \\
&= \frac{1}{24}(e^6 - 1)(e^{38} - e^{22}) \\
&= \frac{1}{24}(e^{44} - e^{38} - e^{28} + e^{22})
\end{aligned}$$