DATA 605 Week 3 Homework

Ilya Kats

September 17, 2017

Problem Set 1

Part (1)

```r
A <- matrix(c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3), nrow=4, byrow=TRUE)

# Consider matrix A
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```r
# RREF of A
library(pracma)
rref(A)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

There are 4 pivot rows, so the rank of A is 4. Double-check with R.

```r
# Rank of A
Rank(A)
```

```
## [1] 4
```

Part (2)

Since rank of a matrix is defined as the number of linearly independent column vectors and is equal to the number of linearly independent row vectors, given an $m \times n$ matrix with $m > n$, the maximum rank is the lower value $rank_{max} = n$. Assuming a non-zero matrix, the rank should be at least 1, so $rank_{min} = 1$.

Part (3)

```r
B <- matrix(c(1,2,1,3,6,3,2,4,2), nrow=3, byrow=TRUE)

# Consider matrix B
B
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
```

```
## [2,]    3    6    3
## [3,]    2    4    2
```

```r
# Rank of B
Rank(B)
```

```
## [1] 1
```

In addition to getting an answer in R, we can see that row 2 and row 3 are multiples of row 1, so rows 1 and 2 and 1 and 3 are linearly dependent. There is only 1 linearly independent row, so $rank(B) = 1$.

Problem Set 2

```r
A <- matrix(c(1,2,3,0,4,5,0,0,6), nrow=3, byrow=TRUE)

# Consider matrix A
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    0    4    5
## [3,]    0    0    6
```

Since A is a **triangular matrix**, its eigenvalues are values on the diagonal, so $\lambda_1 = 1$, $\lambda_2 = 4$ and $\lambda_3 = 6$.

```r
# Double-check eigenvalues in R
eigen(A)$values
```

```
## [1] 6 4 1
```

The **characteristic polynomial** is $p_A(\lambda) = (1-\lambda)(4-\lambda)(6-\lambda)$ or $p_A(\lambda) = 24 - 34\lambda + 11\lambda^2 - \lambda^3$.

If $\lambda = 1$, then $A - 1I_3$ is row-reduced to

```r
rref(A - 1 * diag(3))
```

```
##      [,1] [,2] [,3]
## [1,]    0    1    0
## [2,]    0    0    1
## [3,]    0    0    0
```

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then $v_1 = v_1$ and $v_2 = 0$ and $v_3 = 0$. The **eigenspace** is

$$E_{\lambda=1} = \left\langle \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \right\rangle$$

If $\lambda = 4$, then $A - 4I_3$ is row-reduced to

```r
rref(A - 4 * diag(3))
```

```
##      [,1]       [,2] [,3]
## [1,]    1 -0.6666667    0
## [2,]    0  0.0000000    1
## [3,]    0  0.0000000    0
```

$$\begin{bmatrix} 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then $v_1 - \frac{2}{3}v_2 = 0$ and $v_3 = 0$.

Or $v_1 = v_1$ and $v_2 = \frac{3}{2}v_1 = 1.5v_1$ and $v_3 = 0$.

The **eigenspace** is

$$E_{\lambda=4} = \left\langle \left\{ \begin{bmatrix} 1 \\ 1.5 \\ 0 \end{bmatrix} \right\} \right\rangle$$

Finally, if $\lambda = 6$, then $A - 6I_3$ is row-reduced to

```r
rref(A - 6 * diag(3))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0 -1.6
## [2,]    0    1 -2.5
## [3,]    0    0  0.0
```

$$\begin{bmatrix} 1 & 0 & -1.6 \\ 0 & 1 & -2.5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then $v_1 - 1.6v_3 = 0$ and $v_2 - 2.5v_3 = 0$.

Or $v_1 = 1.6v_3$ and $v_2 = 2.5v_3$ and $v_3 = v_3$.

The **eigenspace** is

$$E_{\lambda=6} = \left\langle \left\{ \begin{bmatrix} 1.6 \\ 2.5 \\ 1 \end{bmatrix} \right\} \right\rangle$$