Multiple linear regression

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Grading the professor

Many college courses conclude by giving students the opportunity to evaluate the course and the instructor anonymously. However, the use of these student evaluations as an indicator of course quality and teaching effectiveness is often criticized because these measures may reflect the influence of non-teaching related characteristics, such as the physical appearance of the instructor. The article titled, "Beauty in the classroom: instructors' pulchritude and putative pedagogical productivity" (Hamermesh and Parker, 2005) found that instructors who are viewed to be better looking receive higher instructional ratings. (Daniel S. Hamermesh, Amy Parker, Beauty in the classroom: instructors pulchritude and putative pedagogical productivity, *Economics of Education Review*, Volume 24, Issue 4, August 2005, Pages 369-376, ISSN 0272-7757, 10.1016/j.econedurev.2004.07.013. http://www.sciencedirect.com/science/article/pii/S0272775704001165.)

In this lab we will analyze the data from this study in order to learn what goes into a positive professor evaluation.

The data

The data were gathered from end of semester student evaluations for a large sample of professors from the University of Texas at Austin. In addition, six students rated the professors' physical appearance. (This is aslightly modified version of the original data set that was released as part of the replication data for *Data Analysis Using Regression and Multilevel/Hierarchical Models* (Gelman and Hill, 2007).) The result is a data frame where each row contains a different course and columns represent variables about the courses and professors.

```
load("more/evals.RData")
```

variable	description
`score`	average professor evaluation score: (1) very unsatisfactory - (5) excellent.
`rank`	rank of professor: teaching, tenure track, tenured.
`ethnicity`	ethnicity of professor: not minority, minority.
`gender`	gender of professor: female, male.

variable	description
`language`	language of school where professor received education: english or non-english.
`age`	age of professor.
`cls_perc_eval`	percent of students in class who completed evaluation.
`cls_did_eval`	number of students in class who completed evaluation.
`cls_students`	total number of students in class.
`cls_level`	class level: lower, upper.
`cls_profs`	number of professors teaching sections in course in sample: single, multiple.
`cls_credits`	number of credits of class: one credit (lab, PE, etc.), multi credit.
`bty_f1lower`	beauty rating of professor from lower level female: (1) lowest - (10) highest.
`bty_f1upper`	beauty rating of professor from upper level female: (1) lowest - (10) highest.
`bty_f2upper`	beauty rating of professor from second upper level female: (1) lowest - (10) highest.

| variable	description
`bty_m1lower`	beauty rating of professor from lower level male: (1) lowest - (10) highest.
`bty_m1upper`	beauty rating of professor from upper level male: (1) lowest - (10) highest.
`bty_m2upper`	beauty rating of professor from second upper level male: (1) lowest - (10) highest.
`bty_avg`	average beauty rating of professor.
`pic_outfit`	outfit of professor in picture: not formal, formal.
`pic_color`	color of professor's picture: color, black & white.

Exploring the data

1. Is this an observational study or an experiment? The original research question posed in the paper is whether beauty leads directly to the differences in course evaluations. Given the study design, is it possible to answer this question as it is phrased? If not, rephrase the question.

This is an observational study. As such it cannot show causation, but only association. Therefore, the original research question cannot be answered with this study. A better question would be "Is there a relationship between beauty and differences in course evaluations?"

2. Describe the distribution of `score`. Is the distribution skewed? What does that tell you about how students rate courses? Is this what you expected to see? Why, or why not?

```
hist(evals$score)
```

Histogram of evals$score
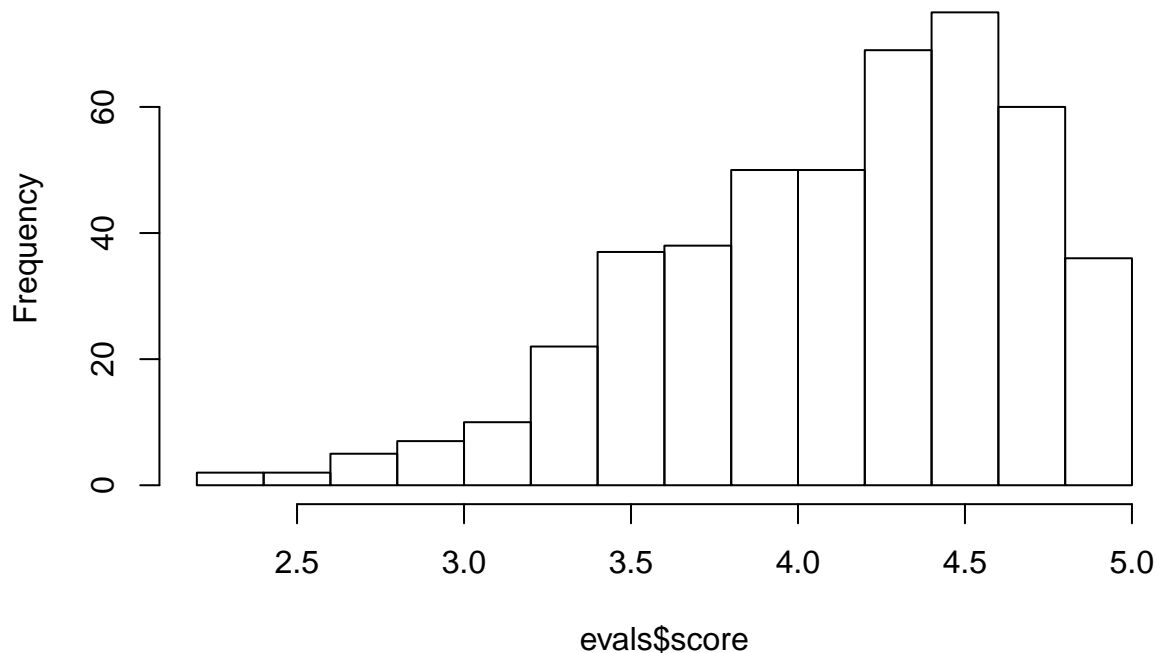
Based on the histogram, the distribution of `score` is unimodal and left-skewed, centered around 4.5. Most students give high marks to courses with fewer students giving low marks. This matches my expectations. If there was a significant number of students dissatisfied with courses, there would be bigger issues to discuss at the University of Texas.

3. Excluding `score`, select two other variables and describe their relationship using an appropriate visualization (scatterplot, side-by-side boxplots, or mosaic plot).

I have chosen `age` and `language`. Since 'age' is numeric and continuous and `language` is categorical with two categories, side-by-side boxplots may be the best visualization to show the relationship (see below). The mean age for two categories is very similar and IQR ranges are almost the same. The age of English-speaking professor reaches higher than the non-English speaking professors, but I believe this is not a significant difference. There doesn't seem to be a significant relationship between these two variables. It is important to note that we have significantly less number of non-English speaking professors (28) than English seaking (435).

```
plot(evals$age ~ evals$language,
     xlab = "Language", ylab = "Age")
```
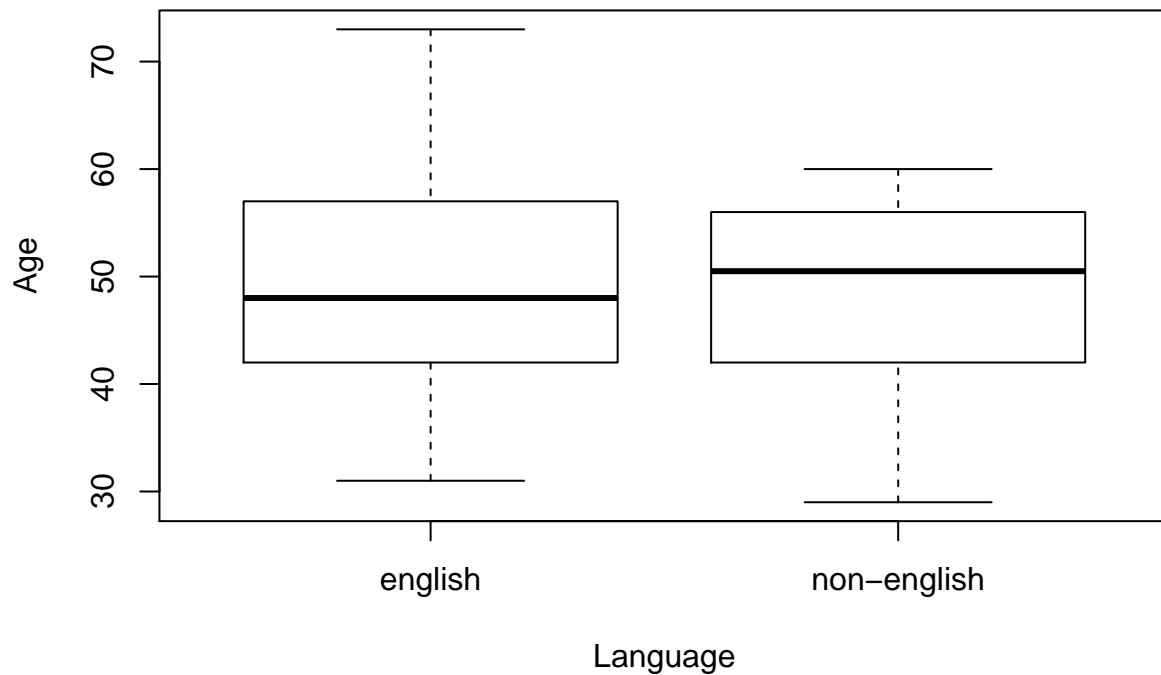
```r
table(evals$language)
```

```
## 
##     english non-english 
##         435          28
```

Simple linear regression

The fundamental phenomenon suggested by the study is that better looking teachers are evaluated more favorably. Let's create a scatterplot to see if this appears to be the case:

```r
plot(evals$score ~ evals$bty_avg)
```

Before we draw conclusions about the trend, compare the number of observations in the data frame with the approximate number of points on the scatterplot. Is anything awry?

4. Replot the scatterplot, but this time use the function `jitter()` on the y- or the x-coordinate. (Use `?jitter` to learn more.) What was misleading about the initial scatterplot?

```r
plot(jitter(evals$score) ~ evals$bty_avg)
```
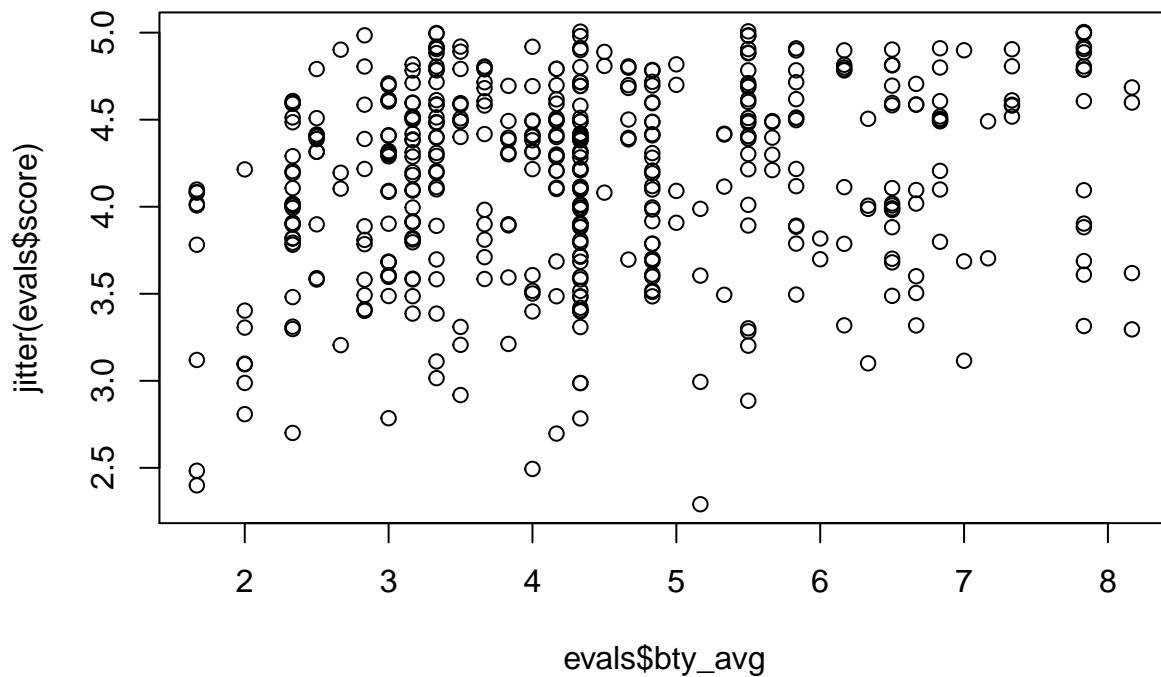
The initial scatterplot displayed significantly less points than we have observations. By utilizing `jitter()` it is shown that this was due to multiple observations having the same `bty_avg` and `score` values, so points were plotted on top of one another.

5. Let's see if the apparent trend in the plot is something more than natural variation. Fit a linear model called `m_bty` to predict average professor score by average beauty rating and add the line to your plot using `abline(m_bty)`. Write out the equation for the linear model and interpret the slope. Is average beauty score a statistically significant predictor? Does it appear to be a practically significant predictor?

```
m_bty <- lm(evals$score ~ evals$bty_avg)
plot(jitter(evals$score) ~ evals$bty_avg)
abline(m_bty)
```
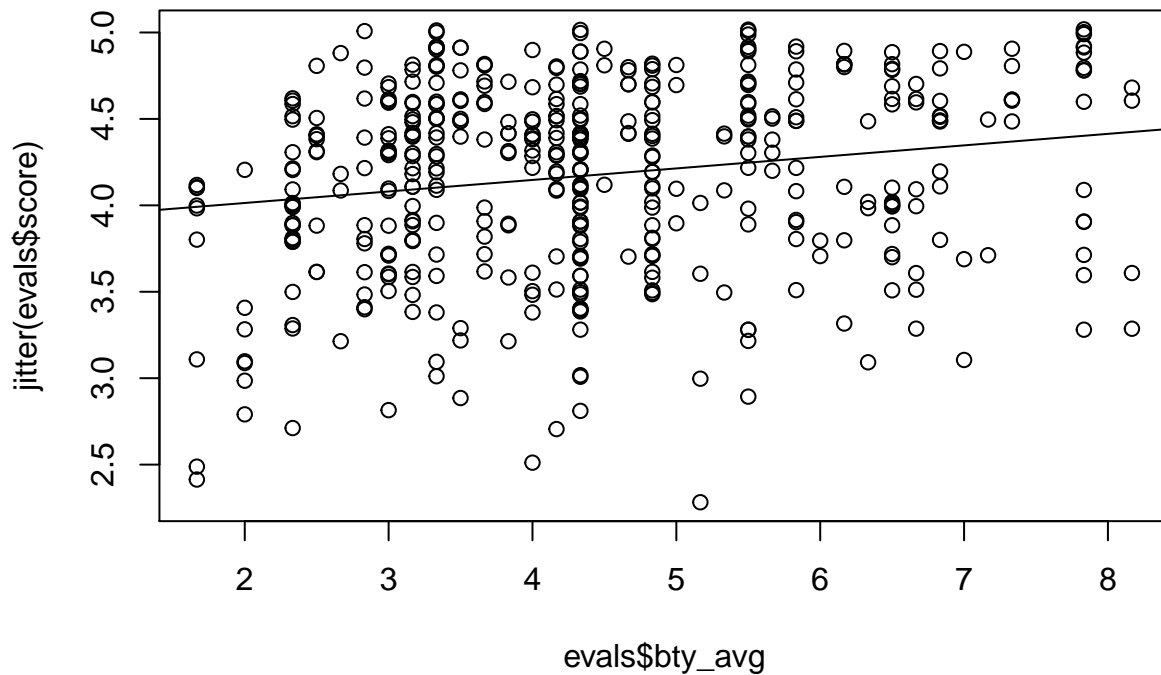
```
summary(m_bty)
```

```
##
## Call:
## lm(formula = evals$score ~ evals$bty_avg)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9246 -0.3690  0.1420  0.3977  0.9309
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3.88034    0.07614   50.96  < 2e-16 ***
## evals$bty_avg  0.06664    0.01629    4.09 5.08e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5348 on 461 degrees of freedom
## Multiple R-squared:  0.03502,    Adjusted R-squared:  0.03293
## F-statistic: 16.73 on 1 and 461 DF,  p-value: 5.083e-05
```

Linear model equation: $\widehat{score} = 3.88 + 0.067 \times avg\ beauty\ rating$

The equation states that for every point in the average beauty rating, the score increases by **0.067.** The $p-value$ from the summary above is close to zero, which means that even at a significance level of **0.001,** the slope is not due to chance and there is a statistically significant relationship. However, it can be argued that practically the increase is so small as to be

insignificant.

6. Use residual plots to evaluate whether the conditions of least squares regression are reasonable. Provide plots and comments for each one (see the Simple Regression Lab for a reminder of how to make these).
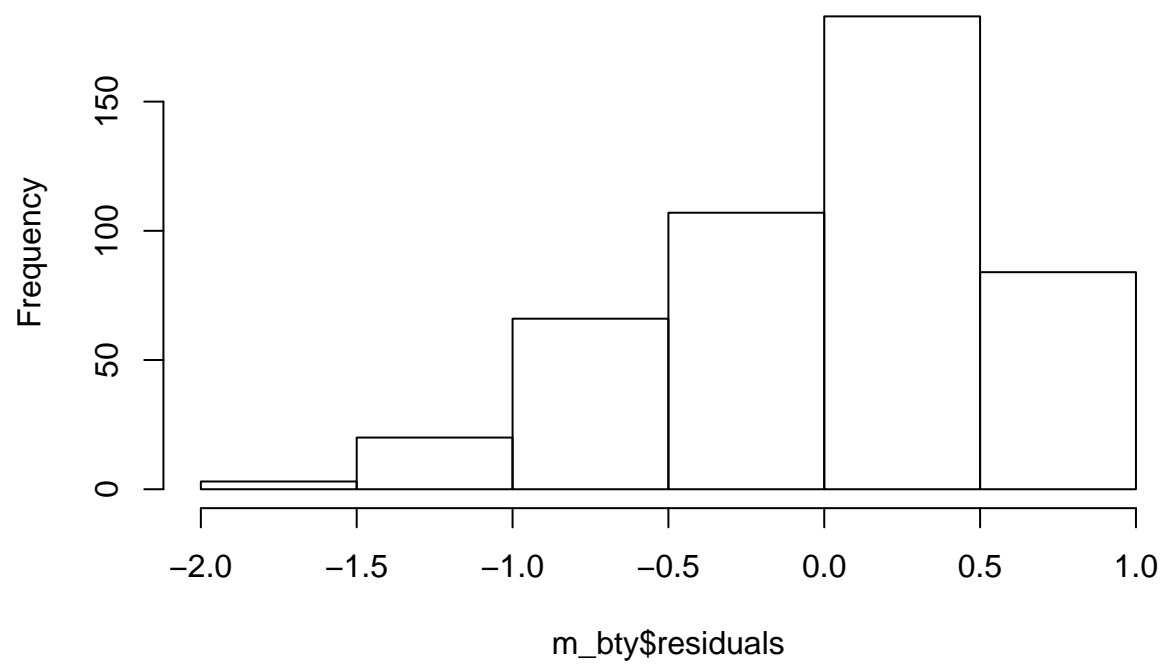
```
plot(m_bty$residuals ~ evals$bty_avg)
abline(h = 0, lty = 3)
```
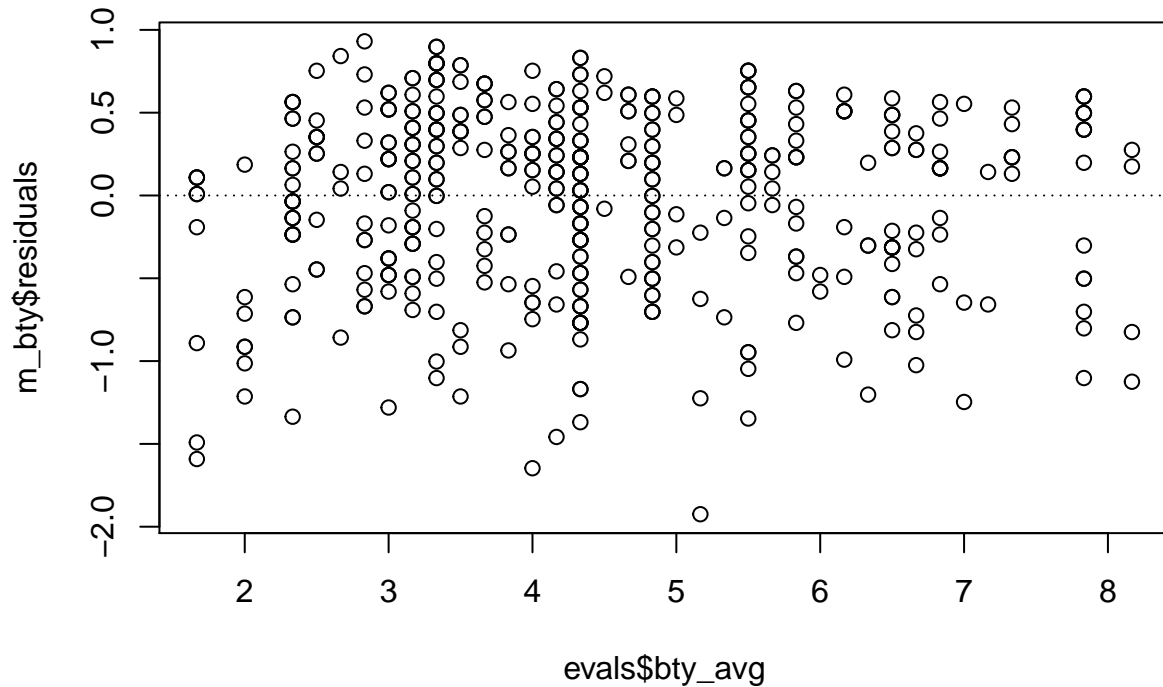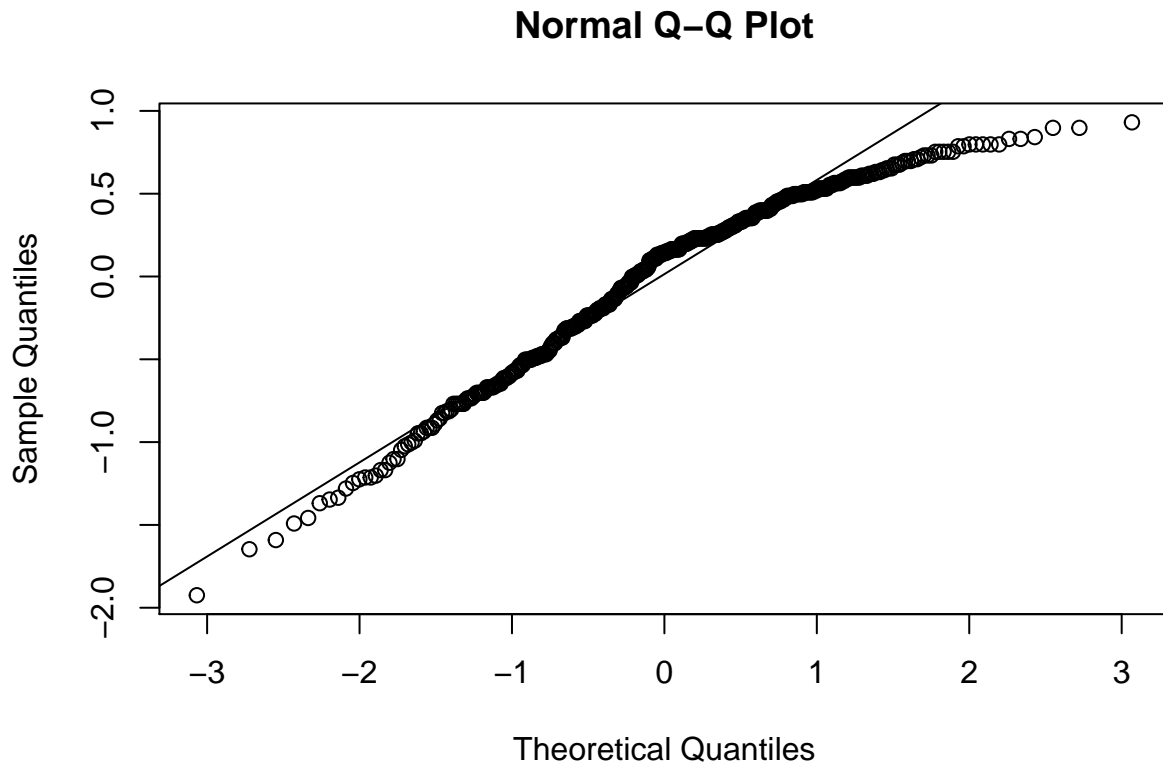

There is no apparent pattern in the residuals plot, so the relationship appears to be linear. This plot also demonstrates that there is constant variability of points.

```
hist(m_bty$residuals)
```

Histogram of m_bty$residuals


```
qqnorm(m_bty$residuals)
qqline(m_bty$residuals)
```

Normal Q–Q Plot

The histogram of residuals appears to be left-skewed. The normal probability plot also deviates from the diagonal line on both ends. I believe it may be reasonably said that the residulas are not normally distributed.

Also, not knowing more about the survey, we have to assume that each student acted independently. It is not unreasonable to think that some students were influenced by their fellow students in scoring either courses or beauty.

Multiple linear regression

The data set contains several variables on the beauty score of the professor: individual ratings from each of the six students who were asked to score the physical appearance of the professors and the average of these six scores. Let's take a look at the relationship between one of these scores and the average beauty score.

```
plot(evals$bty_avg ~ evals$bty_f1lower)
cor(evals$bty_avg, evals$bty_f1lower)
```

As expected the relationship is quite strong - after all, the average score is calculated using the individual scores. We can actually take a look at the relationships between all beauty variables (columns 13 through 19) using the following command:

```
plot(evals[,13:19])
```

These variables are collinear (correlated), and adding more than one of these variables to the model would not add much value to the model. In this application and with these highly-correlated predictors, it is reasonable to use the average beauty score as the single representative of these variables.

In order to see if beauty is still a significant predictor of professor score after we've accounted for the gender of the professor, we can add the gender term into the model.

```
m_bty_gen <- lm(score ~ bty_avg + gender, data = evals)
summary(m_bty_gen)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + gender, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8305 -0.3625  0.1055  0.4213  0.9314
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.74734    0.08466  44.266  < 2e-16 ***
## bty_avg      0.07416    0.01625   4.563 6.48e-06 ***
## gendermale   0.17239    0.05022   3.433 0.000652 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5287 on 460 degrees of freedom
## Multiple R-squared:  0.05912,    Adjusted R-squared:  0.05503
## F-statistic: 14.45 on 2 and 460 DF,  p-value: 8.177e-07
```

7. P-values and parameter estimates should only be trusted if the conditions for the regression are reasonable. Verify that the conditions for this model are reasonable using diagnostic plots.

```
qqnorm(m_bty_gen$residuals)
qqline(m_bty_gen$residuals)
```

Normal Q–Q Plot
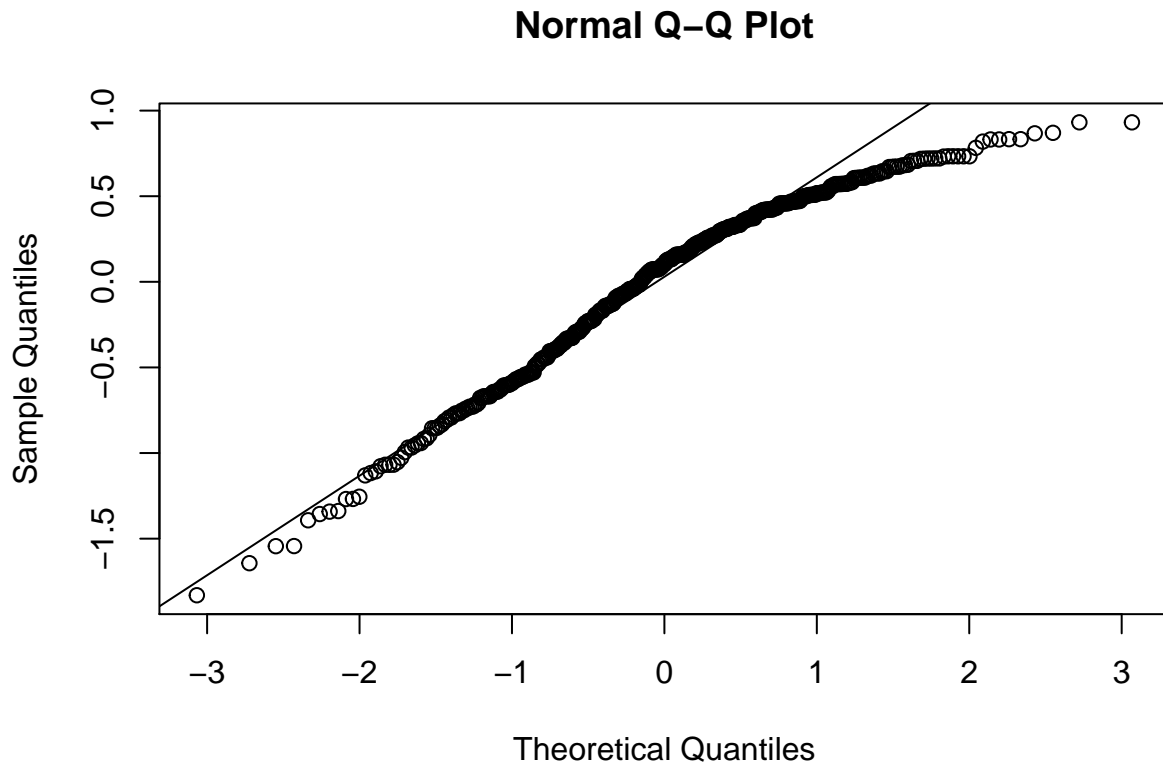
Residuals deviate significantly from the normal probability line and the distribution seems to be left-skewed. I believe that residuals of the model are not nearly normal. For the purposes of this exercise, I will continue with assumption that the distribution of residuals is nearly normal.

```r
plot(abs(m_bty_gen$residuals) ~ m_bty_gen$fitted.values,
     xlab = "Fitted Values", ylab = "Absolute Value of Residuals")
```
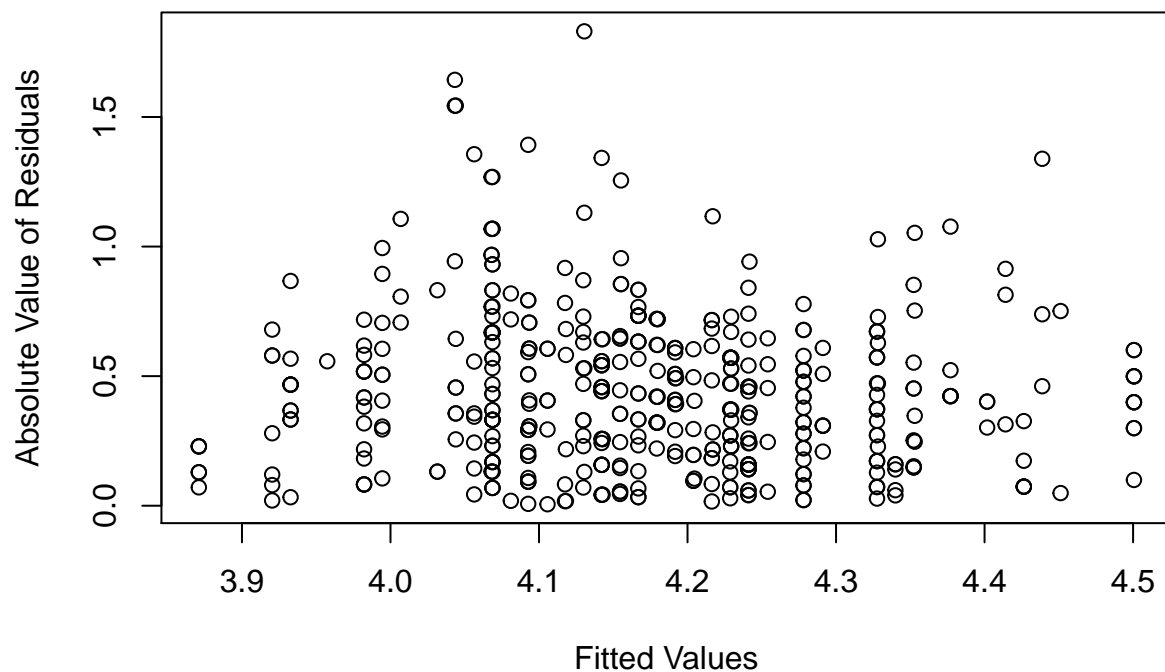
There is limited variability of the residuals on both ends. Given the left-skew of the residuals, I would tend to doubt nearly constant variability as well.

Similar to the first model, I will assume independence of observations.

```
plot(m_bty_gen$residuals ~ evals$gender)
```
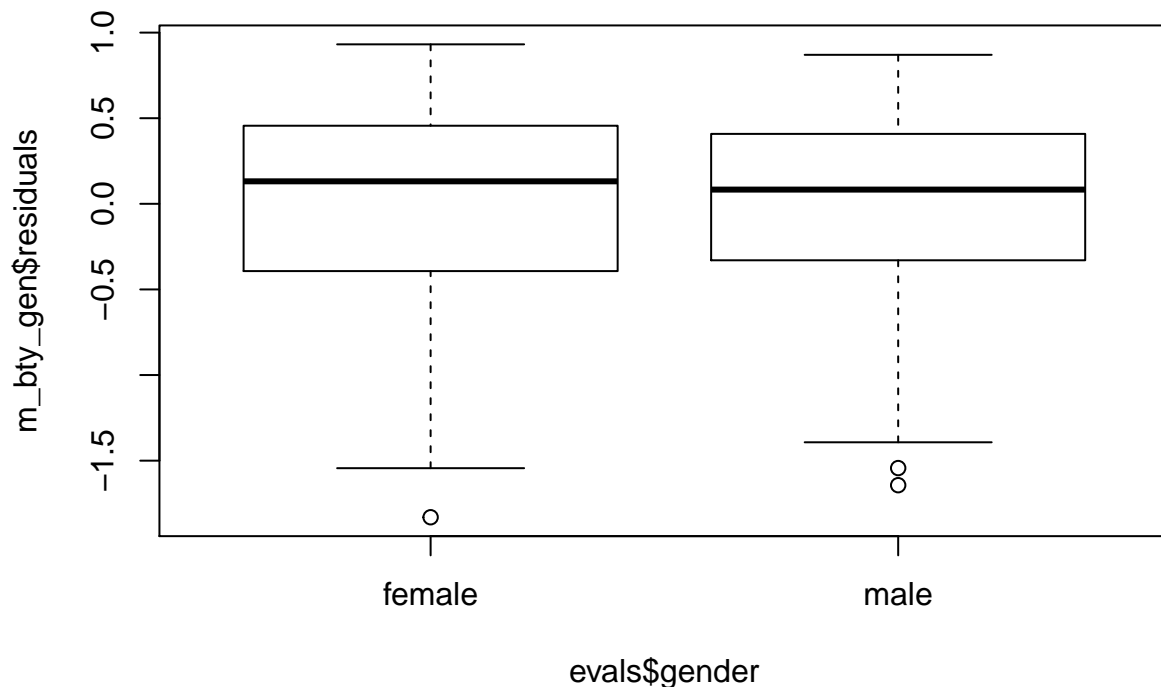
Variability does not change much with the `gender` variable.

I believe that the residuals are not near normal and variability is not necessarily nearly constant. I would be careful about reporting the results of this model.

8. Is `bty_avg` still a significant predictor of `score`? Has the addition of `gender` to the model changed the parameter estimate for `bty_avg`?

The parameter estimate for `bty_avg` has changed with addition of the `gender` variable, but it remains significant predictor of `score`.

Note that the estimate for `gender` is now called `gendermale`. You'll see this name change whenever you introduce a categorical variable. The reason is that R recodes `gender` from having the values of `female` and `male` to being an indicator variable called `gendermale` that takes a value of 0 for females and a value of 1 for males. (Such variables are often referred to as "dummy" variables.)

As a result, for females, the parameter estimate is multiplied by zero, leaving the intercept and slope form familiar from simple regression.

$$\widehat{score} = \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg + \hat{\beta}_2 \times (0)$$
$$= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg$$

We can plot this line and the line corresponding to males with the following custom function.

```
multiLines(m_bty_gen)
```

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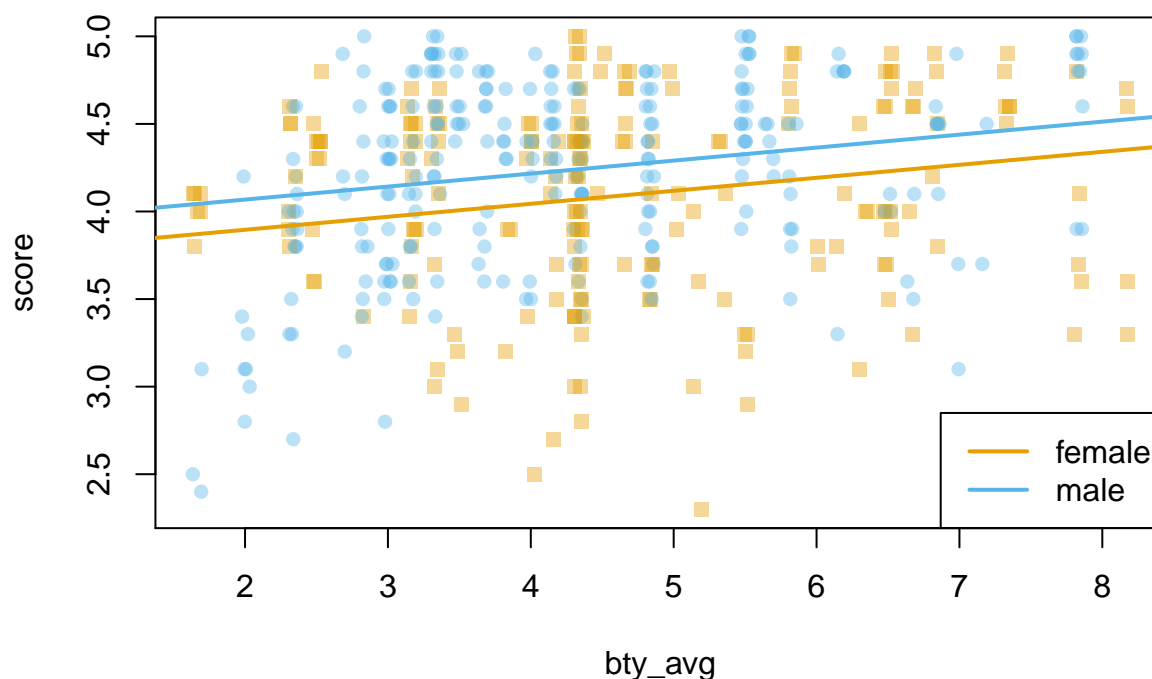
9. What is the equation of the line corresponding to males? (*Hint:* For males, the parameter estimate is multiplied by 1.) For two professors who received the same beauty rating, which gender tends to have the higher course evaluation score?

Equation of the line corresponding to males: $\widehat{score} = \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg + \hat{\beta}_2$

For two professors who received the same beauty rating, evaluation score from males tends to be higher.

The decision to call the indicator variable `gendermale` instead of `genderfemale` has no deeper meaning. R simply codes the category that comes first alphabetically as a 0. (You can change the reference level of a categorical variable, which is the level that is coded as a 0, using the `relevel` function. Use `?relevel` to learn more.)

10. Create a new model called `m_bty_rank` with `gender` removed and `rank` added in. How does R appear to handle categorical variables that have more than two levels? Note that the rank variable has three levels: `teaching`, `tenure track`, `tenured`.

```
m_bty_rank <- lm(score ~ bty_avg + rank, data = evals)
summary(m_bty_rank)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + rank, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8713 -0.3642  0.1489  0.4103  0.9525
##
```

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```
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        3.98155    0.09078  43.860  < 2e-16 ***
## bty_avg            0.06783    0.01655   4.098 4.92e-05 ***
## ranktenure track  -0.16070    0.07395  -2.173   0.0303 *
## ranktenured       -0.12623    0.06266  -2.014   0.0445 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5328 on 459 degrees of freedom
## Multiple R-squared:  0.04652,    Adjusted R-squared:  0.04029
## F-statistic: 7.465 on 3 and 459 DF,  p-value: 6.88e-05
```

R splits the `rank` variable with three levels into two dichotomous variables.

The interpretation of the coefficients in multiple regression is slightly different from that of simple regression. The estimate for `bty_avg` reflects how much higher a group of professors is expected to score if they have a beauty rating that is one point higher *while holding all other variables constant.* In this case, that translates into considering only professors of the same rank with `bty_avg` scores that are one point apart.

The search for the best model

We will start with a full model that predicts professor score based on rank, ethnicity, gender, language of the university where they got their degree, age, proportion of students that filled out evaluations, class size, course level, number of professors, number of credits, average beauty rating, outfit, and picture color.

11. Which variable would you expect to have the highest p-value in this model? Why? *Hint:* Think about which variable would you expect to not have any association with the professor score.

Peeking ahead `cls_profs`, number of professors, has the highest p-value of the full model. Evaluations are for individual professors and their score should not depend on the fact that other sections of the course had other professors.

Let's run the model...

```
m_full <- lm(score ~ rank + ethnicity + gender + language + age + cls_perc_eval
            + cls_students + cls_level + cls_profs + cls_credits + bty_avg
            + pic_outfit + pic_color, data = evals)
summary(m_full)
```

```
##
## Call:
## lm(formula = score ~ rank + ethnicity + gender + language + age +
##     cls_perc_eval + cls_students + cls_level + cls_profs + cls_credits +
##     bty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.77397 -0.32432  0.09067  0.35183  0.95036
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)           4.0952141  0.2905277  14.096  < 2e-16 ***
## ranktenure track     -0.1475932  0.0820671  -1.798  0.07278 .
## ranktenured          -0.0973378  0.0663296  -1.467  0.14295
## ethnicitynot minority 0.1234929  0.0786273   1.571  0.11698
```

```
## gendermale              0.2109481   0.0518230    4.071 5.54e-05 ***
## languagenon-english    -0.2298112   0.1113754   -2.063  0.03965 *
## age                    -0.0090072   0.0031359   -2.872  0.00427 **
## cls_perc_eval           0.0053272   0.0015393    3.461  0.00059 ***
## cls_students            0.0004546   0.0003774    1.205  0.22896
## cls_levelupper          0.0605140   0.0575617    1.051  0.29369
## cls_profssingle        -0.0146619   0.0519885   -0.282  0.77806
## cls_creditsone credit   0.5020432   0.1159388    4.330 1.84e-05 ***
## bty_avg                 0.0400333   0.0175064    2.287  0.02267 *
## pic_outfitnot formal   -0.1126817   0.0738800   -1.525  0.12792
## pic_colorcolor         -0.2172630   0.0715021   -3.039  0.00252 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.498 on 448 degrees of freedom
## Multiple R-squared:  0.1871, Adjusted R-squared:  0.1617
## F-statistic: 7.366 on 14 and 448 DF,  p-value: 6.552e-14
```

12. Check your suspicions from the previous exercise. Include the model output in your response.

As I mentioned in answer to previous question, I peeked ahead.

13. Interpret the coefficient associated with the ethnicity variable.

With all other variables remaining constant, the score will increase by 0.12 if the professor is not part of a minority group.

14. Drop the variable with the highest p-value and re-fit the model. Did the coefficients and significance of the other explanatory variables change? (One of the things that makes multiple regression interesting is that coefficient estimates depend on the other variables that are included in the model.) If not, what does this say about whether or not the dropped variable was collinear with the other explanatory variables?

```
m_no_profs <- lm(score ~ rank + ethnicity + gender + language + age + cls_perc_eval
             + cls_students + cls_level + cls_credits + bty_avg
             + pic_outfit + pic_color, data = evals)
summary(m_no_profs)
```

```
##
## Call:
## lm(formula = score ~ rank + ethnicity + gender + language + age +
##     cls_perc_eval + cls_students + cls_level + cls_credits +
##     bty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.7836 -0.3257  0.0859  0.3513  0.9551
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)          4.0872523  0.2888562  14.150  < 2e-16 ***
## ranktenure track    -0.1476746  0.0819824  -1.801 0.072327 .
## ranktenured         -0.0973829  0.0662614  -1.470 0.142349
## ethnicitynot minority 0.1274458  0.0772887   1.649 0.099856 .
## gendermale           0.2101231  0.0516873   4.065 5.66e-05 ***
## languagenon-english -0.2282894  0.1111305  -2.054 0.040530 *
## age                 -0.0089992  0.0031326  -2.873 0.004262 **
```

```
## cls_perc_eval          0.0052888  0.0015317   3.453 0.000607 ***
## cls_students           0.0004687  0.0003737   1.254 0.210384
## cls_levelupper         0.0606374  0.0575010   1.055 0.292200
## cls_creditsone credit  0.5061196  0.1149163   4.404 1.33e-05 ***
## bty_avg                0.0398629  0.0174780   2.281 0.023032 *
## pic_outfitnot formal  -0.1083227  0.0721711  -1.501 0.134080
## pic_colorcolor        -0.2190527  0.0711469  -3.079 0.002205 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4974 on 449 degrees of freedom
## Multiple R-squared:  0.187,  Adjusted R-squared:  0.1634
## F-statistic: 7.943 on 13 and 449 DF,  p-value: 2.336e-14
```

The coefficients and significance levels changed for all variables with the removel of `cls_profs`.

15. Using backward-selection and p-value as the selection criterion, determine the best model. You do not need to show all steps in your answer, just the output for the final model. Also, write out the linear model for predicting score based on the final model you settle on.

**Based on the significance level of 0.05, the best model is below.**

```
m_best <- lm(score ~ ethnicity + gender + language + age + cls_perc_eval
          + cls_credits + bty_avg + pic_color, data = evals)
summary(m_best)
```

```
##
## Call:
## lm(formula = score ~ ethnicity + gender + language + age + cls_perc_eval +
##     cls_credits + bty_avg + pic_color, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.85320 -0.32394  0.09984  0.37930  0.93610
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           3.771922   0.232053  16.255  < 2e-16 ***
## ethnicitynot minority 0.167872   0.075275   2.230  0.02623 *
## gendermale            0.207112   0.050135   4.131 4.30e-05 ***
## languagenon-english  -0.206178   0.103639  -1.989  0.04726 *
## age                  -0.006046   0.002612  -2.315  0.02108 *
## cls_perc_eval         0.004656   0.001435   3.244  0.00127 **
## cls_creditsone credit 0.505306   0.104119   4.853 1.67e-06 ***
## bty_avg               0.051069   0.016934   3.016  0.00271 **
## pic_colorcolor       -0.190579   0.067351  -2.830  0.00487 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4992 on 454 degrees of freedom
## Multiple R-squared:  0.1722, Adjusted R-squared:  0.1576
## F-statistic:  11.8 on 8 and 454 DF,  p-value: 2.58e-15
```

$\widehat{score} = 3.77 + 0.17 \times ethnicity + 0.21 \times gender - 0.21 \times language - 0.01 \times age + 0.005 \times cls\_perc\_eval + 0.51 \times cls\_credits + 0.05 \times bty\_avg - 0.19 \times pic\_color$

16. Verify that the conditions for this model are reasonable using diagnostic plots.

```r
qqnorm(m_best$residuals)
qqline(m_best$residuals)
```
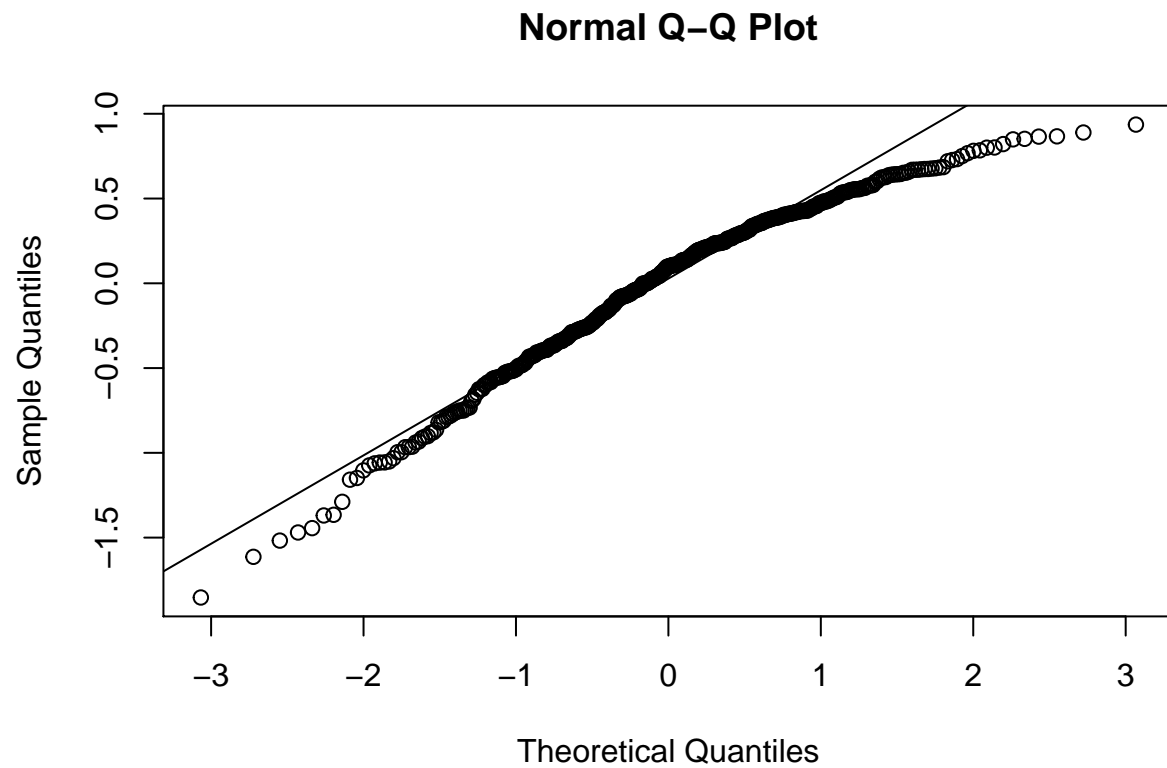
Normal Q–Q Plot


```r
plot(abs(m_bty_gen$residuals) ~ m_bty_gen$fitted.values,
     xlab = "Fitted Values", ylab = "Absolute Value of Residuals")
```
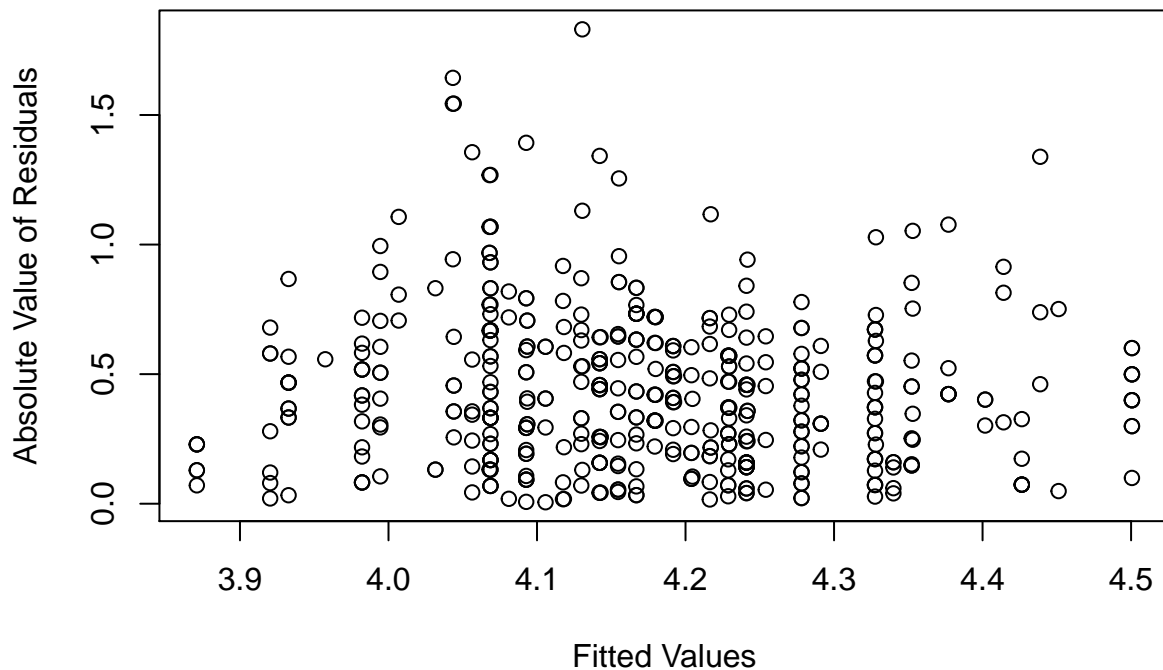
Similar to the beauty ranking/gender model above, I do not believe that the residuals are near normal and I doubt constant variability of the model. I am not sure the conditions are reasonable.

17. The original paper describes how these data were gathered by taking a sample of professors from the University of Texas at Austin and including all courses that they have taught. Considering that each row represents a course, could this new information have an impact on any of the conditions of linear regression?

I think this new information may influence the assumptions concerning independence of observations, but it should not influence other conditions.

18. Based on your final model, describe the characteristics of a professor and course at University of Texas at Austin that would be associated with a high evaluation score.

Based on the model, the highest score will be associated with professors who

- is not part of a minority group,
- is male,
- was taught in an English speaking school,
- on a younger side,
- has a high percentage of students filling out evaluations,
- teaches one credit courses,
- has a high beauty ranking,
- has a black & white picture.

19. Would you be comfortable generalizing your conclusions to apply to professors generally (at any university)? Why or why not?

I would not be comfortable generalizing a model from one university to professors generally.

There may be significant regional differences and other factors (university size, type, ranking, etc.) that influence the score. One university is not a good enough sample for generalization.