# DATA 621 Homework 5

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## Summary

The goal of the project is to test approaches for evaluating the data set containing various wine characteristics and to predict the number of cases ordered. This report includes comparison of linear, poisson, negative binomial regression models (including zero-inflated negative binomial model).

There are some questions about real-world validity of evaluation data since there are a lot of negative values for variables that should not have them. Earlier in the analysis, an attempt was made to use the absolute value for modeling, but that did not improve models and generally does not fit the distribution of variables.

## **Data Exploration**

The data set includes 12,795 observations with 14 variables (excluding the target variable).

#### **Summary of Variables**

The data set includes 14 independent variables:

- AcidIndex: Proprietary method of testing total acidity of wine by using a weighted average.
- Alcohol: Alcohol content of wine.
- Chlorides: Chloride content of wine.
- CitricAcid: Citric acid content of wine.
- Density: Density of wine.
- FixedAcidity: Fixed Acidity of wine.
- FreeSulfurDioxide: Sulfur dioxide content of wine.
- LabelAppeal: Marketing score indicating the appeal of label design for consumers.
- ResidualSugar: Residual sugar of wine.
- STARS: Wine rating by a team of experts. Ranges from 1 (Poor) to 4 (Excellent) stars.
- Sulphates: Sulfate content of wine.
- TotalSulfurDioxide: Total sulfur dDioxide of wine.
- Volatile Acidity: Volatile acid content of wine.
- pH: pH of wine

Dependent variable is TARGET representing number of cases of wine purchased.

The table below shows summary of all variables.

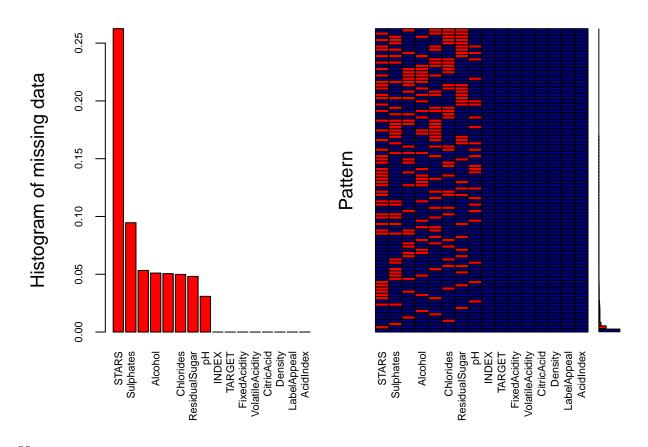
| Variable             | Class   | Min    | Median | Mean      | SD      | Max   |
|----------------------|---------|--------|--------|-----------|---------|-------|
| FixedAcidity         | numeric | -18.1  | 6.9    | 7.076     | 6.318   | 34.4  |
| VolatileAcidity      | numeric | -2.79  | 0.28   | 0.3241    | 0.784   | 3.68  |
| CitricAcid           | numeric | -3.24  | 0.31   | 0.3084    | 0.8621  | 3.86  |
| ResidualSugar        | numeric | -127.8 | 3.9    | 5.419     | 33.75   | 141.2 |
| Chlorides            | numeric | -1.171 | 0.046  | 0.05482   | 0.3185  | 1.351 |
| FreeSulfurDioxide    | numeric | -555   | 30     | 30.85     | 148.7   | 623   |
| TotalSulfurDioxide   | numeric | -823   | 123    | 120.7     | 231.9   | 1057  |
| Density              | numeric | 0.8881 | 0.9945 | 0.9942    | 0.02654 | 1.099 |
| $_{ m pH}$           | numeric | 0.48   | 3.2    | 3.208     | 0.6797  | 6.13  |
| Sulphates            | numeric | -3.13  | 0.5    | 0.5271    | 0.9321  | 4.24  |
| Alcohol              | numeric | -4.7   | 10.4   | 10.49     | 3.728   | 26.5  |
| ${\it Label Appeal}$ | integer | -2     | 0      | -0.009066 | 0.8911  | 2     |
| AcidIndex            | integer | 4      | 8      | 7.773     | 1.324   | 17    |
| STARS                | integer | 1      | 2      | 2.042     | 0.9025  | 4     |

| Variable           | Num of NAs | Num of Zeros | Num of Neg Values |
|--------------------|------------|--------------|-------------------|
| FixedAcidity       | 0          | 39           | 1621              |
| VolatileAcidity    | 0          | 18           | 2827              |
| CitricAcid         | 0          | 115          | 2966              |
| ResidualSugar      | 616        | 6            | 3136              |
| Chlorides          | 638        | 5            | 3197              |
| FreeSulfurDioxide  | 647        | 11           | 3036              |
| TotalSulfurDioxide | 682        | 7            | 2504              |
| Density            | 0          | 0            | 0                 |
| pН                 | 395        | 0            | 0                 |
| Sulphates          | 1210       | 22           | 2361              |
| Alcohol            | 653        | 2            | 118               |
| LabelAppeal        | 0          | 5617         | 3640              |
| AcidIndex          | 0          | 0            | 0                 |
| STARS              | 3359       | 0            | 0                 |

All but three independent variables are continous. Variables LabelAppeal, AcidIndex and STARS are categorical, but represented by numeric values in logical order. The distances between categories/values can be considered equal (i.e. the difference between -2 and -1 for LabelAppeal is the same as the difference between 0 and 1). As such these variables can be used in modeling as numeric variables.

#### Missing Values

Majority variables have negative values. Eight variables have some NA values. The plot and table below show how the missing values are spread out within the data set. About quarter of observations are missing a STARS value. The rest of variables contain missing values for at most 9.5% of observations.

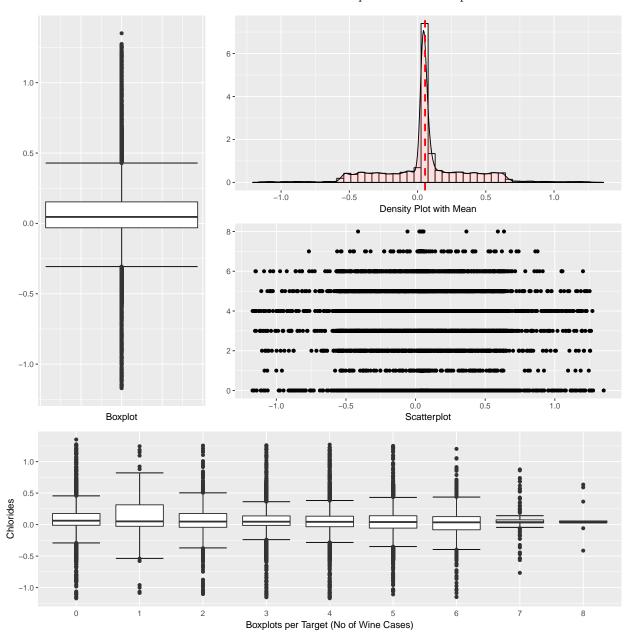


```
##
##
    Variables sorted by number of missings:
##
              Variable
                             {\tt Count}
                  STARS 0.26252442
##
##
             Sulphates 0.09456819
##
    TotalSulfurDioxide 0.05330207
##
                Alcohol 0.05103556
##
     FreeSulfurDioxide 0.05056663
##
             Chlorides 0.04986323
##
         ResidualSugar 0.04814381
##
                     pH 0.03087143
##
                  INDEX 0.0000000
##
                 TARGET 0.0000000
          FixedAcidity 0.00000000
##
       VolatileAcidity 0.00000000
##
            CitricAcid 0.00000000
##
##
               Density 0.00000000
           LabelAppeal 0.00000000
##
             AcidIndex 0.00000000
##
```

### **Exploratory Plots**

All dependent variables were inspected using boxplots, density plots and scatterplots. Distribution is similar for all variables - unimodal and symmetrical. Boxplots are also very similar across all possible outcomes with the exception of the last category - 8 cases purchased. This is due to the fact that the data set contains very few observations with this outcome.

Plots below illustrate results for Chlorides. Other variables produced similar plots.



#### **Correlation Matrix**

Correlation matrix below shows that there is very little correlation between variables. All can contribute in the modeling.

|                 | TARGET | FixedAcidity | VolatileAcidity | CitricAcid | ResidualSugar | Chlorides | FreeSO2 | TotalSO2 | Density | Hq    | Sulphates | Alcohol | LabelAppeal | AcidIndex | STARS |
|-----------------|--------|--------------|-----------------|------------|---------------|-----------|---------|----------|---------|-------|-----------|---------|-------------|-----------|-------|
| TARGET          | 1      | -0.05        | -0.09           | 0.01       | 0.02          | -0.04     | 0.04    | 0.05     | -0.04   | -0.01 | -0.04     | 0.06    | 0.36        | -0.25     | 0.56  |
| FixedAcidity    | -0.05  | 1            | 0.01            | 0.01       | -0.02         | 0         | 0       | -0.02    | 0.01    | -0.01 | 0.03      | -0.01   | 0           | 0.18      | -0.01 |
| VolatileAcidity | -0.09  | 0.01         | 1               | -0.02      | -0.01         | 0         | -0.01   | -0.02    | 0.01    | 0.01  | 0         | 0       | -0.02       | 0.04      | -0.03 |
| CitricAcid      | 0.01   | 0.01         | -0.02           | 1          | -0.01         | -0.01     | 0.01    | 0.01     | -0.01   | -0.01 | -0.01     | 0.02    | 0.01        | 0.07      | 0     |
| ResidualSugar   | 0.02   | -0.02        | -0.01           | -0.01      | 1             | -0.01     | 0.02    | 0.02     | 0       | 0.01  | -0.01     | -0.02   | 0           | -0.01     | 0.02  |
| Chlorides       | -0.04  | 0            | 0               | -0.01      | -0.01         | 1         | -0.02   | -0.01    | 0.02    | -0.02 | 0         | -0.02   | 0.01        | 0.03      | 0     |
| FreeSO2         | 0.04   | 0            | -0.01           | 0.01       | 0.02          | -0.02     | 1       | 0.01     | 0       | 0.01  | 0.01      | -0.02   | 0.01        | -0.04     | -0.01 |
| TotalSO2        | 0.05   | -0.02        | -0.02           | 0.01       | 0.02          | -0.01     | 0.01    | 1        | 0.01    | 0     | -0.01     | -0.02   | -0.01       | -0.05     | 0.01  |
| Density         | -0.04  | 0.01         | 0.01            | -0.01      | 0             | 0.02      | 0       | 0.01     | 1       | 0.01  | -0.01     | -0.01   | -0.01       | 0.04      | -0.02 |
| pН              | -0.01  | -0.01        | 0.01            | -0.01      | 0.01          | -0.02     | 0.01    | 0        | 0.01    | 1     | 0.01      | -0.01   | 0           | -0.06     | 0     |
| Sulphates       | -0.04  | 0.03         | 0               | -0.01      | -0.01         | 0         | 0.01    | -0.01    | -0.01   | 0.01  | 1         | 0       | 0           | 0.03      | -0.01 |
| Alcohol         | 0.06   | -0.01        | 0               | 0.02       | -0.02         | -0.02     | -0.02   | -0.02    | -0.01   | -0.01 | 0         | 1       | 0           | -0.04     | 0.07  |
| LabelAppeal     | 0.36   | 0            | -0.02           | 0.01       | 0             | 0.01      | 0.01    | -0.01    | -0.01   | 0     | 0         | 0       | 1           | 0.02      | 0.33  |
| AcidIndex       | -0.25  | 0.18         | 0.04            | 0.07       | -0.01         | 0.03      | -0.04   | -0.05    | 0.04    | -0.06 | 0.03      | -0.04   | 0.02        | 1         | -0.09 |
| STARS           | 0.56   | -0.01        | -0.03           | 0          | 0.02          | 0         | -0.01   | 0.01     | -0.02   | 0     | -0.01     | 0.07    | 0.33        | -0.09     | 1     |

## Dependent Variable

The dependent variable TARGET ranges from 0 (no cases purchased) to 8 cases of wine purchased. The most common outcome is 4 cases at 25% of all observations followed closely with no purchase (0 cases) at 21%. Not counting the 0 outcome, it seems that the variable has unimodal, symmetrical distribution resembling normal distibution centered around 4.

| Outcome | No of Observations | Percent of Total |
|---------|--------------------|------------------|
| 0       | 2734               | 0.21             |
| 1       | 244                | 0.02             |
| 2       | 1091               | 0.09             |
| 3       | 2611               | 0.2              |
| 4       | 3177               | 0.25             |
| 5       | 2014               | 0.16             |
| 6       | 765                | 0.06             |
| 7       | 142                | 0.01             |
| 8       | 17                 | 0                |

## **Data Preparation**

This data set requires very few transformations as most of the variables appear to be good for analysis as they exist. Two main areas to consider are missing and negative values.

#### Missing Values

The STARS variables contains 3,359 missing values. It represents experts' rating. A missing value - weither it is an intentionally omitted rating or not - may have a meaning for modeling - number of cases purchased for wines without rating is influenced by the fact that there is no rating. For this analysis missing values for STARS have been replaced with 0.

Two other categorical variables - LabelAppeal and AcidIndex - do not have any missing values.

Other variables with missing values are good candidates for imputation. Imputation has been done using the mice R package and its method norm.

#### **Negative Values**

Alcohol variable has a few negative values - only 118 observations. Negative values for this variable are not possible since 0 would be a non-alcoholic beverage. The variable has been transformed by taking absolute value of all observations.

For other variables there is significantly more observations with negative values. Based on some research it seems unlikely that most variables allow for negative values; however, because the distributions for all variables are symmetrical it does not appear that the negative values are data entry or measurement errors. Perhaps, the data was manipulated/shifted or variables were measured using non-standard units. For this analysis, negative values for all variables except STARS remained unchanged.

#### Training/Testing Split

Data set has been split into a training (75% of observations) and testing (25% of observations) sets. Splitting has been accomplished using the caTools R package based on the TARGET variable to make sure that each set has a proportional number of various target classes.

## Modeling: Linear

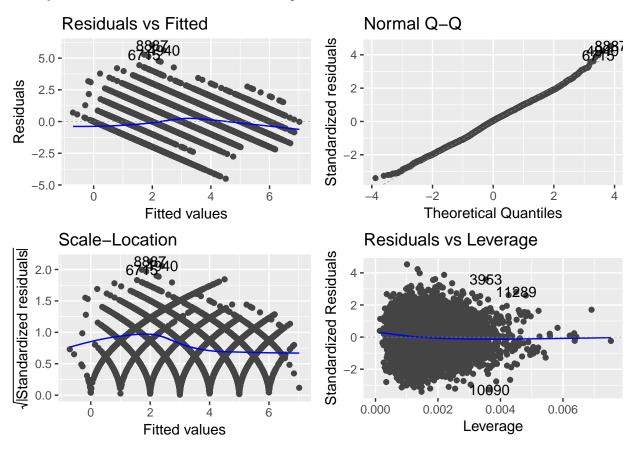
Two main linear models developed were developed and analyzed. The first one included all available variables. It resulted in R<sup>2</sup> of 0.5268, RMSE of 1.3184 and accuracy in predicting the outcomes in the testing set of 0.2853. The second model used stepwise process in both directions to optimize the model (using the stepAIC function). It resulted in R<sup>2</sup> of 0.5266, RMSE of 1.3193 and accuracy of 0.2847. It appears that the full model performed very slightly better than the stepwise model.

Below is the summary of the full model.

```
##
## Call:
## lm(formula = TARGET ~ . - INDEX, data = wineTRAIN)
##
## Residuals:
##
       Min
                1Q
                    Median
                                 3Q
                                        Max
  -4.5069 -0.9507
                    0.0674
                           0.9089
                                     6.0046
##
##
  Coefficients:
##
                         Estimate
                                    Std. Error t value
                                                                    Pr(>|t|)
## (Intercept)
                       3.67369630
                                    0.51997673
                                                 7.065
                                                            0.0000000000172
## FixedAcidity
                       0.00017354
                                    0.00218877
                                                 0.079
                                                                    0.936806
                                                            0.0000013920351
## VolatileAcidity
                      -0.09128121
                                    0.01732010
                                                -5.270
## CitricAcid
                       0.02277522
                                    0.01575707
                                                 1.445
                                                                    0.148379
## ResidualSugar
                       0.00019360
                                    0.00040400
                                                 0.479
                                                                    0.631792
## Chlorides
                      -0.11475363
                                    0.04279211
                                                -2.682
                                                                    0.007338
## FreeSulfurDioxide
                       0.00032030
                                    0.00009067
                                                 3.532
                                                                    0.000414
## TotalSulfurDioxide
                       0.00010772
                                    0.00005848
                                                 1.842
                                                                    0.065493
                                                -1.174
## Density
                      -0.60095976
                                    0.51208962
                                                                    0.240607
## pH
                      -0.02452450
                                   0.01993802
                                                -1.230
                                                                    0.218713
## Sulphates
                      -0.03980772
                                   0.01455681
                                                -2.735
                                                                    0.006256
## Alcohol
                       0.01340460
                                    0.00376286
                                                 3.562
                                                                    0.000369
                                                26.618 < 0.00000000000000002
## LabelAppeal
                       0.41927617
                                    0.01575148
## AcidIndex
                      -0.20162864
                                    0.01068154 -18.876 < 0.00000000000000002
## STARS
                                    0.01199592 81.959 < 0.0000000000000000
                       0.98317365
##
## (Intercept)
## FixedAcidity
## VolatileAcidity
## CitricAcid
## ResidualSugar
## Chlorides
## FreeSulfurDioxide
## TotalSulfurDioxide .
## Density
## pH
## Sulphates
## Alcohol
## LabelAppeal
## AcidIndex
## STARS
##
## Signif. codes:
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.326 on 9580 degrees of freedom
```

```
## Multiple R-squared: 0.5268, Adjusted R-squared: 0.5261
## F-statistic: 761.7 on 14 and 9580 DF, p-value: < 0.000000000000000022</pre>
```

Looking at the diagnostic plots we can see that the model performs reasonably well. Some of the plots are not very suitable for this data set because the dependent variable is a count variable.



The accuracy is fairly low at 28.53%; however, if we examine full confusion matrix below we can see that the model mostly errors only by 1 or 2 cases which may be reasonable enough for a business application. The bigger problem is difficulty in predicting no purchase. There may be significant cost associated with this error.

```
Confusion Matrix and Statistics
##
##
                Reference
##
   Prediction
                   0
                        1
                             2
                                  3
                                            5
                                                 6
                                                      7
                                                           8
##
              0
                  24
                         2
                              1
                                  3
                                       0
                                            0
                                                 0
                                                      0
                                                           0
                                            7
                 313
                       29
                            68
                                 80
                                      22
                                                           0
##
              1
                                                 0
                                                      0
                       26
                          115
                                                10
                                                      2
                                                           0
##
              2
                 256
                                178 109
                                           28
              3
                  86
                        3
                                236 223
                                           71
                                                 9
                                                           0
##
                            74
##
              4
                   5
                            14
                                129 301
                                          197
                                                42
                                                      3
                                                           0
                        1
##
              5
                   0
                        0
                                     128
                                          155
                                                76
                                                     14
                                                           1
##
              6
                   0
                        0
                                      11
                                           45
                                                51
                                                     15
                                                           1
                             0
                                  0
              7
                                                      2
                                                           2
##
                   0
                        0
                                  0
                                       0
                                            1
                                                 3
              8
                   0
                        0
                                  0
                                       0
                                            0
                                                 0
                                                      0
                                                           0
##
                              0
##
##
   Overall Statistics
##
                      Accuracy: 0.2853
##
```

```
##
                    95% CI: (0.2697, 0.3013)
##
       No Information Rate: 0.2481
##
       P-Value [Acc > NIR] : 0.0000008841
##
##
                     Kappa: 0.1642
##
    Mcnemar's Test P-Value : NA
##
##
  Statistics by Class:
##
##
                         Class: 0 Class: 1 Class: 2 Class: 3 Class: 4 Class: 5
## Sensitivity
                         0.035088 0.475410
                                            0.42125
                                                      0.36141
                                                               0.37909
                                                                         0.30754
## Specificity
                                            0.79194
                                                               0.83749
                         0.997615 0.843899
                                                      0.81704
                                                                         0.90838
## Pos Pred Value
                         0.800000 0.055877
                                            0.15884
                                                      0.33618
                                                               0.43497
                                                                         0.38557
## Neg Pred Value
                         0.791798 0.988064
                                            0.93619
                                                      0.83307
                                                               0.80343
                                                                         0.87527
## Prevalence
                         0.213750 0.019062
                                            0.08531
                                                      0.20406
                                                               0.24813
                                                                         0.15750
## Detection Rate
                         0.007500 0.009062
                                            0.03594
                                                      0.07375
                                                               0.09406
                                                                         0.04844
## Detection Prevalence 0.009375 0.162188
                                                      0.21937
                                                               0.21625
                                            0.22625
                                                                         0.12562
## Balanced Accuracy
                         0.516351 0.659655
                                            0.60659
                                                      0.58922
                                                               0.60829
                                                                         0.60796
##
                         Class: 6 Class: 7 Class: 8
## Sensitivity
                          0.26702 0.055556
                                            0.00000
## Specificity
                          0.97607 0.998104
                                            1.00000
## Pos Pred Value
                          0.41463 0.250000
                                                 NaN
## Neg Pred Value
                          0.95450 0.989348
                                            0.99875
## Prevalence
                          0.05969 0.011250
                                            0.00125
## Detection Rate
                          0.01594 0.000625
                                            0.00000
## Detection Prevalence
                         0.03844 0.002500
                                            0.00000
                          0.62154 0.526830
## Balanced Accuracy
                                            0.50000
```

#### Modeling: Poisson

The linear model seemed to perform good with all variables. So for the poisson regression similar strategy was applied - a model with all variables and a model optimized by the stepwise method. There was no considerable imrovement using the stepwise method, so for comparison reasons below is the summary for the full model. RMSE for this model is 1.39855, slightly worse than for the linear model.

```
## Call:
  glm(formula = TARGET ~ . - INDEX, family = poisson, data = wineTRAIN)
##
## Deviance Residuals:
##
       Min
                 1Q
                      Median
                                     3Q
                                             Max
  -2.9561
            -0.7237
                       0.0709
                                0.5750
                                          3.2336
##
## Coefficients:
##
                          Estimate
                                    Std. Error z value
                                                                     Pr(>|z|)
                                    0.22551127
                                                  6.280
                                                                0.0000000034
## (Intercept)
                        1.41611137
                       -0.00028475
                                    0.00095364
                                                 -0.299
                                                                     0.765252
## FixedAcidity
                                                 -4.225
                                                                0.00002386398
## VolatileAcidity
                       -0.03184819
                                    0.00753753
                        0.00849643
                                    0.00682495
## CitricAcid
                                                  1.245
                                                                     0.213166
## ResidualSugar
                        0.00001864
                                    0.00017560
                                                  0.106
                                                                     0.915463
## Chlorides
                                                 -2.111
                       -0.03927542
                                    0.01860154
                                                                     0.034737
## FreeSulfurDioxide
                        0.00013161
                                    0.00003924
                                                  3.354
                                                                     0.000797
## TotalSulfurDioxide 0.00004204
                                    0.00002562
                                                  1.641
                                                                     0.100768
## Density
                       -0.21687294
                                    0.22128775
                                                 -0.980
                                                                     0.327062
```

```
## pH
                    -0.01221488 0.00864782 -1.412
                                                             0.157808
## Sulphates
                    -0.01570441 0.00633604 -2.479
                                                             0.013191
## Alcohol
                     0.00331670 0.00164184
                                            2.020
                                                             0.043371
                     0.12981840 0.00699730 18.553 < 0.0000000000000002
## LabelAppeal
## AcidIndex
                    ## STARS
                     0.31327867  0.00518717  60.395 < 0.0000000000000002
##
## (Intercept)
## FixedAcidity
## VolatileAcidity
## CitricAcid
## ResidualSugar
## Chlorides
## FreeSulfurDioxide
## TotalSulfurDioxide
## Density
## pH
## Sulphates
## Alcohol
## LabelAppeal
## AcidIndex
## STARS
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
##
  (Dispersion parameter for poisson family taken to be 1)
##
      Null deviance: 17142 on 9594 degrees of freedom
## Residual deviance: 11060
                           on 9580 degrees of freedom
## AIC: 35045
##
## Number of Fisher Scoring iterations: 5
```

Confusion matrix below comparing predicted values to the test data shows that the model does not predict no purchase outcome (count is 0). Additionally, it often predicts fewer cases than the test data indicates. Accuracy is lower than for the linear model.

#### ## Accuracy ## 0.233125

| ## | I                  | Refer | ence | •   |     |     |     |    |    |   |   |    |
|----|--------------------|-------|------|-----|-----|-----|-----|----|----|---|---|----|
| ## | ${\tt Prediction}$ | 0     | 1    | 2   | 3   | 4   | 5   | 6  | 7  | 8 | 9 | 10 |
| ## | 0                  | 0     | 0    | 0   | 0   | 0   | 0   | 0  | 0  | 0 | 0 | 0  |
| ## | 1                  | 204   | 19   | 35  | 39  | 6   | 4   | 0  | 0  | 0 | 0 | 0  |
| ## | 2                  | 421   | 38   | 166 | 282 | 178 | 45  | 12 | 2  | 0 | 0 | 0  |
| ## | 3                  | 58    | 4    | 64  | 231 | 305 | 118 | 12 | 1  | 0 | 0 | 0  |
| ## | 4                  | 1     | 0    | 6   | 75  | 186 | 155 | 36 | 2  | 0 | 0 | 0  |
| ## | 5                  | 0     | 0    | 2   | 24  | 94  | 103 | 53 | 11 | 1 | 0 | 0  |
| ## | 6                  | 0     | 0    | 0   | 2   | 18  | 53  | 32 | 6  | 0 | 0 | 0  |
| ## | 7                  | 0     | 0    | 0   | 0   | 5   | 19  | 28 | 9  | 1 | 0 | 0  |
| ## | 8                  | 0     | 0    | 0   | 0   | 2   | 6   | 17 | 4  | 0 | 0 | 0  |
| ## | 9                  | 0     | 0    | 0   | 0   | 0   | 1   | 0  | 1  | 2 | 0 | 0  |
| ## | 10                 | 0     | 0    | 0   | 0   | 0   | 0   | 1  | 0  | 0 | 0 | 0  |

## Modeling: Negative Binomial

Using the MASS R package, negative binomial model was created using all variables. This model turned out to be nearly identical to the poisson model. Because of this details are not included in this report. Code used to create and analyze the model is available in the Appendix B.

Both models exhibited over-dispersion.

## Modeling: Zero-Inflated Negative Binomial

Poisson and negative binomial models do not account for the 0 outcome. So a zero-inflated negative binomial model was attempted using the pscl R package. RMSE for this model is 1.2727, the best one out of all models.

```
##
## Call:
## zeroinfl(formula = TARGET ~ . - INDEX, data = wineTRAIN, dist = "negbin")
##
## Pearson residuals:
                           Median
##
                                         30
         Min
                                                   Max
   -2.105481 -0.406840 -0.007311
                                   0.370580
##
## Count model coefficients (negbin with log link):
##
                          Estimate Std. Error z value
                                                                    Pr(>|z|)
## (Intercept)
                                                               0.0000000257
                        1.38749237
                                    0.23290861
                                                  5.957
## FixedAcidity
                        0.00032298
                                    0.00098200
                                                  0.329
                                                                      0.74223
## VolatileAcidity
                       -0.01187606
                                    0.00778309
                                                 -1.526
                                                                      0.12704
## CitricAcid
                       0.00276410
                                    0.00700564
                                                  0.395
                                                                      0.69317
## ResidualSugar
                       -0.00010593
                                    0.00018122
                                                -0.585
                                                                      0.55887
## Chlorides
                       -0.02437025
                                    0.01911366
                                                -1.275
                                                                      0.20230
## FreeSulfurDioxide
                       0.00001724
                                    0.00003963
                                                 0.435
                                                                      0.66350
## TotalSulfurDioxide -0.00003882
                                    0.00002549
                                                 -1.523
                                                                      0.12771
## Density
                       -0.23328109
                                    0.22802503
                                                -1.023
                                                                      0.30628
## pH
                       0.00388454
                                    0.00891545
                                                  0.436
                                                                      0.66305
                                                -0.168
## Sulphates
                       -0.00109531
                                    0.00652144
                                                                      0.86662
## Alcohol
                        0.00718892
                                    0.00167871
                                                  4.282
                                                               0.00001848844
                                                31.892 < 0.00000000000000002
## LabelAppeal
                        0.23205485
                                    0.00727633
## AcidIndex
                       -0.01776608
                                    0.00566333
                                                 -3.137
## STARS
                        0.10279921
                                    0.00596377
                                                 17.237 < 0.00000000000000002
## Log(theta)
                       17.28537035
                                                  4.891
                                                               0.0000100130
                                    3.53384182
##
## (Intercept)
## FixedAcidity
## VolatileAcidity
## CitricAcid
## ResidualSugar
## Chlorides
## FreeSulfurDioxide
## TotalSulfurDioxide
## Density
## pH
## Sulphates
## Alcohol
## LabelAppeal
                       ***
```

```
## AcidIndex
## STARS
## Log(theta)
##
## Zero-inflation model coefficients (binomial with logit link):
##
                        Estimate Std. Error z value
                                                                Pr(>|z|)
                      -4.4655671 1.5648983 -2.854
## (Intercept)
                                                                0.004323 **
## FixedAcidity
                      -0.0002386
                                 0.0064138
                                            -0.037
                                                                0.970328
## VolatileAcidity
                       0.1521474
                                  0.0500156
                                              3.042
                                                                0.002350 **
## CitricAcid
                      -0.0168663
                                 0.0463038
                                            -0.364
                                                                0.715669
## ResidualSugar
                      -0.0014057
                                  0.0011789
                                             -1.192
                                                                0.233085
## Chlorides
                                                                0.722789
                      -0.0439587
                                  0.1239199
                                             -0.355
## FreeSulfurDioxide -0.0009263 0.0002715
                                            -3.412
                                                                0.000645 ***
## TotalSulfurDioxide -0.0007884
                                 0.0001692
                                            -4.659
                                                              0.00000318 ***
## Density
                                  1.5366213
                       0.6619423
                                              0.431
                                                                0.666630
## pH
                       0.1873466
                                  0.0580398
                                              3.228
                                                                0.001247 **
## Sulphates
                       0.1280083 0.0425673
                                              3.007
                                                                0.002637 **
## Alcohol
                       0.0233231
                                 0.0110155
                                              2.117
                                                                0.034236 *
                       0.7547693  0.0495578  15.230 < 0.0000000000000000 ***
## LabelAppeal
## AcidIndex
                       0.4345200 0.0300666 14.452 < 0.0000000000000000 ***
## STARS
                      -2.3833377  0.0698006  -34.145 < 0.0000000000000000 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Theta = 32132236.8215
## Number of iterations in BFGS optimization: 60
## Log-likelihood: -1.53e+04 on 31 Df
```

This model has the accuracy of 35.78%, again the best one out of all models. It predicts 0 outcomes (not ideally, but perhaps it can be improved with more research).

```
## Accuracy
## 0.3578125
##
```

```
Reference
                            2
                                 3
                                          5
                                                    7
                                                        8
## Prediction
                  0
                       1
                                     4
                                               6
                                 6
                                          3
                                                        0
##
              0 107
                       1
                            0
                                     4
                                               1
                                                    1
                          34
                                                        0
##
              1 323
                      15
                              76
                                    39
                                         16
                                               5
                                                    1
##
              2 138
                      36 125 118
                                    39
                                         11
                                               4
                                                    0
                                                        0
              3 103
                                         64
                                               4
                                                    0
##
                       9 107 308 246
                                                        0
##
              4
                 13
                       0
                            7 140 373 200
                                              33
                                                   3
                                                        0
##
              5
                  0
                       0
                            0
                                 5
                                    81 164
                                              75
                                                  11
                                                        0
##
                  0
                       0
                                 0
                                    12
                                         36
                                                        0
              6
                            0
                                             45
                                                  12
##
              7
                  0
                       0
                            0
                                 0
                                     0
                                         10
                                              20
                                                   8
                                                        4
##
                       0
                            0
                                 0
                                     0
                                          0
                                               4
                                                    0
                                                        0
```

## **Model Comparison**

Considering log-likelihood of all models, it is clear that zero-inflated negative binomial model is the best option. More research in that direction will probably be beneficial.

|         | Log-Likelihood | DF |
|---------|----------------|----|
| Linear  | -16316         | 16 |
| Poisson | -17507         | 15 |
| NB      | -17507         | 16 |
| ZINB    | -15303         | 31 |

Using full models in all methods allows comparison of coefficients. For the most part coefficients are similar in sign and in magnitude. There are a couple of small coefficients that change signs between NB and ZINB models.

|                        | Linear   | Poisson   | NB        | ZINB (Count) |
|------------------------|----------|-----------|-----------|--------------|
| (Intercept)            | 3.674    | 1.416     | 1.416     | 1.387        |
| FixedAcidity           | 0.000174 | -0.000285 | -0.000285 | 0.000323     |
| ${f Volatile Acidity}$ | -0.09128 | -0.03185  | -0.03185  | -0.01188     |
| ${f Citric Acid}$      | 0.02278  | 0.008496  | 0.008497  | 0.002764     |
| ResidualSugar          | 0.000194 | 0.000019  | 0.000019  | -0.000106    |
| Chlorides              | -0.1148  | -0.03927  | -0.03928  | -0.02437     |
| FreeSulfurDioxide      | 0.00032  | 0.000132  | 0.000132  | 0.000017     |
| Total Sulfur Dioxide   | 0.000108 | 0.000042  | 0.000042  | -0.000039    |
| Density                | -0.601   | -0.2169   | -0.2169   | -0.2333      |
| ${f pH}$               | -0.02453 | -0.01222  | -0.01222  | 0.003885     |
| ${f Sulphates}$        | -0.03981 | -0.0157   | -0.01571  | -0.001095    |
| Alcohol                | 0.01341  | 0.003317  | 0.003317  | 0.007189     |
| ${f Label Appeal}$     | 0.4193   | 0.1298    | 0.1298    | 0.2321       |
| ${f AcidIndex}$        | -0.2016  | -0.08412  | -0.08412  | -0.01777     |
| STARS                  | 0.9832   | 0.3133    | 0.3133    | 0.1028       |

## APPENDIX A: Evaluation Data Set

Please note that this appendix includes first 100 observations from the evaluation set.

| Index | Predicted Value | Predicted Outcome |
|-------|-----------------|-------------------|
| 3     | 1.894           | 2                 |
| 9     | 3.826           | 4                 |
| 10    | 2.513           | 3                 |
| 18    | 2.492           | 2                 |
| 21    | 0.7126          | 1                 |
| 30    | 5.695           | 6                 |
| 31    | 3.595           | 4                 |
| 37    | 1.343           | 1                 |
| 39    | 0.2309          | 0                 |
| 47    | 1.468           | 1                 |
| 60    | 2.693           | 3                 |
| 62    | 0.2158          | 0                 |
| 63    | 3.539           | 4                 |
| 64    | 1.329           | 1                 |
| 68    | 1.176           | 1                 |
| 75    | 2.761           | 3                 |
| 76    | 2.517           | 3                 |
| 83    | 0.0539          | 0                 |
| 87    | 3.674           | 4                 |
| 92    | 5.435           | 5                 |
| 98    | 2.317           | 2                 |
| 106   | 1.682           | 2                 |
| 107   | 0.473           | 0                 |
| 113   | 2.537           | 3                 |
| 120   | 3.448           | 3                 |
| 123   | 5.88            | 6                 |
| 125   | 2.838           | 3                 |
| 126   | 5.872           | 6                 |
| 128   | 4.508           | 5                 |
| 129   | 2.449           | 2                 |
| 131   | 4.213           | 4                 |
| 135   | 0.9485          | 1                 |
| 141   | 4.205           | 4                 |
| 147   | 3.249           | 3                 |
| 148   | 1.312           | 1                 |
| 151   | 3.745           | 4                 |
| 156   | 3.179           | 3                 |
| 157   | 3.38            | 3                 |
| 174   | 1.655           | $\overline{2}$    |
| 186   | 0.5061          | 1                 |
| 193   | 2.592           | 3                 |
| 195   | 0.9228          | 1                 |
| 212   | 0.6654          | 1                 |
| 213   | 0.6855          | 1                 |
| 217   | 2.986           | 3                 |
| 223   | 3.834           | $\frac{3}{4}$     |
| 226   | 3.158           | 3                 |
| 228   | 4.544           | 5                 |
| 230   | 4.127           | 4                 |
| _00   |                 | •                 |

| Index | Predicted Value      | Predicted Outcome |
|-------|----------------------|-------------------|
| 241   | 2.628                | 3                 |
| 243   | 3.746                | 4                 |
| 249   | 1.071                | 1                 |
| 281   | 4.079                | 4                 |
| 288   | 0.277                | 0                 |
| 294   | 1.913                | 2                 |
| 295   | 1.963                | 2                 |
| 300   | 5.567                | 6                 |
| 302   | 4.323                | 4                 |
| 303   | 1.856                | 2                 |
| 308   | 1.706                | 2                 |
| 319   | 4.84                 | 5                 |
| 320   | 0.9056               | 1                 |
| 324   | 2.964                | 3                 |
| 331   | 2.665                | 3                 |
| 343   | 2.961                | 3                 |
| 347   | 2.422                | $\frac{3}{2}$     |
| 348   | 3.685                | $\frac{2}{4}$     |
| 350   | 4.667                | 5                 |
| 357   | 1.443                | 1                 |
| 358   | 3.605                | 4                 |
| 360   | 4.18                 | 4                 |
| 366   | $\frac{4.16}{3.558}$ | 4                 |
| 367   | 3.558<br>2.621       | $\frac{4}{3}$     |
| 368   | 2.021<br>4.894       | 5<br>5            |
|       |                      | $\frac{5}{2}$     |
| 376   | 2.318                |                   |
| 380   | 3.275                | 3                 |
| 388   | 0.5946               | 1                 |
| 396   | 4.575                | 5                 |
| 398   | 4.527                | 5                 |
| 403   | 3.921                | 4                 |
| 410   | 2.043                | 2                 |
| 412   | 0.6185               | 1                 |
| 420   | 2.501                | 3                 |
| 434   | 2.495                | 2                 |
| 440   | 3.049                | 3                 |
| 450   | 3.489                | 3                 |
| 453   | 2.763                | 3                 |
| 464   | 4.602                | 5                 |
| 465   | 4.51                 | 5                 |
| 466   | 4.76                 | 5                 |
| 473   | 2.475                | 2                 |
| 476   | 1.708                | 2                 |
| 478   | 1.517                | 2                 |
| 479   | 3.189                | 3                 |
| 493   | 2.733                | 3                 |
| 497   | 3.061                | 3                 |
| 503   | 3.663                | 4                 |
| 504   | 3.592                | 4                 |
| 505   | 2.192                | 2                 |
| 507   | 0.2199               | 0                 |

### APPENDIX B: R Script

```
# Required libraries
library(ggplot2)
                  # plotting
library(dplyr)
                    # data manipulation
library(gridExtra) # display
library(knitr)
                    # display
library(kableExtra) # display
library(mice)
                    # imputation
library(caTools)
                    # train-test split
library(MASS)
                  # boxcox
library(Metrics) # rmse
                  # confusion matrix
library(caret)
library(VIM)
                    # plotting NAs
library(ggfortify) # plotting lm diagnostic
library(car)
                    # VIF
library(pscl)
                    # zero-inflated model
# Import data
wine <- read.csv(url(paste0("https://raw.githubusercontent.com/",
                           "ilyakats/CUNY-DATA621/master/hw5/",
                           "wine-training-data.csv")),
                 na.strings=c("","NA"))
colnames(wine)[1] <- "INDEX"</pre>
# Basic statistic
nrow(wine); ncol(wine)
summary(wine)
# Summary table
sumtbl = data.frame(Variable = character(),
                    Class = character(),
                    Min = integer(),
                    Median = integer(),
                    Mean = double(),
                    SD = double(),
                    Max = integer(),
                    Num_NAs = integer(),
                    Num_Zeros = integer(),
                    Num_Neg = integer())
for (i in c(3:16)) {
  sumtbl <- rbind(sumtbl, data.frame(Variable = colnames(wine)[i],</pre>
                                      Class = class(wine[,i]),
                                      Min = min(wine[,i], na.rm=TRUE),
                                     Median = median(wine[,i], na.rm=TRUE),
                                     Mean = mean(wine[,i], na.rm=TRUE),
                                      SD = sd(wine[,i], na.rm=TRUE),
                                      Max = max(wine[,i], na.rm=TRUE),
                                      Num_NAs = sum(is.na(wine[,i])),
                                     Num_Zeros = length(which(wine[,i]==0)),
                                     Num_Neg = sum(wine[,i]<0 & !is.na(wine[,i]))))</pre>
colnames(sumtbl) <- c("Variable", "Class", "Min", "Median", "Mean", "SD", "Max",</pre>
```

```
"Num of NAs", "Num of Zeros", "Num of Neg Values")
sumtbl
# Categorical variables
table(wine$LabelAppeal)
table(wine$AcidIndex)
table(wine$STARS)
# Exploratory plots
v <- "FixedAcidity"
v <- "VolatileAcidity"
v <- "CitricAcid"
v <- "ResidualSugar"
v <- "Chlorides"
v <- "FreeSulfurDioxide"
v <- "TotalSulfurDioxide"
v <- "Density"
v <- "pH"
v <- "Sulphates"
v <- "Alcohol"
v <- "LabelAppeal"
v <- "AcidIndex"
v <- "STARS"
pd <- as.data.frame(cbind(wine[, v], wine$TARGET)); colnames(pd) <- c("X", "Y")</pre>
bp <- ggplot(pd, aes(x = 1, y = X)) + stat_boxplot(geom ='errorbar') +</pre>
  geom boxplot() +
  xlab("Boxplot") + ylab("") + theme(axis.text.x=element blank(),
                                      axis.ticks.x=element_blank())
hp <- ggplot(pd, aes(x = X)) + geom_histogram(aes(y=..density..),</pre>
                                                colour="black", fill="white") +
  geom_density(alpha=.2, fill="#FF6666") + ylab("") +
  xlab("Density Plot with Mean") +
  geom_vline(aes(xintercept=mean(X, na.rm=TRUE)),
             color="red", linetype="dashed", size=1)
sp <- ggplot(pd, aes(x=X, y=Y)) + geom_point() + xlab("Scatterplot")</pre>
grid.arrange(bp, hp, sp, layout_matrix=rbind(c(1,2,2),c(1,3,3)))
ggplot(wine, aes(x = as.factor(TARGET), y = Chlorides)) +
  stat_boxplot(geom ='errorbar') + geom_boxplot() +
  xlab("Boxplots per No of Wine Cases") + ylab("pH") +
  theme(axis.ticks.x=element_blank())
# Correlation matrix
cm <- cor(wine[,2:16], use="pairwise.complete.obs")</pre>
cm <- round(cm, 2)
cmout <- as.data.frame(cm) %>% mutate_all(function(x) {
  cell_spec(x, "html", color = ifelse(x>0.5 | x<(-0.5), "blue", "black"))
rownames(cmout) <- colnames(cmout)</pre>
cmout %>%
  kable("html", escape = F, align = "c", row.names = TRUE) %>%
  kable_styling("striped", full_width = F)
```

```
# IMPUTATION / TRANSFORMATION
wineOriginal <- wine # Backup of original data
wine $STARS[is.na(wine $STARS)] <- 0 # Missing STARS are 0 score
# Missing values - table
md.pattern(wine)
# Missing values - plot
aggr_plot <- aggr(wine, col=c('navyblue', 'red'),</pre>
                   numbers=FALSE, sortVars=TRUE, labels=names(wine),
                   cex.axis=.7, gap=3,
                   ylab=c("Histogram of missing data", "Pattern"))
# Imputation
wineImputed <- mice(wine, m=5, maxit=20, meth='norm', seed=500)</pre>
summary(wineImputed)
wine <- complete(wineImputed)</pre>
summary(wine)
# Proportion of target variable
table(wine$TARGET)
table(wine$TARGET)/sum(table(wine$TARGET))
wine$Alcohol <- abs(wine$Alcohol)</pre>
# Split into train and validation sets
set.seed(88)
split <- sample.split(wine$TARGET, SplitRatio = 0.75)</pre>
wineTRAIN <- subset(wine, split == TRUE)</pre>
wineTEST <- subset(wine, split == FALSE)</pre>
table(wineTRAIN$TARGET)/sum(table(wineTRAIN$TARGET))
# LINEAR MODEL
# All variables
lmModel <- lm(TARGET ~ .-INDEX,data = wineTRAIN)</pre>
summary(lmModel)
# stepAIC
lmModel <- stepAIC(lmModel, trace=FALSE, direction='both')</pre>
summary(lmModel)
# Model returned by step AIC
lmModel <- lm(TARGET ~ VolatileAcidity + CitricAcid +</pre>
                 Chlorides + FreeSulfurDioxide +
                 TotalSulfurDioxide + Sulphates + Alcohol +
                 LabelAppeal + AcidIndex + STARS,
               data = wineTRAIN)
summary(lmModel)
# Manual variations
lmModel <- lm(TARGET ~ VolatileAcidity + Chlorides +</pre>
                 FreeSulfurDioxide +
                 TotalSulfurDioxide + Sulphates + Alcohol +
                 LabelAppeal + AcidIndex + STARS,
               data = wineTRAIN)
summary(lmModel)
lmModel <- lm(TARGET ~ VolatileAcidity + Chlorides +</pre>
```

```
FreeSulfurDioxide +
                 TotalSulfurDioxide + Alcohol +
                LabelAppeal + AcidIndex + STARS,
              data = wineTRAIN)
summary(lmModel)
# Calculate RMSE
pred <- predict(lmModel, newdata=wineTEST)</pre>
rmse(wineTEST$TARGET, pred)
# Confusion matrix
predRound <- as.factor(round(pred,0))</pre>
table(predRound)
levels(predRound) <- levels(as.factor(wineTEST$TARGET))</pre>
confusionMatrix(predRound, as.factor(wineTEST$TARGET))
autoplot(lmModel)
# Model plots
plot(lmModel$residuals, ylab="Residuals")
abline(h=0)
plot(lmModel$fitted.values, lmModel$residuals,
     xlab="Fitted Values", ylab="Residuals")
abline(h=0)
qqnorm(lmModel$residuals)
qqline(lmModel$residuals)
# POISSON and NB REGRESSION MODEL
# Poisson 1
glmModel <- glm (TARGET ~ .-INDEX, data = wineTRAIN, family = poisson)</pre>
summary(glmModel)
pred <- predict(glmModel, newdata=wineTEST, type='response')</pre>
rmse(wineTEST$TARGET, pred)
predRound <- as.factor(round(pred,0))</pre>
testData <- as.factor(wineTEST$TARGET)</pre>
levels(predRound) <- c("1", "2", "3", "4", "5", "6", "7", "8", "9", "10", "0")
levels(testData) <- c("0", "1", "2", "3", "4", "5", "6", "7", "8", "9", "10")
confusionMatrix(predRound, testData)
# Poisson 2
glmModel2 <- stepAIC(glmModel, trace=FALSE, direction='both')</pre>
summary(glmModel2)
pred <- predict(glmModel2, newdata=wineTEST, type='response')</pre>
rmse(wineTEST$TARGET, pred)
predRound <- as.factor(round(pred,0))</pre>
testData <- as.factor(wineTEST$TARGET)</pre>
levels(predRound) <- c("1", "2", "3", "4", "5", "6", "7", "8", "9", "10", "0")
levels(testData) <- c("0", "1", "2", "3", "4", "5", "6", "7", "8", "9", "10")</pre>
confusionMatrix(predRound, testData)
```

```
# Poisson 3
glmModel3 <- glm(TARGET ~ VolatileAcidity + Chlorides + FreeSulfurDioxide +</pre>
                  Sulphates + Alcohol + LabelAppeal +
                  AcidIndex + STARS, family = poisson, data = wineTRAIN)
summary(glmModel3)
pred <- predict(glmModel3, newdata=wineTEST, type='response')</pre>
rmse(wineTEST$TARGET, pred)
predRound <- as.factor(round(pred,0))</pre>
testData <- as.factor(wineTEST$TARGET)</pre>
levels(predRound) <- c("1", "2", "3", "4", "5", "6", "7", "8", "9", "10", "0")
levels(testData) <- c("0", "1", "2", "3", "4", "5", "6", "7", "8", "9", "10")
confusionMatrix(predRound, testData)
# NB
nbModel <- glm.nb(TARGET ~ .-INDEX, data = wineTRAIN)</pre>
summary(nbModel)
pred <- predict(nbModel, newdata=wineTEST, type='response')</pre>
rmse(wineTEST$TARGET, pred)
predRound <- as.factor(round(pred,0))</pre>
testData <- as.factor(wineTEST$TARGET)</pre>
levels(predRound) <- c("1", "2", "3", "4", "5", "6", "7", "8", "9", "10", "0")
levels(testData) <- c("0", "1", "2", "3", "4", "5", "6", "7", "8", "9", "10")</pre>
confusionMatrix(predRound, testData)
# Zero Inflated
zrModel <- zeroinfl(TARGET ~ .-INDEX, data = wineTRAIN, dist = "negbin")</pre>
summary(zrModel)
pred <- predict(zrModel, newdata=wineTEST, type='response')</pre>
rmse(wineTEST$TARGET, pred)
predRound <- as.factor(round(pred,0))</pre>
testData <- as.factor(wineTEST$TARGET)</pre>
confusionMatrix(predRound, testData)
# Deviance residuals
anova(glmModel, test="Chisq")
anova(glmModel2, test="Chisq")
anova(glmModel3, test="Chisq")
anova(nbModel, test="Chisq")
anova(zrModel, test="Chisq")
# VIF
vif(glmModel)
vif(nbModel)
vif(zrModel)
# Coefficients
coef <- as.data.frame(lmModel$coefficients)</pre>
coef <- cbind(coef, as.data.frame(glmModel$coefficients))</pre>
coef <- cbind(coef, as.data.frame(nbModel$coefficients))</pre>
coef <- cbind(coef, as.data.frame(zrModel$coefficients))</pre>
# Prediction
eval <- read.csv(url(paste0("https://raw.githubusercontent.com/",
```

```
"ilyakats/CUNY-DATA621/master/hw5/",
                             "wine-evaluation-data.csv")),
                  na.strings=c("","NA"))
colnames(eval)[1] <- "INDEX"</pre>
sumtbl = data.frame(Variable = character(),
                     Class = character(),
                     Min = integer(),
                     Median = integer(),
                     Mean = double(),
                     SD = double(),
                     Max = integer(),
                     Num_NAs = integer(),
                     Num_Zeros = integer(),
                     Num_Neg = integer())
for (i in c(3:16)) {
  sumtbl <- rbind(sumtbl, data.frame(Variable = colnames(eval)[i],</pre>
                                       Class = class(eval[,i]),
                                       Min = min(eval[,i], na.rm=TRUE),
                                       Median = median(eval[,i], na.rm=TRUE),
                                       Mean = mean(eval[,i], na.rm=TRUE),
                                       SD = sd(eval[,i], na.rm=TRUE),
                                       Max = max(eval[,i], na.rm=TRUE),
                                       Num_NAs = sum(is.na(eval[,i])),
                                       Num Zeros = length(which(eval[,i]==0)),
                                       Num_Neg = sum(eval[,i]<0 & !is.na(eval[,i]))))</pre>
}
colnames(sumtbl) <- c("Variable", "Class", "Min", "Median", "Mean", "SD", "Max",</pre>
                       "Num of NAs", "Num of Zeros", "Num of Neg Values")
sumtbl
eval$STARS[is.na(eval$STARS)] <- 0</pre>
eval$Alcohol <- abs(eval$Alcohol)</pre>
evalImputed <- mice(eval, m=5, maxit=10, meth='norm', seed=500)
eval <- complete(evalImputed)</pre>
pred <- predict(zrModel, newdata=eval, type="response")</pre>
results <- eval[, c("INDEX")]
results <- cbind(results, prob=round(pred,4))
results <- cbind(results, predict=round(pred,0))</pre>
colnames(results) <- c("Index", "Predicted Value", "Predicted Outcome")</pre>
pander(head(results, 100))
```