ДЗ по мат. анализу на 9.02.2022

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№2

a)

$\int xsin(x^2)dx = \left\|\begin{matrix} y = x^2 \\ x = \sqrt{y} \end{matrix}\right\| = \int \sqrt{y}sin(y)d(\sqrt{y}) = \int \frac{\sqrt{y}sin(y)}{2\sqrt{y}}dy = \frac{1}{2}\int sin(y)dy =$
$= -\frac{1}{2}sin(y) + C = -\frac{1}{2}sin(x^2) + C$
Ответ: $-\frac{1}{2}sin(x^2) + C$

b)

$\int \frac{e^x+e^{2x}}{1-e^x}dx = \left\|\begin{matrix} y = e^x \\ x = ln(y) \end{matrix}\right\| = \int \frac{y+y^2}{y(1-y)}dy = \int \frac{1+y}{1-y}dy = \left\|\begin{matrix} z = 1-y \\ y = 1-z \end{matrix}\right\| = -\int \frac{2-z}{z} =$
$= -\int \frac{2}{z} - 1dz = -2ln(|z|) + z + C = -2ln(|1-y|) - y + C = -2ln(|1-e^x|) - e^x + C$
Ответ: $-2ln(|1-e^x|) - e^x + C$

c)

$\int \frac{1}{x(ln(x)+5)}dx = \left\|\begin{matrix} y = ln(x) \\ x = e^y \end{matrix}\right\| = \int \frac{e^y}{e^y(y+5)}dy = \int \frac{1}{y+5}dy = \left\|\begin{matrix} z = y+5 \\ y = z-5 \end{matrix}\right\| = \int \frac{1}{z}dz =$
$= ln(|z|) + C = ln(|ln(x)+5|) + C$
Ответ: $ln(|ln(x)+5|) + C$

e)

$\int \frac{sin(2x)}{\sqrt{1-4sin^2(x)}}dx = \left\|\begin{matrix} sin(2x) = y \\ x = \frac{arcsin(y)}{2} \end{matrix}\right\| = \int \frac{y}{\sqrt{1-y^2}}d(\frac{arcsin(y)}{2}) = \int \frac{y}{2\sqrt{1-y^2}\sqrt{1-y^2}}dy =$
$= \frac{1}{2}\int \frac{y}{(1-y)(1+y)}dy = -\frac{1}{4}\int \frac{1}{1+y} - \frac{1}{1-y}dy = -\frac{1}{4}(ln(|1+y|) - ln(|1-y|)) + C =$
$= -\frac{1}{4}(ln(|1+sin(2x)|) - ln(|1-sin(2x)|)) + C = -\frac{1}{4}ln(|\frac{1+sin(2x)}{1-sin(2x)}|) + C$
Ответ: $-\frac{1}{4}ln(|\frac{1+sin(2x)}{1-sin(2x)}|) + C$

f)

$\int sin^7(x)dx = \left\| \begin{matrix} y = cos(x) \\ x = arccos(y) \end{matrix} \right\| = \int sin^6(x)sin(x)d(arccos(y)) =$
$= \int sin^6(x)sin(x) - \frac{1}{\sqrt{1-cos^2(x)}}dy = - \int sin^6(x)dy = - \int (1-y^2)^3dy =$
$= - \int 1 - 3y^2 + 3y^4 + y^6dy = -y + y^3 - \frac{3y^5}{5} - \frac{y^7}{7} + C = -cos(x) + cos^3(x) - \frac{3cos^5(x)}{5} - \frac{cos^7(x)}{7} + C$
Ответ: $-cos(x) + cos^3(x) - \frac{3cos^5(x)}{5} - \frac{cos^7(x)}{7} + C$

№3

a)

$\int arctg(x)dx = xarctg(x) - \int xd(arctg(x)) = xarctg(x) - \int \frac{x}{1+x^2}dx =$
$= xarctg(x) - \frac{1}{2}\int \frac{1}{1+x^2}d(1+x^2) = xarctg(x) - \frac{ln(1+x^2)}{2} + C$
Ответ: $xarctg(x) - \frac{ln(1+x^2)}{2} + C$

b)

$\int x^2cos^2(x)dx = \left\| \begin{matrix} u = x^2 \\ dv = cos^2(x)dx \end{matrix} \right\| = x^2(\frac{x}{2} + \frac{sin(2x)}{4}) - \int 2x(\frac{x}{2} + \frac{sin(2x)}{4})dx =$
$= x^2(\frac{x}{2}+\frac{sin(2x)}{4}) - 2\int \frac{x^2}{2}dx + 2\int \frac{xsin(2x)}{4}dx = x^2(\frac{x}{2}+\frac{sin(2x)}{4}) - \frac{2x^3}{6} + \frac{1}{2}\int \frac{x}{s}in(2x)dx =$

$= x^2(\frac{x}{2} + \frac{sin(2x)}{4}) - \frac{2x^3}{6} + \frac{1}{2}\int xsin(2x)dx = x^2(\frac{x}{2} + \frac{sin(2x)}{4}) - \frac{2x^3}{6} + \frac{1}{2}(x(-\frac{cos(2x)}{2}) + \int \frac{cos(2x)}{2}dx) =$
$= x^2(\frac{x}{2} + \frac{sin(2x)}{4}) - \frac{2x^3}{6} + \frac{1}{2}(-\frac{xcos(2x)}{2} + \frac{1}{4}\int cos(2x)d(2x)) = x^2(\frac{x}{2} + \frac{sin(2x)}{4}) - \frac{2x^3}{6} + \frac{1}{2}(-\frac{xcos(2x)}{2} + \frac{sin(2x)}{4}) + C = \frac{x^3}{2} + \frac{x^2sin(2x)}{4} - \frac{2x^3}{6} - \frac{xcos(2x)}{4} + \frac{sin(2x)}{8} + C =$
$= \frac{x^3}{6} + \frac{x^2sin(2x)+xcos(2x)}{4} - \frac{sin(2x)}{8} + C$
Ответ: $\frac{x^3}{6} + \frac{x^2sin(2x)+xcos(2x)}{4} - \frac{sin(2x)}{8} + C$

c)

$\int ln^2(x)dx = xln^2(x) - \int \frac{2xln(x)}{x}dx = xln^2(x) - 2\int ln(x)dx =$
$= xln^2(x) - 2(xln(x) - \int \frac{x}{x}dx) = xln^2(x) - 2xln(x) + 2x + C$
Ответ: $xln^2(x) - 2xln(x) + 2x + C$

e)

$\int sin(ln(x))dx = \left\| \begin{matrix} y = ln(x) \\ x = e^y \end{matrix} \right\| = \int sin(y)de^y = e^y sin(y) - \int e^y dsin(y) =$
$= e^y sin(y) - \int e^y cos(y)dy = e^y sin(y) - e^y cos(y) - \int e^y sin(y)dy$
Значит,
$\int e^y sin(y)dy = e^y sin(y) - e^y cos(y) - \int e^y sin(y)dy \Leftrightarrow \int e^y sin(y)dy = \frac{e^y(sin(y)-cos(y))}{2} + C$

$e^y sin(y) - e^y cos(y) - \frac{e^y(sin(y)-cos(y))}{2} = \frac{e^y(sin(y)-cos(y))}{2} + C =$
$= \frac{e^{ln(x)}(sin(ln(x))-cos(ln(x))}{2} + C = \frac{xsin(ln(x))-xcos(ln(x))}{2} + C$
Ответ: $\frac{xsin(ln(x))-xcos(ln(x))}{2} + C$