ДЗ по мат. анализу на 8.12.2021

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7 декабря 2021 г.

a)

$$\lim_{x\to 0}\frac{\cos x-\cos 3x}{x^2}=\lim_{x\to 0}\frac{1-\frac{x^2}{2}+\overline{o}(x^2)-1+\frac{9x^2}{2}-\overline{o}(9x^2)}{x^2}=\lim_{x\to 0}\frac{4x^2+\overline{o}(x^2)}{x^2}=\lim_{x\to 0}4+\overline{o}(1)=4$$

b)

$$\lim_{x\to 0}\frac{\cos{(a+2x)}-2\cos{(a+x)}+\cos{(a)}}{x^2}=\lim_{x\to 0}\frac{2\cos{(x+a)}\cos{(x)}-2\cos{(a+x)}}{x^2}=$$
$$=\lim_{x\to 0}\frac{2\cos{(x+a)}(\cos{(x)}-1)}{x^2}=\lim_{x\to 0}\frac{2\cos{(x+a)}(-\frac{x^2}{2}+\overline{o}(x^2))}{x^2}=\lim_{x\to 0}2\cos{(x+a)}(-\frac{1}{2}+\overline{o}(1))=$$
$$=-\cos{(a)}$$

c)

$$\lim_{x\to+\infty}(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x})=\lim_{x\to+\infty}((x^3+3x^2)^{\frac{1}{3}}-(x^2-2x)^{\frac{1}{2}})=$$
$$=\lim_{x\to+\infty}(x(1+\frac{3}{x})^{\frac{1}{3}}-x(1-\frac{2}{x})^{\frac{1}{2}})=\left\|\begin{matrix}\frac{3}{x}=y\\ x=\frac{3}{y}\\ y\to 0\end{matrix}\right\|=\lim_{y\to 0}(\frac{3}{y}(1+y)^{\frac{1}{3}}-\frac{3}{y}(1-\frac{2y}{3})^{\frac{1}{2}})=$$
$$=\lim_{y\to 0}(\frac{3}{y}(1+\frac{1}{3}y+\overline{o}(y))-\frac{3}{y}(1-\frac{y}{3}+\overline{o}(-\frac{2y}{3}))=\lim_{y\to 0}(\frac{3}{y}+1+\overline{o}(y)-\frac{3}{y}+1+\overline{o}(\frac{2y}{3}))=$$
$$=\lim_{y\to 0}(2+\overline{o}(y))=2$$

d)

$$\lim_{x\to a}\frac{a^x-x^a}{x-a}=\left\|\begin{matrix}y=x-a\\ x=y+a\\ y\to 0\end{matrix}\right\|=\lim_{y\to 0}\frac{a^{y+a}-(y+a)^a}{y}=\lim_{y\to 0}\frac{a^ya^a-a^a(\frac{y}{a}+1)^a}{y}=a^a\lim_{y\to 0}\frac{a^y-1-y+\overline{o}(\frac{y}{a})}{y}=$$
$$=a^a\frac{\lim\limits_{y\to 0}a^y-1-\lim\limits_{y\to 0}y+\overline{o}(y)}{\lim\limits_{y\to 0}y}=a^a\lim_{y\to 0}\frac{-y+\overline{o}(y)}{y}=a^a\lim_{y\to 0}(-1+\overline{o}(1))=-a^a$$

e)

$$\lim_{x\to a}\frac{ln(x)-ln(a)}{x-a} = \left\|\begin{matrix} y = x-a \\ x = y+a \\ y\to 0\end{matrix}\right\| = \lim_{y\to 0}\frac{ln(y+a)-ln(a)}{y} = \lim_{y\to 0}\frac{ln(\frac{y}{a}+1)}{y} =$$
$$= \lim_{y\to 0}\frac{\frac{y}{a}+\overline{o}(y)}{y} = \frac{1}{a}+\overline{o}(1) = \frac{1}{a}$$

f)

$$\lim_{x\to 0}\frac{ln(x^2+e^x)}{ln(x^4+e^{2x})} = \lim_{x\to 0}\frac{ln(x^2+1+x+\overline{o}(x))}{ln(x^4+1+2x+\overline{o}(x))} = \lim_{x\to 0}\frac{ln(1+x+\overline{o}(x))}{ln(1+2x+\overline{o}(x))} =$$
$$= \lim_{x\to 0}\frac{ln(x+\overline{o}(x+\overline{o}(x)))}{ln(2x+\overline{o}(2x+\overline{o}(x)))} = \lim_{x\to 0}\frac{ln(x+\overline{o}(x))}{ln(2x+\overline{o}(x))} = \frac{1+\overline{o}(1)}{2+\overline{o}(1)} = \frac{1}{2}$$

g)

$$\lim_{x\to 0}(1+tg^2(x))^{\frac{1}{ln(cos(x))}}$$

1)
$$\frac{1}{ln(cos(x))} = \frac{1}{ln(1-\frac{x^2}{2}+\overline{o}(x^2))} = \frac{1}{-\frac{x^2}{2}+\overline{o}(x^2)} = \frac{1}{\overline{o}(x^2)}$$

2)
$$tg^2(x) = (x+\overline{o}(x))^2 = x^2 + x\overline{o}(x) + \overline{o}^2(x) = x^2+\overline{o}(x^2) = \overline{o}(x^2)$$

$$\lim_{x\to 0}(1+tg^2(x))^{\frac{1}{ln(cos(x))}} = \lim_{x\to 0}(1+\overline{o}(x^2))^{\frac{1}{\overline{o}(x^2)}} = \lim_{x\to 0} e^{\frac{1}{\overline{o}(x^2)}\cdot ln(1+\overline{o}(x^2))} = e^{\lim\limits_{x\to 0}\frac{1}{\overline{o}(x^2)}\cdot ln(1+\overline{o}(x^2))} =$$
$$= e^{\lim\limits_{x\to 0}\frac{1}{\overline{o}(x^2)}\cdot\overline{o}(x^2)} = e$$

h)

$$\lim_{x\to 1}(x^2+sin^2(\pi x))^{\frac{1}{ln(x)}} = \left\|\begin{matrix} y = x-1 \\ x = y+1\end{matrix}\right\| = \lim_{y\to 0}((y+1)^2+sin^2(\pi(y+1)))^{\frac{1}{ln(y+1)}} =$$
$$= e^{\lim\limits_{y\to 0}\frac{1}{ln(y+1)}ln((y+1)^2+sin^2(\pi(y+1)))}$$

1) $\frac{1}{ln(y+1)} = \frac{1}{y+\overline{o}(y)}$

2) $ln((y+1)^2+sin^2(\pi(y+1))) = ln((y+1)^2+(\pi(y+1)+\overline{o}(\pi(y+1)))^2) =$
$= ln((y+1)^2+(\pi(y+1)+\overline{o}(1)+\overline{o}(y))) = ln((y+1)^2+\pi(y+1)+\overline{o}(1)+\overline{o}(y)) =$
$ln(y^2+2y+1+\pi y+\pi+\overline{o}(1)+\overline{o}(y)) = ln(2y+1+\pi y+\pi+\overline{o}(y)) = 2y+\pi y+$
$\pi+\overline{o}(y)+\overline{o}(2y+\pi y+\pi+\overline{o}(y)) = 2y+2\overline{o}(y)$

$$e^{\frac{1}{y+\overline{o}(y)}\cdot(2y+2\overline{o}(y))} = e^2$$