ДЗ по мат. анализу на 26.01.2022

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№1

a)

$\int \frac{(x+1)^3}{x^2}dx = \int \frac{x^3}{x^2} + \frac{3x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}dx = \int x + 3 + \frac{3}{x} + \frac{1}{x^2}dx =$
$= \int xdx + \int 3dx + \int \frac{3}{x}dx + \int \frac{1}{x^2}dx = \frac{x^2}{2} + 3x + 3ln(|x|) - \frac{1}{x} + C$
Ответ: $\frac{x^2}{2} + 3x + 3ln(|x|) - \frac{1}{x} + C$

b)

$\int \frac{1}{x^4-1}dx = \int \frac{1}{(x^2-1)(x^2+1)}dx = (*\int \frac{1}{x^2-1}dx - \int \frac{1}{x^2+1}dx) \cdot \frac{1}{2} =$
$= \frac{1}{2}(\frac{1}{2}ln(|\frac{x-1}{x+1}|) - arctg(x)) + C = \frac{1}{4}ln(|\frac{x-1}{x+1}|) - \frac{1}{2}arctg(x) + C$

$*\int \frac{1}{x^2-1}dx = \int \frac{1}{(x-1)(x+1)}dx = \frac{1}{2}(\int \frac{1}{x-1}d(x-1) - \int \frac{1}{x+1}d(x+1)) =$
$= \frac{1}{2}(ln(|x-1|) - ln(|x+1|)) + C = \frac{1}{2}ln(|\frac{x-1}{x+1}|) + C$

Ответ: $\frac{1}{4}ln(|\frac{x-1}{x+1}|) - \frac{1}{2}arctg(x) + C$

c)

$\int \frac{2^{2x-1} - 3^{2x+2}}{6^{2x}}dx = \int \frac{2^{2x}}{2 \cdot 9^x \cdot 2^{2x}} - \frac{9 \cdot 3^{2x}}{4^x \cdot 3^{2x}}dx = \int \frac{1}{2 \cdot 9^x} - \frac{9}{4^x}dx = \frac{1}{2}\int (\frac{1}{9})^x dx - 9\int (\frac{1}{4})^x dx =$
$= \frac{\frac{1}{9^x}}{2ln(\frac{1}{9})} - \frac{\frac{9}{4^x}}{ln(\frac{1}{4})} + C = \frac{\frac{9}{4^x}}{2ln(2)} - \frac{\frac{1}{9^x}}{4ln(3)} + C = \frac{9}{4^x 2ln(2)} - \frac{1}{9^x 4ln(3)} + C$
Ответ: $\frac{9}{4^x 2ln(2)} - \frac{1}{9^x 4ln(3)} + C$

d)

$\int \frac{e^{3x}-1}{e^x-1}dx = \int \frac{(e^x-1)(e^{2x}+e^x+1)}{e^x-1}dx = \int e^{2x} + e^x + 1dx = \int e^{2x}dx + \int e^x dx + \int 1dx =$
$= \frac{e^{2x}}{2} + e^x + x + C$

e)

$\int \frac{1}{sin^2(x)cos^2(x)}dx = \int \frac{1}{sin^2(x)}dx + \int \frac{1}{cos^2(x)}dx = tg(x) - ctg(x) + C$

f)

$$\int \frac{1}{cos(x)+sin(x)}dx = \frac{1}{\sqrt{2}}\int \frac{1}{\frac{1}{\sqrt{2}}cos(x)+\frac{1}{\sqrt{2}}sin(x)}dx = \frac{1}{\sqrt{2}}\int \frac{1}{cos(x)sin(\frac{\pi}{4})+sin(x)cos(\frac{\pi}{4})}dx =$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{sin(x+\frac{\pi}{4})}dx = \frac{1}{\sqrt{2}}ln(|tg(\frac{4x+\pi}{8})|) + C$$