ДЗ по мат. анализу на 15.12.2021

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№2

i)

$$\lim_{x\to\pi}\frac{\cos(x)}{\cos(3x)}^{\left(\frac{1}{(\sqrt{\pi x}-\pi)^2}\right)} = e^{\lim\limits_{x\to\pi}\left(\frac{1}{(\sqrt{\pi x}-\pi)^2}\cdot\ln\left(\frac{\cos(x)}{\cos(3x)}\right)\right)} = \left\|\begin{matrix} y = x-\pi \\ x = y+\pi \end{matrix}\right\|$$

1) $\frac{1}{(\sqrt{\pi(y+\pi)}-\pi)^2} = (\sqrt{\pi(y+\pi)}-\pi)^{-2} = \pi^{-2}((\frac{y}{\pi}+1)^{\frac{1}{2}}-1)^{-2} =$
$= \pi^{-2}(\frac{y}{2\pi}+\overline{o}(y))^{-2} = \frac{1}{\frac{y^2}{4}+\overline{o}(y^2)}$

2) $\ln\left(\frac{\cos(y+\pi)}{\cos(3(y+\pi))}\right) = \ln\left(\frac{\cos(y)}{\cos(3y)}\right) = \ln\left(\frac{1-\frac{y^2}{2}+\overline{o}(y)}{1-\frac{9y^2}{2}+\overline{o}(y)}\right) = \ln(1-\frac{y^2}{2}+\overline{o}(y)) - \ln(1-\frac{9y^2}{2}+\overline{o}(y))$

$= \ln(1+(-\frac{y^2}{2}+\overline{o}(y))) - \ln(1+(-\frac{9y^2}{2}+\overline{o}(y))) =$
$= -\frac{y^2}{2}+\overline{o}(y)+\overline{o}(-\frac{y^2}{2}+\overline{o}(y))+\frac{9y^2}{2}+\overline{o}(y))+\overline{o}(-\frac{9y^2}{2}+\overline{o}(y)) = 4y^2+\overline{o}(y^2)$

$$e^{\lim\limits_{y\to 0}\frac{4y^2+\overline{o}(y^2)}{\frac{y^2}{4}+\overline{o}(y^2)}} = e^{16}$$

j)

$$\lim_{x\to 1} x^{\operatorname{tg}\left(\frac{\pi x}{2}\right)} = \left\|\begin{matrix} y = x-1 \\ x = y+1 \\ y\to 0 \end{matrix}\right\| = e^{\lim\limits_{y\to 0} ln(y+1)\cdot\left(-\frac{1}{tg(\frac{\pi y}{2})}\right)}$$

1) $ln(1+y) = y+\overline{o}(y)$
2) $tg(\frac{\pi y}{2}) = \frac{\pi y}{2}+\overline{o}(\frac{\pi y}{2} = \frac{\pi}{2}(y+\overline{o}(y))$

$$e^{\lim\limits_{y\to 0} -\frac{y+\overline{o}(y)}{\frac{\pi}{2}(y+\overline{o}(y))}} = e^{-\frac{2}{\pi}}$$

№3

a)

$$\lim_{x\to 0}\frac{\sqrt{\cos x}-\sqrt[4]{e^{-x^2}}}{x^4}=\lim_{x\to 0}\frac{\sqrt{1-\frac{x^2}{2!}+\frac{x^4}{4!}+\overline{o}(x^4)}-\sqrt[4]{1-x^2+\frac{x^4}{2}+\overline{o}(x^4)}}{x^4}=$$
$$=\lim_{x\to 0}\frac{(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\overline{o}(x^4))^{\frac{1}{2}}-(1-x^2+\frac{x^4}{2}+\overline{o}(x^4))^{\frac{1}{4}}}{x^4}$$

1) $(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\overline{o}(x^4))^{\frac{1}{2}}=1+\frac{1}{2}(-\frac{x^2}{2!}+\frac{x^4}{4!}+\overline{o}(x^4))+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-\frac{x^2}{2!}+\frac{x^4}{4!}+\overline{o}(x^4))^2+\overline{o}((-\frac{x^2}{2!}+\frac{x^4}{4!}+\overline{o}(x^4))^2)=1-\frac{x^2}{4}+\frac{x^4}{48}-\frac{x^4}{32}+\overline{o}(x^4)=1-\frac{x^2}{4}-\frac{x^4}{96}+\overline{o}(x^4)$

2) $(1-x^2+\frac{x^4}{2}+\overline{o}(x^4))^{\frac{1}{4}}=1+\frac{1}{4}(-x^2+\frac{x^4}{2}+\overline{o}(x^4))+\frac{\frac{1}{4}(\frac{1}{4}-1)}{2}(-x^2+\frac{x^4}{2}+\overline{o}(x^4))^2+\overline{o}(-x^2+\frac{x^4}{2}+\overline{o}(x^4))=1-\frac{x^2}{4}+\frac{x^4}{8}+\overline{o}(x^4)-\frac{3x^4}{32}=1-\frac{x^2}{4}+\frac{x^4}{32}+\overline{o}(x^4)$

$$\lim_{x\to 0}\frac{1-\frac{x^2}{4}-\frac{x^4}{96}+\overline{o}(x^4)-1+\frac{x^2}{4}-\frac{x^4}{32}+\overline{o}(x^4))}{x^4}=\lim_{x\to 0}\frac{-\frac{x^4}{24}+\overline{o}(x^4)}{x^4}=-\frac{1}{24}$$

b)

$$\lim_{x\to 0}\frac{\cos(\sin x)-\sqrt{1-x^2+x^4}}{x^4}$$

1) $cos(sin(x))=1-\frac{1}{2}(x-\frac{x^3}{6}+\overline{o}(x^4))+\frac{1}{24}(x-\frac{x^3}{6}+\overline{o}(x^4))^4+\overline{o}(x^5)=$
$=1-\frac{x^2}{2}+\frac{x^4}{6}+\frac{x^4}{24}+\overline{o}(x^5)=1-\frac{x^2}{2}+\frac{5x^4}{24}+\overline{o}(x^5)$

2) $(1-x^2+x^4)^{\frac{1}{2}}=1+\frac{1}{2}(-x^2+x^4)+\frac{\frac{1}{2}(\frac{1}{2}-1)(-x^2+x^4)}{2}+\overline{o}(x^4)=1-\frac{x^2}{2}+\frac{x^4}{2}-\frac{x^4}{8}+\overline{o}(x^4)=$
$=1-\frac{x^2}{2}+\frac{3x^4}{8}+\overline{o}(x^4)$

$$\lim_{x\to 0}\frac{1-\frac{x^2}{2}+\frac{5x^4}{24}+\overline{o}(x^5)-1+\frac{x^2}{2}-\frac{3x^4}{8}+\overline{o}(x^4))}{x^4}=\lim_{x\to 0}\frac{-\frac{x^4}{6}+\overline{o}(x^4))}{x^4}=-\frac{1}{6}+\overline{o}(1)=-\frac{1}{6}$$