ДЗ по мат. анализу на 16.02.2022

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№2

b)

$$\int\limits_0^{2\pi} \frac{1}{4+cos^2(x)}dx = \int\limits_0^{2\pi} \frac{1}{\frac{4}{cos^2(x)}+1}dtg(x) = \left\| \begin{matrix} & y = tg(x) & \\ & x = arctg(y) & \\ [0, \frac{\pi}{2}) & (\frac{\pi}{2}, \frac{3\pi}{2}) & (\frac{3\pi}{2}, 2\pi] \\ & \downarrow & \\ [0, +\infty) & (-\infty, +\infty) & (-\infty, 0] \end{matrix} \right\| =$$

$$= \int\limits_0^{+\infty} \frac{1}{4y^2+5}dy + \int\limits_{-\infty}^{+\infty} \frac{1}{4y^2+5}dy + \int\limits_{-\infty}^{0} \frac{1}{4y^2+5}dy = \frac{1}{4}(\int\limits_0^{+\infty} \frac{1}{y^2+\frac{4}{5}}dy + \int\limits_{-\infty}^{+\infty} \frac{1}{y^2+\frac{4}{5}}dy + \int\limits_{-\infty}^{0} \frac{1}{y^2+\frac{4}{5}}dy) =$$

$$= \frac{1}{4}(\int\limits_0^{+\infty} \frac{1}{y^2+\frac{4}{5}}dy + \int\limits_{-\infty}^{+\infty} \frac{1}{y^2+\frac{4}{5}}dy + \int\limits_{-\infty}^{0} \frac{1}{y^2+\frac{4}{5}}dy) =$$

$$= \frac{1}{4}(\frac{2}{\sqrt{5}}arctg(\frac{2y}{\sqrt{5}})|_0^{+\infty} + \frac{2}{\sqrt{5}}arctg(\frac{2y}{\sqrt{5}})|_{-\infty}^{+\infty} + \frac{2}{\sqrt{5}}arctg(\frac{2y}{\sqrt{5}})|_{-\infty}^{0}) =$$

$$= \frac{1}{4}(\frac{2}{\sqrt{5}(\frac{\pi}{2}+\frac{\pi}{2}-(-\frac{\pi}{2})+0-(-\frac{\pi}{2}))}) = \frac{\pi}{\sqrt{5}}$$

Ответ: $\frac{\pi}{\sqrt{5}}$

№3

a)

$$\int\limits_0^{\infty} \frac{xln(x)}{(1+x^2)^2}dx = \lim_{\delta\to\infty}(F(\delta) - F(0))$$

$$F(x) = \int \frac{xln(x)}{(1+x^2)^2}dx = -\frac{ln(x)}{2(x^2+1)} + \int \frac{1}{2x(x^2+1)}dx$$

$$\int \frac{1}{2x(x^2+1)}dx = \frac{1}{2}\int \frac{1}{x(x^2+1)}dx = \left\| \begin{matrix} y = x^2+1 \\ x = \sqrt{y-1} \end{matrix} \right\| = \frac{1}{2}\int \frac{1}{\sqrt{y-1}y}\frac{1}{2\sqrt{y-1}}dy = \frac{1}{4}\int \frac{1}{y(y-1)} =$$

$$= \frac{1}{4}\int \frac{1}{y}dy - \int \frac{1}{y-1}dy = \frac{1}{4}(ln(|y|) - ln(|y-1|)) = \frac{ln(|\frac{y}{y-1}|)}{4} = \frac{ln(|\frac{x^2+1}{x^2}|)}{4}$$

$$F(X) = -\frac{ln(x)}{2(x^2+1)} + \frac{ln(|\frac{x^2+1}{x^2}|)}{4}$$

$F(\infty) - F(0) = 0 - 0 = 0$
Ответ: 0

b)

$\int\limits_0^\infty \frac{1}{(x^2+x+1)^2}dx = \lim\limits_{\delta\to\infty}(F(\delta) - F(0))$

$F(x) = \int \frac{1}{(x^2+x+1)^2}dx = \int \frac{1}{((x+\frac{1}{2})^2+\frac{3}{4})^2} = \left\| \begin{matrix} y = arctg(\frac{2x+1}{\sqrt{3}}) \\ x = \frac{\sqrt{3}tg(y)-1}{2} \end{matrix} \right\| = \int \frac{8cos^2(y)}{3\sqrt{3}}dy =$
$= \frac{8}{3\sqrt{3}} \int \frac{cos(2y)+1}{2}dy = \frac{8}{6\sqrt{3}}(\int cos(2y)dy + \int 1dy) = \frac{4}{3\sqrt{3}}(\frac{sin(2y)}{2} + y) =$
$= \frac{2sin(2y)}{3\sqrt{3}} + \frac{4y}{3\sqrt{3}} = \frac{2sin(2arctg(\frac{2x+1}{\sqrt{3}}))+4arctg(\frac{2x+1}{\sqrt{3}})}{3\sqrt{3}}$

$F(\infty) = \frac{2sin(\pi)+2\pi}{3\sqrt{3}} = \frac{2\pi}{3\sqrt{3}}$
$F(0) = \frac{2sin(\frac{\pi}{6})+\frac{2\pi}{3}}{3\sqrt{3}} = \frac{1}{3} + \frac{2\pi}{9\sqrt{3}}$
$F(\infty) - F(0) = \frac{2\pi}{3\sqrt{3}} - \frac{1}{3} - \frac{2\pi}{9\sqrt{3}} = \frac{4\pi}{9\sqrt{3}} - \frac{1}{3}$
Ответ: $\frac{4\pi}{9\sqrt{3}} - \frac{1}{3}$

c)

$\int\limits_0^\infty e^{-ax}sin(bx)dx = -\frac{e^{-ax}}{a}sin(bx)|_0^\infty + \frac{b}{a}\int\limits_0^\infty e^{-ax}cos(bx)dx =$
$= -\frac{e^{-ax}}{a}sin(bx)|_0^\infty + \frac{b}{a}(-\frac{e^{-ax}}{a}cos(bx))|_0^\infty - \frac{b^2}{a^2}\int_0^\infty e^{-ax}sin(bx)dx$
Мы пришли к тому же, а, значит, верно следующее:
$I = -\frac{e^{-ax}}{a}sin(bx)|_0^\infty - \frac{b}{a^2}e^{-ax}cos(bx)|_0^\infty - \frac{b^2}{a^2}I$
$I(1 + \frac{b^2}{a^2}) = 0 - 0 - \frac{b}{a^2}\cdot 0 + \frac{b}{a^2}\cdot 1 = \frac{b}{a^2}$
$I = \frac{\frac{b}{a^2}}{1+\frac{b^2}{a^2}} = \frac{b}{a^2+b^2}$
Ответ: $\frac{b}{a^2+b^2}$

d)

$\int\limits_0^1 cos^2(ln(x))dx = \lim\limits_{\delta\to\infty}(F(\delta) - F(0))$
$F(x) = \int cos^2(ln(x))dx = \left\| \begin{matrix} y = ln(x) \\ x = e^y \end{matrix} \right\| = \int e^y cos^2(y)dy = \int \frac{e^y(cos(2y)+1)}{2}dy =$
$= \frac{1}{2}\int e^y cos(2y) + e^y dy = \frac{1}{2}(e^y cos(2y) - \int 2e^y(-sin(2y)dy) =$
$= \frac{1}{2}(e^y cos(2y) + 2\int e^y sin(2y)dy) = \frac{1}{2}(e^y cos(2y) + 2(e^y sin(2y) - 2\int e^y cos(2y)dy))$
Отсюда следует следующее:
$\int e^y cos(2y)dy = e^y cos(2y) + 2(e^y sin(2y) - 2\int e^y cos(2y)dy)$
$5\int e^y cos(2y)dy = e^y cos(2y) + e^y 2sin(2y)$

$\int e^y cos(2y)dy = \frac{e^y cos(2y)+2e^y sin(2y)}{5}$

$\int cos^2(ln(x))dx = \frac{1}{2}(\frac{e^y cos(2y)+2e^y sin(2y)}{5} + e^y) = \frac{xcos(2ln(x))+2xsin(2ln(x))+5x}{10}$
$F(1) = \frac{3}{5}$
$F(0) = 0$
$F(1) - F(0) = \frac{3}{5}$
Ответ: $\frac{3}{5}$