ДЗ по мат. анализу на 16.02.2022

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№2

a)

$$\int \frac{1}{3+sin(x)}dx = \left\| \begin{matrix} y = tg(\frac{x}{2}) \\ dy = \frac{1}{2cos^2(\frac{x}{2})}dx \\ sin(x) = \frac{2y}{y^2+1} \end{matrix} \right\| = \int \frac{2}{3y^2+2y+3}dy = \frac{2}{3}\int \frac{1}{(y+\frac{1}{3})^2+\frac{8}{9}}dy =$$

$$= \frac{2}{3} \cdot \frac{3}{2\sqrt{2}} \cdot arctg(\frac{3y+1}{2\sqrt{2}}) + C = \frac{arctg(\frac{3y+1}{2\sqrt{2}})}{\sqrt{2}} + C = \frac{arctg(\frac{3tg(\frac{x}{2})+1}{2\sqrt{2}})}{\sqrt{2}} + C$$

Ответ: $\frac{arctg(\frac{3tg(\frac{x}{2})+1}{2\sqrt{2}})}{\sqrt{2}} + C$

b)

$$\int \frac{1}{2sin(x)+3cos(x)+5}dx = \int \frac{1}{4sin(\frac{x}{2})cos(\frac{x}{2})+6cos^2(\frac{x}{2})-3+5}dx =$$

$$= \int \frac{1}{3cos^2(\frac{x}{2})+2sin(\frac{x}{2})cos(\frac{x}{2})+sin^2(\frac{x}{2})+cos^2(\frac{x}{2})}d\frac{x}{2} = \int \frac{1}{cos^2(\frac{x}{2})(tg^2(\frac{x}{2})+2tg(\frac{x}{2})+4)}d\frac{x}{2} =$$

$$= \int \frac{1}{tg^2(\frac{x}{2})+2tg(\frac{x}{2})+4}dtg(\frac{x}{2}) = \left\| \begin{matrix} y = tg(\frac{x}{2}) \\ x = 2arctg(y) \end{matrix} \right\| = \int \frac{1}{y^2+2y+4}dy = \int \frac{1}{(y+1)^2+3}dy =$$

$$= \cdot\frac{1}{\sqrt{3}} \cdot arctg(\frac{y+1}{\sqrt{3}}) + C = \frac{arctg(\frac{y+1}{\sqrt{3}})}{\sqrt{3}} + C = \frac{arctg(\frac{tg(\frac{x}{2})+1}{\sqrt{3}})}{\sqrt{3}} + C$$

Ответ: $\frac{arctg(\frac{tg(\frac{x}{2})+1}{\sqrt{3}})}{\sqrt{3}} + C$

c)

$$\int \frac{1}{2sin^2(x)+3cos^2(x)}dx = \int \frac{1}{cos^2(x)(2tg^2(x)+3)}dx = \int \frac{1}{2tg^2(x)+3}dtg(x) = \left\| \begin{matrix} y = tg(x) \\ x = arctg(y) \end{matrix} \right\| =$$

$$= \int \frac{1}{2y^2+3}dy = \frac{1}{2}\int \frac{1}{y^2+\frac{3}{2}}dy = \frac{1}{2} \cdot \sqrt{\frac{2}{3}} \cdot arctg(\frac{\sqrt{2}y}{\sqrt{3}}) + C = \frac{arctg(\frac{\sqrt{2}y}{\sqrt{3}})}{\sqrt{6}} + C =$$

$$= \frac{arctg(\frac{\sqrt{2}tg(x)}{\sqrt{3}})}{\sqrt{6}} + C$$

Ответ: $\frac{arctg(\frac{\sqrt{2}tg(x)}{\sqrt{3}})}{\sqrt{6}} + C$

d)

$$\int \frac{cos(x)}{sin(x)-5cos(x)}dx = \int \frac{5sin(x)+cos(x)-5(sin(x)-5cos(x))}{26(sin(x)-5cos(x))}dx = \int \frac{5sin(x)+cos(x)}{26(sin(x)-5cos(x))} - \frac{5}{26}dx =$$

$$= \frac{1}{26}\int \frac{5sin(x)+cos(x)}{sin(x)-5cos(x)}dx - \frac{5}{26}x + C = \frac{1}{26}\int \frac{1}{sin(x)-5cos(x)}d(sin(x)-5cos(x)) - \frac{5}{26}x + C =$$

$$= \frac{1}{26}ln(|sin(x)-5cos(x)|) - \frac{5}{26}x + C$$

Ответ: $\frac{1}{26}ln(|sin(x)-5cos(x)|) - \frac{5}{26}x + C$

e)

$$\int \frac{1}{\sqrt[3]{(x+1)^2(x-1)^7}}dx = \int \frac{1}{(x-1)^2(x+1)\sqrt[3]{\frac{x-1}{x+1}}}dx = \left\| \begin{matrix} y = \sqrt[3]{\frac{x-1}{x+1}} \\ x = \frac{1+y^3}{1-y^3} \end{matrix} \right\| =$$

$$= \int \frac{1}{(\frac{1+y^3}{1-y^3}-1)^2(\frac{1+y^3}{1-y^3}+1)y} \cdot \frac{6y^2}{(1-y^3)^2}dy = \int \frac{6y^2}{y(\frac{1+y^3}{1-y^3}-1)^2(\frac{1+y^3}{1-y^3}+1)(1-y^3)^2}dy = \int \frac{6y^2}{4y^7(\frac{1+y^3}{1-y^3}+1)}dy =$$

$$= \frac{3}{2}\int \frac{1}{y^5(\frac{1+y^3}{1-y^3}+1)}dy = \frac{3}{2}\int \frac{1-y^3}{2y^5}dy = \frac{3}{4}\int \frac{1-y^3}{y^5}dy = \frac{3}{4}(-\frac{1}{4y^4} + \frac{1}{y}) + C =$$

$$= -\frac{3}{16y^4} + \frac{3}{4y} + C = -\frac{3}{16\sqrt[3]{\frac{x-1}{x+1}}^4} + \frac{3}{4\sqrt[3]{\frac{x-1}{x+1}}} + C = -\frac{3(x+1)\sqrt[3]{x+1}}{16(x-1)\sqrt[3]{x-1}} + \frac{3\sqrt[3]{x+1}}{4\sqrt[3]{x-1}} + C$$

Ответ: $-\frac{3(x+1)\sqrt[3]{x+1}}{16(x-1)\sqrt[3]{x-1}} + \frac{3\sqrt[3]{x+1}}{4\sqrt[3]{x-1}} + C$

№1 (доп)

a)

$$\int \frac{x^7}{(x^4+1)^2}dx = \left\| \begin{matrix} y = x^4+1 \\ x = \sqrt[4]{y-1} \end{matrix} \right\| = \int \frac{(y-1)\sqrt[4]{y-1}^3}{y^2}\frac{1}{4\sqrt[4]{(x-1)^3}}dy = \frac{1}{4}\int \frac{y-1}{y^2}dy =$$

$$= \frac{1}{4}(\int \frac{1}{y}dy - \int \frac{1}{y^2}dy) = \frac{1}{4}ln(|y|) + \frac{1}{4y} + C = \frac{1}{4}ln(|x^4+1|) + \frac{1}{4x^4+4} + C$$

Ответ: $\frac{1}{4}ln(|x^4+1|) + \frac{1}{4x^4+4} + C$

b)

$$\int \frac{x^2-1}{x^4+x^2+1}dx = \int \frac{x^2-1}{(x^2-1)^2-x^2}dx = \int \frac{x^2-1}{(x^2-1-x)(x^2-1+x)}dx = \frac{1}{2}\int \frac{2x-1}{x^2-1-x} + \frac{2x+1}{x^2-1+x}dx =$$

$$= \left\| \begin{matrix} y = x^2-1-x \\ z = x^2-1+x \end{matrix} \right\| = \frac{1}{2}\int \frac{1}{y}dy + \frac{1}{2}\int \frac{1}{z}dz = \frac{1}{2}ln(|x^2-x+1|) - \frac{1}{2}ln(|x^2+x+1|) + C$$

Ответ: $\frac{1}{2}ln(|x^2-x+1|) - \frac{1}{2}ln(|x^2+x+1|) + C$

c)

$\int \frac{cos(x)}{cos^2(x)-5cos(x)+6}dx = \left\| \begin{matrix} y = tg(\frac{x}{2}) \\ x = arccos(\frac{1-y^2}{y^2+1}) \end{matrix} \right\| = \int \frac{\frac{1-y^2}{1+y^2}}{(\frac{1-y^2}{1+y^2})^2-5(\frac{1-y^2}{1+y^2})+6} \cdot \frac{1}{\frac{1}{2}(1+y^2)}dy =$
$= \int \frac{\frac{1-y^2}{1+y^2}}{\frac{1-2y^2+y^4-5+5y^4+6+6y^2+6t^4}{(1+y^2)^2}} \cdot \frac{1}{\frac{1}{2}(1+y^2)}dy = \int \frac{2-2y^2}{1-2y^2+y^4-5+5y^4+6+12y^2+6y^4}dy = \int \frac{1-y^2}{1+5y^2+6y^4}dy =$
$= \int \frac{1-y^2}{(2y^2+1)(3y^2+1)}dy = -\frac{3}{2}\int \frac{1}{y^2+\frac{1}{2}}dy + \frac{4}{3}\int \frac{1}{y^2+\frac{1}{3}} + C = -\frac{3\sqrt{2}arctg(\sqrt{2}y)}{2} + \frac{4\sqrt{3}arctg(\sqrt{3}y)}{3} +$
$C =$
$= -\frac{3\sqrt{2}arctg(\sqrt{2}tg(\frac{x}{2}))}{2} + \frac{4\sqrt{3}arctg(\sqrt{3}tg(\frac{x}{2}))}{3} + C$
Ответ: $-\frac{3\sqrt{2}arctg(\sqrt{2}tg(\frac{x}{2}))}{2} + \frac{4\sqrt{3}arctg(\sqrt{3}tg(\frac{x}{2}))}{3} + C$

e)

$\int \frac{1}{\sqrt{x}(1+\sqrt[3]{x})}dx = \left\| \begin{matrix} y = \sqrt[3]{x} \\ x = y^3 \end{matrix} \right\| = \int \frac{3y^2}{\sqrt{y^3}(1+y)}dy = \int \frac{3\sqrt{y}}{1+y}dy = \left\| \begin{matrix} z = \sqrt{y} \\ y = z^2 \end{matrix} \right\| = 6\int \frac{z^2}{1+z^2}dz =$
$= 6(\int 1dz - \frac{1}{1+z^2}dz) = 6z - 6arctg(z) + C = 6\sqrt{y} - 6arctg(\sqrt{y}) + C =$
$= 6\sqrt[6]{x} - 6arctg(\sqrt[6]{x}) + C$
Ответ: $6\sqrt[6]{x} - 6arctg(\sqrt[6]{x}) + C$