ДЗ по мат. анализу на 16.02.2022

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№1

a)

$\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{k}{k^2+n^2} = \lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{k}{\frac{k^2}{n}+n}\cdot\frac{1}{n}$
$a = \lim\limits_{n\to\infty}\xi 1 = \lim\limits_{n\to\infty}\frac{1}{n} = 0$
$b = \lim\limits_{n\to\infty}\xi n = \lim\limits_{n\to\infty}\frac{n}{n} = 1$
$\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{k}{\frac{k^2}{n}+n}\cdot\frac{1}{n} = \lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{\frac{k^2}{n^2}+1}\cdot\frac{k}{n}\cdot\frac{1}{n} = \int\limits_0^1\frac{x}{x^2+1}dx = \left\|\begin{matrix} y = x^2+1 \\ x = \sqrt{y-1}\end{matrix}\right\| = \int\limits_0^1\frac{\sqrt{y-1}}{y}\frac{1}{2\sqrt{y-1}}dy =$
$= \int\limits_0^1\frac{1}{2y}dy = \frac{ln(|y|)}{2}|_0^1 = \frac{ln(|x^2+1|)}{2}|_0^1 = \frac{ln(2)}{2}$
Ответ: $\frac{ln(2)}{2}$

№2

a)

$\int_0^{2\pi} sin^4(x)dx = \int_0^{2\pi}(\frac{1-cos(2x)}{2})^2dx = \frac{1}{4}\int_0^{2\pi} 1-2cos(2x)+cos^2(2x)dx =$
$= \frac{1}{4}(x - sin(2x) + \frac{1}{4}\int 1+cos(4x)dx)|_0^{2\pi} = \frac{1}{4}(x-sin(2x)+\frac{x}{2}+\frac{sin(4x)}{8})|_0^{2\pi} =$
$= \frac{3x}{8} - \frac{sin(2x)}{4} + \frac{sin(4x)}{32}|_0^{2\pi} = \frac{6\pi}{8} - 0 + 0 - 0 + 0 - 0 = \frac{3\pi}{4}$

Ответ: $\frac{3\pi}{4}$

b)

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{4+cos^2(x)}dx = \left\|\begin{matrix} y = tg(x) \\ cos^2(x) = \frac{1}{y^2+1}\end{matrix}\right\| = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{4y^2+5}dy = \frac{1}{4}\int\frac{1}{y^2+\frac{5}{4}}dy|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} =$
$= \frac{1}{4\sqrt{\frac{5}{4}}}arctg(\frac{y}{\sqrt{\frac{5}{4}}})|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{arctg(\frac{2y}{\sqrt{5}})}{2\sqrt{5}}|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{arctg(\frac{2tg(x)}{\sqrt{5}})}{2\sqrt{5}}|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{arctg(\frac{2}{\sqrt{5}})}{2\sqrt{5}} + \frac{arctg(\frac{2}{\sqrt{5}})}{2\sqrt{5}} =$

$= \frac{arctg(\frac{2}{\sqrt{5}})}{\sqrt{5}}$
Ответ: $\frac{arctg(\frac{2}{\sqrt{5}})}{\sqrt{5}}$

c)

$\int_{-1}^{1} x^3 e^{x^2} dx$
$\int x^3 e^{x^2} dx = \int x^2 \cdot x e^{x^2} dx = x^2 \cdot \frac{e^{x^2}}{2} - \int \frac{e^{x^2}}{2} 2x dx = x^2 \frac{e^{x^2}}{2} - \int e^{x^2} x dx =$
$= x^2 \frac{e^{x^2}}{2} - \int \frac{1}{2} d(e^{x^2}) = x^2 \frac{e^{x^2}}{2} - \frac{e^{x^2}}{2} = \frac{e^{x^2}(x^2-1)}{2}$
$\frac{e^{x^2}(x^2-1)}{2}|_{-1}^{1} = 0$
Ответ: 0

d)

$\int_{0}^{\sqrt{3}} xarctg(x) dx$
$\int xarctg(x) dx = arctg(x) \frac{x^2}{2} - \int \frac{x^2}{2} \frac{1}{1+x^2} dx = arctg(x) \frac{x^2}{2} - \int \frac{x^2}{2+2x^2} dx =$
$= arctg(x) \frac{x^2}{2} - \frac{1}{2} \int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1} dx = arctg(x) \frac{x^2}{2} - \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{1}{x^2+1} dx =$
$= arctg(x) \frac{x^2}{2} - \frac{x}{2} + \frac{arctg(x)}{2} dx = \frac{arctg(x)x^2 - x + arctg(x)}{2}$
$\frac{arctg(x)x^2 - x + arctg(x)}{2}|_{0}^{\sqrt{3}} = \frac{arctg(\sqrt{3})3 - \sqrt{3} + arctg(\sqrt{3})}{2} - \frac{arctg(0)0 - 0 + arctg(0)}{2} =$
$= \frac{\pi - \sqrt{3} + \frac{\pi}{3}}{2} - 0 = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$
Ответ: $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

e)

$\int_{-10}^{10} sin(x) arctg(x^2) dx$
Проверим функцию на четность.
1) $f(-x) = -sin(x) arctg(x^2)$
2) $-f(x) = -sin(x) arctg(x^2)$
Выходит, что функция нечетная, а, значит, $\int_{-10}^{10} sin(x) arctg(x^2) dx = 0$
Ответ: 0