ДЗ по мат. анализу на 16.02.2022

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№2

d)

$\int \frac{1}{cos(x)}dx = \int \frac{cos(x)}{cos^2(x)}dx = \int \frac{cos(x)}{1-sin^2(x)}dx = \left\| \begin{matrix} y = sin(x) \\ x = arcsin(x) \end{matrix} \right\| = \int \frac{\sqrt{1-y^2}}{1-y^2} \cdot \frac{1}{\sqrt{1-y^2}}dy =$
$= \int \frac{1}{(1-y)(1+y)}dy = \frac{1}{2}\int \frac{1}{1-y}dy + \frac{1}{2}\int \frac{1}{1+y}dy = \frac{1}{2}(-ln(|1-y|) + ln(|1+y|)) + C =$
$= \frac{1}{2}ln(|\frac{1+y}{1-y}|) + C = \frac{1}{2}ln(|\frac{1+sin(x)}{1-sin(x)}|) + C$
Ответ: $\frac{1}{2}ln(|\frac{1+sin(x)}{1-sin(x)}|) + C$

g)

$\int x^2\sqrt{1-x^2}dx = \left\| \begin{matrix} y = arcsin(x) \\ x = siny \end{matrix} \right\| = \int sin^2(y)\sqrt{1-sin^2(y)}d(sin(y)) =$
$= \int \frac{1}{4}(2sin(y)cos(y))^2dy = \frac{1}{4}\int sin^2(2y)dy = \frac{1}{8}\int 1 - cos(4y)dy = \frac{1}{8}(\int 1dy - \int cos(4y)dy) = \frac{1}{8}(y - \frac{sin(4y)}{4}) + C = \frac{y}{8} - \frac{sin(4y)}{32} + C = \frac{arcsin(x)}{8} - \frac{sin(4arcsin(x))}{32} + C$
Ответ: $\frac{arcsin(x)}{8} - \frac{sin(4arcsin(x))}{32} + C$

h)

$\int \frac{1}{(a^2-x^2)^{\frac{3}{2}}}dx = \left\| \begin{matrix} y = arcsin(\frac{x}{a}) \\ x = asin(y) \end{matrix} \right\| = \int \frac{1}{(a^2-a^2sin^2(y))^{\frac{3}{2}}} \cdot acos(y)dy = \int \frac{acos(y)}{a^3(1-sin^2(y))^{\frac{3}{2}}}dy =$
$= \frac{a}{a^3}\int \frac{cos(y)}{cos^3(y)}dy = \frac{1}{a^2}\int \frac{1}{cos^2(y)}dy = \frac{1}{a^2}tg(y) + C = \frac{1}{a^2}tg(arcsin(\frac{x}{a})) + C =$
$= \frac{1}{a^2}\frac{sin(arcsin(\frac{x}{a}))}{cos(arcsin(\frac{x}{a}))} + C = \frac{1}{a^2}\frac{\frac{x}{a}}{\frac{\sqrt{a^2-x^2}}{a}} + C = \frac{x}{a^2\sqrt{a^2-x^2}} + C$

№3

d)

$\int x^2 ln(1+x)dx = \frac{x^3 ln(x+1)}{3} - \int \frac{x^3}{3(x+1)}dx = \left\| \begin{matrix} y = x+1 \\ x = y-1 \end{matrix} \right\| = \frac{x^3 ln(x+1)}{3} - \int \frac{x^3}{3(x+1)}dx =$
$= \frac{x^3 ln(x+1)}{3} - \frac{1}{3}\int \frac{(y-1)^3}{y}dy = \frac{x^3 ln(x+1)}{3} - \frac{1}{3}\int \frac{y^3-3y^2+3y-1}{y}dy =$
$= \frac{x^3 ln(x+1)}{3} - \frac{1}{3}(\int y^2 dy - \int 3ydy + \int 3dy - \int \frac{1}{y}dy) =$
$= \frac{x^3 ln(x+1)}{3} - (-\frac{ln(|y|)}{3} + \frac{y^3}{9} - \frac{y^2}{2} + y) = \frac{x^3 ln(x+1)}{3} + \frac{ln(|x+1|)}{3} - \frac{(x+1)^3}{9} + \frac{(x+1)^2}{2} - x + C$
Ответ: $\frac{x^3 ln(x+1)}{3} + \frac{ln(|x+1|)}{3} - \frac{(x+1)^3}{9} + \frac{(x+1)^2}{2} - x + C$

№1 (Лист 3)

a)

$\int \frac{1}{x(x+1)(x+2)}dx = \int \frac{a}{x}dx + \int \frac{b}{x+1}dx + \int \frac{c}{x+2}dx$
$x^2 : 0 = a + b + c$
$x^1 : 0 = 3a + 2b + c$
$x^0 : 1 = 2a$
Отсюда,

$$\begin{cases} a = \dfrac{1}{2} \\ b = -1 \\ c = \dfrac{1}{2} \end{cases}$$

$\frac{1}{2}\int \frac{1}{x}dx - \int \frac{1}{x+1}dx + \frac{1}{2}\int \frac{1}{x+2}dx = \frac{1}{2}ln(|x|) - ln(|x+1|) + \frac{1}{2}ln(|x+2|) + C = ln(|\frac{\sqrt{x^2+2x}}{x+1}|) + C$
Ответ: $ln(|\frac{\sqrt{x^2+2x}}{x+1}|) + C$

b)

$\int \frac{x^2+5x+4}{x^4+5x^2+4}dx = \int \frac{x^2+5x+4}{(x^2+4)(x^2+1)}dx = \int \frac{ax+c}{x^2+4}dx + \int \frac{bx+d}{x^2+1}dx$
$x^3 : 0 = a + b$
$x^2 : 1 = c + d$
$x^1 : 5 = a + 4b$
$x^0 : 4 = c + 4d$

Отсюда,

$$\begin{cases} a = -\dfrac{5}{3} \\ b = \dfrac{5}{3} \\ c = 0 \\ d = 1 \end{cases}$$

$-\frac{5}{3}\int \frac{x}{x^2+4}dx + \frac{5}{3}\int \frac{x+1}{x^2+1}dx = -\frac{5}{6}ln(|x^2+4|) + \frac{1}{3}(\int \frac{5x}{x^2+1}dx + \int \frac{3}{x^2+1}dx) =$
$= -\frac{5}{6}ln(|x^2+4|) + \frac{5}{6}ln(|x^2+1|) + arctg(x) + C$
Ответ: $-\frac{5}{6}ln(|x^2+4|) + \frac{5}{6}ln(|x^2+1|) + arctg(x) + C$