FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION
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Report

on the practical task 4

"Algorithms for unconstrained nonlinear optimization. Stochastic and metaheuristic algorithms"

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Goal

The use of stochastic and metaheuristic algorithms (Simulated Annealing, Differential Evolution, Particle Swarm Optimization) in the tasks of unconstrained nonlinear optimization and the experimental comparison of them with Nelder-Mead and Levenberg-Marquardt algorithms

Problems and methods

I. *Generate the noisy data* (x_k, y_k), where $k = 0, \dots, 1000$, *according to the rule:*

$$y_k = \begin{cases} -100 + \delta_k, f(x_k) < -100 \\ f(x_k) + \delta_k, -100 <= f(x_k) <= 100, \\ 100 + \delta_k, f(x_k) > 100 \end{cases} \qquad x_k = \frac{3k}{1000},$$

$$f(x) = 1 / (x^2 - 3x + 2),$$

Where $\delta_k \sim N(0, 1)$ *are values of random variable with standard normal distribution. Approximate the data by the rational function*

$$F(x, a, b, c, d) = (ax + b) / (x^2 + cx + d)$$

by means of least squares through the numerical minimization of the following function:

$$D(a, b, c, d) = \sum_{k=0}^{1000} (F(x_k, a, b, c, d) - y_k)^2.$$

*To solve the minimization problem, use Nelder-Mead algorithm, Levenberg Marquardt algorithm and **at least two** of the methods among Simulated Annealing, Differential Evolution and Particle Swarm Optimization. If necessary, set the initial approximations and other parameters of the methods. Use eps* = 0.001 *as the precision; at most 1000 iterations are allowed. Visualize the data and the approximants obtained **in a single plot**. Analyze and compare the results obtained (in terms of number of iterations, precision, number of function evaluations, etc.).*

II. *Choose at least 15 cities in the world having land transport connections between them. Calculate the distance matrix for them and then apply the Simulated Annealing method to solve the corresponding Travelling Salesman Problem. Visualize the results at the first and the last iteration. If necessary, use the city dataset from* [*https://people.sc.fsu.edu/~jburkardt/datasets/cities/cities.html*](https://people.sc.fsu.edu/~jburkardt/datasets/cities/cities.html)

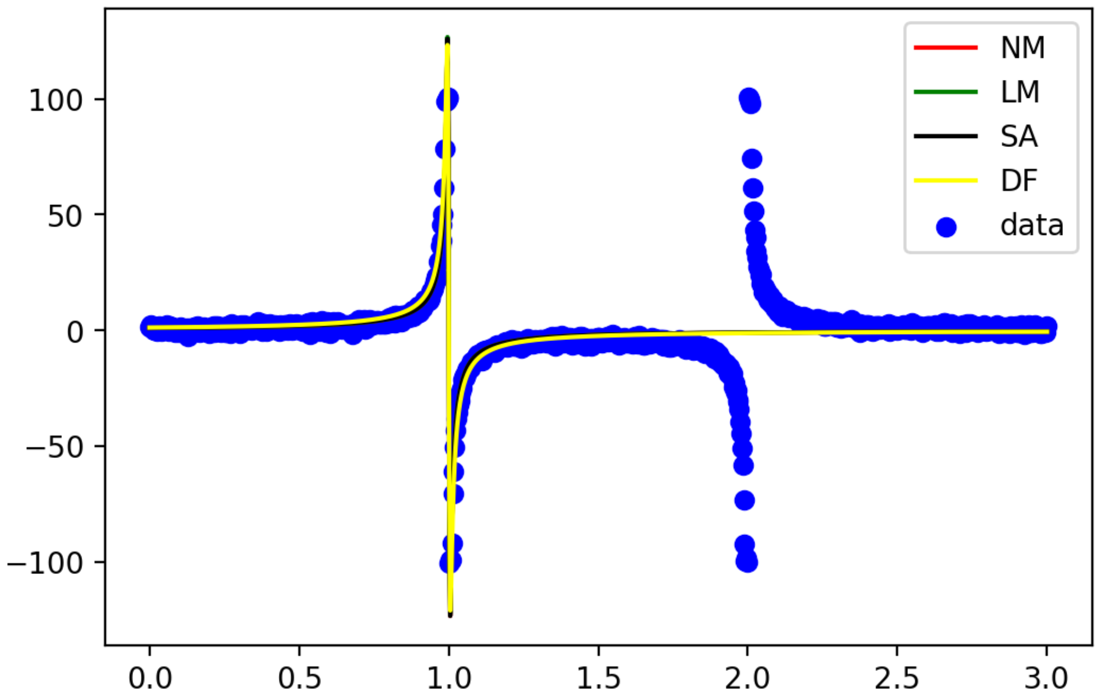
Brief theoretical part

1) Simulated annealing method:
   Let a_0 be an initial approximation. At each iteration k:
   1) we choose $a * \in Neighbours(a_k)$, where *Neighbours* is a custom searching rule;
   2) if $f(a*) < f(a_k)$, then $a_{k+1} = a*$; if $f(a*) > f(a_k)$, then $a_{k+1} = a*$ with probability
   $exp(-\frac{f(a*) - f(a_k)}{T_k})$, where T_k is decreasing temperature sequence.
   Process stops, when $T_k = 0$. Then a_k is our minimization point for *f*

2) Differential evolution method:
   1) Randomly pick *N* agents (the population).
   2) for every $\boldsymbol{x}$ in the population pick 3 distinct agents from the population *a, b, c*
   3) If $r_i < p$, then $y_i = a_i + w(b_i - c_i)$, otherwise $y_i = x_i$. $r_i \sim U(0, 1)$
   4) If $f(y) > f(x)$, then replace x with y in the population
   Pick the best agent from the population (i. e. $f(x) \rightarrow \min x \in Q$) when a certain number of iterations have passed

Results

I. Plot with generated noisy data and 4 approximative curves obtained by four optimization methods:



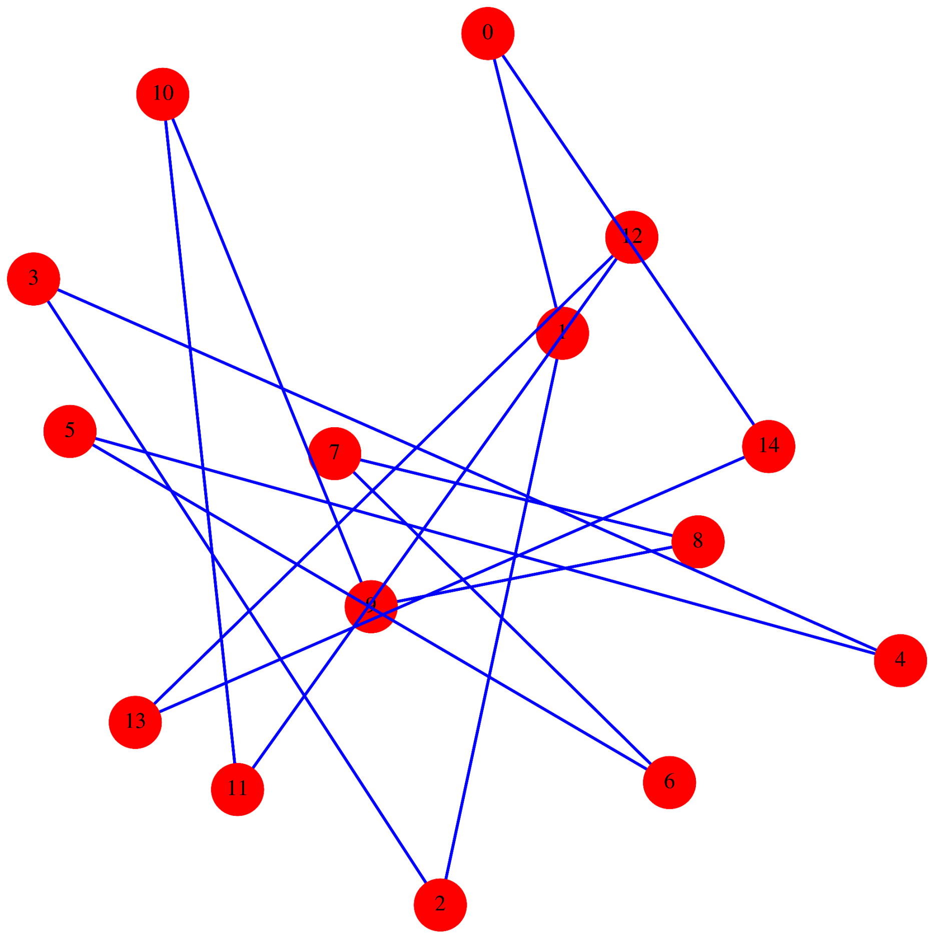
We used Dual Annealing method that is an improved version of SA as it's more likely to escape the local minimum and converges faster than SA.

Comparison of optimization algorithms:

Method	Number of iterations	$D(a_{min}, b_{min}, c_{min, }d_{min})$	Number of f-calculations	Execution time, seconds
NM	454	135934.479	469	0.563
LM	135	135934.479	135	0.003
SA	1000	135934.482	8966	6.639
DF	4	138048.131	1150	0.868

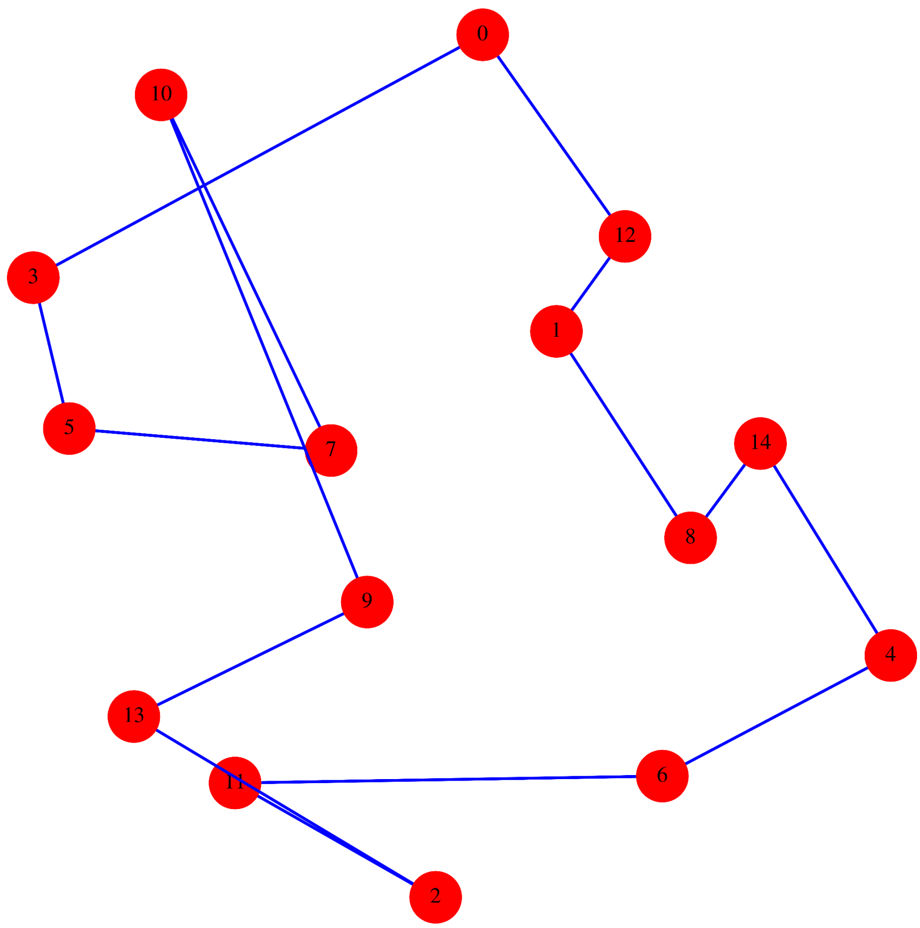
(a_{min}, b_{min}) is the obtained solution for minimization task. NM and LM methods are faster than the stochastic algorithms. NM and LM methods are also more accurate than stochastic ones in terms of cost function value at minimization point. SA has the longest execution's time. However, with our initial parameters it achieved more accurate result than the DF method. Also SA's initial parameters may be tweaked to reduce the overall execution time.

II. We used 15x15 distance matrix (*https://github.com/ilyalinov/Algorithms/blob/master/4/dist.txt*) to pose the travelling salesman problem. Visualized input cycle for simulated annealing algorithm (distances between cities preserved):



$cost = 817$ (initial route's cost); $min\_cost = 291$ (known cost of the opitimal route).

Visualized final cycle obtained by simulated annealing method (distances between cities preserved used):



$cost\_SA = 398$ (final cost of the route obtained by simulated annealing algorithm for 1000 iterations)

Conclusion

We used LM, NM and stochastic and metaheuristic algorithms (Simulated Annealing, Differential Evolution) in the task of unconstrained nonlinear optimization. We analyzed and compared the characteristics of these methods. We visualized 4 obtained approximations. We solved the travelling salesman problem for 15 cities using SA method. We visualized results at first and last iterations.

Appendix

https://github.com/ilyalinov/Algorithms