FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION

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Report

on the practical task 2

“Algorithms for unconstrained nonlinear optimization. Direct methods”

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Task 2. Algorithms for unconstrained nonlinear optimization. Direct methods

Goal

The use of direct methods (one-dimensional methods of exhaustive search, dichotomy, golden section search; multidimensional methods of exhaustive search, Gauss (coordinate descent), Nelder-Mead) in the tasks of unconstrained nonlinear optimization.

Problems and methods

I. *Use the one-dimensional methods of exhaustive search, dichotomy and golden section search to find an approximate (with precision* eps = 0.001*) solution* x: $f(x) \;-> \; min$ *for the following functions and domains:*

1. $f(x) = x^{3}$, $x \in [0, 1]$;
2. $f(x) = |x - 0.2|$, $x \in [0, 1]$;
3. $f(x) = x * sin(1/x)$, $x \in [0.01, 1]$.

Calculate the number of f -calculations and the number of iterations performed in each method and analyze the results. Explain differences (if any) in the results obtained.

II. *Generate random numbers* $\alpha \in (0,1)$ *and* $\beta \in (0,1)$. *Furthermore, generate the noisy data* $\{x_k, y_k\}$, *where* $k = 0, \ldots, 100$, *according to the following rule:*

$$y_k = \alpha x_k + \beta + \delta_k, \quad x_k = (1/100) * k$$

where $\delta_k \sim N(0, 1)$ *are values of a random variable with standard normal distribution. Approximate the data by the following linear and rational functions:*

1. $F(x, a, b) = ax + b$ *(linear approximant),*
2. $F(x, a, b) = a / (1 + bx)$ *(rational approximant),*

by means of least squares through the numerical minimization (with precision eps = 0.001*) of the following function:*

$$D(a, b) = \sum_{k=0}^{100} (F(x_k, a, b) - y_k)^2$$

To solve the minimization problem, use the methods of exhaustive search, Gauss and Nelder-Mead. If necessary, set the initial approximations and other parameters of the methods. Visualize the data and the approximants obtained in a plot separately for each type of ***approximant*** *so that one can compare the results for the numerical* *methods used. Analyze the results obtained (in terms of number of iterations, precision, number of function evaluations, etc.).*

Results

I. Analysis of optimization methods for single variable functions:

Method	Number of iterations	Number of f-calculations
Exhaustive search	1001	1001
Dichotomy	11	22
Golden section search	15	16

The above table is valid for all three functions. The segment considered at each iteration decreases faster in the dichotomy method. That is why there are the least number of iterations. But at each step it is necessary to calculate the value of *f* twice. For the Golden Section method, the value of *f* is calculated n + 1 times, where n is the number of iterations of the method. Number of iterations and *f*-calculations for exhaustive search equals $\lceil h \,/\, eps\ +\ 1\rceil$, where h is initial the length of the segment of the function domain.

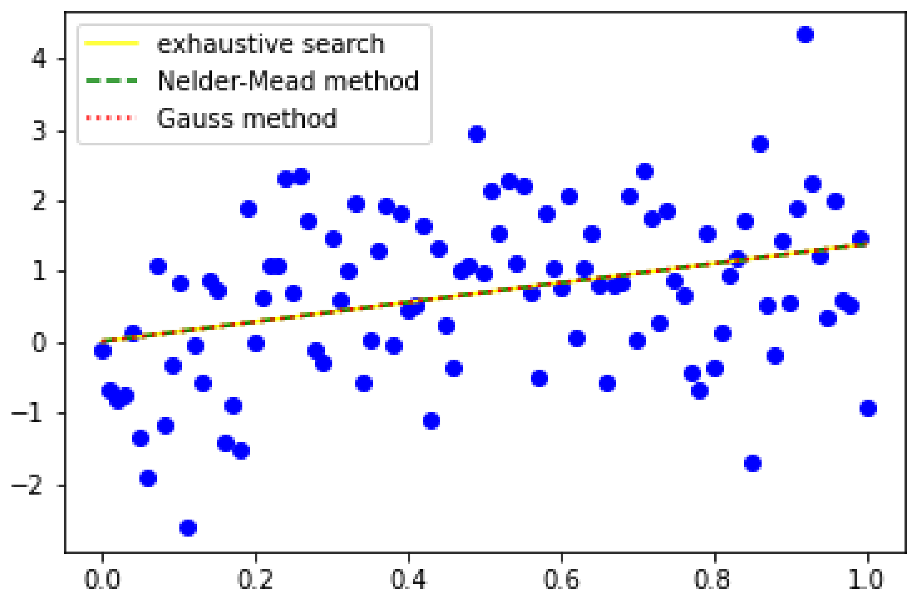
II.

Analysis of optimization methods for functions of multiple variables:

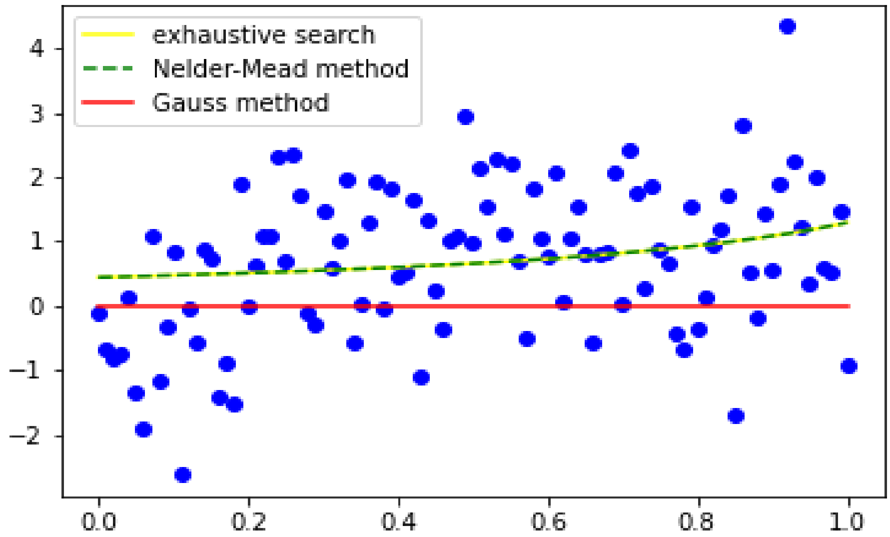
Method	Number of iterations (linear approximant)	Number of iterations (rational approximant)	Number of f-calculations (rational approximant)	Number of f-calculations (rational approximant)
Exhaustive search	4001^2	$4001\ *\ 2001$	4001^2	$4001\ *\ 2001$
Nelder-Mead method	56	107	112	205
Gauss method	20	2	20 * 2 * 4001	2 * 2 * 4001

For exhaustive search we used a grid of 2001 *x* 4001 points for rational approximation and 4001 *x* 4001 grid for linear approximation. We picked (0, 0) as a starting point for the Gauss and Nelder-Mead methods. We used the brute force (for 4001 points for each *f*) method to minimize two single variable functions at each iteration of the Gauss method. Interesting to note that Gauss method converged in 2 iterations. However, the approximation isn't as good as the approximations of exhaustive search and Nelder-Mead methods. Formally, $D(a_g, b_g) > D(a_{NM}, b_{NM})$ and $D(a_g, b_g) > D(a_{exh}, b_{exh})$, where $(a_g, b_g)$is the solution of minimization task using the Gauss method. $(a_{exh}, b_{exh})$and (a_{NM}, b_{NM}) are solutions of the same task using the brute force and Nelder-Mead methods, respectively. The stopping criterion for the Nelder-Mead method was as follows: $eps > std(s)$, where eps = 0.001, s is an array with values of our current simplex. The stopping criterion for the Gauss method was as follows: $eps > \sqrt{(xc[0] - xp[0])^2 + (xc[1] - xp[1])^2}$, where xc is a point of the current iteration and xp is the point obtained at the previous iteration.

Plot for the linear approximation:



Plot for the rational approximations:



Conclusion

We used one-dimensional methods of exhaustive search, dichotomy, golden section search to solve the task of unconstrained minimization; we used multidimensional Gauss (coordinate descent), Nelder-Mead and exhaustive search methods to solve the tasks of unconstrained optimization. We analyzed obtained results in terms of number of iterations, number of *f*-calculations and stopping criterion. We visualized obtained linear and rational approximations.

Appendix

https://github.com/ilyalinov/Algorithms