Eddington luminosity or Eddington limit

Ilya Mandel
ilya.mandel@monash.edu

Pretend for simplicity that there is spherically symmetric accretion and spherically symmetric radiative energy outflow. [This is obviously a simplifying assumption, and we will discuss alternatives later.]

Consider a thin spherical shell at radius r of mass dm around a central object of mass M (which could be a star, a black hole, etc.). As a limiting case of accretion, this shell is in hydrostatic equilibrium. In that case, the force of gravity is

$$F_G = \frac{GMdm}{r^2}. \tag{1}$$

The radiation pressure force is given by the rate of momentum deposition into the shell. The momentum per unit time flowing through the shell in radiation is L/c, where L is the luminosity and c is the speed of light (recall that photon energy is pc, where p is momentum). However, only a fraction of photons will interact with the material in the shell. That fraction is the net scattering cross-section of all baryons in the shell, κdm (recall that the opacity κ has units of area per unit mass) divided by the total shell surface area $4\pi r^2$. Thus, the outward radiation pressure force on the shell is

$$F_\gamma = \frac{L}{c}\frac{\kappa dm}{4\pi r^2}. \tag{2}$$

In hydrostatic equilibrium, $F_G = F_\gamma$. Solving for L, we find a maximum (Eddington) luminosity

$$L_{\rm Edd} = \frac{4\pi GMc}{\kappa}. \tag{3}$$

The actual value of κ depends on what is responsible for the opacity. For example, in the outer regions of massive stars, it is generally ionized hydrogen; scattering is due to the electrons with Thomson cross-section σ_T, while the mass is set by protons, so $\kappa = \sigma_T/m_p$. In that case,

$$L_{\rm Edd} \approx 3.2\times 10^4 \left(\frac{M}{M_\odot}\right) L_\odot \approx 10^{38}\frac{\rm erg}{\rm s}\left(\frac{M}{M_\odot}\right). \tag{4}$$

Note that 10^{39} erg s^{-1} is the lower limit for an X-ray binary to be defined as an ultra-luminous X-ray source (ULX) – i.e., objects above this limit must either be more massive than the usual stellar-mass black holes, or must be radiation at super-Eddington luminosities (NS ULXs).

However, quite different values of opacity, and hence Eddington luminosity, are possible. E.g., in low-mass cool stars, opacity is given by Kramer's law, $\kappa \propto \rho T^{-3.5}$, while in very hot environments, positron-electron plasma can be created, with twice the scattering particles at $\sim 1/1000$ of the mass, i.e., the Eddington limit will be a factor fo ~ 2000 lower. The most important bit for us is that the luminosity scales linearly with the mass.

Two examples:

1. Most massive stars asymptote to a fixed lifetime. Stars must get rid of $0.007 m_p c^2$ of energy per hydrogen atom when fusing hydrogen into helium on the main sequence; in practice, not all hydrogen is fused, so approximate this as $0.002Mc^2$. Assume the star cannot go above Eddington luminosity (actually, it can do this in regions within the star, if it's able to transport energy efficiently through convection, and in fact, reaching the Eddington limit can be thought of as demanding convection). Then the maximum lifetime is

$$\tau \sim \frac{0.002Mc^2}{L_{\rm Edd}} \approx 3\times 10^{-8}\frac{M_\odot c^2}{L_\odot} \approx 2\times 10^{13}{\rm s} \approx 1{\rm Myr}. \tag{5}$$

2. Eddington-limited black holes double in size on a fixed timescale. If you imagine that black holes accrete from a thin disk which efficiently cools through radiation, then the net energy radiated per unit mass is the specific energy at the stellar surface (for a neutron star) or the innermost stable circular orbit (ISCO) for a black hole. [We will discuss the validity of this another time, when we talk about accretion disk physics, including advection dominated accretion flows.] In other words, $\dot{E} = \epsilon \dot{M} c^2$. For a Schwarzschild black hole, the ISCO is at $r = 6GMc^{-2}$, so the specific energy there (ignoring GR) is $GM/(2r) = (1/12)c^2$. It is therefore common to use $\epsilon \approx 0.1$. Then we can write

$$\dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{\epsilon c^2} \approx 10^{-8}\left(\frac{M}{M_\odot}\right)\frac{M_\odot}{\rm yr}. \tag{6}$$

Thus, black holes double their mass in 100 million years when maximally accreting, regardless of their actual mass. Note that this presents a challenge for forming massive black holes from light seeds at high redshift, as observed via distant quasars: it would take 25 doubling times, or 2.5 billion years, to grow a 30 solar mass black hole into a $10^9 M_\odot$ black hole, which is much longer than the age of the Universe at $z \gtrsim 7$, when quasars are already observed.

Note that there are really two Eddington limits: the Eddington limit for mass accretion, and the Eddington limit for luminosity. The first is easy to overcome: the growth grate of a black hole during a stellar collapse is many orders of magnitude super-Eddington. Excess energy can be transported out through neutrinos rather than photons (though there is eventually a neutrino Eddington limit), or carried away mechanically by winds, or advected inside the horizon altogether. The second is more challenging to overcome, but possible: inflows and outflows are not spherical, and funnelling in material in one direction while collimating the outflows in another (perhaps with the aid of magnetic fields) is one possibility (in general, if outflows are collimated, it may be hard to measure the total output luminosity, only its isotropic equivalent); porosity is another aspect of this.