Heuristic Analysis

Ilya Nikokoshev

January 1, 2018

Abstract

As part of the Project 2 in the Artificial Intelligence Nanodegree Program, we implement three heuristics for score computation.

Contents

1 Heuristic Descriptions

All of our heuristics attempt to express the goodness of a position as a value that would be larger whenever there seem to be more choices for the first player (denoted A) and fewer choices for the opponent (denoted B).

1.1 Distance Heuristic $D(z, \alpha, f)$

1.1.1 Definition

For each blank space s on the board, we find whether it is accessible from the current positions of players A and B. If it is not accessible by either, this square does not contribute to the heuristic value.

Otherwise, let d_s^A and d_s^B denote the minimum number of steps required to move to this square from the positions of, respectively, player A and B, or $+\infty$ if this square is inaccessible. We set the contribution of s to the heuristic then as $\alpha f(d_s^B - d_s^A)$, where f is a suitably chosen monotonic function with the property P that $f(+\infty) = 1$ and $f(-\infty) = -1$.

The total value of the heuristic will be therefore computed as

$$D(z, \alpha, f) = z + \alpha \sum_{s \in A} f(d_s^B - d_s^A),$$

where A is the set of accessible spaces on the board, d_s^P is defined above, and z provides the initial value.

1.1.2 Theoretical analysis

We will select either arctan or sigmoid function $(1 + e^{-x})^{-1}$ as the function f (both of them, of course, suitably rescaled).

The condition P ensures that if the board is divided into two disjoint parts, so that A only has access to n_A cells, and B only has access to some other n_B cells, the value of the heuristic $D = z + \alpha(n_A - n_B)$. Naively, such a position is likely to be winning for A if and only if $n_A > n_B$. Moreover, for the cases when $n_A = n_B = 0$ (A loses immediately), and $n_A = 1, n_B = 0$ (B loses after A's move), the utility is known to be equal to -1 and 1. If we want the value of D to be equal to the utility in these two cases, we should select $\alpha = 2, z = -1/2$.

1.1.3 Validating the parameter choices

For comparison, we provide results of the competition of matches between selected distance heuristics with different values of parameters and `improved_score` heuristic in Table 1. In the table, cells corresponds to the percentage of wins of an alphabeta search agent with a row heuristic against the column heuristic in 100 matches, and the average is computed from rows as well as columns (thus, it is an average win in 700 parties against other opponents).

Table 1: Distance heuristics competition

	z	α	f	Wins against 1	2	3	4	Improved	Average
Variant 1	0	1	arctan	52%	52%	51%	58%	57%	53.0%
Variant 2	-1/2	1	arctan	50%	52%	51%	47%	45%	49.3%
Variant 3	-1/2	2	arctan	48%	51%	55%	48%	51%	48.9%
Variant 4	-1/2	2	sigmoid	49%	45%	54%	49%	49%	49.1%

The table suggests that, contrary to the theoretical analysis in 1.1.2, the Variant 1 heuristic is the strongest, and 3 and 4 are the weakest.

Unfortunately, the standard deviation for the case of tossing a perfect coin 700 times comprises about 2%, which means that the results in the average column cannot rule out the hypothesis that all of the opponents have equal strength. Thus the confidence in the results above is not very high.

In an attempt to further examine the question of parameters, we put Variant 1 against Variant 4 in two other competitions with 500 matches, not only on a standard 7×7, but also on a 13×13 boards. The results, presented in Table 2, do not strongly support selecting one choice over the other.

In combination with the theoretical analysis, we therefore make a choice to take both $D(0, 1, \text{arctan})$ and $D(-1/2, 2, \text{sigmoid})$ as the preferred distance heuristics.

Table 2: Further comparison of distance heuristic variants 1 and 4

Board size	Wins of 4 over 1
7×7	52.6%
13×13	50.6%

1.2 Free Moves Heuristic $F(d, u, \beta)$

2 Final Choices

We combine the results of 1.1.3 to select three heuristic functions and present the results of a competition between them and `improved_score` heuristics in Table 3.

Table 3: Final heuristics competition

	Function	Heuristic	1	2	3	Improved
1	`custom_score`	$D(0, 1, \text{arctan})$				
2	`custom_score_2`	$D(-1/2, 2, \text{sigmoid})$				
3	`custom_score_3`	??				