1 Exercises

Exercise 1.1.1 (ex. 9)**.**

$$\begin{aligned}
|S| &= 25 \\
|A| &= 40 \\
|S \cap A| &= 10 \\
|S \cup A| &=?
\end{aligned}$$

Solution.

$$\begin{aligned}
|S| &= 25 \\
|A| &= 40 \\
|S \cap A| &= 10 \\
|S \cup A| &= |S| + |A| - |S \cap A| = 25 + 40 - 10 \\
|S \cup A| &= 55
\end{aligned}$$

□

Exercise 1.1.2 (ex. 10)**.**

$$\begin{aligned}
|BUS| &= 30 \\
|TRAIN| &= 35 \\
|AUTO| &= 100 \\
|BUS \cap TRAIN| &= 15 \\
|BUS \cap AUTO| &= 15 \\
|TRAIN \cap AUTO| &= 20 \\
|BUS \cap TRAIN \cap AUTO| &= 5 \\
|BUS \cup TRAIN \cup AUTO| &=?
\end{aligned}$$

Solution.

$$\begin{aligned}
|BUS \cup TRAIN \cup AUTO| &= |BUS| + |TRAIN| + |AUTO| \\
&- |BUS \cap TRAIN| - |BUS \cap AUTO| - |TRAIN \cap AUTO| \\
&+ |BUS \cap TRAIN \cap AUTO| \\
&= 30 + 35 + 100 - 15 - 15 - 20 + 5 \\
&= 120
\end{aligned}$$

□

2 Problems

Problem 1.2.1.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup B =$ {a, b, c, d, e, f, g}

(b) $B \cup C =$ {a, c, d, e, f, g}

(c) $A \cap C =$ {a, c}

(d) $B \cap D =$ {f}

(e) $(A \cup B) - C =$ {b, d, e, g}

(f) $A - B =$ {a, b, c}

(g) $\overline{A} =$ {d, e, f, h, k}

(h) $A \oplus B =$ {a, b, c} $\cup$ {d, e, f} = {a, b, c, d, e, f}

(i) $A \oplus C =$ {b, g} $\cup$ {f} = {b, g, f}

(j) $(A \cap B) - C =$ {g} - {a, c, f} = {g}

Problem 1.2.2.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup D =$ {a, b, c , f, g, h, k}

(b) $B \cup D =$ {d, e, f, g, h, k}

(c) $C \cap D =$ {f}

(d) $A \cap D = \varnothing$

(e) $(A \cup B) - (C \cup B) =$ {a, b, c, d, e, f, g} - {a, c, d, e, f, g} = {b}

(f) $B - C =$ {d, e, g}

(g) $\overline{B}$ = {a, b, c, h, k}

(h) $C - B$ = {a, c}

(i) $C \oplus D$ = {a, c} $\cup$ {h, k} = {a, c, h, k}

(j) $(A \cap B) - (B \cap D)$ = {g} - {f} = {g}

Problem 1.2.3.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup B \cup C$ = {a, b, c, d, e, f, g}

(b) $A \cap B \cap C = \varnothing$

(c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ = {g} $\cup$ {a, c} = {a, c, g}

(d) $(A \cup B) \cap C = (C \cap A) \cup (C \cap B)$ = {a, c} $\cup$ {f} = {a, c, f}

(e) $\overline{A \cup B}$ = {d, e, f, h, k} $\cap$ {a, b, c, h, k} = {h, k}

(f) $\overline{A \cap B}$ = {d, e, f, h, k} $\cup$ {a, b, c, h, k} = {a, b, c, d, e, f, h, k}

Problem 1.2.4.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup \varnothing$ = A

(b) $A \cup U$ = U

(c) $B \cup B$ = B

(d) $C \cap \varnothing = \varnothing$

(e) $\overline{C \cup D}$ = {b, d, e, g, h, k} $\cap$ {a, b, c, d, e, g} = {b, d, e, g}

(f) $\overline{C \cap D}$ = {b, d, e, g, h, k} $\cup$ {a, b, c, d, e, g} = {a, b, c, d, e, g, h, k}

Problem 1.2.5.

$$
\begin{aligned}
U &= \{1,2,3,4,5,6,7,8,9\} \\
A &= \{1,2,4,6,8\} \\
B &= \{2,4,5,9\} \\
C &= \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1,2,3,4\} \\
D &= \{7,8\}
\end{aligned}
$$

Compute

(a) $A \cup B = \{1,\ 2,\ 4,\ 5,\ 6,\ 8,\ 9\}$

(b) $A \cup C = \{1,\ 2,\ 3,\ 4,\ 6,\ 8\}$

(c) $A \cup D = \{1,\ 2,\ 4,\ 6,\ 7,\ 8\}$

(d) $B \cup C = \{1,\ 2,\ 3,\ 4,\ 5,\ 9\}$

(e) $A \cap C = \{1,\ 2,\ 4\}$

(f) $A \cap D = \{8\}$

(g) $B \cap C = \{2,\ 4\}$

(h) $C \cap D = \varnothing$

Problem 1.2.6.

$$
\begin{aligned}
U &= \{1,2,3,4,5,6,7,8,9\} \\
A &= \{1,2,4,6,8\} \\
B &= \{2,4,5,9\} \\
C &= \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1,2,3,4\} \\
D &= \{7,8\}
\end{aligned}
$$

Compute

(a) $A - B = \{1,\ 6,\ 8\}$

(b) $B - A = \{5,\ 9\}$

(c) $C - D = \{1,\ 2,\ 3,\ 4\}$

(d) $\overline{C} = \{5,\ 6,\ 7,\ 8,\ 9\}$

(e) $\overline{A} = \{3,\ 5,\ 7,\ 9\}$

(f) $A \oplus B = \{1,\ 6,\ 8\} \cup \{5,\ 9\} = \{1,\ 5,\ 6,\ 8,\ 9\}$

(g) $C \oplus D = \{1,\ 2,\ 3,\ 4,\ 7,\ 8\}$

(h) $B \oplus C = \{1,\ 3,\ 5,\ 9\}$

Problem 1.2.7.

$$\begin{aligned}
&U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}\\
&A = \{1, 2, 4, 6, 8\}\\
&B = \{2, 4, 5, 9\}\\
&C = \{x \mid x \in Z^+ \ \wedge x^2 \le 16\} = \{1, 2, 3, 4\}\\
&D = \{7, 8\}
\end{aligned}$$

Compute

(a) $A \cup B \cup C = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 8,\ 9\}$

(b) $A \cap B \cap C = \{2,\ 4\}$

(c) $A \cap (B \cup C) = \{1,\ 2,\ 4\}$

(d) $(A \cup B) \cap D = \{8\}$

(e) $\overline{A \cup B} = \{3,\ 5,\ 7,\ 9\} \cap \{1,\ 3,\ 6,\ 7,\ 8\} = \{3,\ 7\}$

(f) $\overline{A \cap B} = \{3,\ 5,\ 7,\ 9\} \cup \{1,\ 3,\ 6,\ 7,\ 8\} = \{1,\ 3,\ 5,\ 6,\ 7,\ 8,\ 9\}$

Problem 1.2.8.

$$\begin{aligned}
&U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}\\
&A = \{1, 2, 4, 6, 8\}\\
&B = \{2, 4, 5, 9\}\\
&C = \{x \mid x \in Z^+ \ \wedge x^2 \le 16\} = \{1, 2, 3, 4\}\\
&D = \{7, 8\}
\end{aligned}$$

Compute

(a) $B \cup C \cup D = \{1,\ 2,\ 3,\ 4,\ 5,\ 7,\ 8,\ 9\}$

(b) $B \cap C \cap D = \varnothing$

(c) $A \cup A = \{1,\ 2,\ 4,\ 6,\ 8\}$

(d) $A \cap \overline{A} = \varnothing$

(e) $A \cup \overline{A} = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9\}$

(f) $A \cap (\overline{C} \cup D) = \{6,\ 8\} \cup \{8\} = \{6,\ 8\}$

Problem 1.2.9.

$$\begin{aligned}U &= \{a, b, c, d, e, f, g, h\}\\ A &= \{a, c, f, g\}\\ B &= \{a, e\}\\ C &= \{b, h\}\end{aligned}$$

Compute

(a) $\overline{A} =$ {b, d, e, h}

(b) $\overline{B} =$ {b, c, d, f, g, h}

(c) $\overline{A \cup B} =$ {b, d, h}

(d) $\overline{A \cap B} =$ {b, c, d, e, f, g, h}

(e) $\overline{U} = \varnothing$

(f) $A - B =$ {c, f, g}

Problem 1.2.10.

$$\begin{aligned}U &= \{a, b, c, d, e, f, g, h\}\\ A &= \{a, c, f, g\}\\ B &= \{a, e\}\\ C &= \{b, h\}\end{aligned}$$

Compute

(a) $\overline{A} \cap \overline{B} =$ {b, d, e, h} $\cap$ {b, c, d, f, g, h} = {b, d, h}

(b) $\overline{B} \cup \overline{C} =$ {b, c, d, f, g, h} $\cup$ {a, c, d, e, f, g} = {a, b, c, d, e , f, g, h}

(c) $\overline{A \cup A} = \overline{A} =$ {b, d, e, h}

(d) $\overline{C \cap C} = \overline{C} =$ {a, c, d, e, f, g}

(e) $A \oplus B =$ {c, e, f, g}

(f) $B \oplus C =$ {a, e, b, h}

Problem 1.2.11.

$$\begin{aligned}&U = R\\ &A = \{x \mid x \text{ is a solution to } x^2 - 1 = 0\} = \{-1, 1\}\\ &B = \{-1, 4\}\end{aligned}$$

Compute

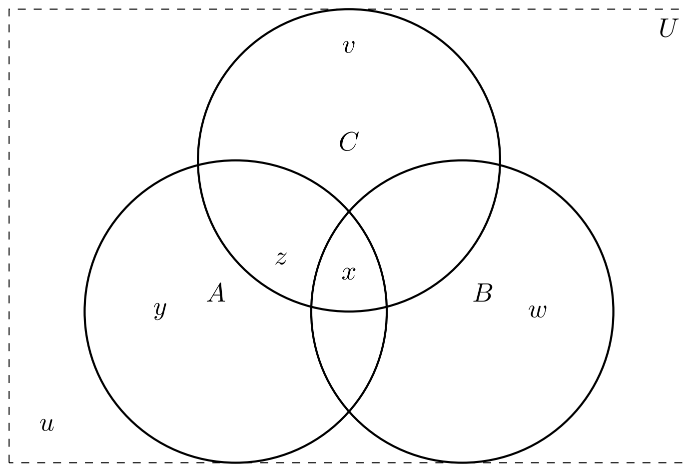
(a) $\overline{A} = \{x \mid x \in (-\infty, -1) \vee (-1, 1) \vee x \in (1, \infty)\}$

(b) $\overline{B} = \{x \mid x \in (-\infty, -1) \vee x \in (-1, 4) \vee x \in (4, \infty)\}$

(c) $\overline{A \cup B} = \{x \mid x \in (-\infty, -1) \vee (-1, 1) \vee (1, 4) \vee x \in (4, \infty)\}$

(d) $\overline{A \cap B} = \{x \mid x \in (-\infty, -1) \vee (-1, \infty)\}$

Problem 1.2.12.



Compute

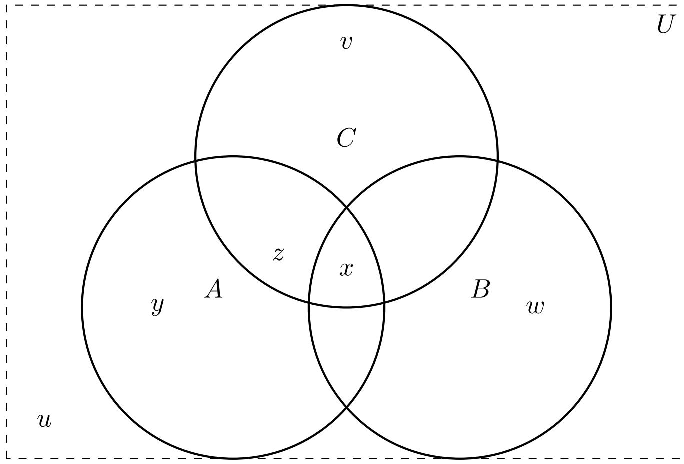
(a) $y \in A \cap B =$ False: y $\notin$ B

(b) $x \in B \cup C =$ True: x $\in$ B $\wedge$ x $\in$ C

(c) $w \in B \cap C =$ False: w $\notin$ C

(d) $u \notin C =$ True: u $\in \overline{A \cup B \cup C}$

Problem 1.2.13.



Compute

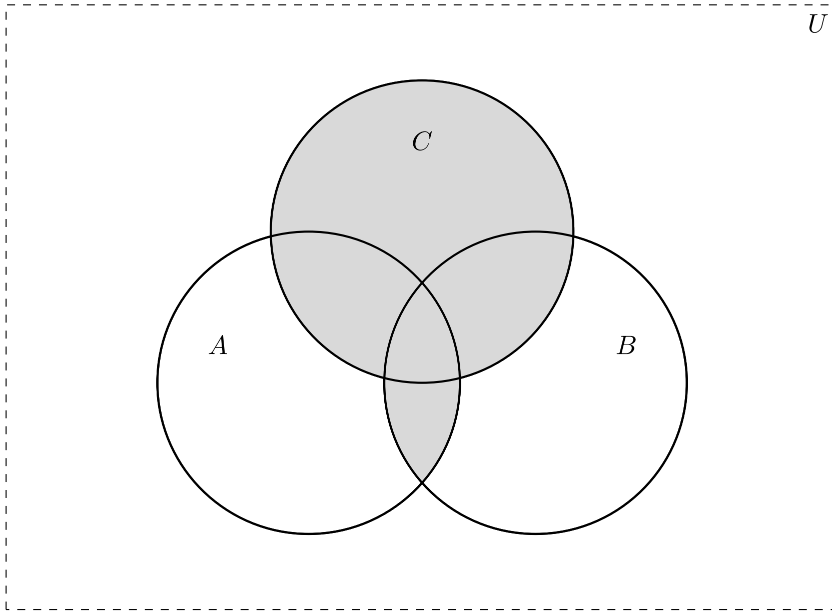
(a) $x \in A \cap B \cap C$ = True: x ∈ A ∧ x ∈ B ∧ x ∈ C

(b) $y \in A \cup B \cup C$ = True: y ∈ A

(c) $z \in A \cap C$ = True: z ∈ A ∧ z ∈ C

(d) $v \in B \cap C$ = False: v ∈ C ∧ v ∉ B

Problem 1.2.14.



Describe shaded region

$$(A \cap B) \cup C$$

Problem 1.2.15.

$$\begin{aligned}
|A| &= 6 \\
|B| &= 8 \\
|C| &= 6 \\
|A \cup B \cup C| &= 11 \\
|A \cap B| &= 3 \\
|A \cap C| &= 2 \\
|B \cap C| &= 5 \\
|A \cap B \cap C| &= ?
\end{aligned}$$

Solution

$$\begin{aligned}
|A \cup B \cup C| = {} & |A| + |B| + |C| \\
& - |A \cap B| - |A \cap C| - |B \cap C| \\
& + |A \cap B \cap C|
\end{aligned}$$

$$\begin{aligned}
11 &= 6 + 8 + 6 - 3 - 2 - 5 + x \\
x &= 1
\end{aligned}$$

Problem 1.2.16.

$A \cap B = B \cap A$

Validate

(a) $A = \{1, 2, 3, 4\}, B = \{2, 3, 5, 6, 8\}$

$$\{x \mid x \in A \wedge x \in B\} = \{2, 3\}$$
$$\{x \mid x \in B \wedge x \in A\} = \{2, 3\}$$

(b) $A = \{1, 2, 3, 4\}, B = \{5, 6, 7, 8, 9\}$

$$\{x \mid x \in A \wedge x \in B\} = \varnothing$$
$$\{x \mid x \in B \wedge x \in A\} = \varnothing$$

Problem 1.2.17.

$A \cap B = B \cap A$

Validate

(a) $A = \{a, b, c, d, e, f\}, B = \{a, c, f, g, h, i, r\}$

$$\{x \mid x \in A \wedge x \in B\} = \{a, c, f\}$$
$$\{x \mid x \in B \wedge x \in A\} = \{a, c, f\}$$

(b) $A = \{a, b, c, d, e\}, B = \{f, g, r, s, t, u\}$

$$\{x \mid x \in A \wedge x \in B\} = \varnothing$$
$$\{x \mid x \in B \wedge x \in A\} = \varnothing$$

Problem 1.2.18.

$A \cap B = B \cap A$

Validate

(a) $A = \{x \mid x \in Z^+ \wedge x < 8\} = \{1, 2, 3, 4, 5, 6, 7\}$
$B = \{x \mid x \in Z \wedge 2 \leq x \leq 5\} = \{2, 3, 4, 5\}$

$$\{x \mid x \in A \wedge x \in B\} = \{2, 3, 4, 5\}$$
$$\{x \mid x \in B \wedge x \in A\} = \{2, 3, 4, 5\}$$

(b) $A = \{x \mid x \in Z^+ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$
$B = \{x \mid x \in Z^- \wedge x^2 \leq 25\} = \{-1, -2, -3, -4, -5\}$

$$\{x \mid x \in A \wedge x \in B\} = \varnothing$$
$$\{x \mid x \in B \wedge x \in A\} = \varnothing$$

Problem 1.2.19.

$$if A \cap B = \varnothing \wedge |A \cup B| = |A|$$

what is true about B?

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$|A \cup B| = |A| \implies |B| = 0 \wedge |A \cap B| = 0$$
$$|B| = \varnothing \implies B = \varnothing$$

Problem 1.2.20.

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

Plain English Explanation:

To determine the elements in the universal set that do not belong to $A \cup B$, first identify the elements that are not in A and the elements that are not in B. Then, take the intersection of these two sets. In other words, an element is not in $A \cup B$ if and only if it is neither in A nor in B.

For example, imagine a fruit basket where A represents apples and B represents oranges. To find the fruits that are neither apples nor oranges, list all fruits that are not apples and all fruits that are not oranges, then select the fruits that appear on both lists.

Problem 1.2.21.

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

Plain English Explanation:

To determine the elements in the universal set that do not belong to $A \cap B$, first identify the elements that are not in A and the elements that are not in B. Then, take the union of these two sets. In other words, an element is not in $A \cap B$ if and only if it is either not in A or not in B.

For example, consider a classroom where A represents the students who submitted their math homework, and B represents the students who submitted their science homework. The intersection $A \cap B$ contains the students who submitted both assignments. The complement $\overline{A \cap B}$ then includes all students who failed to submit at least one of the assignments. This is equivalent to taking the union of the students who did not submit math homework ($\overline{A}$) and those who did not submit science homework ($\overline{B}$).

Problem 1.2.22.

$$A = \{a, b, c, d, e\}$$
$$B = \{d, e, f, g, h, i, k\}$$
$$C = \{a, c, d, e, k, r, s, t\}$$

Validate

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$\begin{aligned} ? &= 5 + 7 + 8 - |\{d, e\}| - |\{d, e, k\}| - |\{a, c, d, e\}| + |\{d, e\}| \\ &= 20 - 2 - 3 - 4 + 2 \\ &= 20 - 7 = 13 \end{aligned}$$

Problem 1.2.23.

$$A = \{1, 2, 3, 4, 5, 6\}$$
$$B = \{2, 4, 7, 8, 9\}$$
$$C = \{1, 2, 4, 7, 10, 12\}$$

Validate

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$\begin{aligned} ? &= 6 + 5 + 6 - |\{2, 4\}| - |\{2, 4, 7\}| - |\{1, 2, 4\}| + |\{2, 4\}| \\ &= 17 - 2 - 3 - 3 + 2 \\ &= 11 \end{aligned}$$

Problem 1.2.24.

$$A = \{x \mid x \in Z^+ \wedge x < 8\} = \{1, 2, 3, 4, 5, 6, 7\}$$
$$B = \{x \mid x \in Z \wedge 2 \leq x \leq 4\} = \{2, 3, 4\}$$
$$C = \{x \mid x \in Z \wedge x^2 < 16 \ = \{-3, -2, -1, 0, 1, 2, 3\}$$

Validate

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$\begin{aligned} ? &= 7 + 3 + 7 - |\{2,3,4\}| - |\{2,3\}| - |\{1,2,3\}| + |\{2,3\}| \\ &= 17 - 8 + 2 = 11 \end{aligned}$$

Problem 1.2.25.

$$\begin{aligned} |U| &= 260 \\ |MATH| &= 64 \\ |CS| &= 94 \\ |BUS| &= 58 \\ |MATH \cap BUS| &= 28 \\ |MATH \cap CS| &= 26 \\ |CS \cap BUS| &= 22 \\ |MATH \cap CS \cap BS| &= 14 \end{aligned}$$

(a) How many students were surveyed who had taken none of the three types of courses?
$|(\overline{MATH \cup CS \cup BUS})| = ?$

$$\begin{aligned} |(\overline{MATH \cup CS \cup BUS})| &= |U| - |MATH \cup CS \cup BUS| \\ |MATH \cup CS \cup BUS| &= 64 + 94 + 48 - 28 - 26 - 22 + 14 \\ &= 216 - 76 + 14 = 230 - 76 = 154 \\ |U| - 144 &= 116 \end{aligned}$$

(b) Of the students surveyed, how many had taken only a computer science course?
$|CS| - |MATH \cap CS| - |CS \cap BUS| + |MATH \cap CS \cap BUS| = ?$

since CS intersects both and if we subtracted both intersections, the tripple intersection subset would've been counted twice

$94 - 26 - 22 + 14 = 60$

Problem 1.2.26.

$$
\begin{aligned}
|U| &= 500 \\
|FB| &= 285 \\
|HK| &= 195 \\
|BB| &= 115 \\
|FB \cap BB| &= 45 \\
|FB \cap HK| &= 70 \\
|HK \cap BB| &= 50 \\
|(\overline{FB \cup HK \cup BB})| &= 50
\end{aligned}
$$

(a) How many people in the survey watch all three kinds of games?
$|FB \cap HK \cap BB| = ?$

$$
\begin{aligned}
|FB \cup HK \cup BB| &= |FB| + |BB| + |HK| \\
&\quad - |FB \cap BB| - |FB \cap HK| - |HK \cap BB| \\
&\quad + |FB \cap BB \cap HK| \\
&= 285 + 195 + 115 - 45 - 70 - 50 + X \\
&= 595 - 165 + X
\end{aligned}
$$

$$
\begin{aligned}
|FB \cup HK \cup BB| &= |U| - |\overline{FB \cup HK \cup BB}| \\
&= 500 - 50 = 450
\end{aligned}
$$

$$
\begin{aligned}
450 &= 595 - 165 + X \\
450 &= 430 + X \\
X &= 20 \\
|FB \cap HK \cap BB| \ &= 20
\end{aligned}
$$

(b) How many people watch exactly one of the sports?

$$
\begin{aligned}
|FB| - |HK \cap FB| - |FB \cap BB| + |HK \cap FB \cap BB| \ &=? \\
|HK| - |HK \cap FB| - |HK \cap BB| + |HK \cap FB \cap BB| \ &=? \\
|BB| - |BB \cap FB| - |HK \cap BB| + |HK \cap FB \cap BB| \ &=?
\end{aligned}
$$

285 - 70 - 45 + 20 = 190
195 - 70 - 50 + 20 = 95
115 - 45 - 50 + 20 = 40
190 + 95 + 40 = 325

Problem 1.2.27.

$$\begin{aligned}
|U| &= 166\\
|TV| &= 88\\
|PP| &= 73\\
|MAG| &= 46\\
|TV \cap PP| &= 34\\
|TV \cap MG| &= 16\\
|PP \cap MG| &= 12\\
|PP \cap TV \cap MG| &= 5
\end{aligned}$$

(a) How many use none of three media
$|(\overline{TV \cup PP \cup MAG})| = ?$

$$\begin{aligned}
|TV \cup PP \cup MAG| &= |PP| + |TV| + |MAG|\\
&- |PP \cap TV| - |PP \cap MAG| - |MAG \cap TV|\\
&+ |PP \cap TV \cap MAG|\\
&= 88 + 73 + 46 - 34 - 16 - 12 + 5\\
&= 207 - 62 + 5\\
&= 150
\end{aligned}$$

$$|\overline{TV \cup PP \cup MAG}| = |U| - 150 = 16$$

(b) How many obtain their news from a news magazine exclusively?
$|MAG| - |TV \cap MAG| - |MAG \cap PP| + |TV \cap PP \cap MAG| = ?$

$$46 - 16 - 12 + 5 = 23$$

Problem 1.2.28.

$$\begin{aligned}
|U| &= 100\\
|FR| &= 37\\
|VG| &= 33\\
|CH| - |CH \cap VG| - |CH \cap FR| + |CH \cap FR \cap VG| &= 12\\
|CH \cap FR| &= 9\\
|CH \cap VG| &= 12\\
|FR \cap VG| &= 10\\
|FR \cap CH \cap VG| &= 3
\end{aligned}$$

(a) How many people surveyed eat cheese
$|CH| = ?$

$$\begin{aligned}
|CH| - |CH \cap VG| - |CH \cap FR| + |CH \cap FR \cap VG| &= 12\\
|CH| - 12 - 9 + 3 &= 12\\
|CH| &= 30
\end{aligned}$$

$$\begin{aligned}
|\overline{CH \cup VG \cup FR}| &=?\\
&= |U| - |CH \cup VG \cup FR|
\end{aligned}$$

(b) How many do not eat any of the offerings?

$$\begin{aligned}
|CH \cup VG \cup FR| &= |CH| + |VG| + |FR|\\
&- |CH \cap VG| - |CH \cap FR| - |VG \cap FR|\\
&+ |CH \cap VG \cap FR|\\
&= 30 + 37 + 33 - 9 - 12 - 10 + 3\\
&= 100 - 28\\
&= 72
\end{aligned}$$

$$\begin{aligned}
&|U| - |CH \cup VG \cup FR|\\
&\quad = 100 - 72\\
&\quad = 28
\end{aligned}$$

Problem 1.2.29. Consider the following table:

	Endo	Ecto	Meso
Male	72	54	36
Female	62	64	38

Classification of subjects by body type and gender.

(a) $|MAL| = |ENDO| + |ECTO| + |MESO| = 72 + 54 + 36 = 162$

(b) $|ECTO| = |MAL \cap ECTO| + |FEM \cap ECTO| = 54 + 64 = 118$

(c) $|FEM \cup ENDO| = |F| + |M \cap ENDO| = 62+64+38+72 = 110+126 = 236$

(d) $|\overline{MAL \cap MESO}| = |F| + |M \cap ENDO| + |M \cap ECTO| = 164 + 72 + 54 = 290$

(e) $|MAL \cup ECTO \cup MESO| = |ECTO| + |MESO| + |M \cap ENDO| = 264$

Problem 1.2.30. Consider the following table:

	DE	UD
Sophomore	143	289
Junior	245	158
Senior	392	36

information about the sophomore, junior, and senior classes at Old U

(a) $|D \cap J| = 245$ Juniors who declared major

(b) $|\overline{U \cup R}| = 143+245 = 388$ everyone who's neither undeclared nor a senior

(c) $|(D \cup S) \cap \overline{R}| = 143+289+245 = 677$ Everyone who's either declared or sophmore excluding seniors

Problem 1.2.31. Diagram of the previous problem's table. Mutually exclusive and collectively exhaustive.

	DE	UD
Sophomore	143	289
Junior	245	158
Senior	392	36

Problem 1.2.32. Complete the following proof that $A \subseteq A \cup B$. Suppose $x \in A$. Then $x \in A \cup B$, because x by definition is an element of A and $\cup B$ condition is automatically satisfied, as with any union. Thus by the definition of subset $A \subseteq A \cup B$

Problem 1.2.33. Choose $x \in A \cap B$, classify whether each statement is true, false or not possible to identify

(a) $x \in A =$ True

(b) $x \in B =$ True

(c) $x \notin A =$ False since $x \in A \cap B \implies x \in A \wedge x \in B$

(d) $x \notin B =$ False, same as c

Problem 1.2.34. Choose $y \in A \cup B$, classify whether each statement is true, false or not possible to identify

(a) $y \in A =$ True

(b) $y \in B =$ True

(c) $y \notin A =$ False

(d) $y \notin B =$ False

(e) $y \in A \cap B =$ impossible to determine - can be $y \in A \wedge y \notin B$

(f) $y \notin A \cap B =$ impossible to determine, same as 'e'

Problem 1.2.35. Choose $z \in A \cup (B \cap C)$, classify whether each statement is true, false or not possible to identify

(a) $z \in A =$ impossible to determine, could be $z \in A \vee z \in B \cap C$

(b) $z \in B =$ impossible to determine, could be $z \in A \vee z \in B \cap C$

(c) $z \in C =$ impossible to determine, could be $z \in A \vee z \in B \cap C$

(d) $z \in B \cap C =$ impossible to determine, could be $z \in A \vee z \in B \cap C$

(e) $z \notin A =$ impossible to determine, could be $z \in A \vee z \in B \cap C$

(f) $z \notin C =$ impossible to determine, could be $z \in A \vee z \in B \cap C$

Problem 1.2.36. Choose $w \in D \cap (E \cup F)$, classify whether each statement is true, false or not possible to identify

(a) $w \in D =$ True

(b) $w \in E =$ impossible to determine, $w \in E \vee w \in F$

(c) $w \in F =$ impossible to determine, same as above

(d) $w \notin D =$ False

(e) $w \in F \cup E =$ True

(f) $w \in (D \cap E) \cup (D \cap F) =$ True, due to distributive property

Problem 1.2.37. Choose $t \in \overline{D \cap E}$, classify whether each statement is true, false or not possible to identify

(a) $t \in D =$ impossible to determine, could belong to D, E or $\overline{D \cup E}$

(b) $t \in E =$ impossible to determine, same as above

(c) $t \notin D =$ impossible to determine

(d) $t \notin E =$ impossible to determine

(e) $t \in D \cup E =$ impossible to determine

Problem 1.2.38. Choose $x \in \overline{A} \cup (B \cap C)$, classify whether each statement is true, false or not possible to identify

(a) $x \in A =$ impossible to determine as we don't know much about $B \cap C$

(b) $x \in B =$ impossible to determine, could also be in whatever $\overline{A}$ is

(c) $x \in C =$ impossible to determine, same as above

(d) $x \in A \cup B =$ impossible to determine

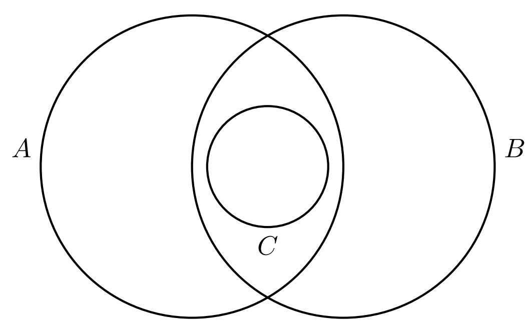
(e) $x \in (\overline{A} \cup B) \cap (\overline{A} \cup C)$

$$x \in (\overline{A} \cup B) \cap (\overline{A} \cup C)$$

$\overline{A} \cup (B \cap C) =$ True by definition

Problem 1.2.39. Complete the proof that $A \cap B \subseteq A$
Suppose that $x \in A \cap B$. Then $x \in A \wedge x \in B$. Thus $A \cap B \subseteq A$

Problem 1.2.40. (a) Draw Venn diagram to represent C $\subseteq A \wedge C \subseteq B$



(b) To prove $C \subseteq A \cup B$ we should choose an element from which set? Set C because we're proving that all points of C either belong to A or B

(c) Prove that $C \subseteq A \wedge C \subseteq B \implies C \subseteq A \cup B$

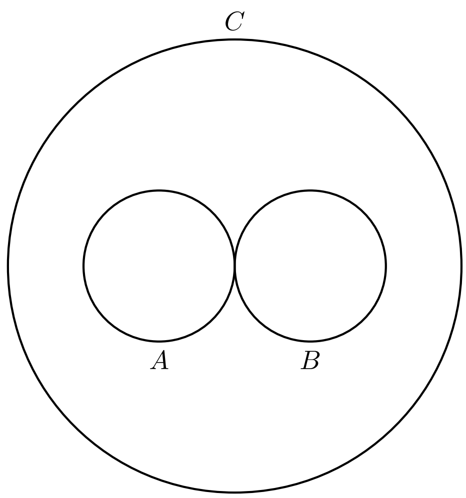
$x -$ arbitrary element of C

$C \subseteq A \implies x \in A$

$x \in A \implies x \in A \cup B$

then $C \subseteq A \cup B$

Problem 1.2.41. (a) Draw Venn diagram to represent $A \subseteq C \wedge B \subseteq C$



(b) To prove $A \cup B \subseteq C$ we should choose an element from which set? From either A or B

(c) Prove that if $A \subseteq C \wedge B \subseteq C \implies A \cup B \subseteq C$

$x -$ arbitrary element of $A \cup B$

$x \in A \vee / \wedge x \in B$

$x \in A \implies x \in C$ since $A \subseteq C$

$x \in B \implies x \in C$ since $B \subseteq C$

$x \in C \implies A \cup B \subseteq C$