1 Exercises

Exercise 1.1.1 (ex. 9)**.**

$$\begin{aligned} |S| &= 25 \\ |A| &= 40 \\ |S \cap A| &= 10 \\ |S \cup A| &= ? \end{aligned}$$

Solution.

$$\begin{aligned} |S| &= 25 \\ |A| &= 40 \\ |S \cap A| &= 10 \\ |S \cup A| &= |S| + |A| - |S \cap A| = 25 + 40 - 10 \\ |S \cup A| &= 55 \end{aligned}$$

□

Exercise 1.1.2 (ex. 10)**.**

$$\begin{aligned} |BUS| &= 30 \\ |TRAIN| &= 35 \\ |AUTO| &= 100 \\ |BUS \cap TRAIN| &= 15 \\ |BUS \cap AUTO| &= 15 \\ |TRAIN \cap AUTO| &= 20 \\ |BUS \cap TRAIN \cap AUTO| &= 5 \\ |BUS \cup TRAIN \cup AUTO| &= ? \end{aligned}$$

Solution.

$$\begin{aligned} |BUS \cup TRAIN \cup AUTO| &= |BUS| + |TRAIN| + |AUTO| \\ &\quad - |BUS \cap TRAIN| - |BUS \cap AUTO| - |TRAIN \cap AUTO| \\ &\quad + |BUS \cap TRAIN \cap AUTO| \\ &= 30 + 35 + 100 - 15 - 15 - 20 + 5 \\ &= 120 \end{aligned}$$

□

2 Problems

Problem 1.2.1.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup B = \{a, b, c, d, e, f, g\}$

(b) $B \cup C = \{a, c, d, e, f, g\}$

(c) $A \cap C = \{a, c\}$

(d) $B \cap D = \{f\}$

(e) $(A \cup B) - C = \{b, d, e, g\}$

(f) $A - B = \{a, b, c\}$

(g) $\overline{A} = \{d, e, f, h, k\}$

(h) $A \oplus B = \{a, b, c\} \cup \{d, e, f\} = \{a, b, c, d, e, f\}$

(i) $A \oplus C = \{b, g\} \cup \{f\} = \{b, g, f\}$

(j) $(A \cap B) - C = \{g\}$ - $\{a, c, f\} = \{g\}$

Problem 1.2.2.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup D = \{a, b, c , f, g, h, k\}$

(b) $B \cup D = \{d, e, f, g, h, k\}$

(c) $C \cap D = \{f\}$

(d) $A \cap D = \varnothing$

(e) $(A \cup B) - (C \cup B) = \{a, b, c, d, e, f, g\}$ - $\{a, c, d, e, f, g\} = \{b\}$

(f) $B - C = \{d, e, g\}$

(g) $\overline{B} = \{a, b, c, h, k\}$

(h) $C - B = \{a, c\}$

(i) $C \oplus D = \{a, c\} \cup \{h, k\} = \{a, c, h, k\}$

(j) $(A \cap B) - (B \cap D) = \{g\}$ - $\{f\} = \{g\}$

Problem 1.2.3.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup B \cup C = \{a, b, c, d, e, f, g\}$

(b) $A \cap B \cap C = \varnothing$

(c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \{g\} \cup \{a, c\} = \{a, c, g\}$

(d) $(A \cup B) \cap C = (C \cap A) \cup (C \cap B) = \{a, c\} \cup \{f\} = \{a, c, f\}$

(e) $\overline{A \cup B} = \{d, e, f, h, k\} \cap \{a, b, c, h, k\} = \{h, k\}$

(f) $\overline{A \cap B} = \{d, e, f, h, k\} \cup \{a, b, c, h, k\} = \{a, b, c, d, e, f, h, k\}$

Problem 1.2.4.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup \varnothing = A$

(b) $A \cup U = U$

(c) $B \cup B = B$

(d) $C \cap \varnothing = \varnothing$

(e) $\overline{C \cup D} = \{b, d, e, g, h, k\} \cap \{a, b, c, d, e, g\} = \{b, d, e, g\}$

(f) $\overline{C \cap D} = \{b, d, e, g, h, k\} \cup \{a, b, c, d, e, g\} = \{a, b, c, d, e, g, h, k\}$

Problem 1.2.5.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$A = \{1, 2, 4, 6, 8\}$$
$$B = \{2, 4, 5, 9\}$$
$$C = \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$$
$$D = \{7, 8\}$$

Compute

(a) $A \cup B = \{1, 2, 4, 5, 6, 8, 9\}$

(b) $A \cup C = \{1, 2, 3, 4, 6, 8\}$

(c) $A \cup D = \{1, 2, 4, 6, 7, 8\}$

(d) $B \cup C = \{1, 2, 3, 4, 5, 9\}$

(e) $A \cap C = \{1, 2, 4\}$

(f) $A \cap D = \{8\}$

(g) $B \cap C = \{2, 4\}$

(h) $C \cap D = \varnothing$

Problem 1.2.6.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$A = \{1, 2, 4, 6, 8\}$$
$$B = \{2, 4, 5, 9\}$$
$$C = \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$$
$$D = \{7, 8\}$$

Compute

(a) $A - B = \{1, 6, 8\}$

(b) $B - A = \{5, 9\}$

(c) $C - D = \{1, 2, 3, 4\}$

(d) $\overline{C} = \{5, 6, 7, 8, 9\}$

(e) $\overline{A} = \{3, 5, 7, 9\}$

(f) $A \oplus B = \{1, 6, 8\} \cup \{5, 9\} = \{1, 5, 6, 8, 9\}$

(g) $C \oplus D = \{1, 2, 3, 4, 7, 8\}$

(h) $B \oplus C = \{1, 3, 5, 9\}$

Problem 1.2.7.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$A = \{1, 2, 4, 6, 8\}$$
$$B = \{2, 4, 5, 9\}$$
$$C = \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$$
$$D = \{7, 8\}$$

Compute

(a) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$

(b) $A \cap B \cap C = \{2, 4\}$

(c) $A \cap (B \cup C) = \{1, 2, 4\}$

(d) $(A \cup B) \cap D = \{8\}$

(e) $\overline{A \cup B} = \{3, 5, 7, 9\} \cap \{1, 3, 6, 7, 8\} = \{3, 7\}$

(f) $\overline{A \cap B} = \{3, 5, 7, 9\} \cup \{1, 3, 6, 7, 8\} = \{1, 3, 5, 6, 7, 8, 9\}$

Problem 1.2.8.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$A = \{1, 2, 4, 6, 8\}$$
$$B = \{2, 4, 5, 9\}$$
$$C = \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$$
$$D = \{7, 8\}$$

Compute

(a) $B \cup C \cup D = \{1, 2, 3, 4, 5, 7, 8, 9\}$

(b) $B \cap C \cap D = \varnothing$

(c) $A \cup A = \{1, 2, 4, 6, 8\}$

(d) $A \cap \overline{A} = \varnothing$

(e) $A \cup \overline{A} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(f) $A \cap (\overline{C} \cup D) = \{6, 8\} \cup \{8\} = \{6, 8\}$

Problem 1.2.9.

$$U = \{a, b, c, d, e, f, g, h\}$$
$$A = \{a, c, f, g\}$$
$$B = \{a, e\}$$
$$C = \{b, h\}$$

Compute

(a) $\overline{A} = \{\text{b, d, e, h}\}$

(b) $\overline{B} = \{\text{b, c, d, f, g, h}\}$

(c) $\overline{A \cup B} = \{\text{b, d, h}\}$

(d) $\overline{A \cap B} = \{\text{b, c, d, e, f, g, h}\}$

(e) $\overline{U} = \varnothing$

(f) $A - B = \{\text{c, f, g}\}$

Problem 1.2.10.

$$U = \{a, b, c, d, e, f, g, h\}$$
$$A = \{a, c, f, g\}$$
$$B = \{a, e\}$$
$$C = \{b, h\}$$

Compute

(a) $\overline{A} \cap \overline{B} = \{\text{b, d, e, h}\} \cap \{\text{b, c, d, f, g, h}\} = \{\text{b, d, h}\}$

(b) $\overline{B} \cup \overline{C} = \{\text{b, c, d, f, g, h}\} \cup \{\text{a, c, d, e, f, g}\} = \{\text{a, b, c, d, e , f, g, h}\}$

(c) $\overline{A \cup A} = \overline{A} = \{\text{b, d, e, h}\}$

(d) $\overline{C \cap C} = \overline{C} = \{\text{a, c, d, e, f, g}\}$

(e) $A \oplus B = \{\text{c, e, f, g}\}$

(f) $B \oplus C = \{\text{a, e, b, h}\}$

Problem 1.2.11.

$$U = R$$
$$A = \{x \mid x \text{ is a solution to } x^2 - 1 = 0\} = \{-1, 1\}$$
$$B = \{-1, 4\}$$

Compute

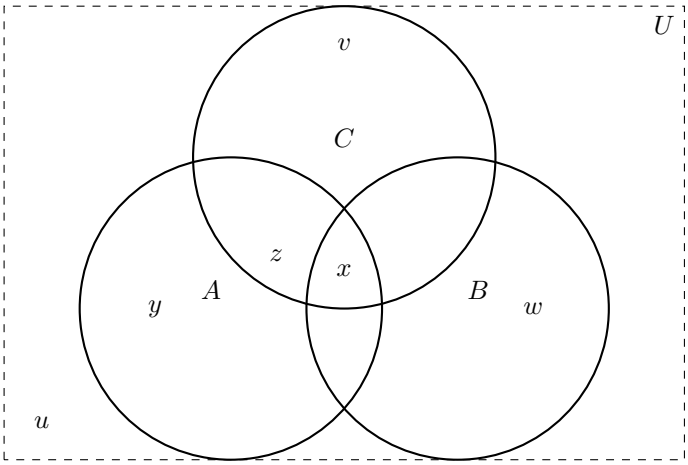
(a) $\overline{A} = \{x \mid x \in (-\infty, -1) \vee (-1, 1) \vee x \in (1, \infty)\}$

(b) $\overline{B} = \{x \mid x \in (-\infty, -1) \vee x \in (-1, 4) \vee x \in (4, \infty)\}$

(c) $\overline{A \cup B} = \{x \mid x \in (-\infty, -1) \vee (-1, 1) \vee (1, 4) \vee x \in (4, \infty)\}$

(d) $\overline{A \cap B} = \{x \mid x \in (-\infty, -1) \vee (-1, \infty)\}$

Problem 1.2.12.



Compute

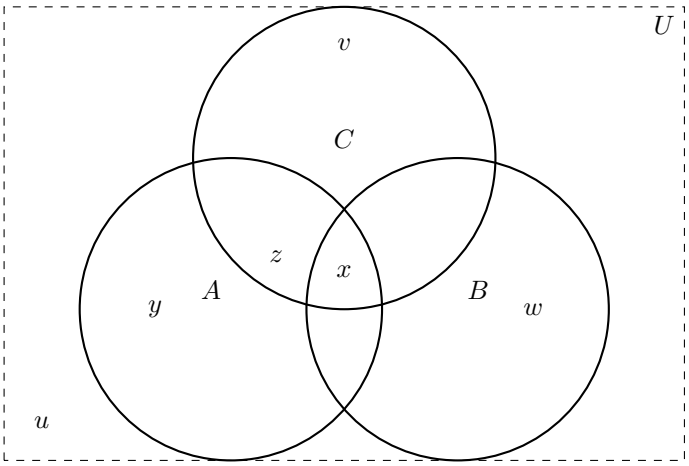
(a) $y \in A \cap B$ = False: y $\notin$ B

(b) $x \in B \cup C$ = True: x $\in$ B $\wedge$ x $\in$ C

(c) $w \in B \cap C$ = False: w $\notin$ C

(d) $u \notin C$ = True: u $\in \overline{A \cup B \cup C}$

Problem 1.2.13.



Compute

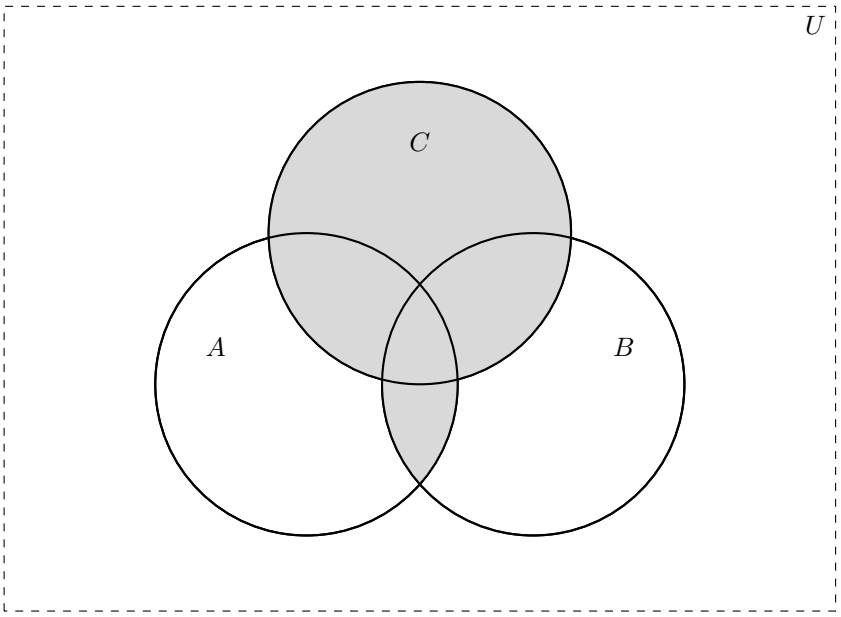
(a) $x \in A \cap B \cap C$ = True: x ∈ A ∧ x ∈ B ∧ x ∈ C

(b) $y \in A \cup B \cup C$ = True: y ∈ A

(c) $z \in A \cap C$ = True: z ∈ A ∧ z ∈ C

(d) $v \in B \cap C$ = False: v ∈ C ∧ v ∉ B

Problem 1.2.14.



Describe shaded region

$$(A \cap B) \cup C$$

Problem 1.2.15.

$$\begin{aligned} |A| &= 6 \\ |B| &= 8 \\ |C| &= 6 \\ |A \cup B \cup C| &= 11 \\ |A \cap B| &= 3 \\ |A \cap C| &= 2 \\ |B \cap C| &= 5 \\ |A \cap B \cap C| &= ? \end{aligned}$$

Solution

$$\begin{aligned} |A \cup B \cup C| = |A| + |B| + |C| \\ - |A \cap B| - |A \cap C| - |B \cap C| \\ + |A \cap B \cap C| \end{aligned}$$

$$\begin{aligned} 11 &= 6 + 8 + 6 - 3 - 2 - 5 + x \\ x &= 1 \end{aligned}$$

Problem 1.2.16.

$A \cap B = B \cap A$

Validate

(a) $A = \{1, 2, 3, 4\}, B = \{2, 3, 5, 6, 8\}$

$$\{x \mid x \in A \wedge x \in B\} = \{2, 3\}$$
$$\{x \mid x \in B \wedge x \in A\} = \{2, 3\}$$

(b) $A = \{1, 2, 3, 4\}, B = \{5, 6, 7, 8, 9\}$

$$\{x \mid x \in A \wedge x \in B\} = \varnothing$$
$$\{x \mid x \in B \wedge x \in A\} = \varnothing$$

Problem 1.2.17.

$A \cap B = B \cap A$

Validate

(a) $A = \{a, b, c, d, e, f\}, B = \{a, c, f, g, h, i, r\}$

$$\{x \mid x \in A \wedge x \in B\} = \{a, c, f\}$$
$$\{x \mid x \in B \wedge x \in A\} = \{a, c, f\}$$

(b) $A = \{a, b, c, d, e\}, B = \{f, g, r, s, t, u\}$

$$\{x \mid x \in A \wedge x \in B\} = \varnothing$$
$$\{x \mid x \in B \wedge x \in A\} = \varnothing$$

Problem 1.2.18.

$A \cap B = B \cap A$

Validate

(a) $A = \{x \mid x \in Z^+ \wedge x < 8\} = \{1, 2, 3, 4, 5, 6, 7\}$
$B = \{x \mid x \in Z \wedge 2 \leq x \leq 5\} = \{2, 3, 4, 5\}$

$$\{x \mid x \in A \wedge x \in B\} = \{2, 3, 4, 5\}$$
$$\{x \mid x \in B \wedge x \in A\} = \{2, 3, 4, 5\}$$

(b) $A = \{x \mid x \in Z^+ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$
$B = \{x \mid x \in Z^- \wedge x^2 \leq 25\} = \{-1, -2, -3, -4, -5\}$

$$\{x \mid x \in A \wedge x \in B\} = \varnothing$$
$$\{x \mid x \in B \wedge x \in A\} = \varnothing$$

Problem 1.2.19.

$$if A \cap B = \varnothing \wedge |A \cup B| = |A|$$

what is true about B?

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$|A \cup B| = |A| \implies |B| = 0 \wedge |A \cap B| = 0$$
$$|B| = \varnothing \implies B = \varnothing$$

Problem 1.2.20.

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

Plain English Explanation:

To determine the elements in the universal set that do not belong to $A \cup B$, first identify the elements that are not in A and the elements that are not in B. Then, take the intersection of these two sets. In other words, an element is not in $A \cup B$ if and only if it is neither in A nor in B.

For example, imagine a fruit basket where A represents apples and B represents oranges. To find the fruits that are neither apples nor oranges, list all fruits that are not apples and all fruits that are not oranges, then select the fruits that appear on both lists.

Problem 1.2.21.

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

Plain English Explanation:

To determine the elements in the universal set that do not belong to $A \cap B$, first identify the elements that are not in A and the elements that are not in B. Then, take the union of these two sets. In other words, an element is not in $A \cap B$ if and only if it is either not in A or not in B.

For example, consider a classroom where A represents the students who submitted their math homework, and B represents the students who submitted

their science homework. The intersection $A \cap B$ contains the students who submitted both assignments. The complement $\overline{A \cap B}$ then includes all students who failed to submit at least one of the assignments. This is equivalent to taking the union of the students who did not submit math homework ($\overline{A}$) and those who did not submit science homework ($\overline{B}$).

Problem 1.2.22.

$A = \{a, b, c, d, e\}$
$B = \{d, e, f, g, h, i, k\}$
$C = \{a, c, d, e, k, r, s, t\}$

Validate

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$\begin{aligned} ? &= 5 + 7 + 8 - |\{d, e\}| - |\{d, e, k\}| - |\{a, c, d, e\}| + |\{d, e\}| \\ &= 20 - 2 - 3 - 4 + 2 \\ &= 20 - 7 = 13 \end{aligned}$$

Problem 1.2.23.

$A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 7, 8, 9\}$
$C = \{1, 2, 4, 7, 10, 12\}$

Validate

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$\begin{aligned} ? &= 6 + 5 + 6 - |\{2, 4\}| - |\{2, 4, 7\}| - |\{1, 2, 4\}| + |\{2, 4\}| \\ &= 17 - 2 - 3 - 3 + 2 \\ &= 11 \end{aligned}$$

Problem 1.2.24.

$A = \{x \mid x \in Z^+ \wedge x < 8\} = \{1, 2, 3, 4, 5, 6, 7\}$
$B = \{x \mid x \in Z \wedge 2 \leq x \leq 4\} = \{2, 3, 4\}$
$C = \{x \mid x \in Z \wedge x^2 < 16 \ = \{-3, -2, -1, 0, 1, 2, 3\}$

Validate

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$\begin{aligned} ? &= 7 + 3 + 7 - |\{2,3,4\}| - |\{2,3\}| - |\{1,2,3\}| + |\{2,3\}| \\ &= 17 - 8 + 2 = 11 \end{aligned}$$

Problem 1.2.25.

$$\begin{aligned} |U| &= 260 \\ |MATH| &= 64 \\ |CS| &= 94 \\ |BUS| &= 58 \\ |MATH \cap BUS| &= 28 \\ |MATH \cap CS| &= 26 \\ |CS \cap BUS| &= 22 \\ |MATH \cap CS \cap BS| &= 14 \end{aligned}$$

(a) How many students were surveyed who had taken none of the three types of courses?
$|(\overline{MATH \cup CS \cup BUS})| = ?$

$$\begin{aligned} |(\overline{MATH \cup CS \cup BUS})| &= |U| - |MATH \cup CS \cup BUS| \\ |MATH \cup CS \cup BUS| &= 64 + 94 + 48 - 28 - 26 - 22 + 14 \\ &= 216 - 76 + 14 = 230 - 76 = 154 \\ |U| - 144 &= 116 \end{aligned}$$

(b) Of the students surveyed, how many had taken only a computer science course?
$|CS| - |MATH \cap CS| - |CS \cap BUS| + |MATH \cap CS \cap BUS| = ?$

since CS intersects both and if we subtracted both intersections, the tripple intersection subset would've been counted twice

$94 - 26 - 22 + 14 = 60$

Problem 1.2.26.

$$
\begin{aligned}
|U| &= 500 \\
|FB| &= 285 \\
|HK| &= 195 \\
|BB| &= 115 \\
|FB \cap BB| &= 45 \\
|FB \cap HK| &= 70 \\
|HK \cap BB| &= 50 \\
|(\overline{FB \cup HK \cup BB})| &= 50
\end{aligned}
$$

(a) How many people in the survey watch all three kinds of games?
$|FB \cap HK \cap BB| = ?$

$$
\begin{aligned}
|FB \cup HK \cup BB| &= |FB| + |BB| + |HK| \\
&\quad - |FB \cap BB| - |FB \cap HK| - |HK \cap BB| \\
&\quad + |FB \cap BB \cap HK| \\
&= 285 + 195 + 115 - 45 - 70 - 50 + X \\
&= 595 - 165 + X
\end{aligned}
$$

$$
\begin{aligned}
|FB \cup HK \cup BB| &= |U| - |\overline{FB \cup HK \cup BB}| \\
&= 500 - 50 = 450
\end{aligned}
$$

$$
\begin{aligned}
450 &= 595 - 165 + X \\
450 &= 430 + X \\
X &= 20 \\
|FB \cap HK \cap BB| &= 20
\end{aligned}
$$

(b) How many people watch exactly one of the sports?

$$
\begin{aligned}
|FB| - |HK \cap FB| - |FB \cap BB| + |HK \cap FB \cap BB| &= ? \\
|HK| - |HK \cap FB| - |HK \cap BB| + |HK \cap FB \cap BB| &= ? \\
|BB| - |BB \cap FB| - |HK \cap BB| + |HK \cap FB \cap BB| &= ?
\end{aligned}
$$

285 - 70 - 45 + 20 = 190
195 - 70 - 50 + 20 = 95
115 - 45 - 50 + 20 = 40
190 + 95 + 40 = 325

Problem 1.2.27.

$$\begin{aligned}
|U| &= 166 \\
|TV| &= 88 \\
|PP| &= 73 \\
|MG| &= 46 \\
|TV \cap PP| &= 34 \\
|TV \cap MG| &= 16 \\
|PP \cap MG| &= 12 \\
|PP \cup TV \cup MG| &= 5
\end{aligned}$$

(a) How many use none of three media
$|(\overline{TV \cup PP \cup MAG})| = ?$

(b) How many obtain their news from a news magazine exclusively?
$|PP| - |TV \cap PP| - |MG \cap PP| + |TV \cap PP \cap MG| = ?$

since CS intersects both and if we subtracted both intersections, the tripple intersection subset would've been counted twice

$94 - 26 - 22 + 14 = 60$