1 Exercises

Exercise 1.1.1 (ex. 9)**.**

$$\begin{aligned} |S| &= 25 \\ |A| &= 40 \\ |S \cap A| &= 10 \\ |S \cup A| &= ? \end{aligned}$$

Solution.

$$\begin{aligned} |S| &= 25 \\ |A| &= 40 \\ |S \cap A| &= 10 \\ |S \cup A| &= |S| + |A| - |S \cap A| = 25 + 40 - 10 \\ |S \cup A| &= 55 \end{aligned}$$

□

Exercise 1.1.2 (ex. 10)**.**

$$\begin{aligned} |BUS| &= 30 \\ |TRAIN| &= 35 \\ |AUTO| &= 100 \\ |BUS \cap TRAIN| &= 15 \\ |BUS \cap AUTO| &= 15 \\ |TRAIN \cap AUTO| &= 20 \\ |BUS \cap TRAIN \cap AUTO| &= 5 \\ |BUS \cup TRAIN \cup AUTO| &= ? \end{aligned}$$

Solution.

$$\begin{aligned} |BUS \cup TRAIN \cup AUTO| &= |BUS| + |TRAIN| + |AUTO| \\ &\quad - |BUS \cap TRAIN| - |BUS \cap AUTO| - |TRAIN \cap AUTO| \\ &\quad + |BUS \cap TRAIN \cap AUTO| \\ &= 30 + 35 + 100 - 15 - 15 - 20 + 5 \\ &= 120 \end{aligned}$$

□

2 Problems

Problem 1.2.1.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup B = \{a, b, c, d, e, f, g\}$

(b) $B \cup C = \{a, c, d, e, f, g\}$

(c) $A \cap C = \{a, c\}$

(d) $B \cap D = \{f\}$

(e) $(A \cup B) - C = \{b, d, e, g\}$

(f) $A - B = \{a, b, c\}$

(g) $\overline{A} = \{d, e, f, h, k\}$

(h) $A \oplus B = \{a, b, c\} \cup \{d, e, f\} = \{a, b, c, d, e, f\}$

(i) $A \oplus C = \{b, g\} \cup \{f\} = \{b, g, f\}$

(j) $(A \cap B) - C = \{g\}$ - $\{a, c, f\} = \{g\}$

Problem 1.2.2.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup D = \{a, b, c , f, g, h, k\}$

(b) $B \cup D = \{d, e, f, g, h, k\}$

(c) $C \cap D = \{f\}$

(d) $A \cap D = \varnothing$

(e) $(A \cup B) - (C \cup B) = \{a, b, c, d, e, f, g\}$ - $\{a, c, d, e, f, g\} = \{b\}$

(f) $B - C = \{d, e, g\}$

(g) $\overline{B} = \{a, b, c, h, k\}$

(h) $C - B = \{a, c\}$

(i) $C \oplus D = \{a, c\} \cup \{h, k\} = \{a, c, h, k\}$

(j) $(A \cap B) - (B \cap D) = \{g\}$ - $\{f\} = \{g\}$

Problem 1.2.3.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup B \cup C = \{a, b, c, d, e, f, g\}$

(b) $A \cap B \cap C = \varnothing$

(c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \{g\} \cup \{a, c\} = \{a, c, g\}$

(d) $(A \cup B) \cap C = (C \cap A) \cup (C \cap B) = \{a, c\} \cup \{f\} = \{a, c, f\}$

(e) $\overline{A \cup B} = \{d, e, f, h, k\} \cap \{a, b, c, h, k\} = \{h, k\}$

(f) $\overline{A \cap B} = \{d, e, f, h, k\} \cup \{a, b, c, h, k\} = \{a, b, c, d, e, f, h, k\}$

Problem 1.2.4.

$$U = \{a, b, c, d, e, f, g, h, k\}$$
$$A = \{a, b, c, g\}$$
$$B = \{d, e, f, g\}$$
$$C = \{a, c, f\}$$
$$D = \{f, h, k\}$$

Compute

(a) $A \cup \varnothing = A$

(b) $A \cup U = U$

(c) $B \cup B = B$

(d) $C \cap \varnothing = \varnothing$

(e) $\overline{C \cup D} = \{b, d, e, g, h, k\} \cap \{a, b, c, d, e, g\} = \{b, d, e, g\}$

(f) $\overline{C \cap D} = \{b, d, e, g, h, k\} \cup \{a, b, c, d, e, g\} = \{a, b, c, d, e, g, h, k\}$

Problem 1.2.5.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$A = \{1, 2, 4, 6, 8\}$$
$$B = \{2, 4, 5, 9\}$$
$$C = \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$$
$$D = \{7, 8\}$$

Compute

(a) $A \cup B = \{1, 2, 4, 5, 6, 8, 9\}$

(b) $A \cup C = \{1, 2, 3, 4, 6, 8\}$

(c) $A \cup D = \{1, 2, 4, 6, 7, 8\}$

(d) $B \cup C = \{1, 2, 3, 4, 5, 9\}$

(e) $A \cap C = \{1, 2, 4\}$

(f) $A \cap D = \{8\}$

(g) $B \cap C = \{2, 4\}$

(h) $C \cap D = \varnothing$

Problem 1.2.6.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$A = \{1, 2, 4, 6, 8\}$$
$$B = \{2, 4, 5, 9\}$$
$$C = \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$$
$$D = \{7, 8\}$$

Compute

(a) $A - B = \{1, 6, 8\}$

(b) $B - A = \{5, 9\}$

(c) $C - D = \{1, 2, 3, 4\}$

(d) $\overline{C} = \{5, 6, 7, 8, 9\}$

(e) $\overline{A} = \{3, 5, 7, 9\}$

(f) $A \oplus B = \{1, 6, 8\} \cup \{5, 9\} = \{1, 5, 6, 8, 9\}$

(g) $C \oplus D = \{1, 2, 3, 4, 7, 8\}$

(h) $B \oplus C = \{1, 3, 5, 9\}$

Problem 1.2.7.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$A = \{1, 2, 4, 6, 8\}$$
$$B = \{2, 4, 5, 9\}$$
$$C = \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$$
$$D = \{7, 8\}$$

Compute

(a) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$

(b) $A \cap B \cap C = \{2, 4\}$

(c) $A \cap (B \cup C) = \{1, 2, 4\}$

(d) $(A \cup B) \cap D = \{8\}$

(e) $\overline{A \cup B} = \{3, 5, 7, 9\} \cap \{1, 3, 6, 7, 8\} = \{3, 7\}$

(f) $\overline{A \cap B} = \{3, 5, 7, 9\} \cup \{1, 3, 6, 7, 8\} = \{1, 3, 5, 6, 7, 8, 9\}$

Problem 1.2.8.

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$A = \{1, 2, 4, 6, 8\}$$
$$B = \{2, 4, 5, 9\}$$
$$C = \{x \mid x \in Z^+ \ \wedge x^2 \leq 16\} = \{1, 2, 3, 4\}$$
$$D = \{7, 8\}$$

Compute

(a) $B \cup C \cup D = \{1, 2, 3, 4, 5, 7, 8, 9\}$

(b) $B \cap C \cap D = \varnothing$

(c) $A \cup A = \{1, 2, 4, 6, 8\}$

(d) $A \cap \overline{A} = \varnothing$

(e) $A \cup \overline{A} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(f) $A \cap (\overline{C} \cup D) = \{6, 8\} \cup \{8\} = \{6, 8\}$

Problem 1.2.9.

$$U = \{a, b, c, d, e, f, g, h\}$$
$$A = \{a, c, f, g\}$$
$$B = \{a, e\}$$
$$C = \{b, h\}$$

Compute

(a) $\overline{A} = \{\text{b, d, e, h}\}$

(b) $\overline{B} = \{\text{b, c, d, f, g, h}\}$

(c) $\overline{A \cup B} = \{\text{b, d, h}\}$

(d) $\overline{A \cap B} = \{\text{b, c, d, e, f, g, h}\}$

(e) $\overline{U} = \varnothing$

(f) $A - B = \{\text{c, f, g}\}$

Problem 1.2.10.

$$U = \{a, b, c, d, e, f, g, h\}$$
$$A = \{a, c, f, g\}$$
$$B = \{a, e\}$$
$$C = \{b, h\}$$

Compute

(a) $\overline{A} \cap \overline{B} = \{\text{b, d, e, h}\} \cap \{\text{b, c, d, f, g, h}\} = \{\text{b, d, h}\}$

(b) $\overline{B} \cup \overline{C} = \{\text{b, c, d, f, g, h}\} \cup \{\text{a, c, d, e, f, g}\} = \{\text{a, b, c, d, e , f, g, h}\}$

(c) $\overline{A \cup A} = \overline{A} = \{\text{b, d, e, h}\}$

(d) $\overline{C \cap C} = \overline{C} = \{\text{a, c, d, e, f, g}\}$

(e) $A \oplus B = \{\text{c, e, f, g}\}$

(f) $B \oplus C = \{\text{a, e, b, h}\}$

Problem 1.2.11.

$$U = R$$
$$A = \{x \mid x \text{ is a solution to } x^2 - 1 = 0\} = \{-1, 1\}$$
$$B = \{-1, 4\}$$

Compute

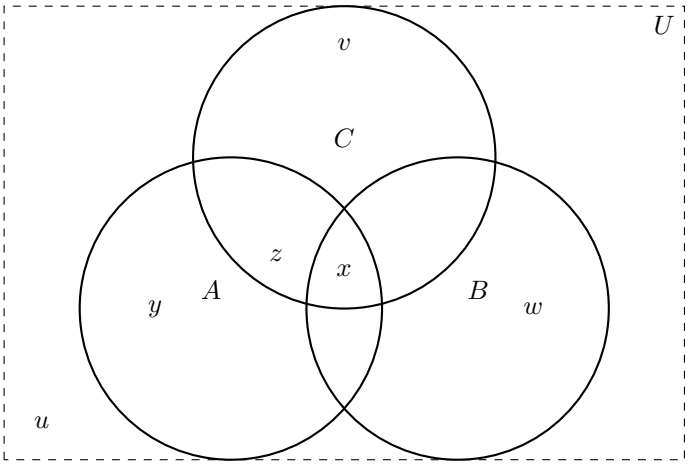
(a) $\overline{A} = \{x \mid x \in (-\infty, -1) \vee (-1, 1) \vee x \in (1, \infty)\}$

(b) $\overline{B} = \{x \mid x \in (-\infty, -1) \vee x \in (-1, 4) \vee x \in (4, \infty)\}$

(c) $\overline{A \cup B} = \{x \mid x \in (-\infty, -1) \vee (-1, 1) \vee (1, 4) \vee x \in (4, \infty)\}$

(d) $\overline{A \cap B} = \{x \mid x \in (-\infty, -1) \vee (-1, \infty)\}$

Problem 1.2.12.



Compute

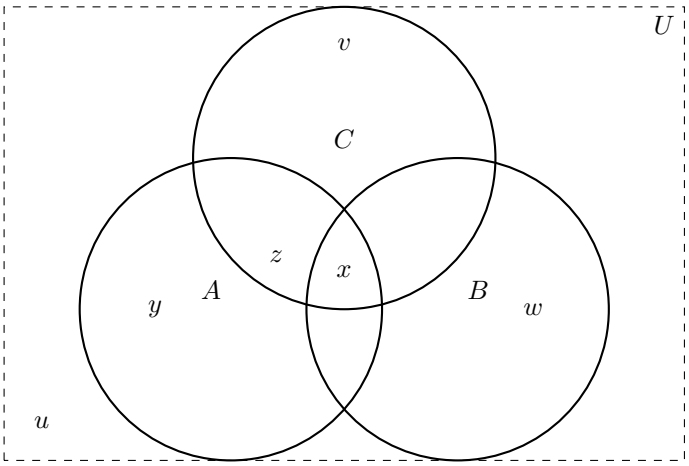
(a) $y \in A \cap B$ = False: y $\notin$ B

(b) $x \in B \cup C$ = True: x $\in$ B $\wedge$ x $\in$ C

(c) $w \in B \cap C$ = False: w $\notin$ C

(d) $u \notin C$ = True: u $\in \overline{A \cup B \cup C}$

Problem 1.2.13.



Compute

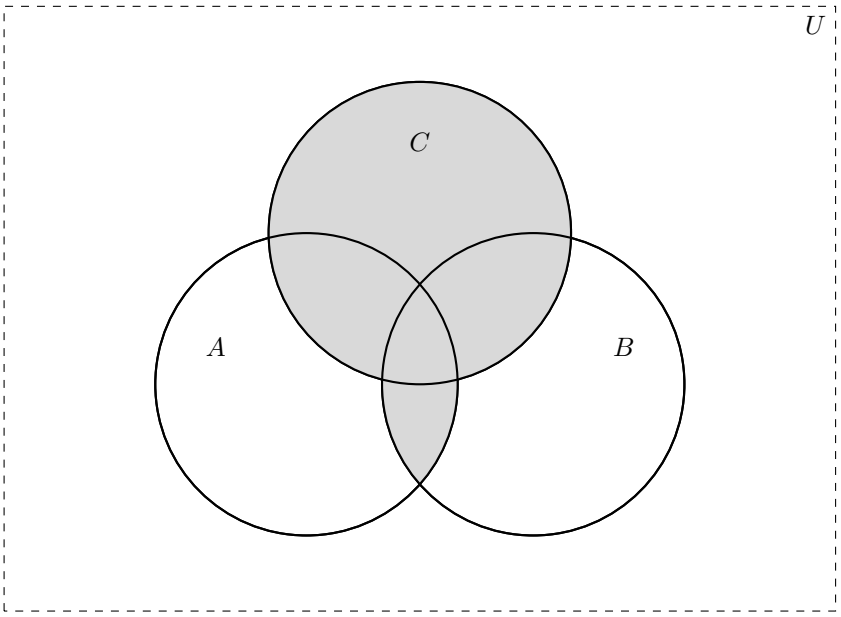
(a) $x \in A \cap B \cap C$ = True: x ∈ A ∧ x ∈ B ∧ x ∈ C

(b) $y \in A \cup B \cup C$ = True: y ∈ A

(c) $z \in A \cap C$ = True: z ∈ A ∧ z ∈ C

(d) $v \in B \cap C$ = False: v ∈ C ∧ v ∉ B

Problem 1.2.14.



Describe shaded region

$$(A \cap B) \cup C$$