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Nonlinear Decoherence in the Presence of Space Charge

(Dated: October 25, 2017)

I. INTRODUCTION

A. Nonlinear Integrable Optics for Single Particles

B. Impact of Space Charge

II. CREATING A MATCHED BUNCH

For a periodic accelerator matching of the bunch to the lattice usually entails finding a periodically constant set of Twiss parameters. The statistical properties of the distribution are then set equal to the Twiss parameters of the lattice. Thus matched such a bunch will remain stable in time.

For matching of a bunch to a lattice including an elliptic nonlinear element we use two versions of the lattice. The first is the base lattice without any nonlinear elements, using this lattice we can obtain the matched Twiss parameters that will be needed to construct the bunch. For the actual construction of the bunch we make use of the nonlinear potential that will be created based on the parameters of the nonlinear element. It has been found that such a matching procedure is necessary to prevent beam loss [1].

For an idealized Kapchinskij-Vladimirskij (KV) distribution all particles will have an identical value for the Hamiltonian

$$H = \frac{\hat{p}_x^2}{2} + \frac{\hat{p}_y^2}{2} + \frac{\hat{x}^2}{2} + \frac{\hat{y}^2}{2} + tU(\hat{x}, \hat{y}). \tag{1}$$

So that all particles lie on a single hyper-ellipsoid in the transverse phase space. While the KV distribution is useful due to its linear space charge forces which means that all particles experience a single tune depression it is also prone to producing numerical effects and has been found to not produce long-term stable results in these efforts to simulate non

III. SIMULATIONS OF IOTA

A. Simulation Descriptions

Simulations of IOTA were performed with the tracking code Synergia [2]. Space charge forces were calculated using a self-consistent 2.5D model that slices the beam longitudinally and then applies transverse kicks to the bunch calculated for each slice. As we are primarily concerned with transverse effects a very long bunch $\sigma_z \gg \sigma_{x,y}$ was used with zero initial momentum spread. Under these conditions the space charge kick should be almost entirely uniform along the bunch reducing the space charge model to 2D.

The lattice elements are all modeled using first-order maps to prevent mismatch produced from higher-order effect creating a loss of integrability beyond that which will be produced by space charge. The exception to this is the nonlinear magnet which is modeled using a second order drift kick approach. The elliptic potential is a function of a strength parameter t and geometric parameter c. For invariance to be maintained these parameters must scale with the β-function as β^{-1} and $\sqrt{\beta}$ respectively along the nonlinear magnet. For construction of the physical magnet smooth scaling is not realistically achievable due to engineering constraints and the magnet is broken into 20 thin slices [**?**]. In simulation, it has been seen that 20 slices with the second order drift-kick scheme is sufficient to provide convergence. In our simulations with the inclusion of space charge we use 60 slices to ensure good convergence.

B. Simulation Results

To examine the operation of the nonlinear element we start the simulation with a bunch that has been matched into the elliptic potential as previous described, but after the matching procedure the bunch centroid is displaced from zero in x by $100\,\mu$m. A comparison of bunch centroid motion with and without the noninear element is shown in Fig. 1 With a purely linear lattice this displaced bunch exhibits coherent oscillations of the centroid around zero indefinitely. With the nonlinear element turned on these oscillations rapidly damp due to the tune spread created by the nonlinear magnet.

The tune spread induced for the zero current case is shown in Fig. **??**. Because the multipole expansion of the elliptic potential has a quadrupole term in the lowest order there is a splitting of the horizontal and vertical tunes, as well as the spread from the higher-order terms.

1. Nonlinear Decoherence with Space Charge

We now illustrate what happens when space charge is included in the simulation. The space charge model used in Synergia was previously discussed. For all simulations with space charge a tune depression of $0.03 \times 2\pi$ / turn is maintained. For a waterbag bunch distribution an asymmetric distribution of tunes, shifted from the lat-

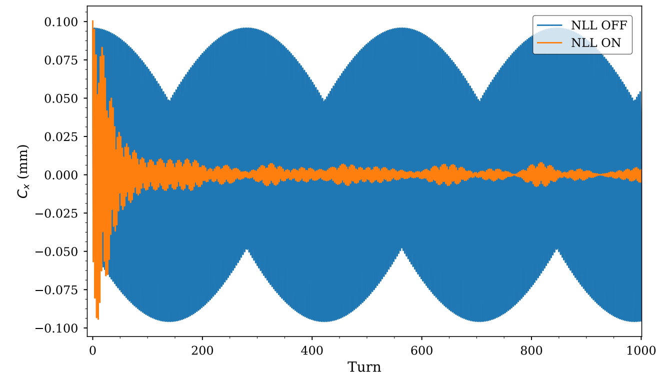

FIG. 1. Centroid (C_x) motion for a matched bunch displaced horizontally by 100 μm. Motion with the nonlinear element off is shown in blue and with nonlinear element on in orange. Space charge is not included in the simulation.

TABLE I. Characteristics of the bunch and lattice for simulations.

Quantity	Value	Units
Beam Parameters		
dQ_{SC}	0.03	-
ε_0	4, 8, 12, 16	$mm - mrad$
I	0.2056, 0.4113, 0.6169, 0.8225	mA
K	2.5	MeV
Δx	100	μm
Nonlinear Magnet Parameters		
t	0.2/0.4	-
c	0.1	$m^{1/2}$
ψ_{nll}	0.3	2π

tice design point, occurs. To match the beam current to the ideal tune depression as closely as possible a scan over the current is performed. The ideal current is selected to that which maximizes the number of particles in the bunch that have a phase advance from the exit to the entrance of the nonlinear element within one standard deviation of zero. Values for various emittances are shown in Table. I.

For a bunch with an initial emittance of 8 mm − mrad and with a nonlinear magnet strength $t = 0.4$ the turn-to-turn centroid motion is shown in Fig. 2. It is seen that with space charge now included the damping time is greatly increased and one thousand turns is no longer sufficient to see the centroid motion decrease appreciably. The main culprit for this loss in effectiveness of nonlinear decoherence is the space charge induced tune spread. The assumption of time-invariance created by the ideal T-insert is broken for particles that do not maintain the design tune.

[1] S. Webb, D. Bruhwiler, V. Danilov, R. Kishek, S. Nagaitsev, and A. Valishev, in *Proceedings, 6th International Particle Accelerator Conference (IPAC 2015): Richmond, Virginia, USA, May 3-8, 2015* (2015) p. MOPMA029.

[2] "Synergia simulation package," `https://cdcvs.fnal.gov/redmine/projects/synergia2`.

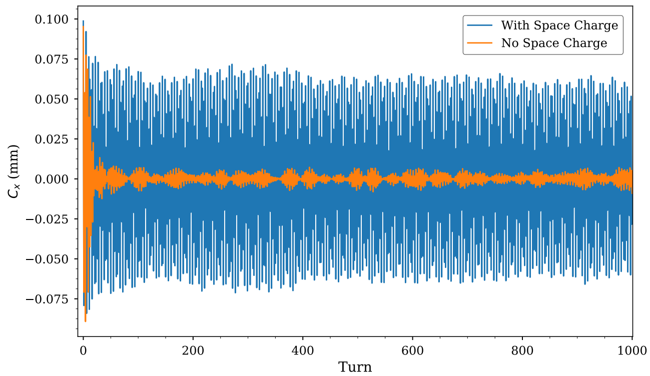

FIG. 2. Centroid (C_x) motion for a matched bunch displaced horizontally by $100\,\mu$m. Only single particle dynamics are considered for the simulation in orange. Space charge is included for the simulation shown in blue.