

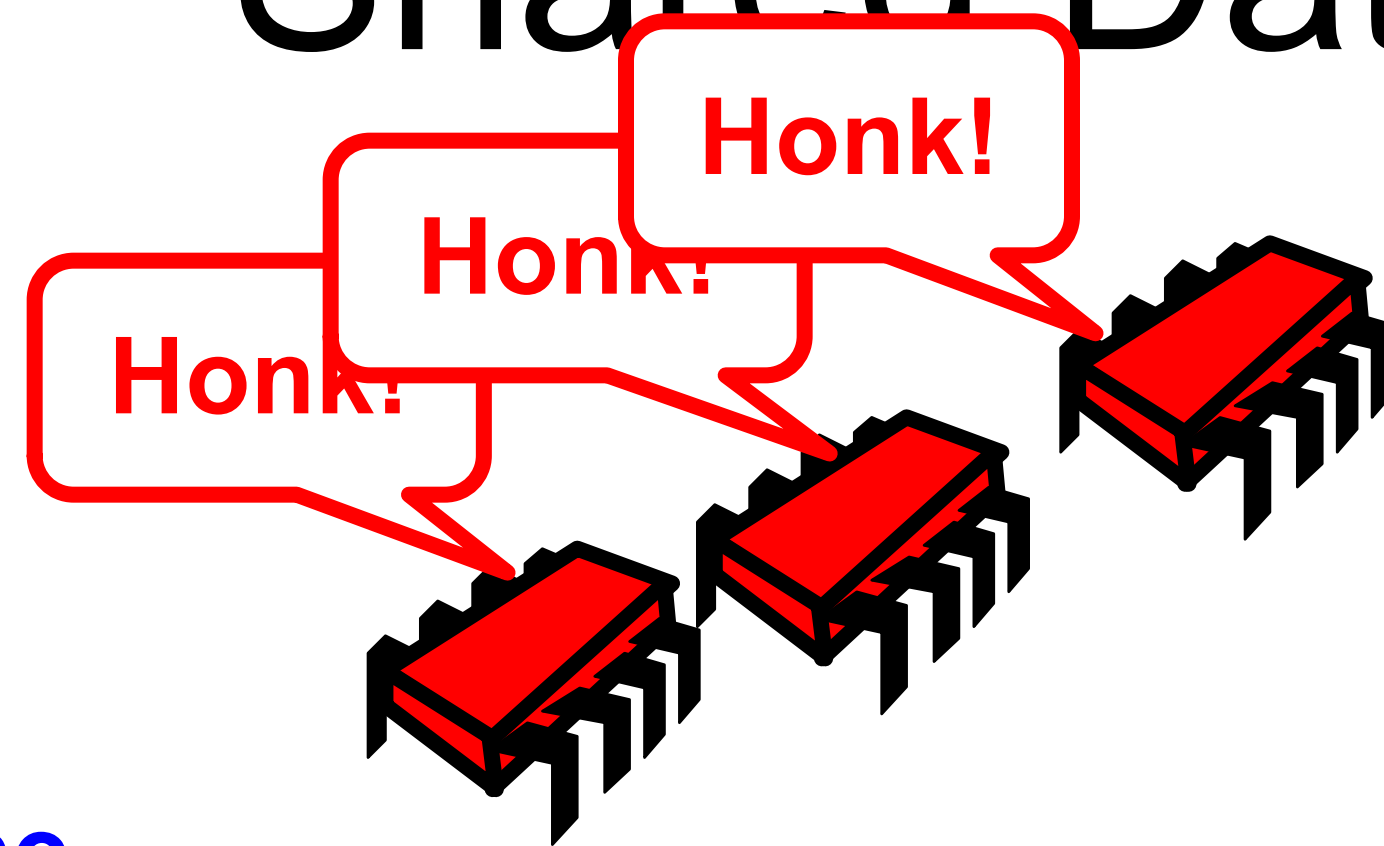
# YSC4231: Parallel, Concurrent and Distributed Programming

Mutual Exclusion

# Review: Amdahl's Law

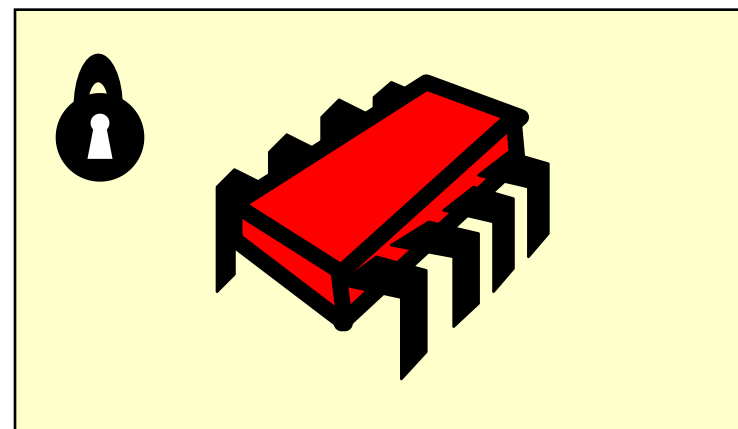
$$\text{Speedup} = \frac{1}{1 - p + \frac{p}{n}}$$

# Shared Data Structures

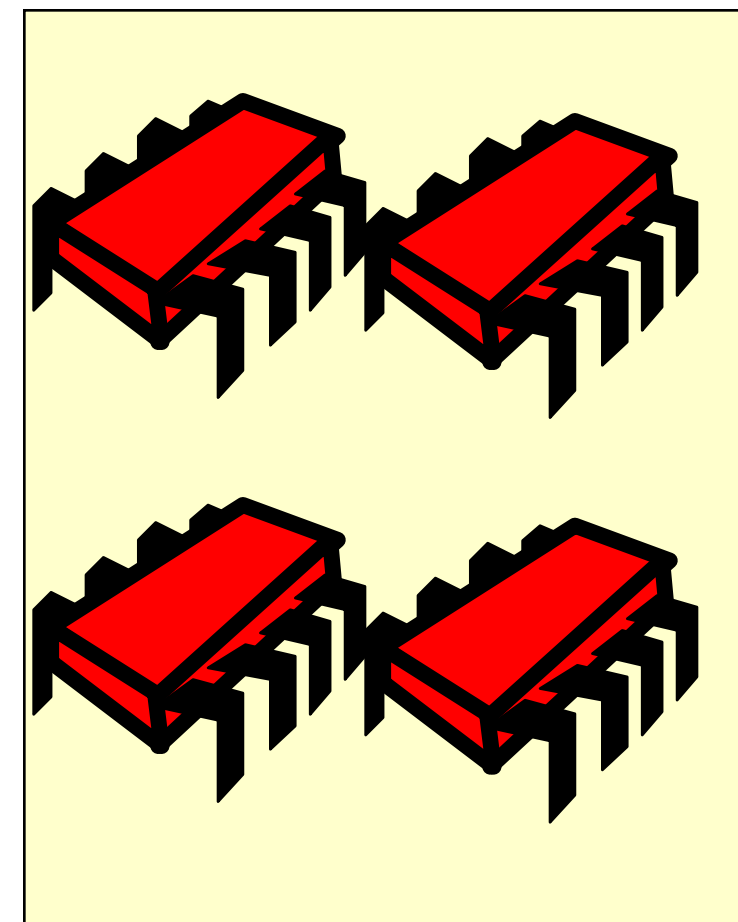


**Why fine-grained  
parallelism matters**

**Coarse  
Grained**

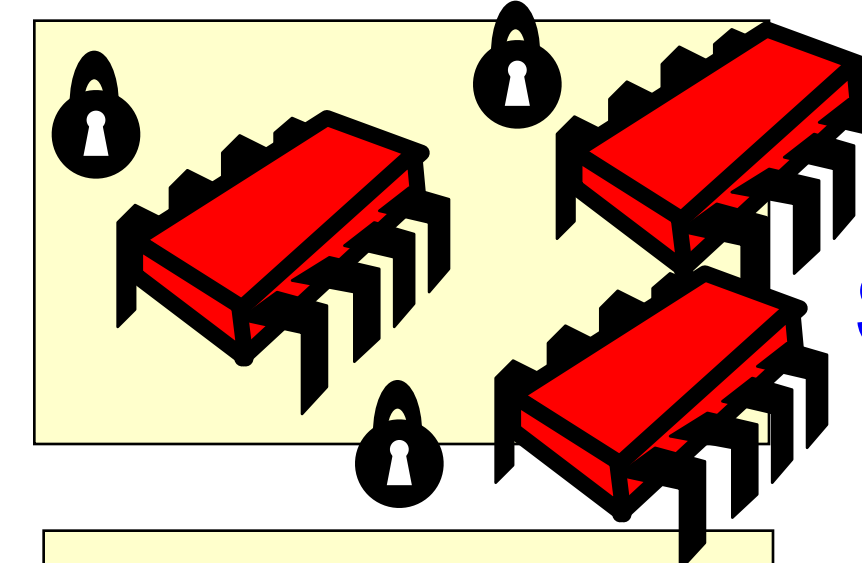


**25%  
Shared**

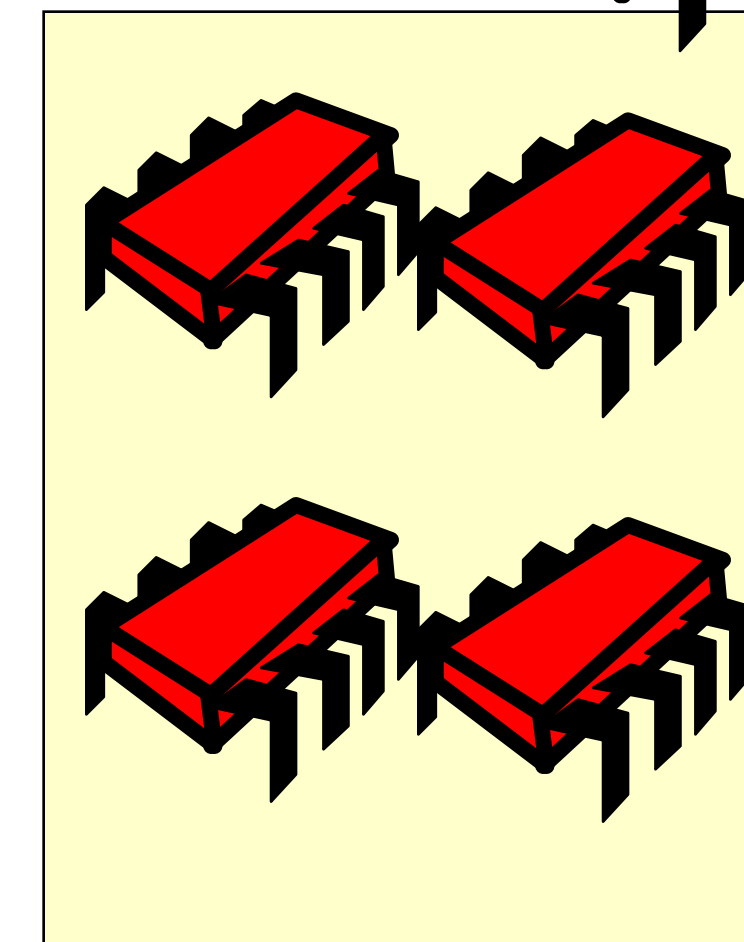


**75%  
Unshared**

**Fine  
Grained**



**25%  
Shared**



**75%  
Unshared**

# Example Synchronization Paradigms

- Mutual exclusion
- Readers-Writers
- Producer-Consumer

# Mutual Exclusion



- We will clarify our understanding of mutual exclusion
- We will also see how to *reason* about various properties in an asynchronous concurrent setting

# Mutual Exclusion



In his 1965 paper E. W. Dijkstra wrote:

"Given in this paper is a solution to a problem which, to the knowledge of the author, has been an open question since at least 1962, irrespective of the solvability. [...]"

Although the setting of the problem might seem somewhat academic at first, the author trusts that anyone familiar with the logical problems that arise in computer coupling will appreciate the significance of the fact that this problem indeed can be solved."

# Mutual Exclusion



- Formal problem definitions
- Solutions for 2 threads
- Solutions for  $n$  threads
- Fair solutions
- Inherent costs

# Warning

- You will *never* use these protocols
  - Get over it
- You are advised to understand them
  - The same issues show up everywhere
  - Except hidden and more complex



# Why is Concurrent Programming so Hard?

- Try preparing a seven-course banquet
  - By yourself
  - With one friend
  - With twenty-seven friends ...
- Before we can talk about programs
  - Need a language
  - Describing time and concurrency

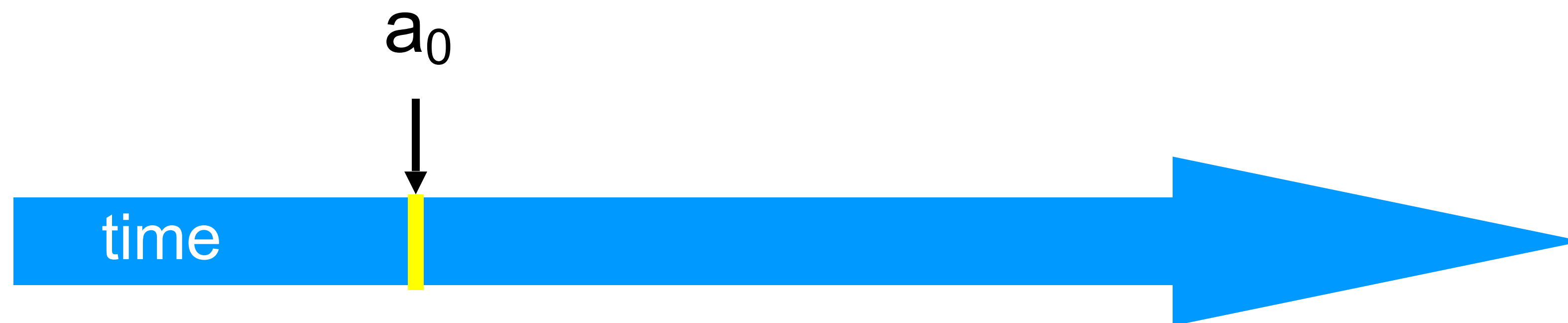
# Time

- *“Absolute, true and mathematical time, of itself and from its own nature, flows equably without relation to anything external.”* (Isaac Newton, 1689)
- *“Time is what keeps everything from happening at once.”* (Ray Cummings, 1922)



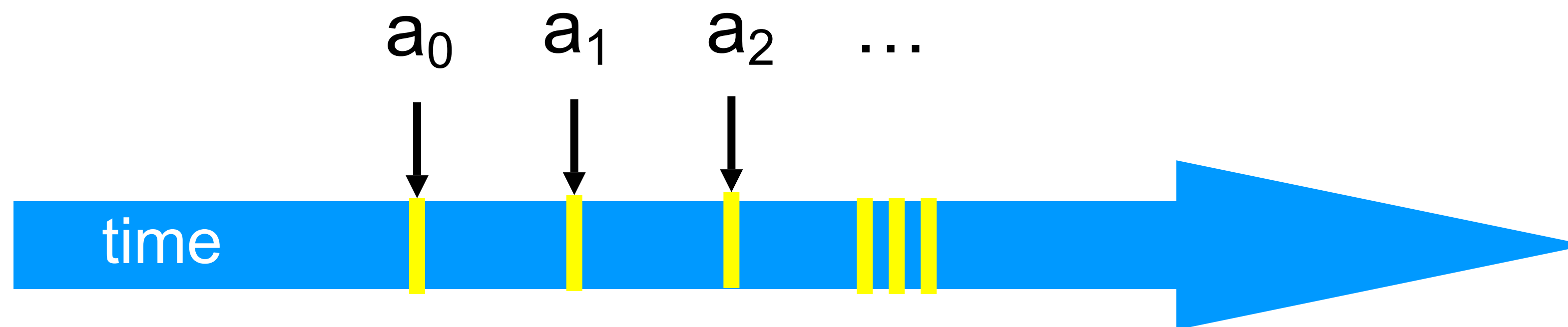
# Events

- An *event*  $a_0$  of thread  $A$  is
  - Instantaneous
  - No simultaneous events (break ties)



# Threads

- A *thread*  $A$  is (formally) a sequence  $a_0, a_1, \dots$  of events
  - “Trace” model
  - Notation:  $a_0 \rightarrow a_1$  indicates order

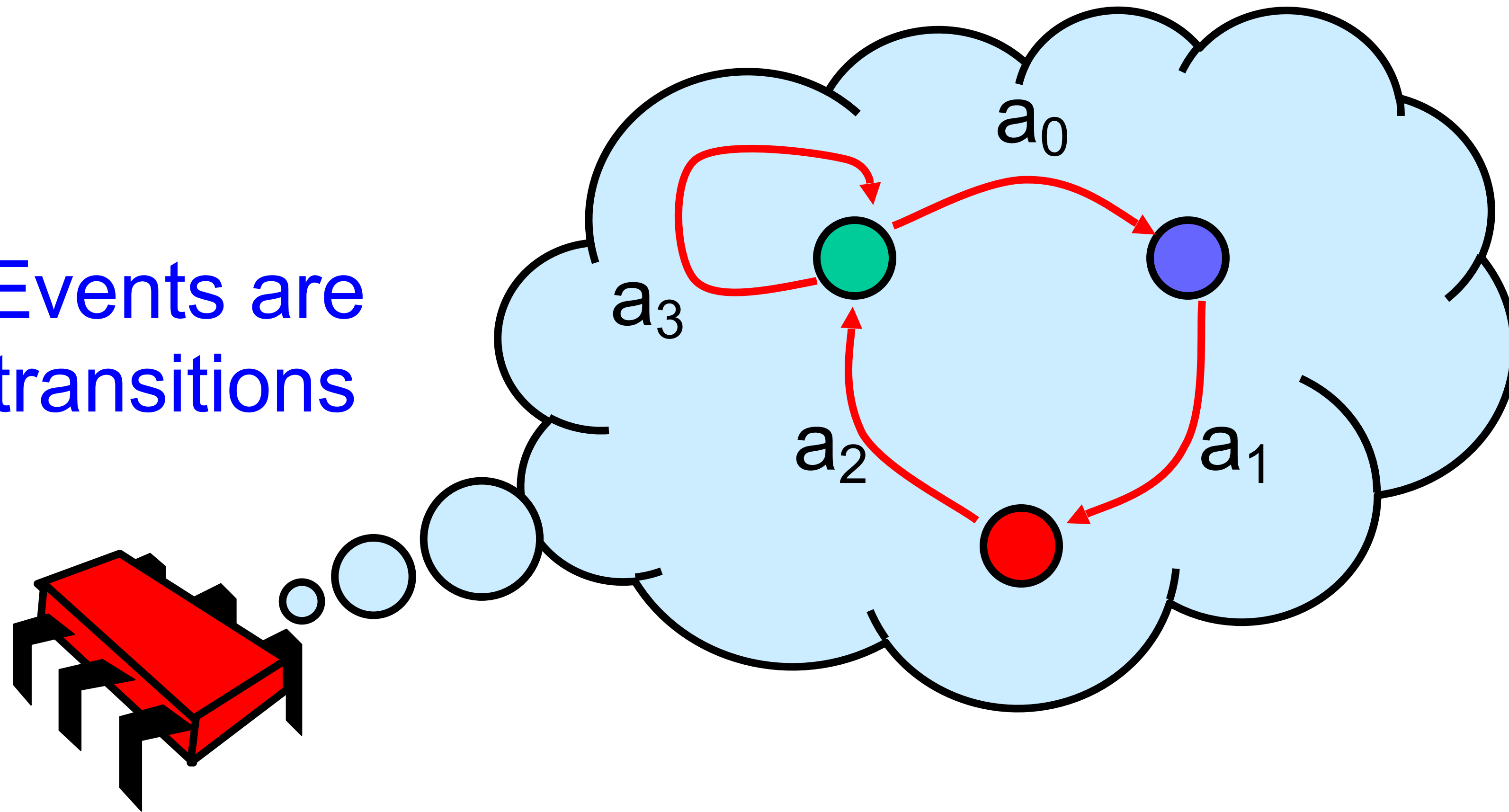


# Example Thread Events

- Assign to shared variable
- Assign to local variable
- Invoke method
- Return from method
- Lots of other things ...

# Threads are State Machines

Events are  
transitions



# States

- Thread State
  - Program counter
  - Local variables
- System state
  - Object fields (shared variables)
  - Union of thread states

# Concurrency

- Thread A





# Concurrency

- Thread A

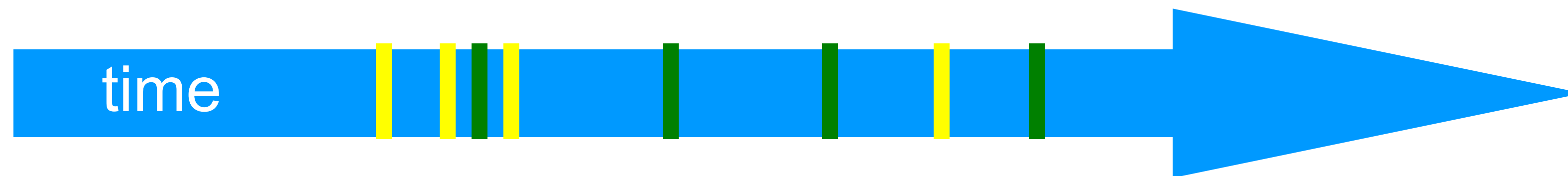


- Thread B



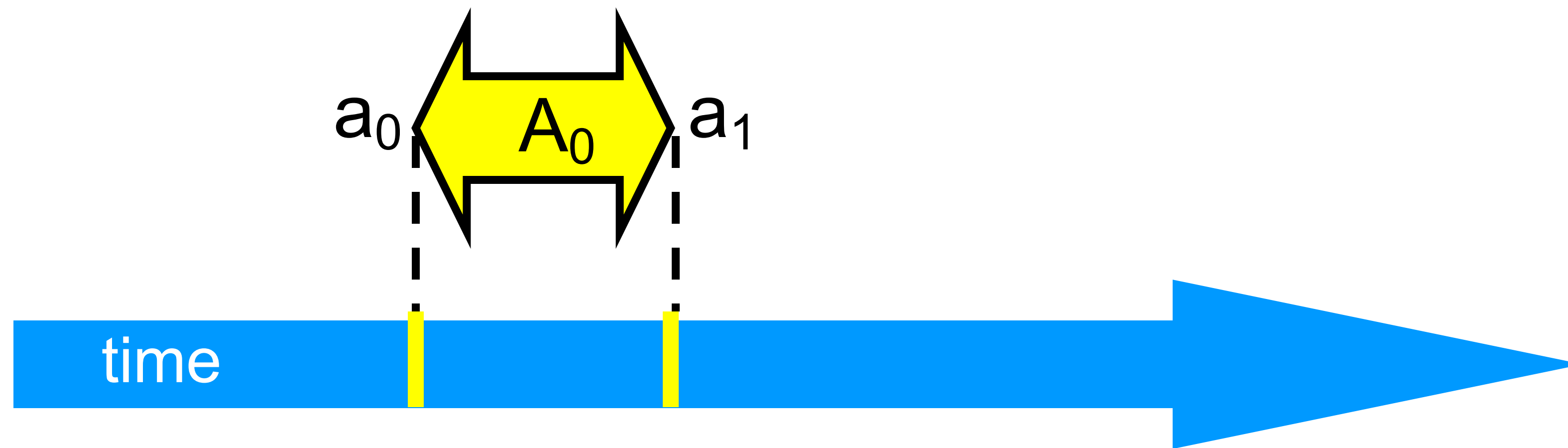
# Interleavings

- Events of two or more threads
  - Interleaved
  - Not necessarily independent (why?)

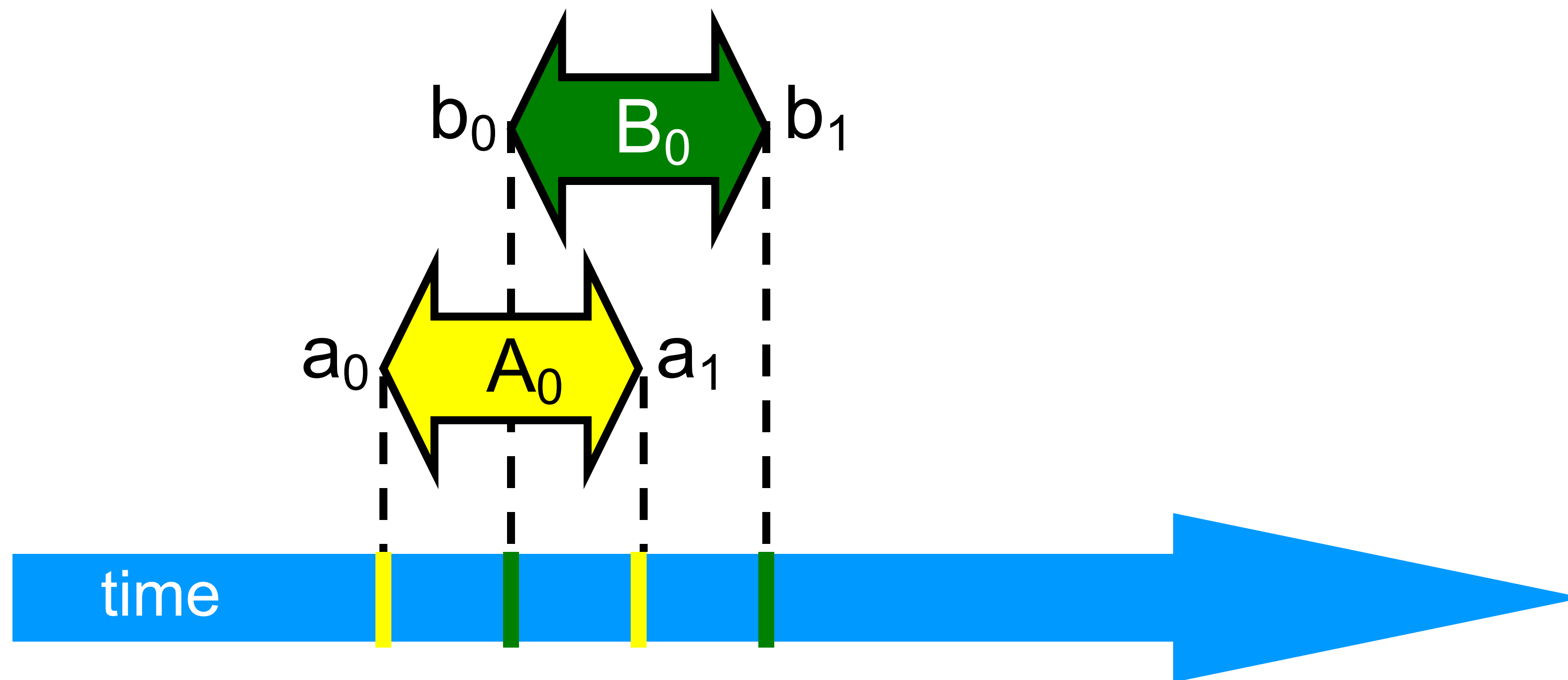


# Intervals

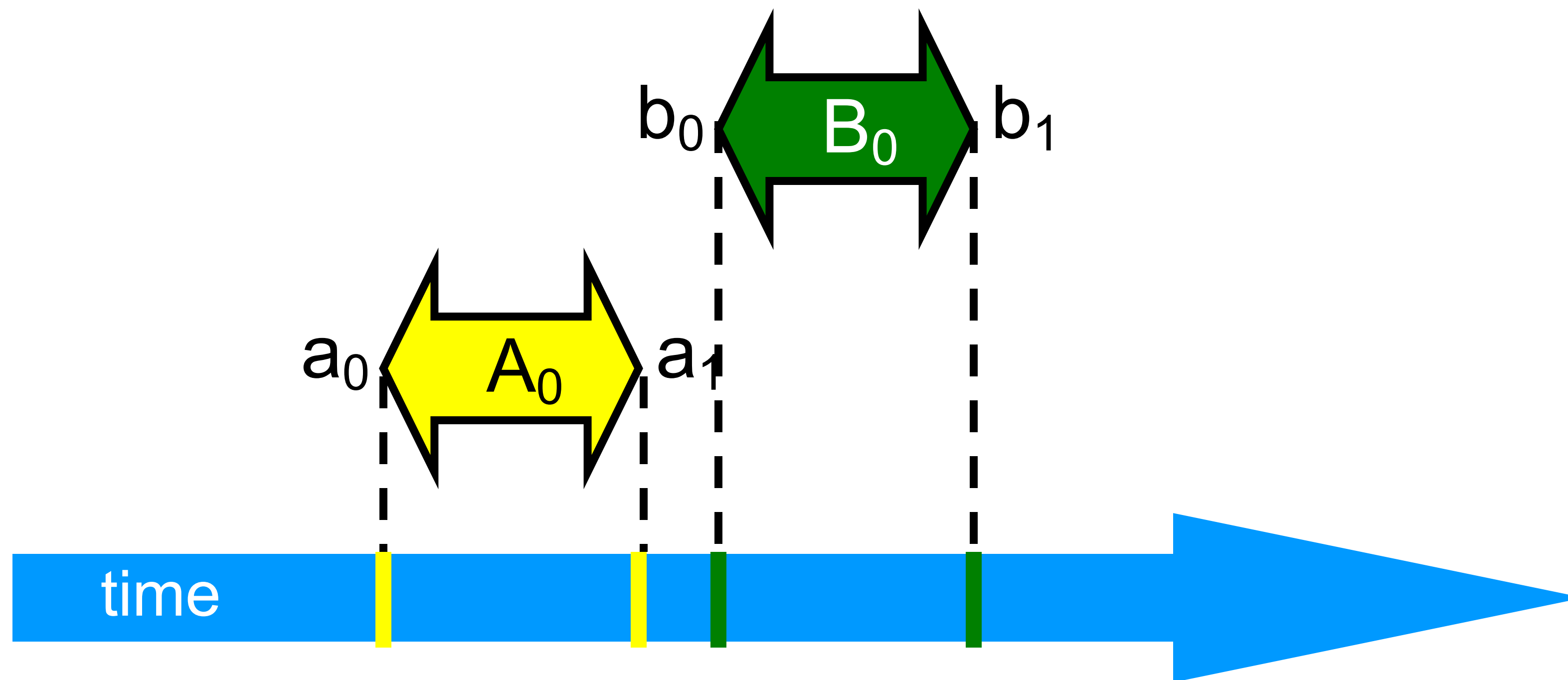
- An *interval*  $A_0 = (a_0, a_1)$  is
  - Time between events  $a_0$  and  $a_1$



# Intervals may Overlap

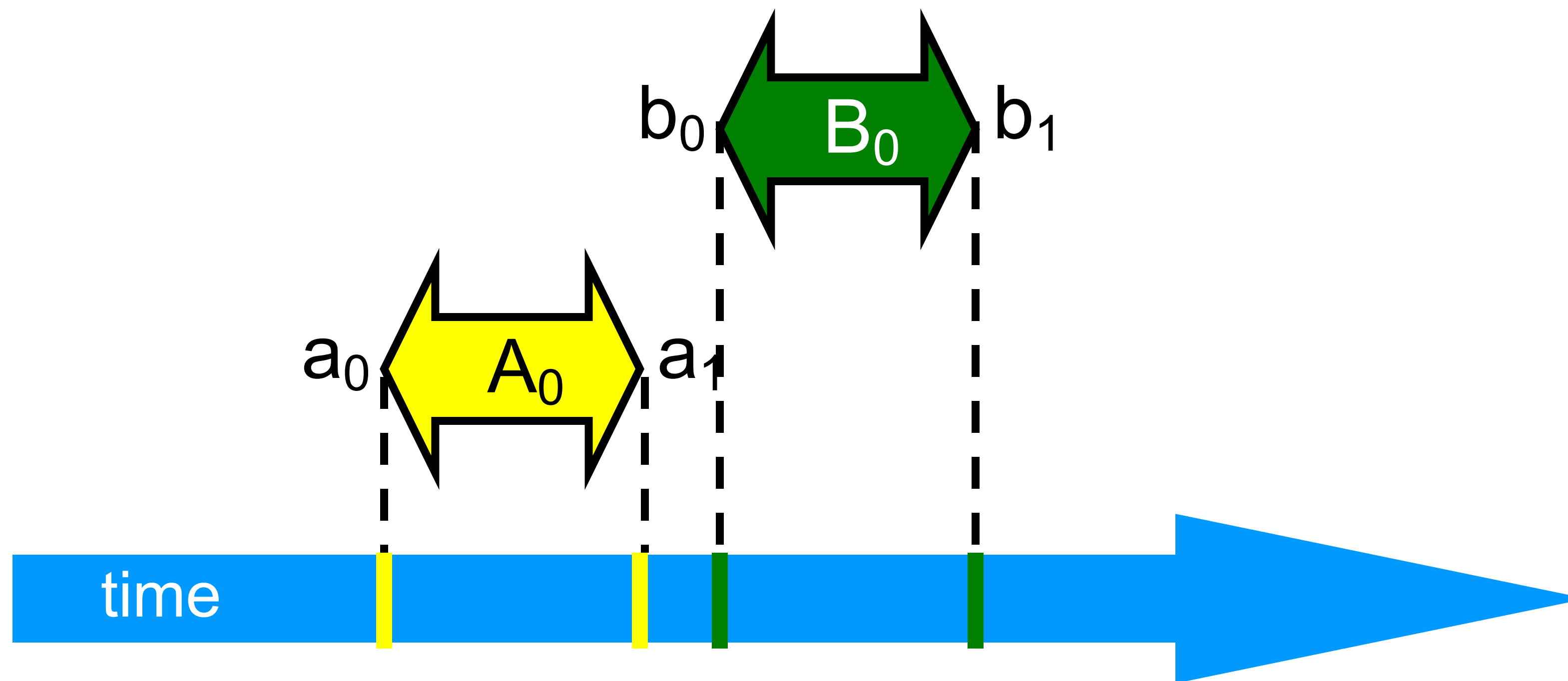


# Intervals may be Disjoint

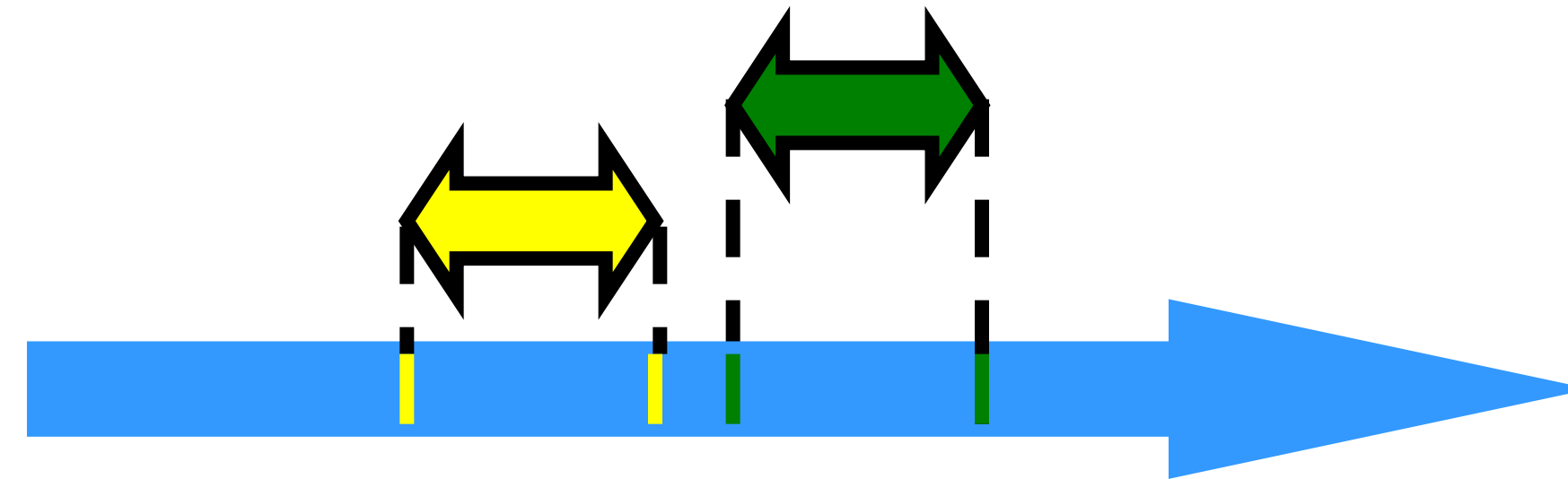


# Precedence

Interval  $A_0$  precedes interval  $B_0$

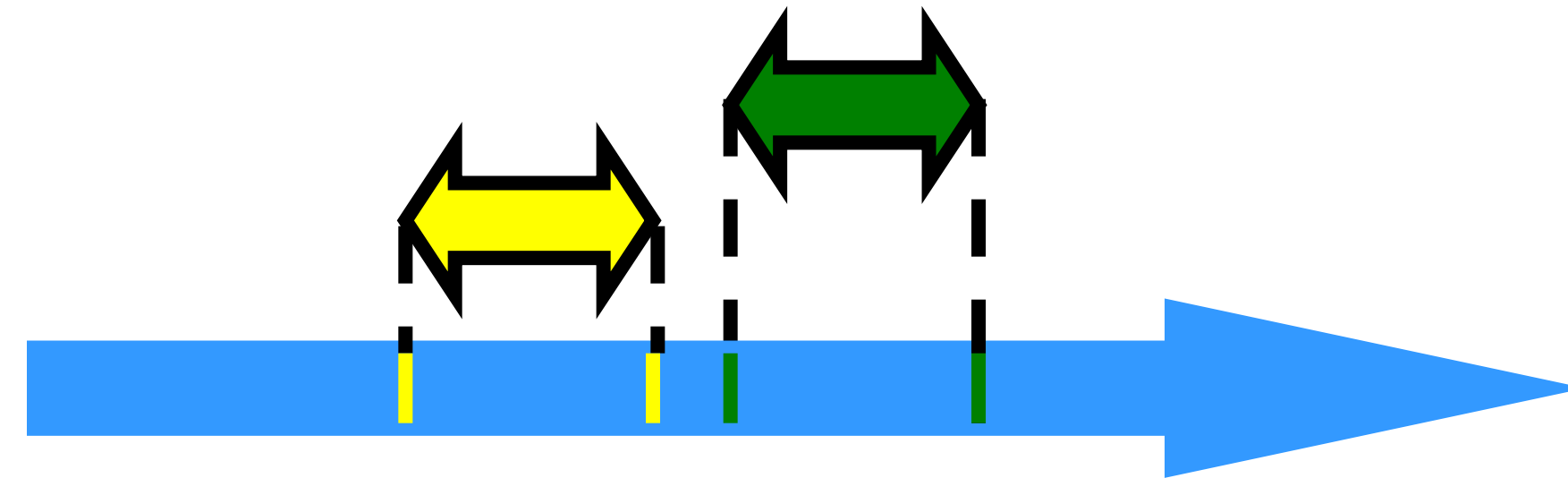


# Precedence



- Notation:  $A_0 \rightarrow B_0$
- Formally,
  - End event of  $A_0$  before start event of  $B_0$
  - Also called “happens before” or “precedes”

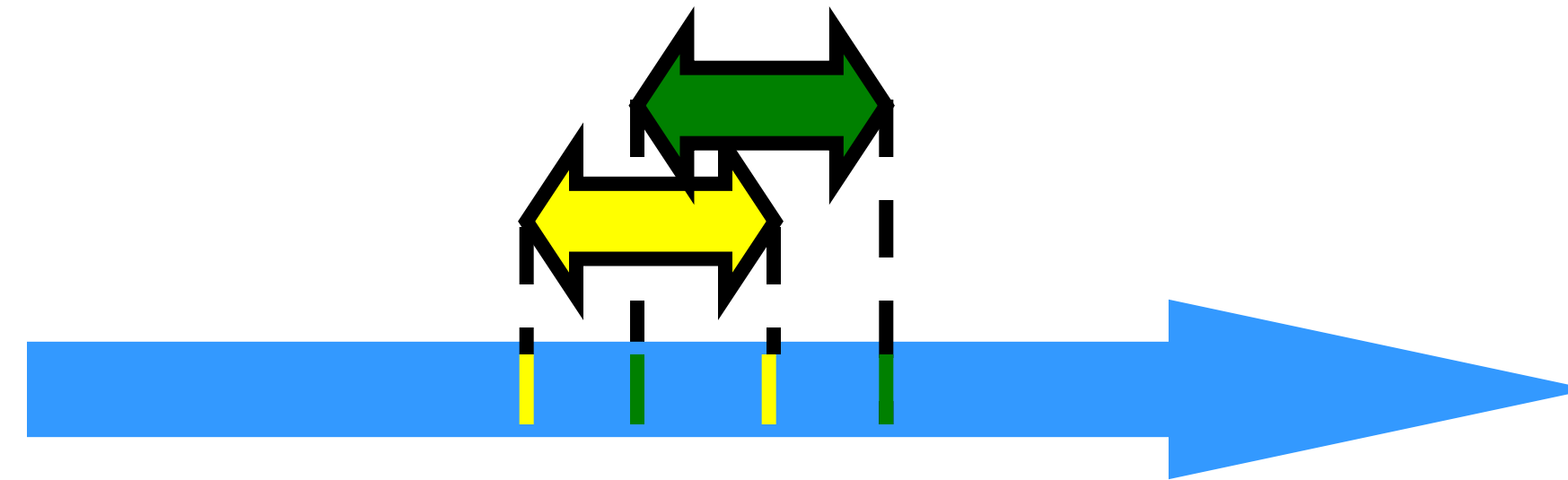
# Precedence Ordering



- Remark:  $A_0 \rightarrow B_0$  is just like saying
  - 1066 AD  $\rightarrow$  1492 AD,
  - Middle Ages  $\rightarrow$  Renaissance,
- Oh wait,
  - what about this week vs this month?



# Precedence Ordering



- Never true that  $A \rightarrow A$
- If  $A \rightarrow B$  then not true that  $B \rightarrow A$
- If  $A \rightarrow B$  &  $B \rightarrow C$  then  $A \rightarrow C$
- Funny thing:  $A \rightarrow B$  &  $B \rightarrow A$  might both be false!

# Partial Orders

(review)

- Irreflexive:
  - Never true that  $A \rightarrow A$
- Antisymmetric:
  - If  $A \rightarrow B$  then not true that  $B \rightarrow A$
- Transitive:
  - If  $A \rightarrow B$  &  $B \rightarrow C$  then  $A \rightarrow C$

# Total Orders

(review)

- Also
  - Irreflexive
  - Antisymmetric
  - Transitive
- Except that for every distinct  $A, B$ ,
  - Either  $A \rightarrow B$  or  $B \rightarrow A$

# Repeated Events

```
while (mumble) {  
    a0; a1;  
}
```

*k*-th occurrence of  
event  $a_0$

$a_0^k$

$A_0^k$

*k*-th occurrence of  
interval  $A_0 = (a_0, a_1)$

# Implementing a Counter

```
class Counter {  
  private var count = 0  
  
  def getAndIncrement: Int = {  
    val tmp = count  
    count = tmp + 1  
    tmp  
  }  
}
```

Make these steps  
*indivisible* using locks

# Locks (Mutual Exclusion)

```
trait Lock {  
  
  def lock(): Unit  
  
  def unlock(): Unit  
}
```

# Locks (Mutual Exclusion)

```
trait Lock {
```

```
  def lock() : Unit
```

**acquire lock**

```
  def unlock() : Unit
```

```
}
```

# Locks (Mutual Exclusion)

```
trait Lock {
```

```
  def lock(): Unit
```

**acquire lock**

```
  def unlock(): Unit
```

**release lock**

```
}
```

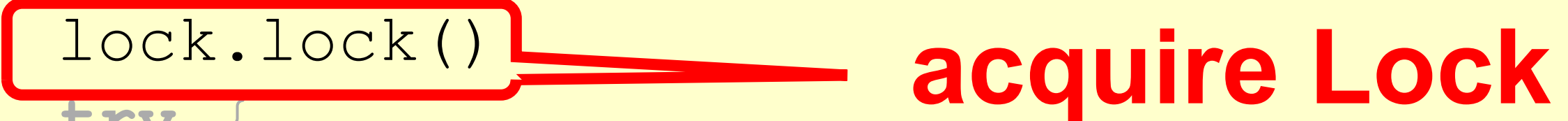


# Using Locks

```
class Counter {  
  private var count = 0  
  private val lock : Lock = ...  
  
  def getAndIncrement: Int = {  
    var tmp = 0  
    lock.lock()  
    try {  
      tmp = count  
      count = tmp + 1  
    } finally {  
      lock.unlock()  
      tmp  
    }  
  }  
}
```

# Using Locks

```
class Counter {  
  private var count = 0  
  private val lock : Lock = ...  
  
  def getAndIncrement: Int = {  
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      count = tmp + 1  
    } finally {  
      lock.unlock()  
      tmp  
    }  
  }  
}
```



**acquire Lock**

# Using Locks

```
class Counter {  
  private var count = 0  
  private val lock : Lock = ...  
  
  def getAndIncrement: Int = {  
    var tmp = 0  
    lock.lock()  
    try {  
      tmp = count  
      count = tmp + 1  
    } finally {  
      lock.unlock()  
      tmp  
    }  
  }  
}
```

Release lock  
(no matter what)

# Using Locks

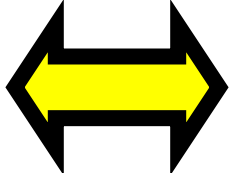
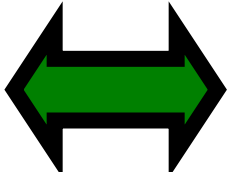
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  private var count = 0  
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  def getAndIncrement: Int = {  
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    lock.lock()  
    try {  
      tmp = count  
      count = tmp + 1  
    } finally {  
      lock.unlock()  
      tmp  
    }  
  }  
}
```

critical section

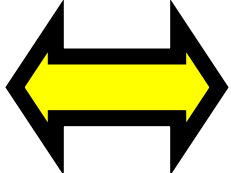
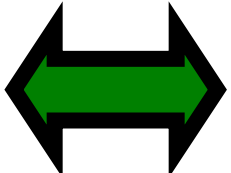
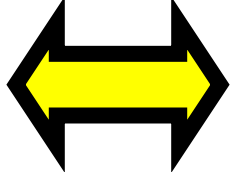
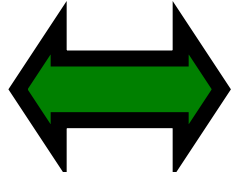
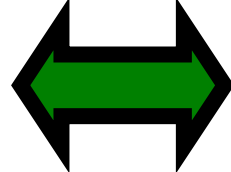
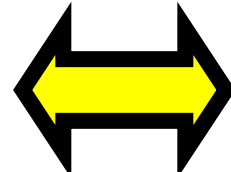
# Mutual Exclusion

- Let  $CS_i^k$   be thread i's k-th critical section execution

# Mutual Exclusion

- Let  $CS_i^k$   be thread i's k-th critical section execution
- And  $CS_j^m$   be thread j's m-th critical section execution

# Mutual Exclusion

- Let  $CS_i^k$   be thread i's k-th critical section execution
- And  $CS_j^m$   be j's m-th execution
- Then either
  -   or  

# Mutual Exclusion

- Let  $CS_i^k \iff$  be thread  $i$ 's  $k$ -th critical section execution
- And  $CS_j^m \iff$  be  $j$ 's  $m$ -th execution
- Then either

–  $\iff \iff$  or  $\iff \iff$

$CS_i^k \rightarrow CS_j^m$



# Mutual Exclusion

- Let  $CS_i^k \iff$  be thread  $i$ 's  $k$ -th critical section execution
- And  $CS_j^m \iff$  be  $j$ 's  $m$ -th execution
- Then either

–  $\iff \iff$  or  $\iff \iff$

$CS_i^k \rightarrow CS_j^m$

$CS_j^m \rightarrow CS_i^k$

# Deadlock-Free



- If some thread calls **lock()**
  - And never returns (fails to acquire the lock)
  - Then other threads must complete **lock()** and **unlock()** calls infinitely often
- System as a whole makes progress
  - Even if individuals starve

# Starvation-Free



- If some thread calls `lock()`
  - It will eventually return
- Individual threads make progress

# Two-Thread vs $n$ -Thread Solutions

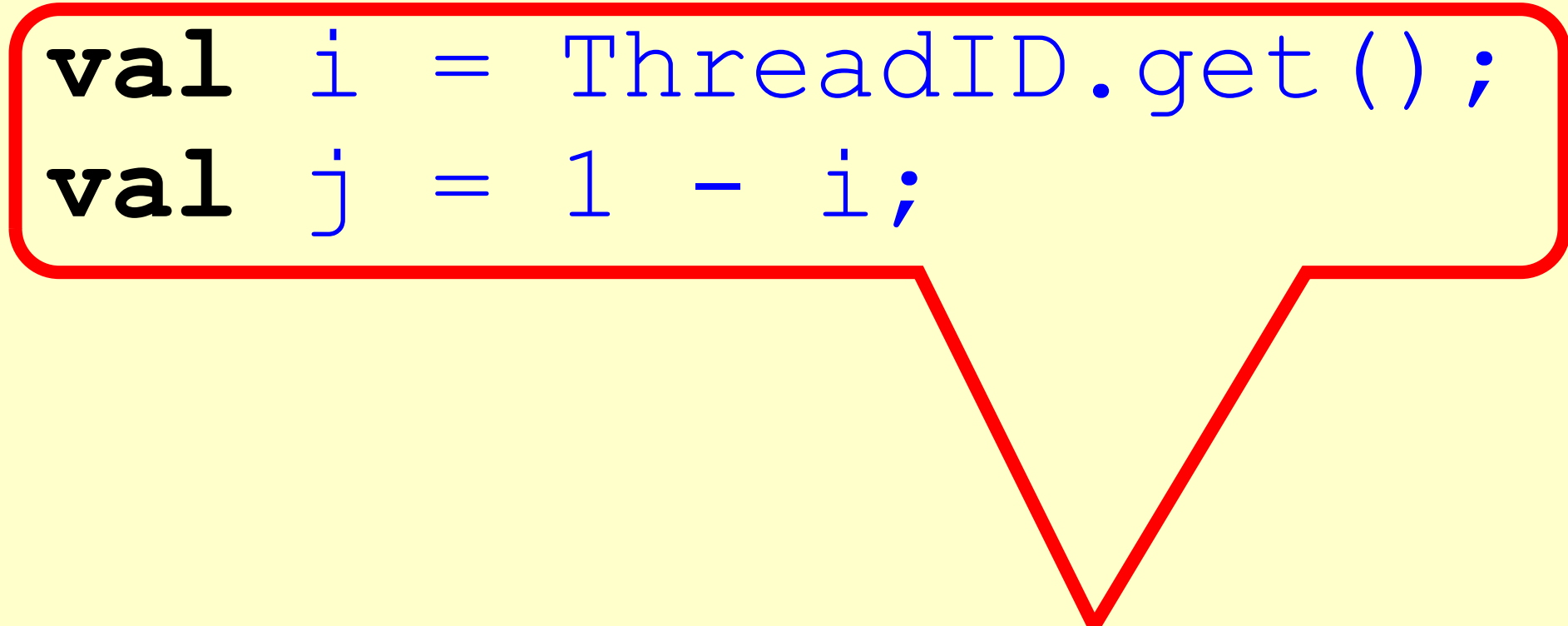
- 2-thread solutions first
  - Illustrate most basic ideas
  - Fits on one slide
- Then  $n$ -thread solutions

# Two-Thread Conventions

```
class ... extends Lock {  
  ...  
  // thread-local index, 0 or 1  
  def lock(): Unit = {  
    val i = ThreadID.get();  
    val j = 1 - i;  
    ...  
  }  
}
```

# Two-Thread Conventions

```
class ... extends Lock {  
  ...  
  // thread-local index, 0 or 1  
  def lock(): Unit = {  
    val i = ThreadID.get();  
    val j = 1 - i;  
    ...  
  }  
}
```



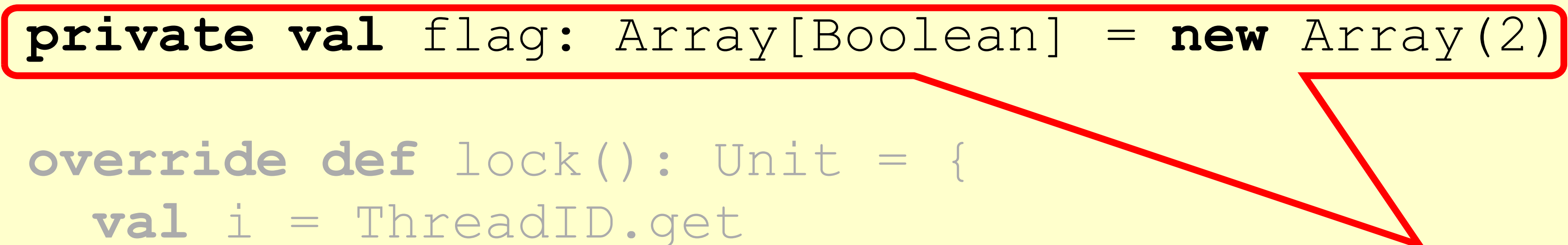
Henceforth: **i** is current  
thread, **j** is other thread

# LockOne

```
class LockOne extends Lock {  
  private val flag: Array[Boolean] = new Array(2)  
  
  override def lock(): Unit = {  
    val i = ThreadID.get  
    val j = 1 - i  
    flag(i) = true  
    while (flag(j)) {} // spin  
  }  
  ...
```

# LockOne

```
class LockOne extends Lock {  
  private val flag: Array[Boolean] = new Array(2)  
  
  override def lock(): Unit = {  
    val i = ThreadID.get  
    val j = 1 - i  
    flag(i) = true  
    while (flag(j)) {} // spin  
  }  
  ...  
}
```



**Each thread has flag**

\* In JVM reality, using an array this way is not quite right, but we'll gloss over it for now...



# LockOne

```
class LockOne extends Lock {  
  private val flag: Array[Boolean] = new Array(2)  
  
  override def lock(): Unit = {  
    val i = ThreadID.get  
    val j = 1 - i  
    flag(i) = true  
    while (flag(j)) {} // spin  
  }  
  ...  
}
```

**Set my flag**

# LockOne

```
class LockOne extends Lock {  
  private val flag: Array[Boolean] = new Array(2)  
  
  override def lock(): Unit = {  
    val i = ThreadID.get  
    val j = 1 - i  
    flag(i) = true  
    while (flag(j)) {} // spin  
  }  
  ...  
}
```

**Wait for other flag to  
become false**

# LockOne Satisfies Mutual Exclusion

- Assume  $CS_A^j$  overlaps  $CS_B^k$
- Consider each thread's last
  - $(j^{th}$  and  $k^{th})$  read and write ...
  - in **lock** () before entering
- Derive a contradiction

# From the Code

- **write<sub>A</sub>(flag[A]=true) →  
read<sub>A</sub>(flag[B]==false) → CS<sub>A</sub>**
- **write<sub>B</sub>(flag[B]=true) →  
read<sub>B</sub>(flag[A]==false) → CS<sub>B</sub>**

```
override def lock(): Unit = {  
  ...  
  flag(i) = true  
  while (flag(j)) {} // spin  
}
```

# From the Assumption

Since A is in the CS it did not see B's flag and vice versa.

- **read<sub>A</sub>(flag[B]==false) → write<sub>B</sub>(flag[B]=true)**
- **read<sub>B</sub>(flag[A]==false) → write<sub>A</sub>(flag[A]=true)**

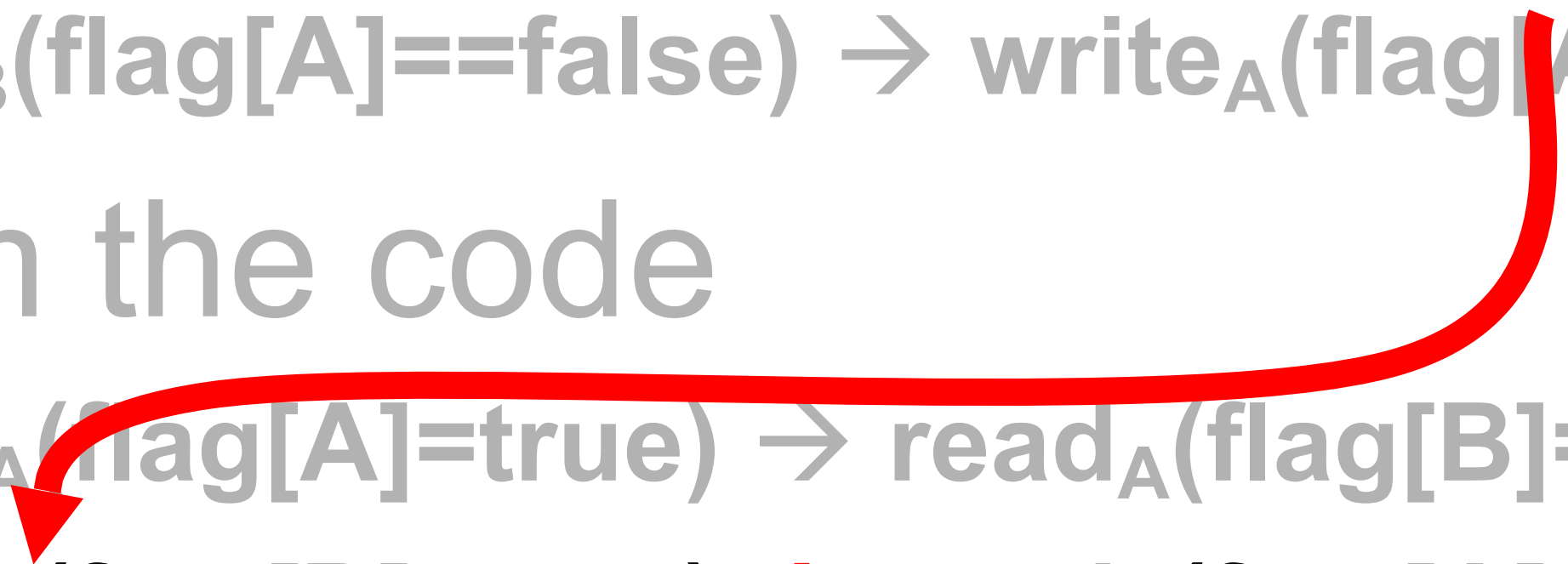
```
override def lock(): Unit = {  
  ...  
  flag(i) = true  
  while (flag(j)) {} // spin  
}
```

# Combining

- Assumptions:
  - $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$
  - $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$
- From the code
  - $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$
  - $\text{write}_B(\text{flag}[B] = \text{true}) \rightarrow \text{read}_B(\text{flag}[A] == \text{false})$

```
override def lock(): Unit = {  
  ...  
  flag(i) = true  
  while (flag(j)) {} // spin  
}
```

# Combining

- Assumptions:
    - **read<sub>A</sub>(flag[B]==false) → write<sub>B</sub>(flag[B]=true)**
    - read<sub>B</sub>(flag[A]==false) → write<sub>A</sub>(flag[A]=true)
  - From the code
    - write<sub>A</sub>(flag[A]=true) → read<sub>A</sub>(flag[B]==false)
    - **write<sub>B</sub>(flag[B]=true) → read<sub>B</sub>(flag[A]==false)**
- 

```
override def lock(): Unit = {  
  ...  
  flag(i) = true  
  while (flag(j)) {} // spin  
}
```

# Combining

- Assumptions:

- $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$

- $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$

- From the code

- $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$

- $\text{write}_B(\text{flag}[B] = \text{true}) \rightarrow \text{read}_B(\text{flag}[A] == \text{false})$

```
override def lock(): Unit = {  
  ...  
  flag(i) = true  
  while (flag(j)) {} // spin  
}
```



# Combining

- Assumptions:

- $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$

- $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$

- From the code

- $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$

- $\text{write}_B(\text{flag}[B] = \text{true}) \rightarrow \text{read}_B(\text{flag}[A] == \text{false})$

```
override def lock(): Unit = {  
  ...  
  flag(i) = true  
  while (flag(j)) {} // spin  
}
```

# Combining

- Assumptions:

- $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$

- $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$

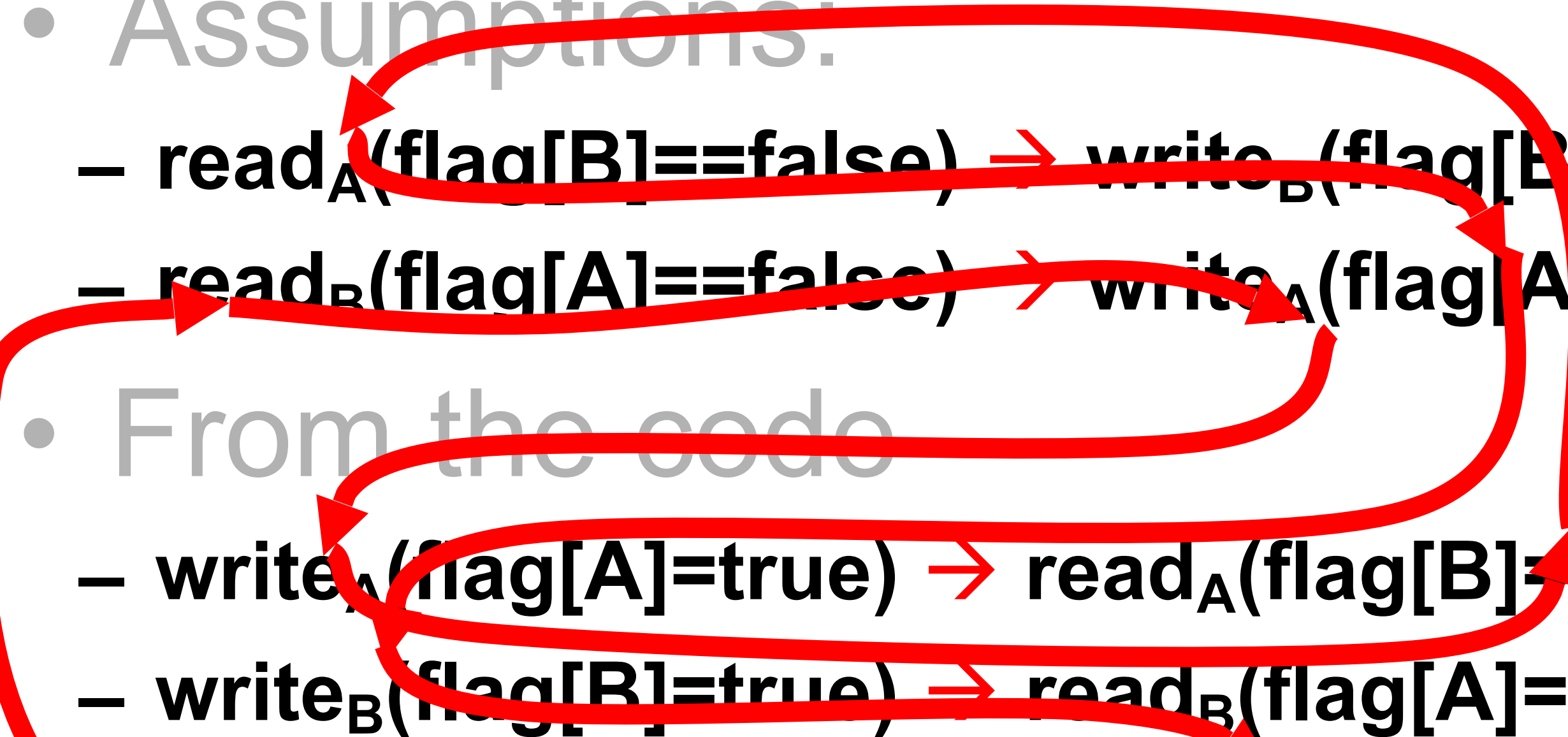
- From the code

- $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$

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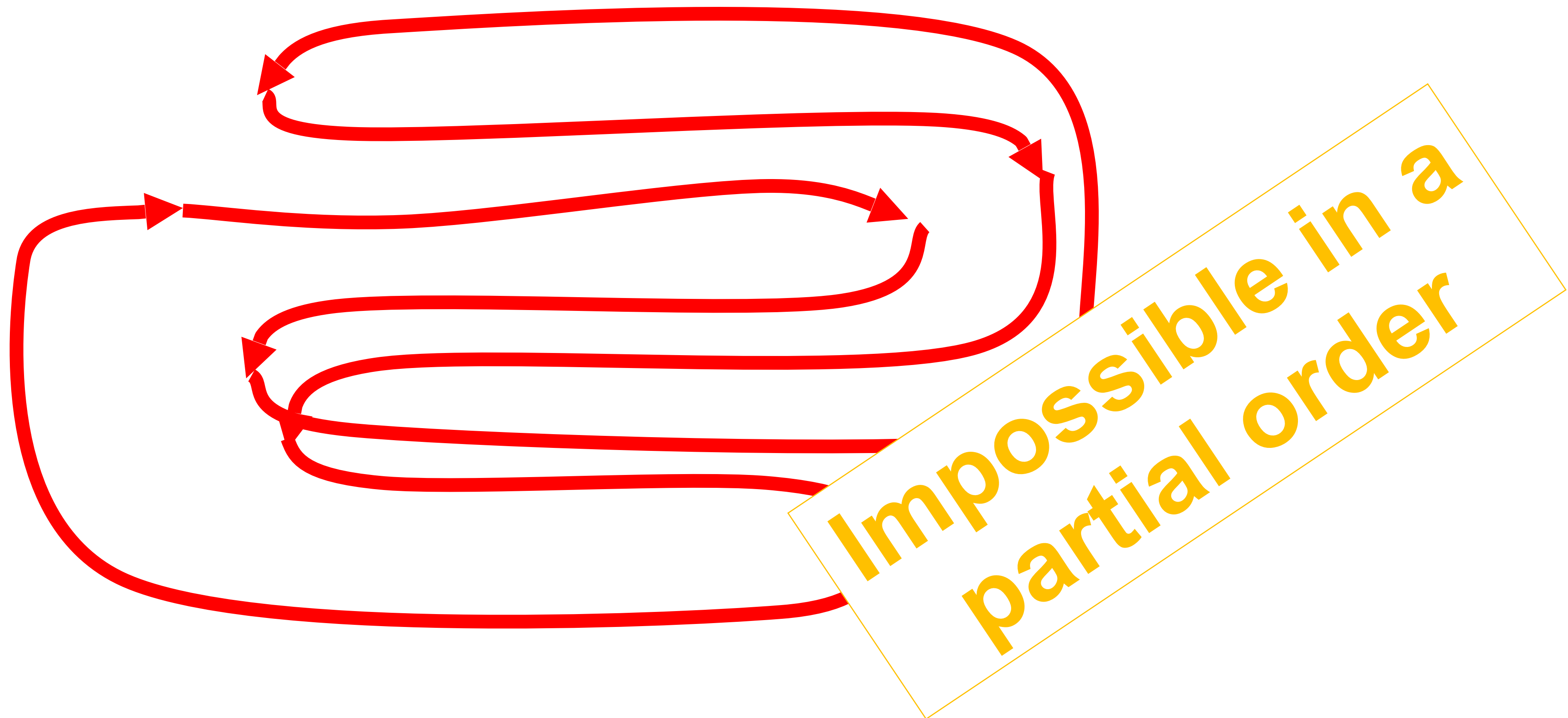
```
override def lock(): Unit = {  
  ...  
  flag(i) = true  
  while (flag(j)) {} // spin  
}
```

# Combining

- Assumptions:
    - $\text{read}_A(\text{flag}[B] == \text{false}) \Rightarrow \text{write}_B(\text{flag}[B] = \text{true})$
    - $\text{read}_B(\text{flag}[A] == \text{false}) \Rightarrow \text{write}_A(\text{flag}[A] = \text{true})$
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    - $\text{write}_B(\text{flag}[B] = \text{true}) \Rightarrow \text{read}_B(\text{flag}[A] == \text{false})$
- 

```
override def lock(): Unit = {  
  ...  
  flag(i) = true  
  while (flag(j)) {} // spin  
}
```

# Cycle!



# Demo: Testing Locks

Is LockOne Deadlock-free?

# Deadlock Freedom

- LockOne Fails deadlock-freedom
  - Concurrent execution can deadlock

```
flag(i) = true    flag(j) = true  
while (flag(j)) {} while (flag(i)) {}
```

- Sequential executions OK

# LockTwo

```
class LockTwo extends Lock {  
  private var victim: Int = 0  
  
  override def lock(): Unit = {  
    val i = ThreadID.get  
    victim = i  
    while (victim == i) {}  
  }  
  ...  
}
```



# LockTwo

```
class LockTwo extends Lock {  
  private var victim: Int = 0  
  
  override def lock(): Unit = {  
    val i = ThreadID.get  
    victim = i  
    while (victim == i) {}  
  }  
  ...  
}
```

**Let other go first**

# LockTwo

```
class LockTwo extends Lock {  
  private var victim: Int = 0  
  
  override def lock(): Unit = {  
    val i = ThreadID.get  
    victim = i  
    while (victim == i) {}  
  }  
}
```

...

**Wait for  
permission**



# LockTwo

```
class LockTwo extends Lock {  
  private var victim: Int = 0  
  
  override def lock(): Unit = {  
    val i = ThreadID.get  
    victim = i  
    while (victim == i) {}  
  }
```

```
  override def unlock(): Unit = {}  
  ...
```

Nothing to do

# LockTwo Claims

- Satisfies mutual exclusion
  - If thread **i** in CS
  - Then **victim == j**
  - Cannot be both 0 and 1
- Not deadlock free
  - Sequential execution deadlocks
  - Concurrent execution does not

```
def lock() {  
    victim = i; // my ThreadID  
    while (victim == i) {};  
}
```

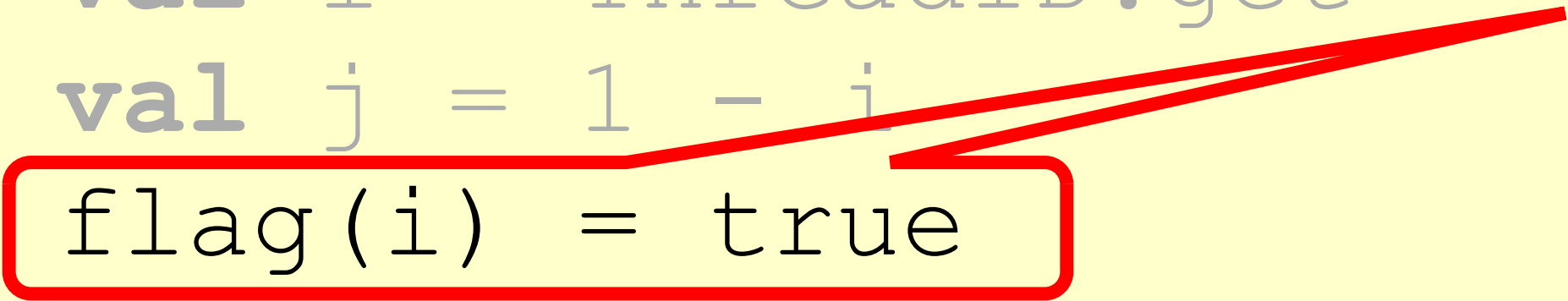
# Peterson's Algorithm

```
def lock(): Unit = {  
    val i = ThreadID.get  
    val j = 1 - i  
    flag(i) = true  
    victim = i  
    while (flag(j) && victim == i) {}  
}  
  
def unlock(): Unit = {  
    val i = ThreadID.get  
    flag(i) = false  
}
```

# Peterson's Algorithm

```
def lock(): Unit = {  
    val i = ThreadID.get  
    val j = 1 - i  
    flag(i) = true  
    victim = i  
    while (flag(j) && victim == i) {}  
}  
  
def unlock(): Unit = {  
    val i = ThreadID.get  
    flag(i) = false  
}
```

**Announce I'm interested**



# Peterson's Algorithm

```
def lock(): Unit = {  
    val i = ThreadID.get  
    val j = 1 - i  
    flag(i) = true  
    victim = i  
    while (flag(j) && victim == i) {}  
}  
  
def unlock(): Unit = {  
    val i = ThreadID.get  
    flag(i) = false  
}
```

**Announce I'm interested**

**Defer to other**

# Peterson's Algorithm

```
def lock(): Unit = {  
  val i = ThreadID.get  
  val j = 1 - i
```

**Announce I'm  
interested**

```
    flag(i) = true
```

```
    victim = i
```

**Defer to other**

```
    while (flag(j) && victim == i) {}  
}
```

**Wait while other  
interested & I'm the  
victim**

```
def unlock(): Unit = {  
  val i = ThreadID.get  
  flag(i) = false  
}
```



# Peterson's Algorithm

```
def lock(): Unit = {  
  val i = ThreadID.get  
  val j = 1 - i
```

**Announce I'm  
interested**

```
    flag(i) = true
```

```
    victim = i
```

**Defer to other**

```
    while (flag(j) && victim == i) {}
```

```
def unlock(): Unit = {
```

```
  val i = ThreadID.get
```

```
  flag(i) = false
```

```
}
```

**Wait while other  
interested & I'm the  
victim**

**No longer  
interested**

# Mutual Exclusion

(1)  $\text{write}_B(\text{Flag}[B]=\text{true}) \rightarrow \text{write}_B(\text{victim}=B)$

```
def lock() {  
    flag(i) = true  
    victim = i  
    while (flag(j) && victim == i) {}  
}
```

From the Code

# Also from the Code

(2)  $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$   
 $\rightarrow \text{read}_A(\text{victim})$

```
def lock() {  
    flag(i) = true  
    victim = i  
    while (flag(j) && victim == i) {}  
}
```

# Assumption

(3)  $\text{write}_B(\text{victim}=B) \rightarrow \text{write}_A(\text{victim}=A)$

W.L.O.G. assume **A** is the last  
thread to write **victim**

```
def lock() {  
    flag(i) = true  
    victim = i  
    while (flag(j) && victim == i) {}  
}
```

# Combining Observations

(1)  $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow \text{write}_B(\text{victim}=B)$

(3)  $\text{write}_B(\text{victim}=B) \rightarrow \text{write}_A(\text{victim}=A)$

(2)  $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$   
 $\rightarrow \text{read}_A(\text{victim})$

```
def lock() {  
    flag(i) = true  
    victim = i  
    while (flag(j) && victim == i) {}  
}
```

# Combining Observations

(1)  $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow$

(3)  $\text{write}_B(\text{victim}=B) \rightarrow$

(2)  $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$   
 $\rightarrow \text{read}_A(\text{victim})$

```
def lock() {  
    flag(i) = true  
    victim = i  
    while (flag(j) && victim == i) {}  
}
```

# Combining Observations

```
def lock() {  
    flag(i) = true  
    victim = i  
    while (flag(j) && victim == i) {}  
}
```

(1) write<sub>B</sub>(flag[B]=true)→

(3) write<sub>B</sub>(victim=B)→

(2) write<sub>A</sub>(victim=A)→read<sub>A</sub>(flag[B])

→ read<sub>A</sub>(victim)

A read flag[B] == true and victim == A, so it  
could not have entered the CS (QED)

# Deadlock Free

```
def lock() {  
    ...  
    while (flag(j) && victim == i) {};
```

- Thread blocked
  - only at **while** loop
  - only if other's flag is true
  - only if it is the victim
- Solo: other's flag is false
- Both: one or the other not the victim



# Starvation Free

- Thread *i* blocked only if *j* repeatedly re-enters so that  
`flag(j) == true and victim == i`
- When *j* re-enters
  - it sets `victim` to *j*.
  - So *i* gets in

```
def lock() {  
    flag(i) = true  
    victim  = i  
    while (flag(j) && victim == i) {};  
}  
  
public void unlock() {  
    flag(i) = false  
}
```

# Demo: Peterson Lock

# Bounded Waiting

- Want *stronger* fairness guarantees
- Thread not “overtaken” too much
- If **A** starts before **B**, then **A** enters before **B**?
- But what does “start” mean?
- Need to adjust definitions .....

# Bounded Waiting

- Divide **lock ()** method into 2 parts:
  - Doorway interval:
    - Written  **$D_A$**
    - always finishes in finite steps
  - Waiting interval:
    - Written  **$W_A$**
    - may take unbounded steps

# First-Come-First-Served

- For threads A and B:
  - If  $D_A^k \rightarrow D_B^j$ 
    - A's k-th doorway precedes B's j-th doorway
  - Then  $CS_A^k \rightarrow CS_B^j$ 
    - A's k-th critical section precedes B's j-th critical section
    - B cannot overtake A

# Bakery Algorithm

- Provides First-Come-First-Served for  $n$  threads
- How?
  - Take a “number”
  - Wait until lower numbers have been served
- Lexicographic order
  - $(a,i) > (b,j)$ 
    - If  $a > b$ , or  $a = b$  and  $i > j$

# Bakery Algorithm

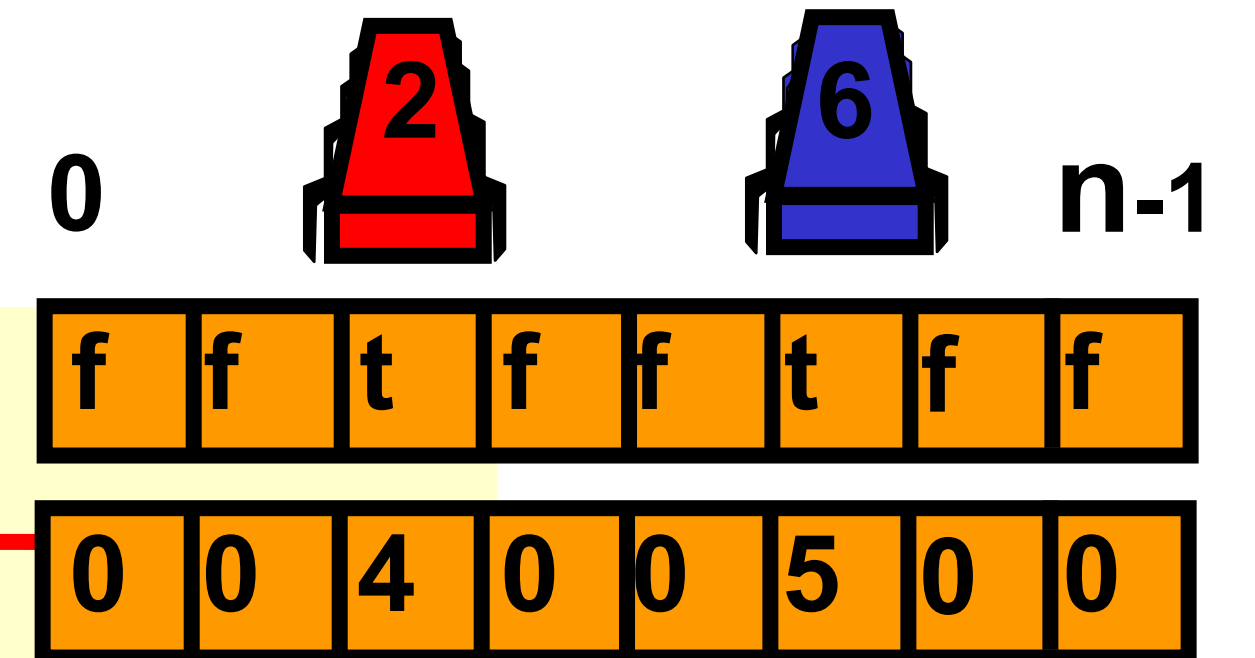
```
class BakeryLock(val threads: Int) extends Lock {  
  
    private val label: Array[Label] =  
        Array.fill(threads)(new Label())  
  
    private val flag: Array[Boolean] = new Array(threads)
```

# Bakery Algorithm

```
class BakeryLock(val threads: Int) extends Lock {
```

```
  private val label: Array[Label] =  
    Array.fill(threads)(new Label())
```

```
  private val flag: Array[Boolean] = new Array(threads)
```



CS



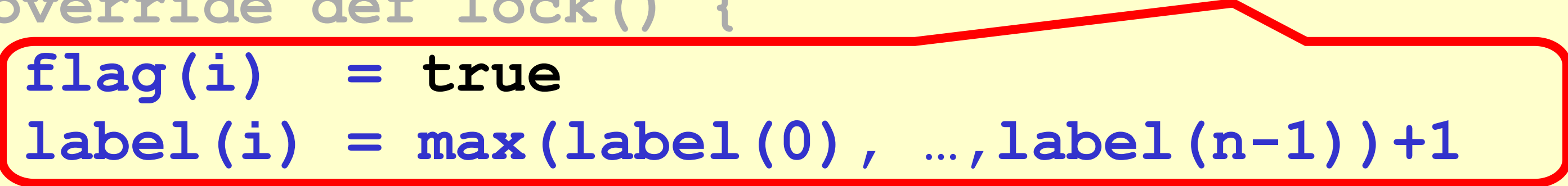
# Bakery Algorithm

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while ( $\exists k$  flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

# Bakery Algorithm

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while ( $\exists k$  flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

**Doorway**



# Bakery Algorithm

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while ( $\exists k$  flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

**I'm interested**

# Bakery Algorithm

**Take increasing label  
(read labels in some  
arbitrary order)**

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while ( $\exists k$  flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

# Bakery Algorithm

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while ( $\exists k$  flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

**Someone is  
interested**

# Bakery Algorithm

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while ( $\exists k$  flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

**Someone is  
interested**

**... whose (label,i) in  
lexicographic order is lower**

# Bakery Algorithm

```
class BakeryLock extends Lock {  
    ...  
  
    override def unlock() {  
        flag(i) = false;  
    }  
}
```

# Bakery Algorithm

```
class BakeryLock extends Lock {  
    ...  
  
    override def unlock() {  
        flag(i) = false;  
    }  
}
```

**No longer interested**

**labels are always increasing**



# No Deadlock

- There is always one thread with earliest label
- Ties are impossible (why?)

# First-Come-First-Served

- If  $D_A \rightarrow D_B$  then
  - A's label is smaller
- And:
  - $\text{write}_A(\text{label}[A]) \rightarrow$
  - $\text{read}_B(\text{label}[A]) \rightarrow$
  - $\text{write}_B(\text{label}[B]) \rightarrow \text{read}_B(\text{flag}[A])$
- So B sees
  - smaller label for A
  - locked out while  $\text{flag}[A]$  is true

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while ( $\exists k$  flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

# Mutual Exclusion

- Suppose A and B in CS together
- Suppose A has earlier label
- When B entered, it must have seen
  - flag[A] is *false*, or
  - label[A] > label[B]

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while ( $\exists k$  flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

# Mutual Exclusion

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while ( $\exists k$  flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

- Labels are strictly increasing so
- B must have seen flag[A] == false

# Mutual Exclusion

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while (∃k flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

- Labels are strictly increasing so
- B must have seen  $\text{flag}[A] == \text{false}$
- $\text{Labeling}_B \rightarrow \text{read}_B(\text{flag}[A]) \rightarrow$   
 $\text{write}_A(\text{flag}[A]) \rightarrow \text{Labeling}_A$

# Mutual Exclusion

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while (∃k flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

- Labels are strictly increasing so
- B must have seen  $\text{flag}[A] == \text{false}$
- $\text{Labeling}_B \rightarrow \text{read}_B(\text{flag}[A]) \rightarrow \text{write}_A(\text{flag}[A]) \rightarrow \text{Labeling}_A$
- Which contradicts the assumption that A has an earlier label

Any issues with BackeryLock?

# Bakery Y2<sup>32</sup>K Bug

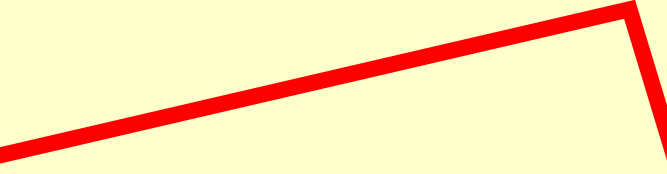
```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
  
        while (∃k flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```



# Bakery Y2<sup>32</sup>K Bug

```
class BakeryLock extends Lock {  
    ...  
    override def lock() {  
        flag(i) = true  
        label(i) = max(label(0), ..., label(n-1)) + 1  
        while (∃k flag(k)  
                && (label(i), i) > (label(k), k)) {}  
    }  
}
```

**Mutex breaks if  
label[i] overflows**



# Does Overflow Actually Matter?

- Yes
  - Y2K
  - 18 January 2038 (Unix `time_t` rollover)
  - 16-bit counters
- No
  - 64-bit counters
- Maybe
  - 32-bit counters

# Deep Philosophical Question

- The Bakery Algorithm is
  - Succinct,
  - Elegant, and
  - Fair.
- Q: So why isn't it practical?
- A: Well, you have to read **N** distinct variables

# Shared Memory

- Shared read/write memory locations called *Registers* (historical reasons)
- Come in different flavors
  - Multi-Reader-Single-Writer (**flag**)
  - Multi-Reader-Multi-Writer (**victim**)
  - Not that interesting: SRMW and SRSW

# Theorem

At least **N** MRSW (multi-reader/single-writer) registers are needed to solve deadlock-free mutual exclusion.

**N** registers such as **flag()** ...

# Real-Life Implementations

- Demo

# Summary of the last two lectures

- We have seen several impractical examples of implementing *mutual exclusion* algorithms.
- We learned how to reason about their claims: *mutual exclusion, deadlock freedom, etc.*
- Today we know how to solve the *First-Come-First-Served*  $N$  thread mutual exclusion using  $2N$  RW-Registers

# Summary of the last two lectures

- N RW-Registers inefficient
  - Because writes “**cover**” older writes
- Need stronger hardware operations
  - that do not have the “**covering problem**”
- In next lectures - understand what these operations are...



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