

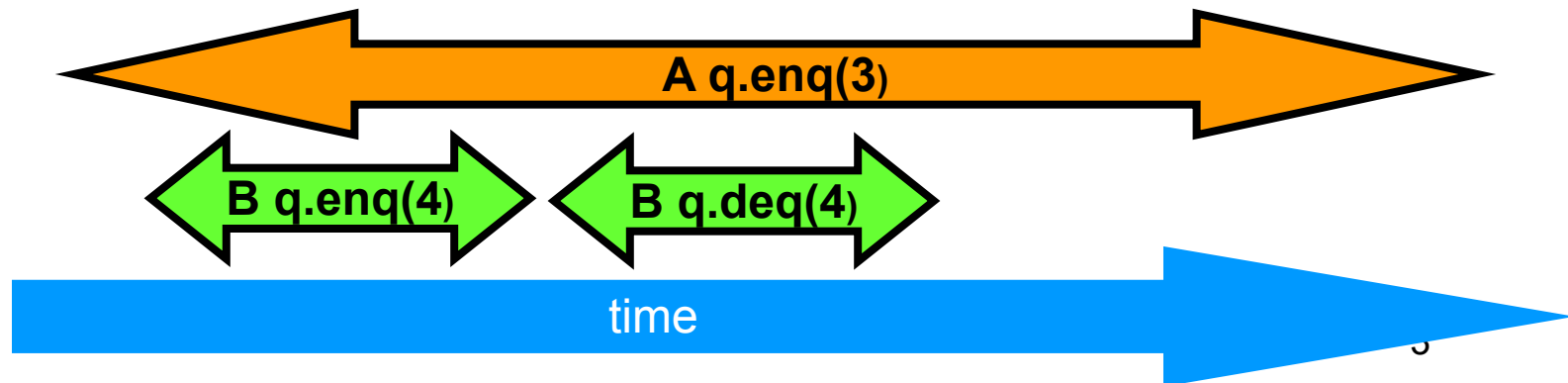
YSC3248: Parallel, Concurrent and Distributed Programming

Wait-Free Implementations

Last Week: Linearizability

Example

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void



Example

A q.enq(3)

B q.enq(4)

B q:void

B q.deq()

B q:4

A q:void

B q.enq(4)

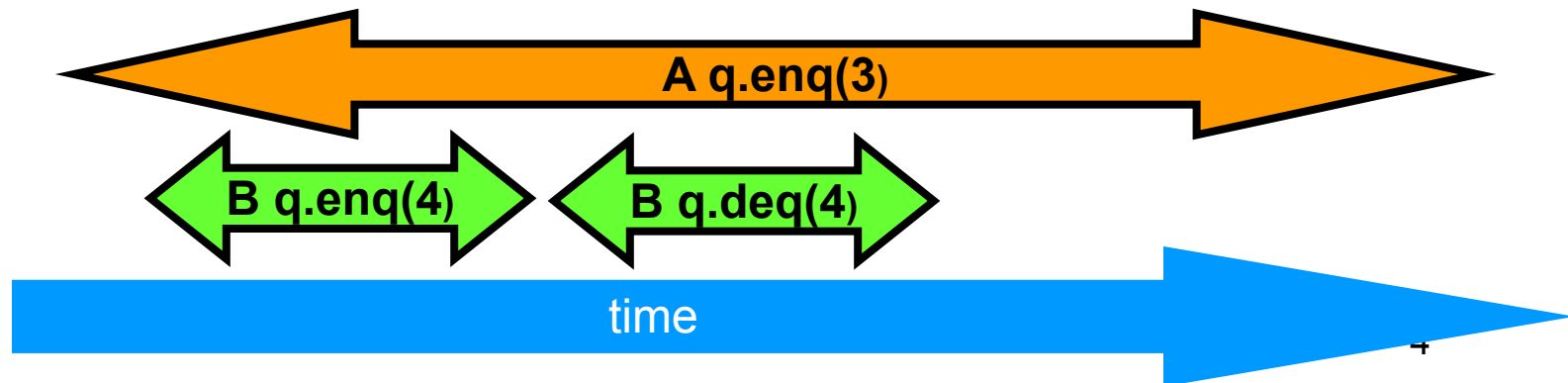
B q:void

A q.enq(3)

A q:void

B q.deq()

B q:4

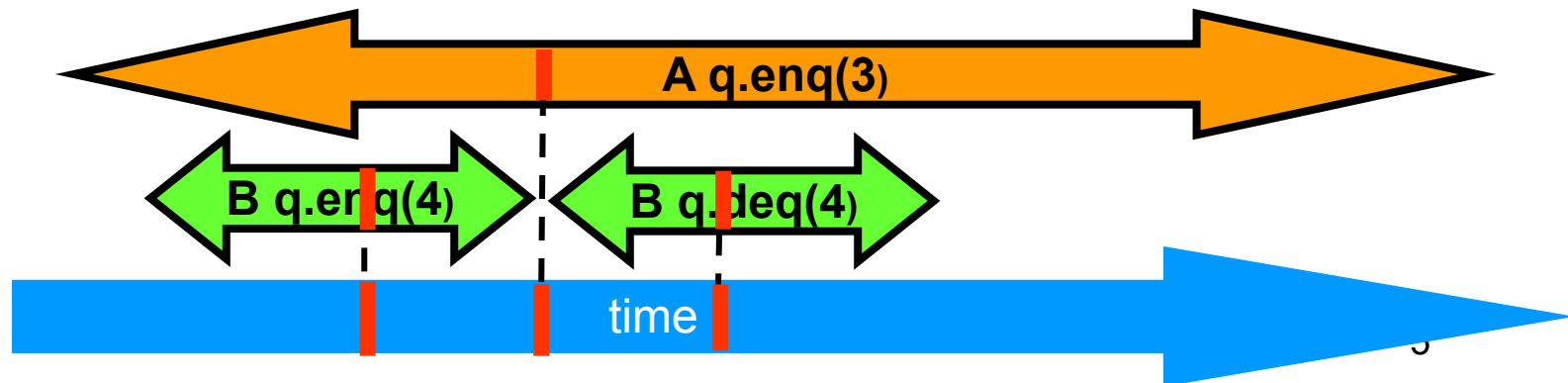


Example

Equivalent sequential history

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void

B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4



Linearizability

- History H is *linearizable* if it can be extended to G by
 - Appending zero or more responses to pending invocations
 - Discarding other pending invocations
- So that G is equivalent to
 - Legal sequential history S
 - where $\rightarrow_G \subset \rightarrow_S$

Alternative: Sequential Consistency

- History H is **Sequentially Consistent** if it can be extended to G by
 - Appending zero or more responses to pending invocations
 - Discarding other pending invocations
- So that G is equivalent to a
 - Legal sequential history S
 - ~~– Where $\rightarrow G \subseteq \rightarrow S$~~

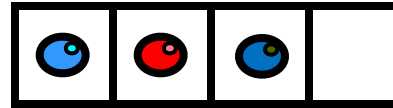
Differs from
linearizability



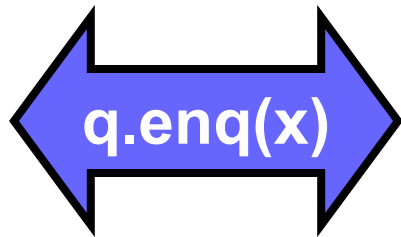
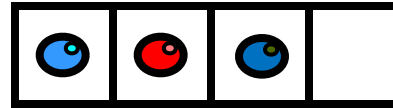
Sequential Consistency

- No need to preserve real-time order
 - Cannot re-order operations done by the same thread
 - Can re-order non-overlapping operations done by different threads
- Often used to describe multiprocessor memory architectures

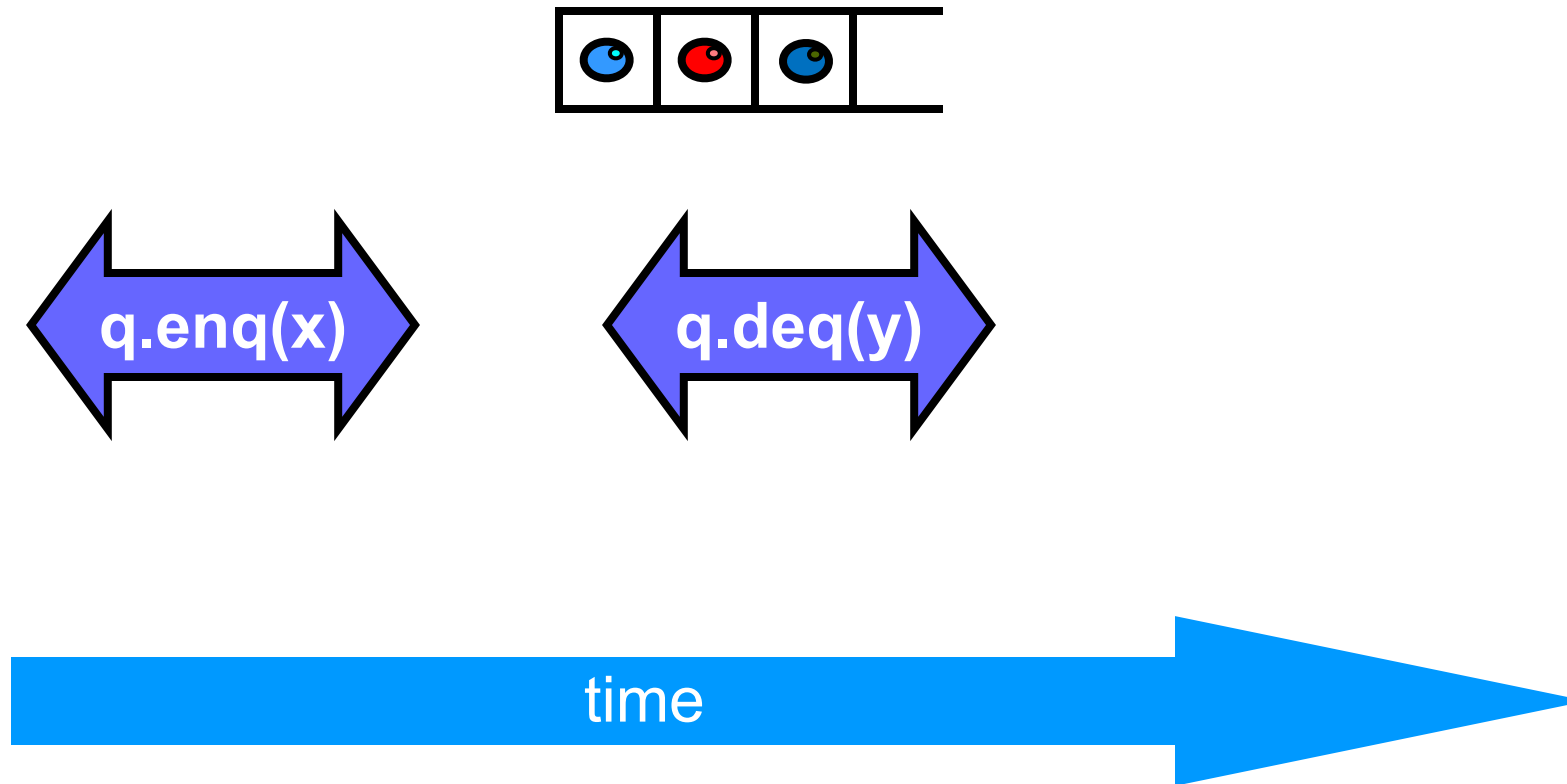
Example



Example

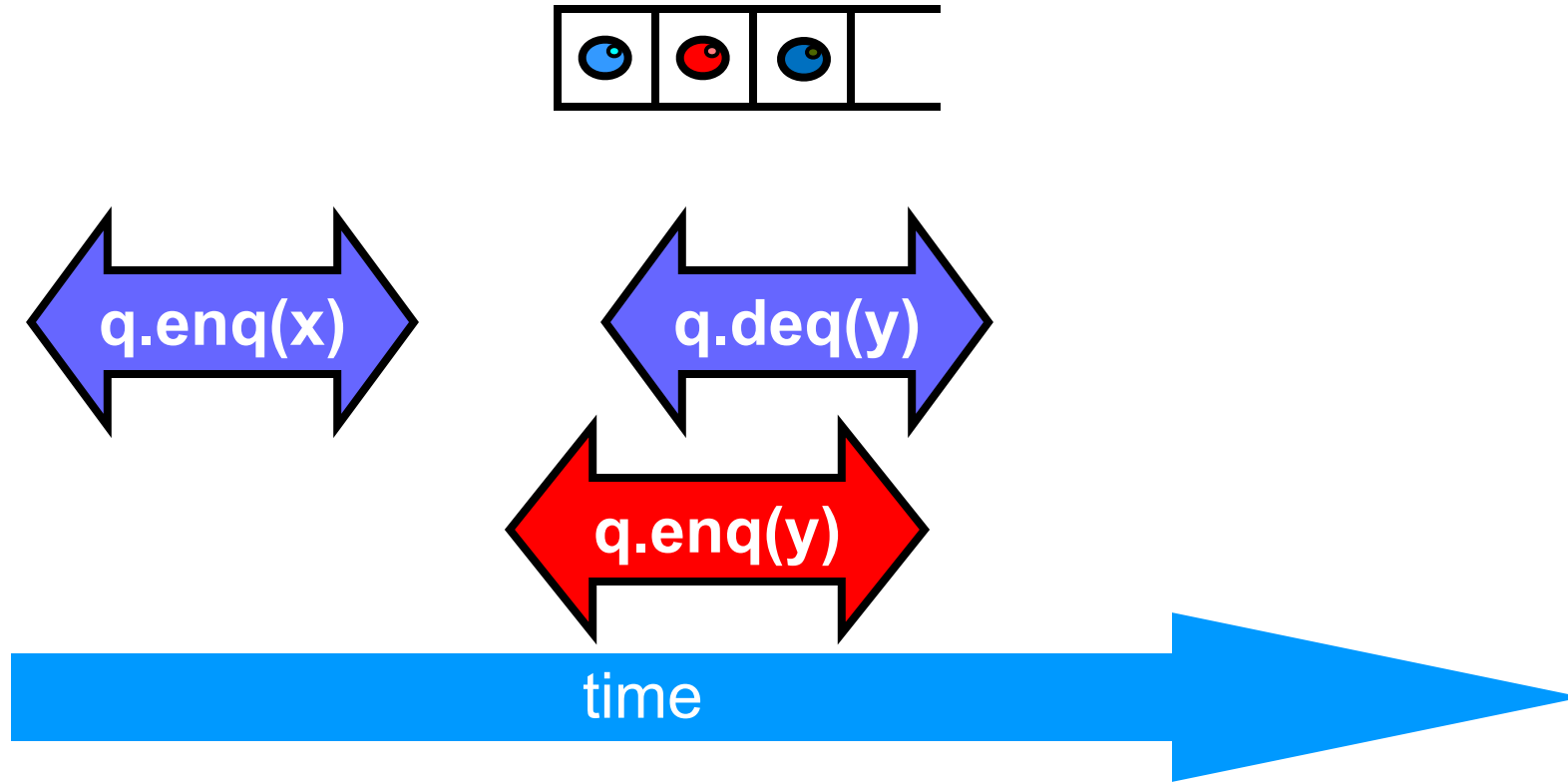


Example



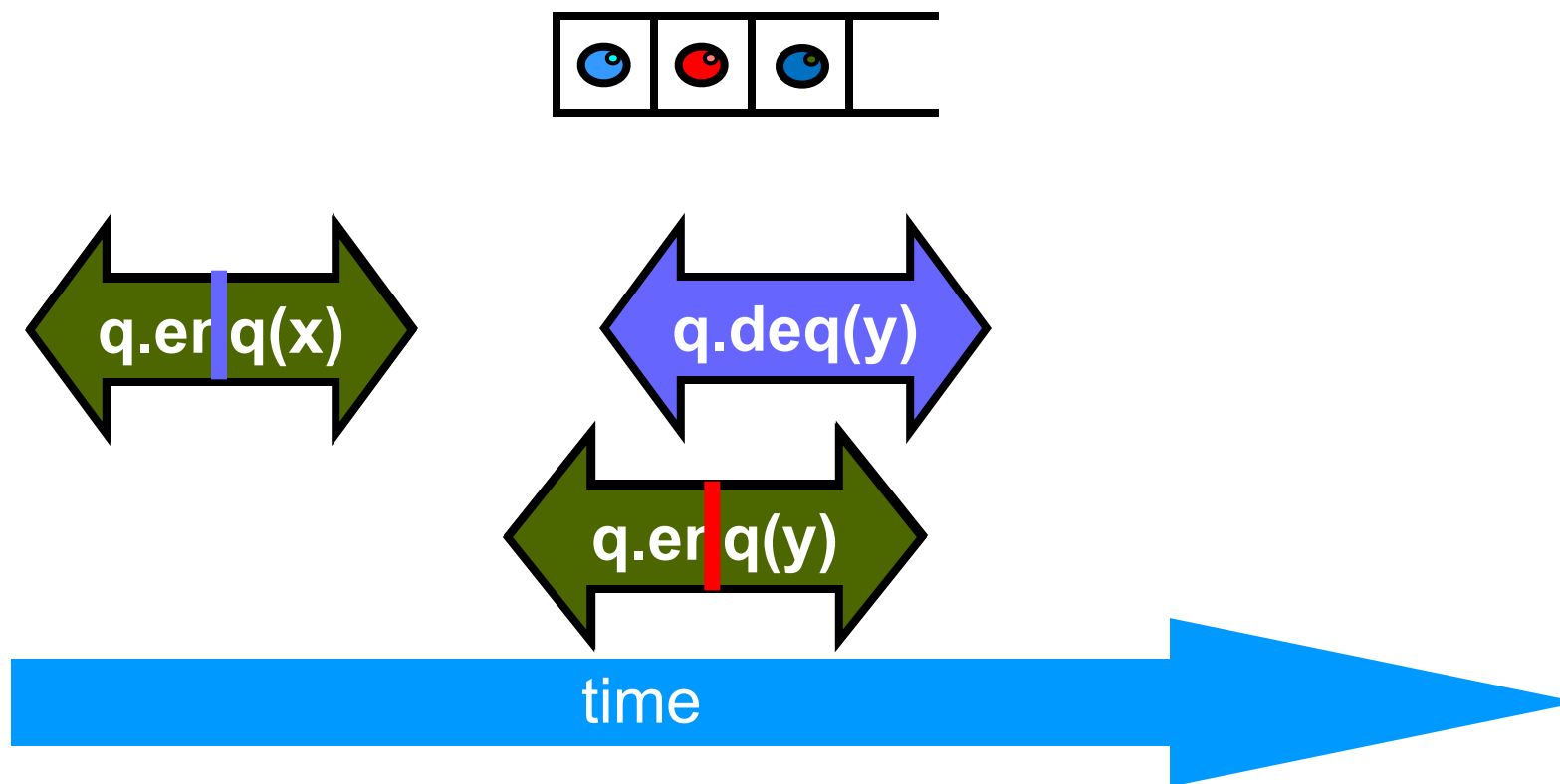


Example



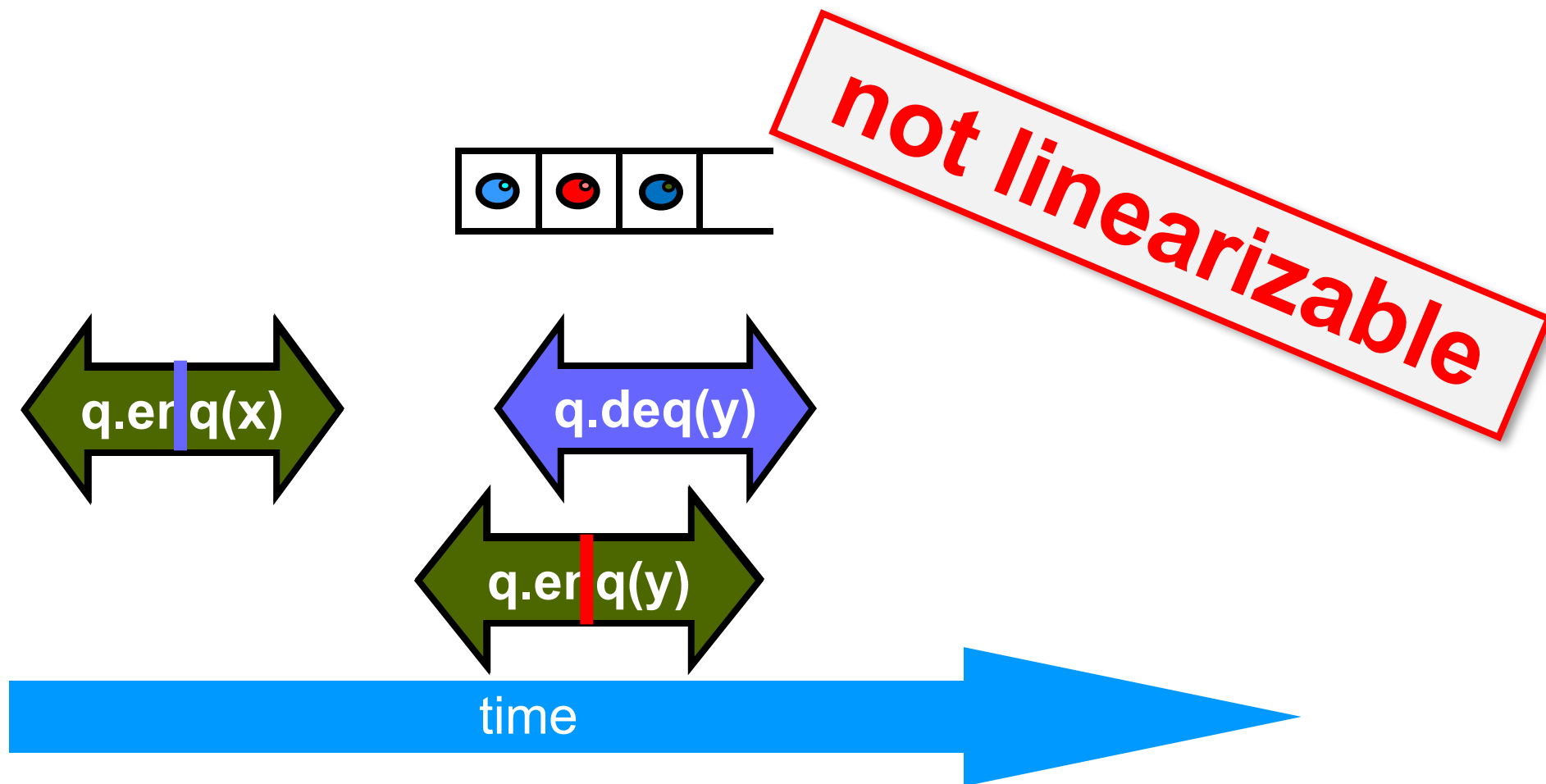


Example





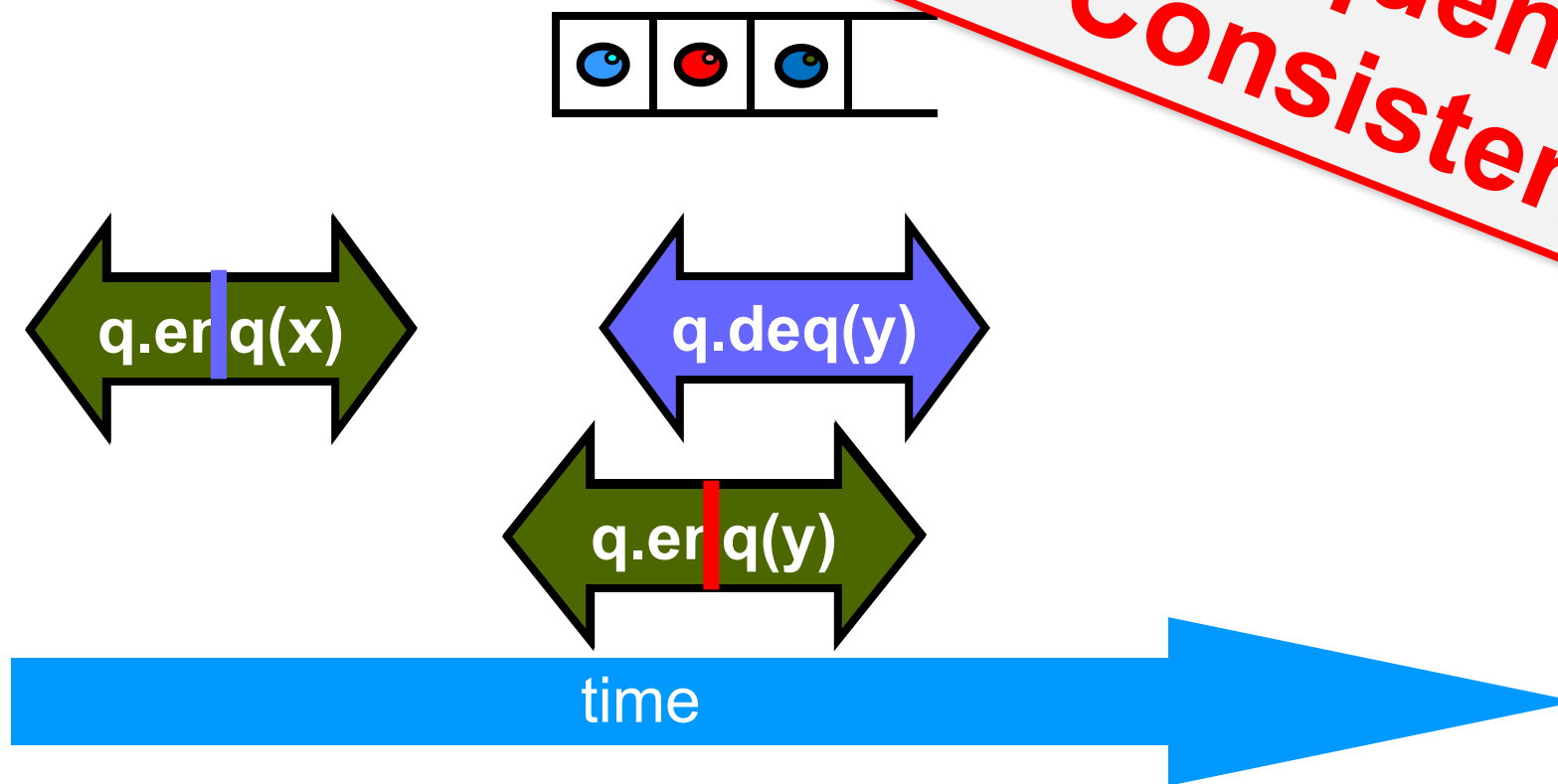
Example





Example

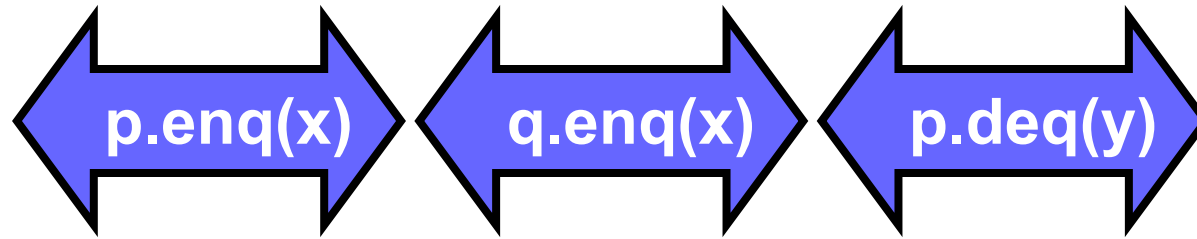
Yet Sequentially Consistent



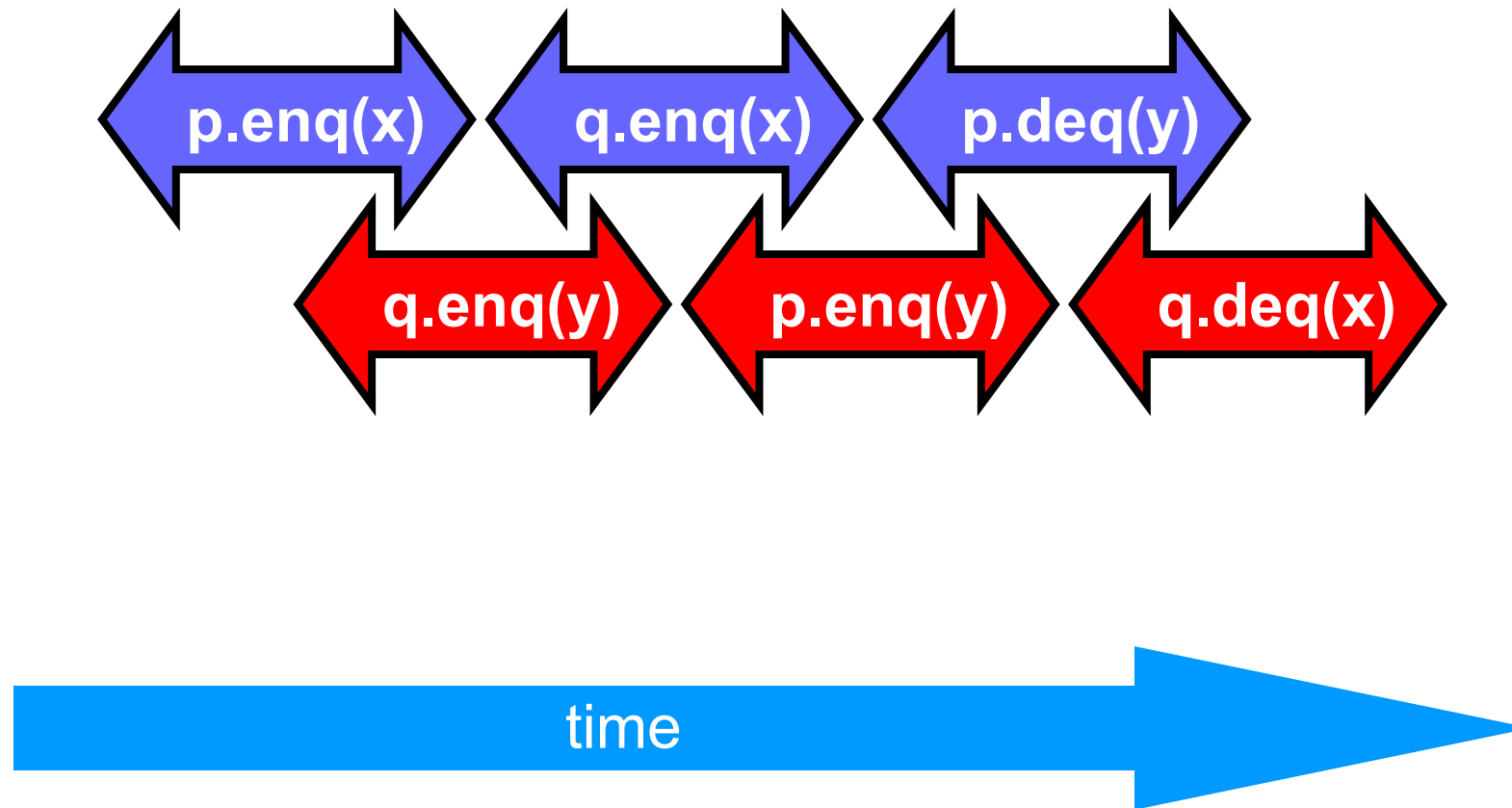
Theorem

Sequential Consistency is not composable

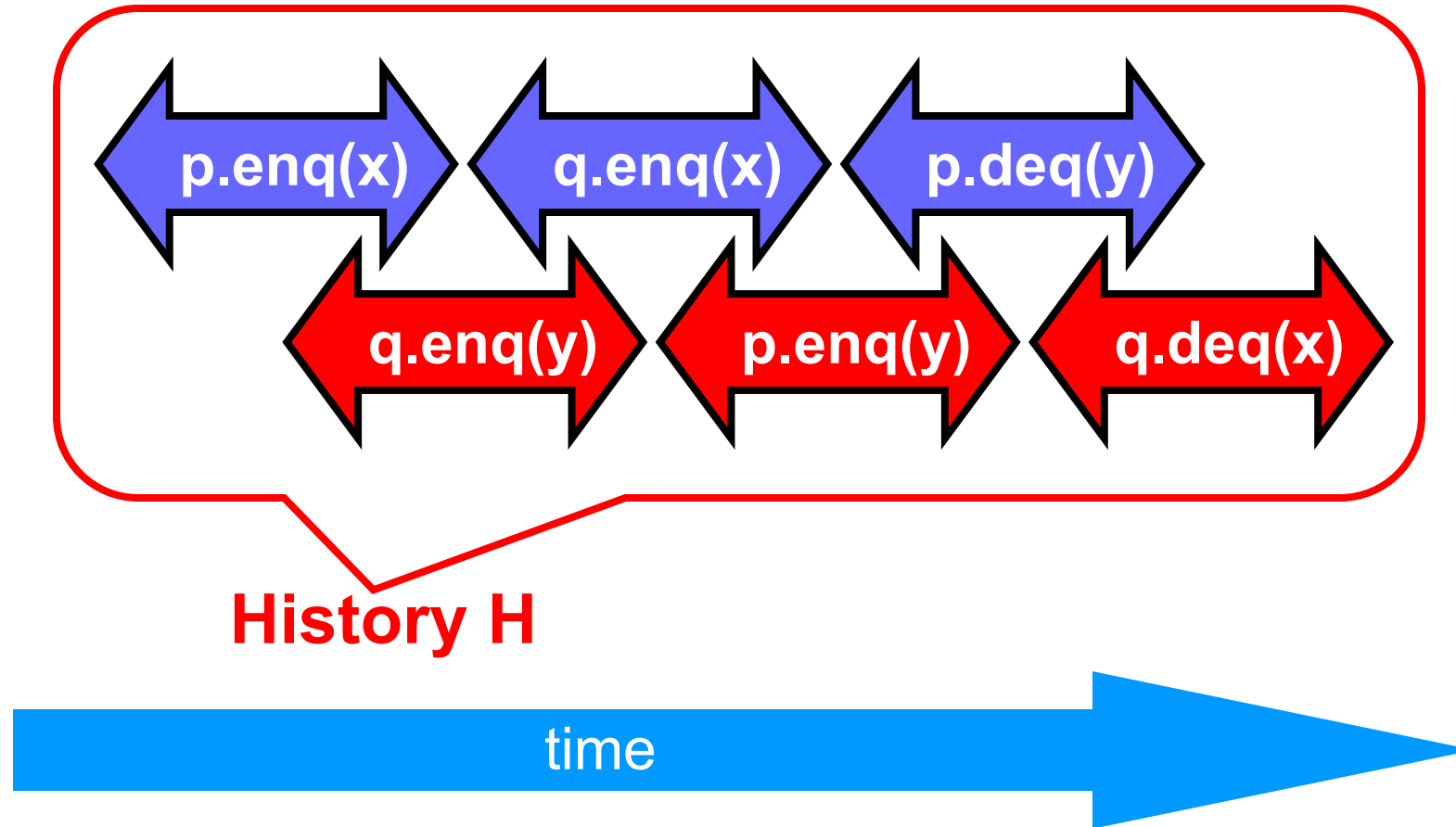
FIFO Queue Example



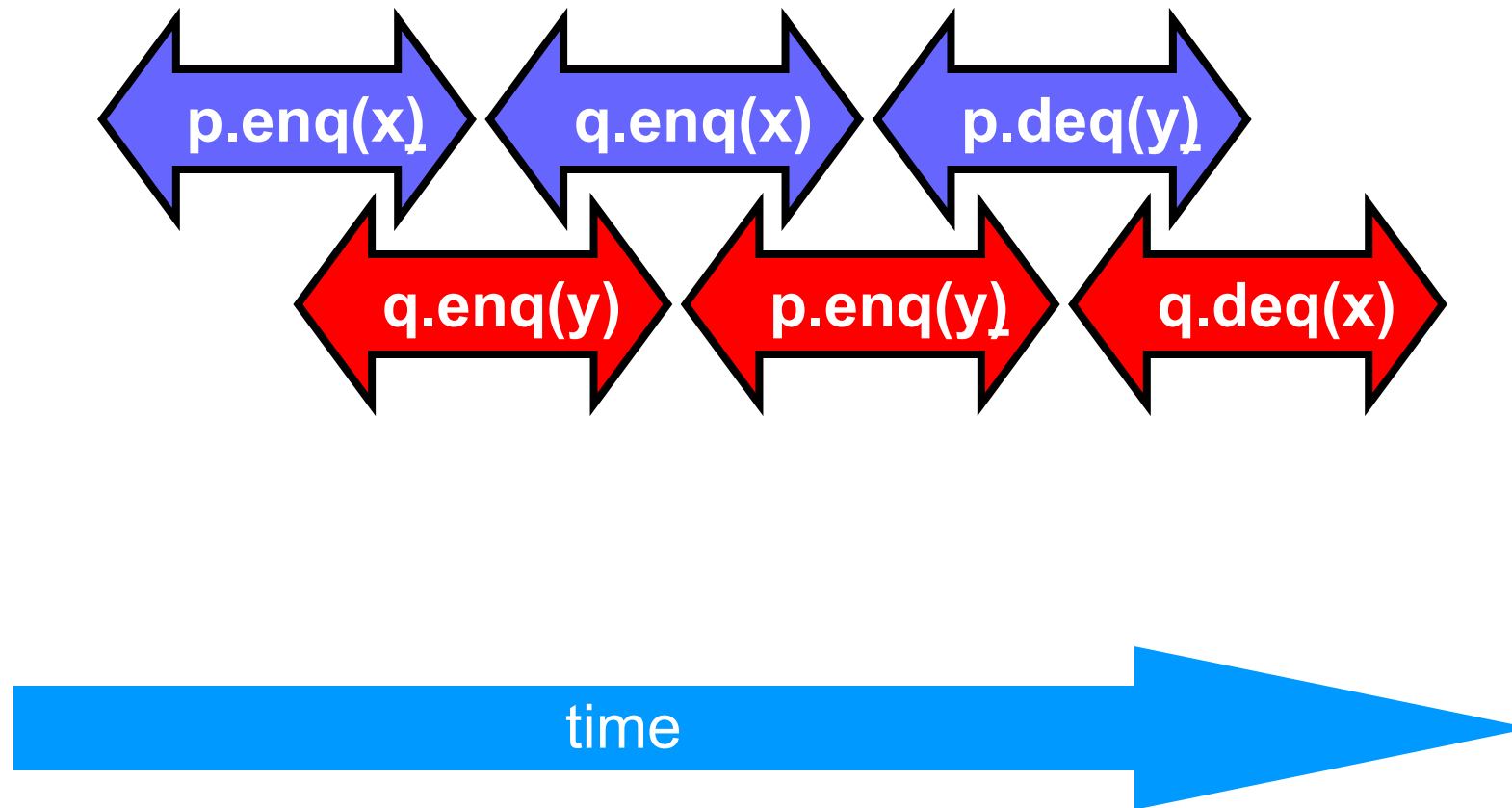
FIFO Queue Example



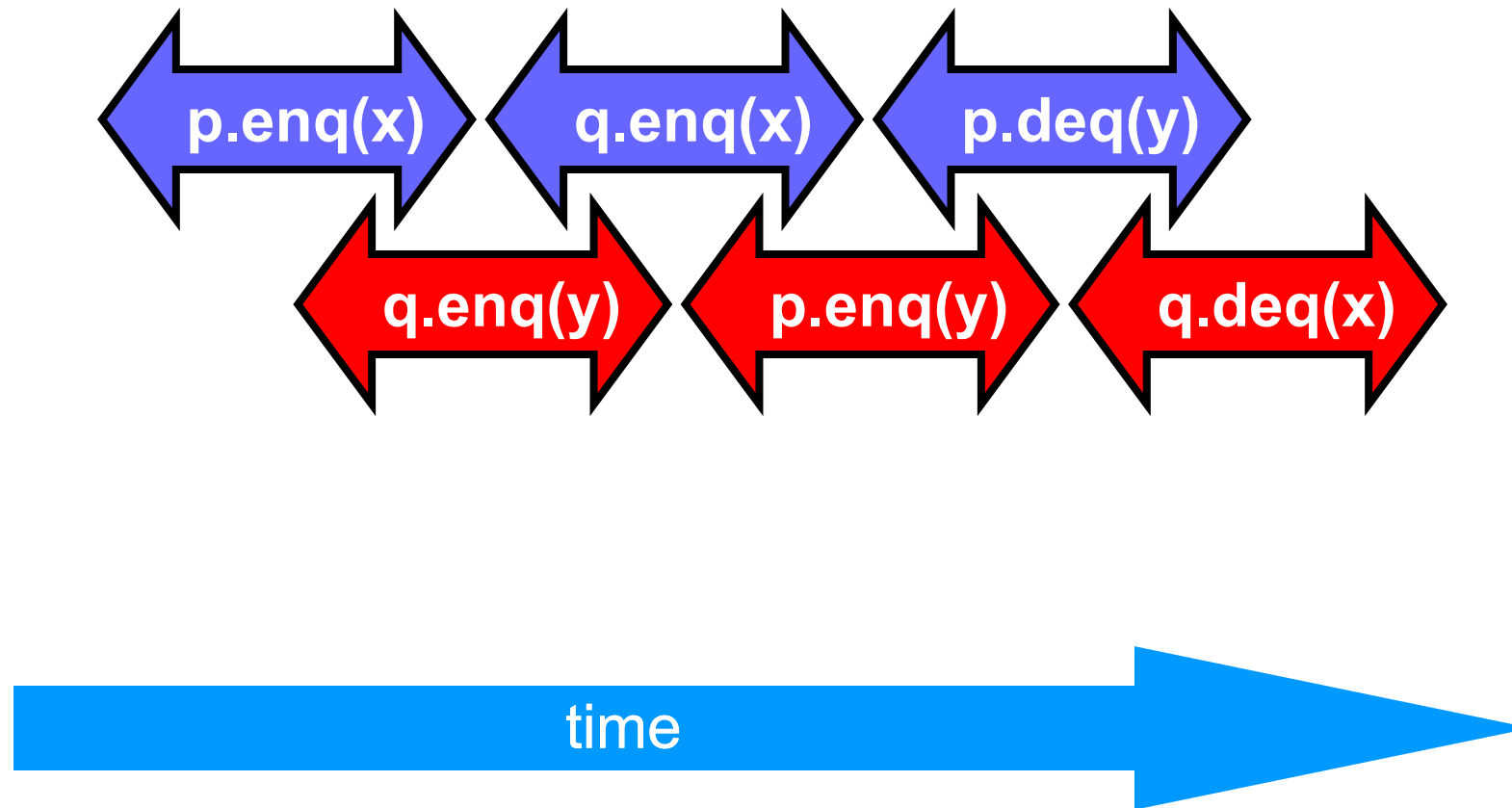
FIFO Queue Example



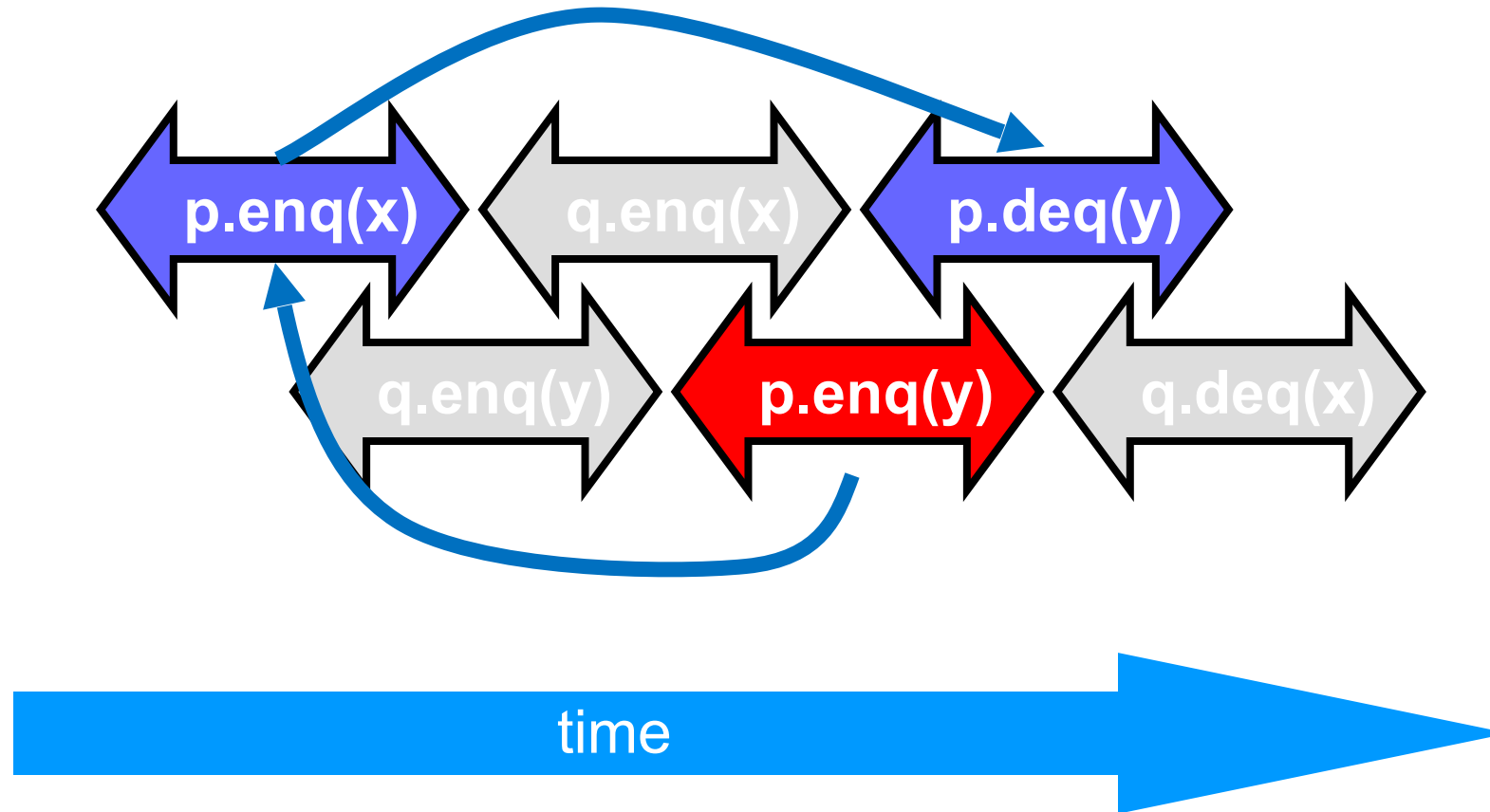
H/p Sequentially Consistent



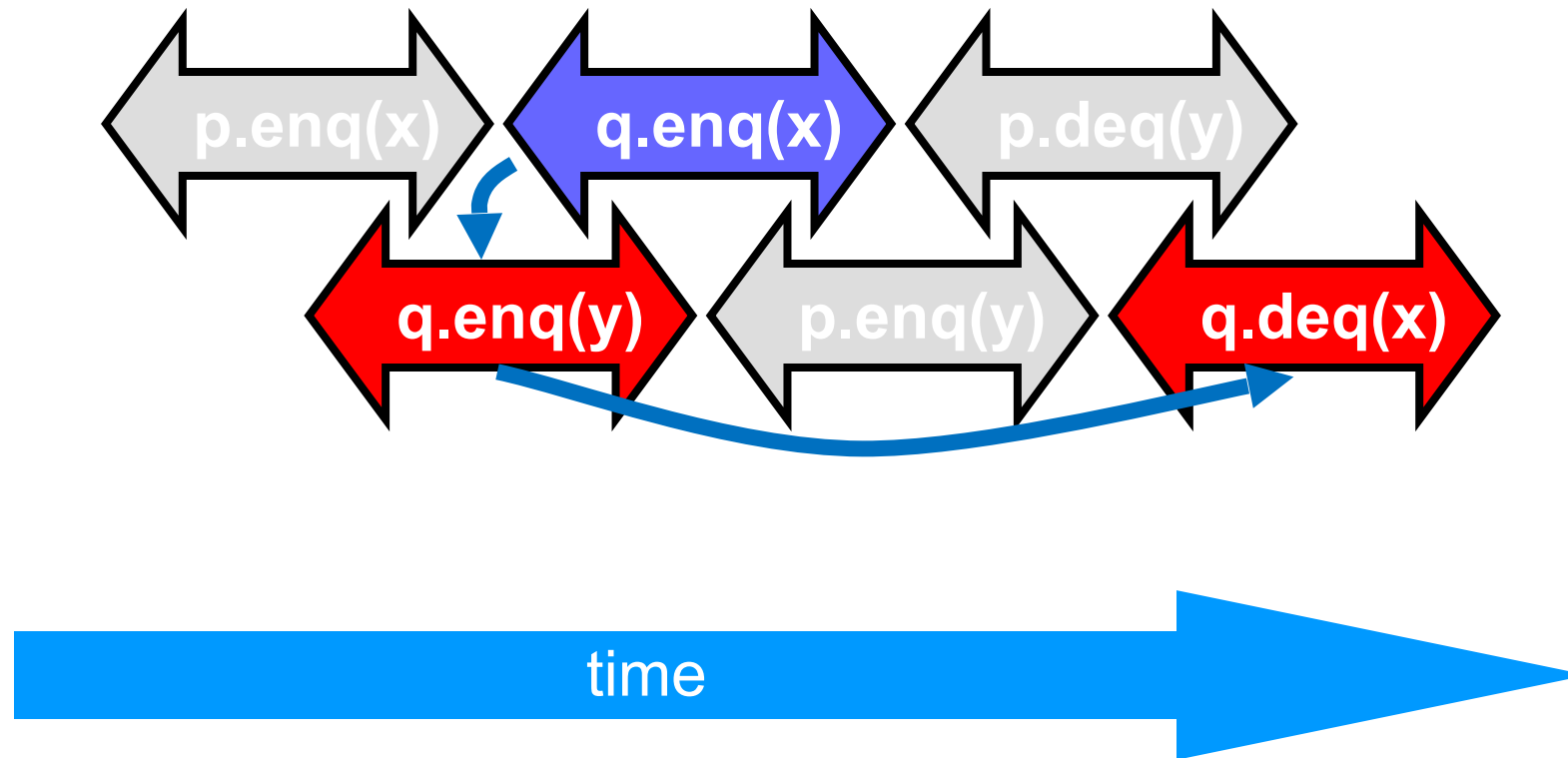
H/q Sequentially Consistent



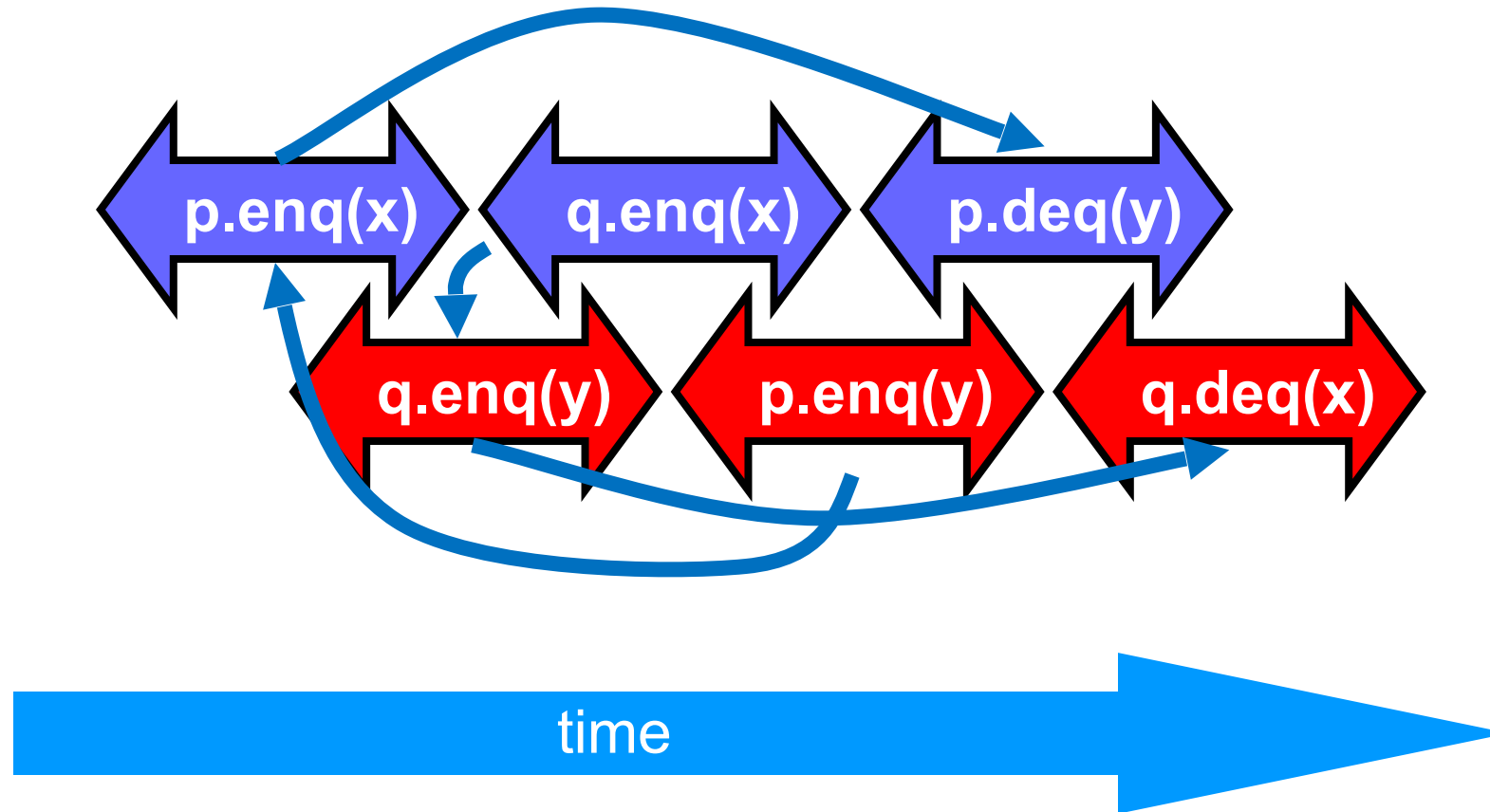
Ordering imposed by p



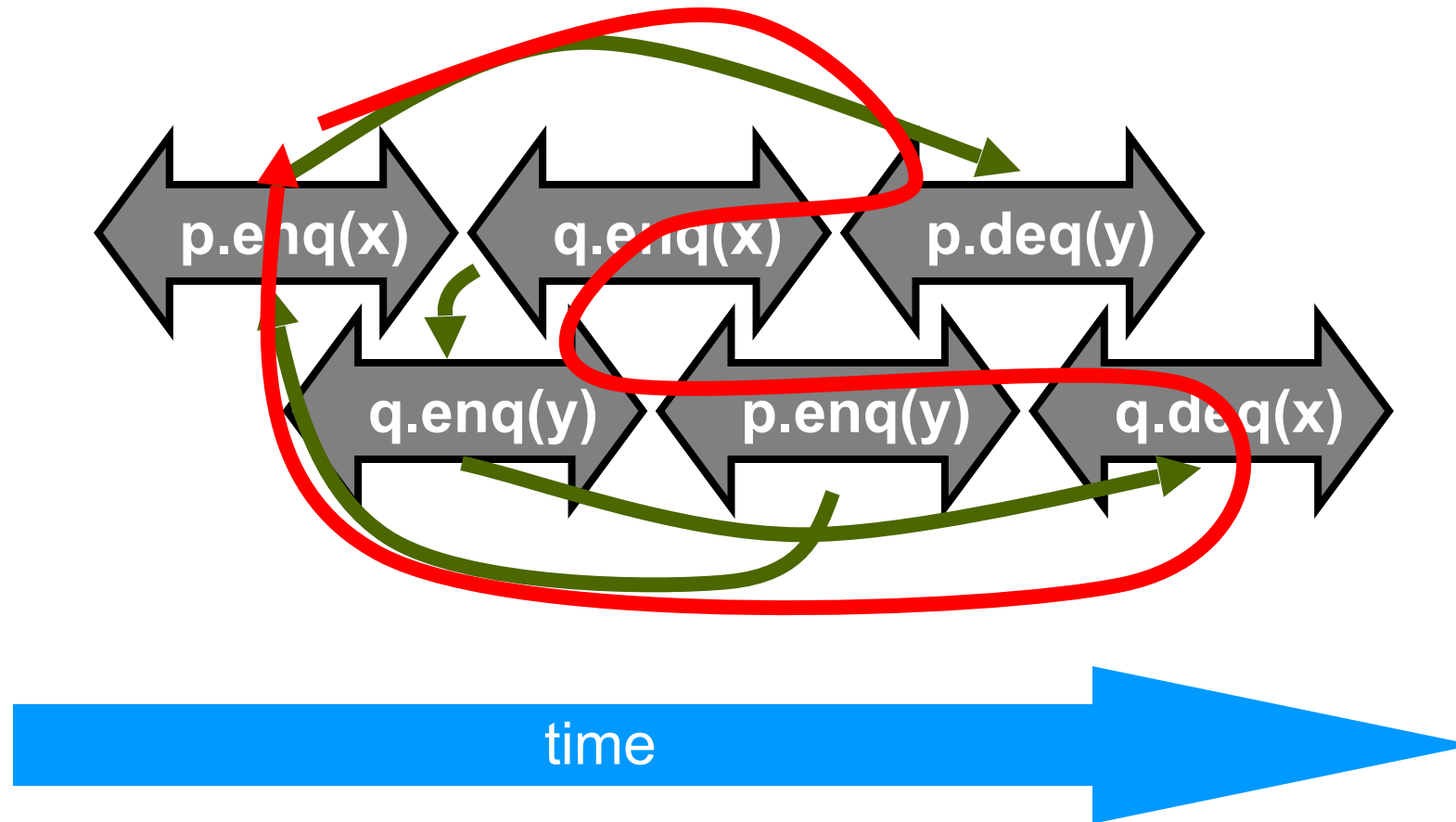
Ordering imposed by q



Ordering imposed by both



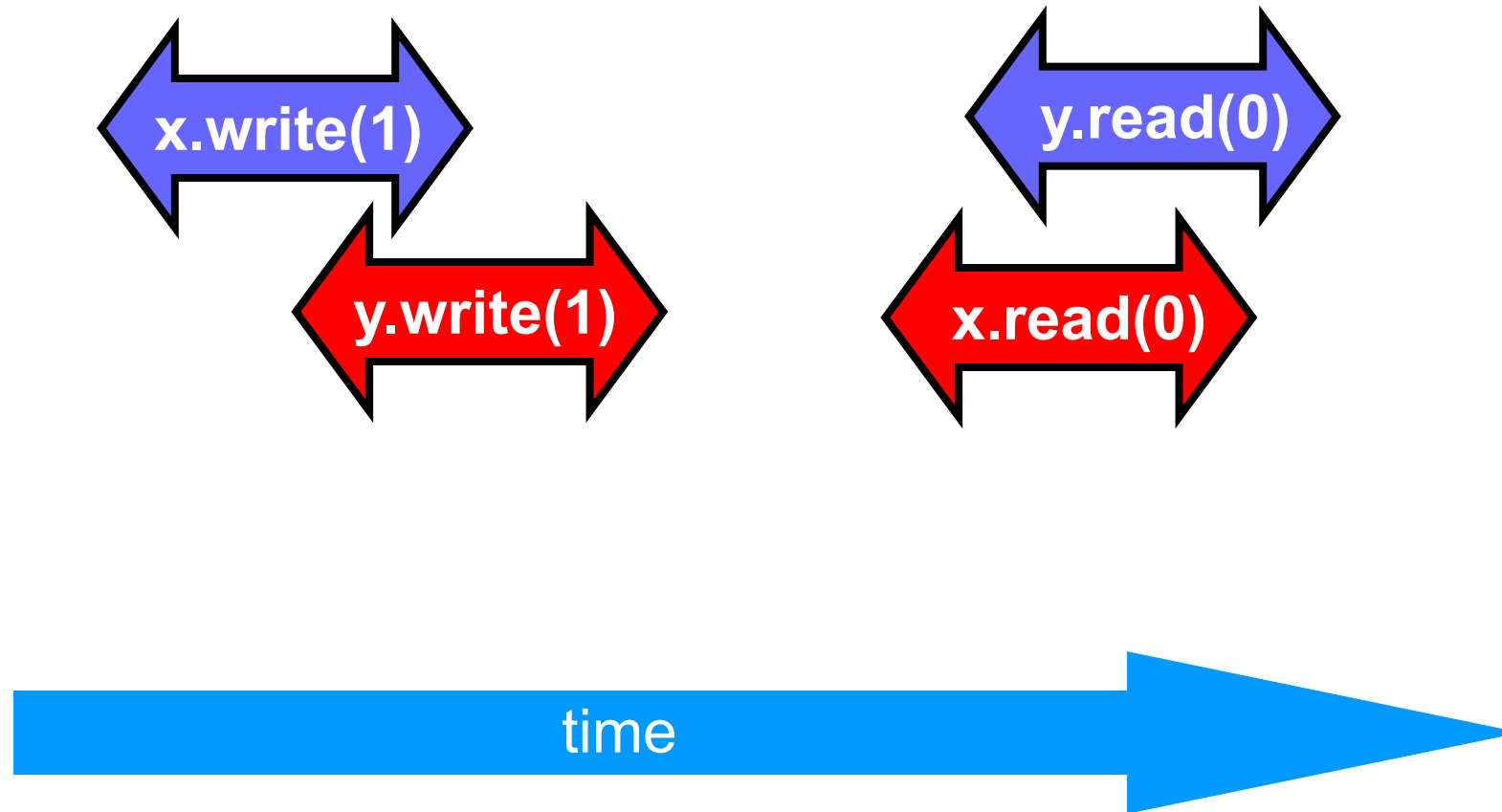
Combining orders



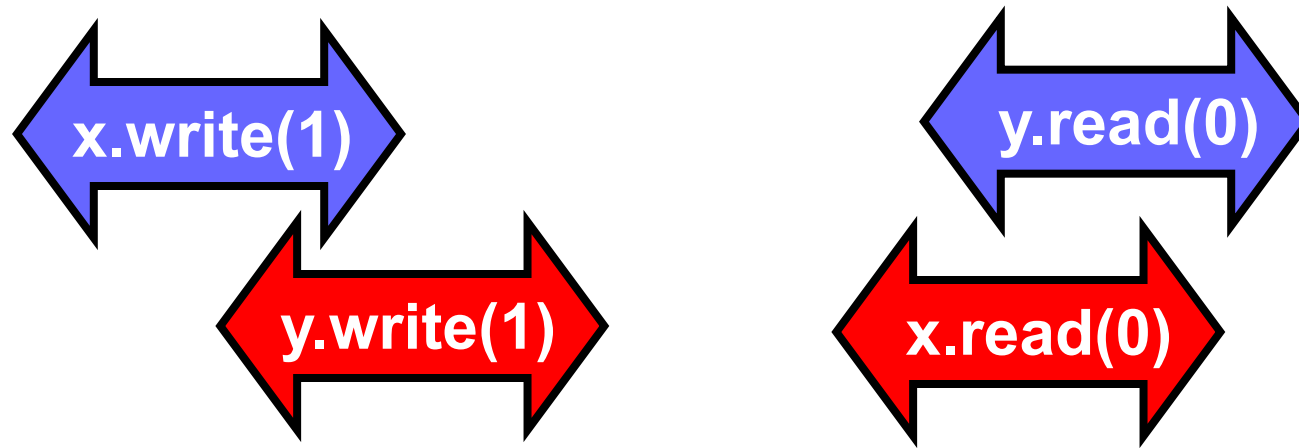
Fact

- Most hardware architectures don't even support sequential consistency
- Because they think it's too strong
- Here's another story ...

The Flag Example

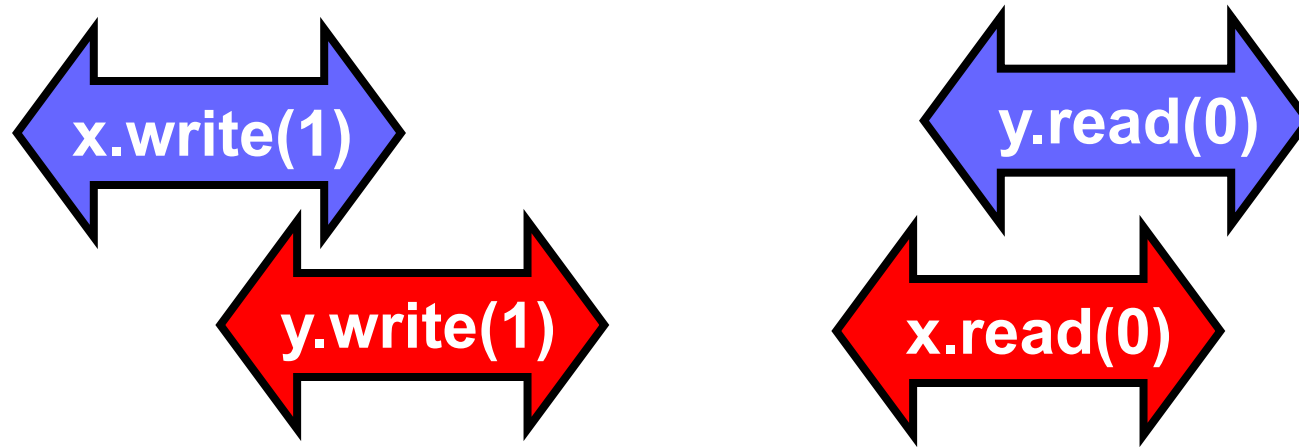


The Flag Example



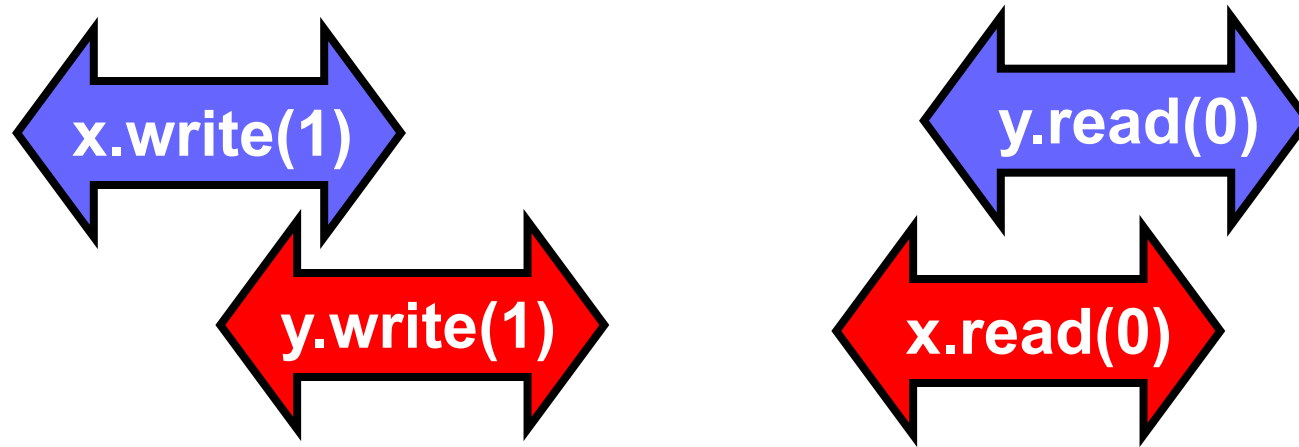
- Each thread's view is sequentially consistent
 - It went first

The Flag Example



- Entire history isn't sequentially consistent
 - Can't *both* go first
 - Petersen's lock now got a problem!

The Flag Example



- Is this behavior really so wrong?
 - We can argue either way ...

Opinion: It's Wrong

- This pattern
 - Write mine, read yours
- Is exactly the flag principle
 - Beloved of Alice and Bob
 - Heart of mutual exclusion
 - Peterson
 - Bakery, etc.
- It's non-negotiable!

Peterson's Algorithm

```
def lock(): Unit = {  
    flag(i) = true  
    victim = i  
    while (flag(1 - i) && victim == i) {}  
}  
  
def unlock(): Unit = {  
    val i = ThreadID.get  
    flag(i) = false  
}
```


Crux of Peterson Proof

(1) $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow$

(3) $\text{write}_B(\text{victim}=B) \rightarrow$

(2) $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$
 $\rightarrow \text{read}_A(\text{victim})$

Crux of Peterson Proof

(1) $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow$

(3) $\text{write}_B(\text{victim}=B) \rightarrow$

(2) $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$
 $\rightarrow \text{read}_A(\text{victim})$

Observation: proof relied on fact that if a location is stored, a later load by some thread will return this or a later stored value.

Opinion: But It Feels So Right ...

- Many hardware architects think that sequential consistency is too strong
- Too expensive to implement in modern hardware
- OK if flag principle
 - violated by default
 - Honored by explicit request

Hardware Consistency

Initially, $a = b = 0$.

Processor 0

```
mov 1, a    ;Store  
mov b, %ebx ;Load
```

Processor 1

```
mov 1, b    ;Store  
mov a, %eax ;Load
```

What are the final possible values of `%eax` and `%ebx` after both processors have executed?

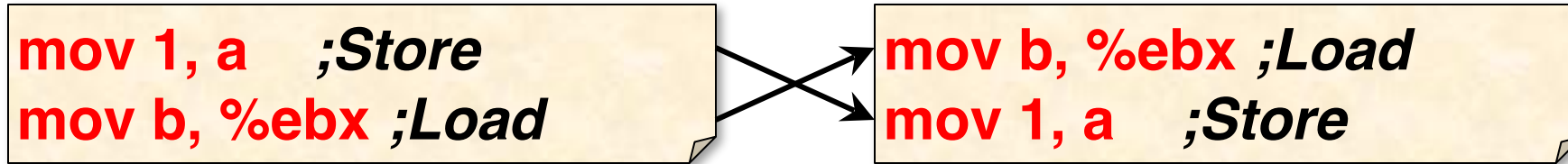
Sequential consistency implies that no execution ends with `%eax = %ebx = 0`

Hardware Consistency

- No modern-day processor implements sequential consistency.
- Hardware actively reorders instructions.
- Compilers may reorder instructions, too.
- Why?
- Because most of performance is derived from a single thread's unsynchronized execution of code!

This is known as **Weak (Relaxed) Memory Semantics**

Weak-Memory Instruction Reordering

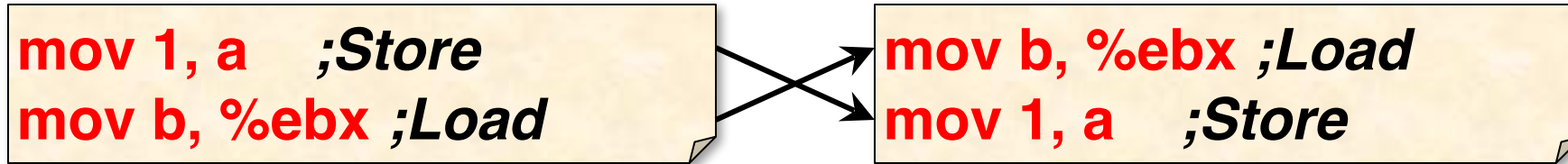


Program Order

Execution Order

- Q. Why might the hardware or compiler decide to reorder these instructions?
- A. To obtain higher performance by covering load latency — *instruction-level parallelism*.

Weak-Memory Instruction Reordering



Program Order

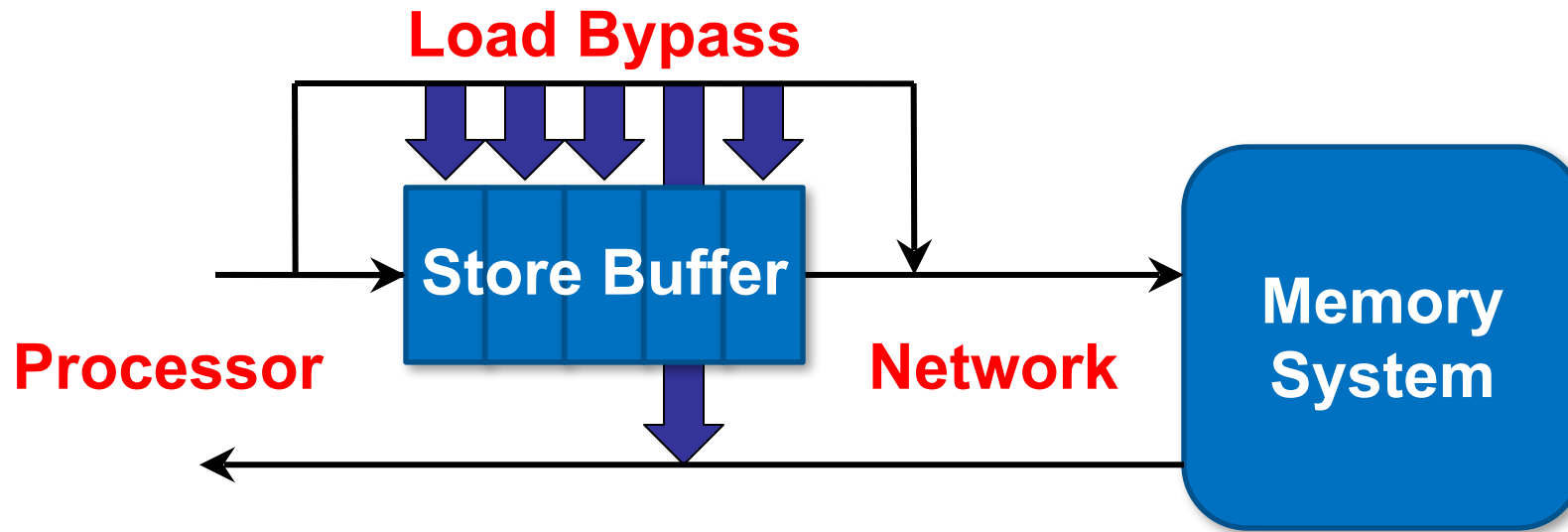
Execution Order

Q. When is it safe for the hardware or compiler to perform this reordering?

A. When $a \neq b$.

A'. And there's no concurrency.

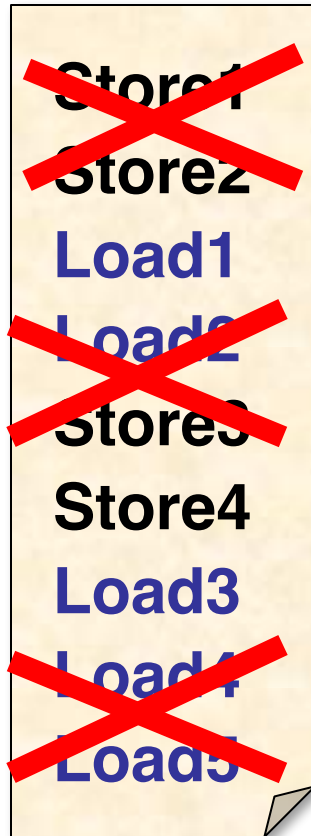
Hardware Reordering



- Processor can issue stores faster than the network can handle them \Rightarrow store buffer.
- Loads take priority, bypassing the store buffer.
- Except if a load address matches an address in the store buffer, the store buffer returns the result.

X86 Relaxed Memory Model

Thread's Code



~~Store1~~
~~Store2~~
Load1
~~Load2~~
~~Store3~~
Store4
Load3
~~Load4~~
~~Load5~~

1. Loads *are not* reordered with loads.
2. Stores *are not* reordered with stores.
3. Stores *are not* reordered with prior loads.
4. A load *may* be reordered with a prior store to a different location *but not* with a prior store to the same location.
5. Stores to the same location *respect a global total order*.

X86 Relaxed Memory Model

Thread's Code

Store1
Store2
Load1
Load2
Store3
Store4
Load3
Load4
Load5

↑
L
O
A
D
S

1. Loads *are not* reordered with loads.
2. Stores *are not* reordered with stores.
3. **Total Store Ordering (TSO)** ...weaker than sequential consistency
4. *with a prior store to the same location.*
5. Stores to the same location *respect a global total order.*

OK!

Memory Barriers (Fences)

- *A memory barrier (or memory fence) is a hardware action that enforces an ordering constraint between the instructions before and after the fence.*
- *A memory barrier can be issued explicitly as an instruction (x86: mfence)*
- *The typical cost of a memory fence is comparable to that of an L2-cache access.*

X86 Relaxed Memory Model

Thread's Code

Store1

Store2

Load1

Load2

Store3

Store4

Barrier

Load3

Load4

Load5

1. Loads
2. Store
3. Store loads
4. A load store with a prior store to the same location.
5. Stores to the same location respect a global total order.

Total Store Ordering +
properly placed memory barriers
= sequential consistency

Memory Barriers

- Explicit Synchronization
- Memory barrier will
 - Flush write buffer
 - Bring caches up to date
- Compilers often do this for you
 - Entering and leaving critical sections

Java/Scala Volatile Variables

- In Java, can ask compiler to keep a variable up-to-date by declaring it **volatile**
- In Scala, use **@volatile** annotation
- Adds a memory barrier after each store
- Inhibits reordering, removing from loops, & other “compiler optimizations”

Summary: Real-World

- Hardware is weaker than sequential consistency
- Can get sequential consistency at a price
- Linearizability better fit for high-level software

Linearizability

- Linearizability
 - Operation takes effect instantaneously between invocation and response
 - Uses sequential specification, locality implies composability

Summary: Correctness

- Sequential Consistency
 - Not composable
 - Harder to work with
 - Good way to think about hardware models
- We will use *linearizability* as our consistency condition in the remainder of this course unless stated otherwise

<A good place for a break>

Checkpoint

- Defined concurrent objects using linearizability and sequential consistency
- Fact: implemented linearizable objects (Two thread FIFO Queue) in read-write memory without mutual exclusion
- Fact: hardware does not provide linearizable read-write memory

Fundamentals

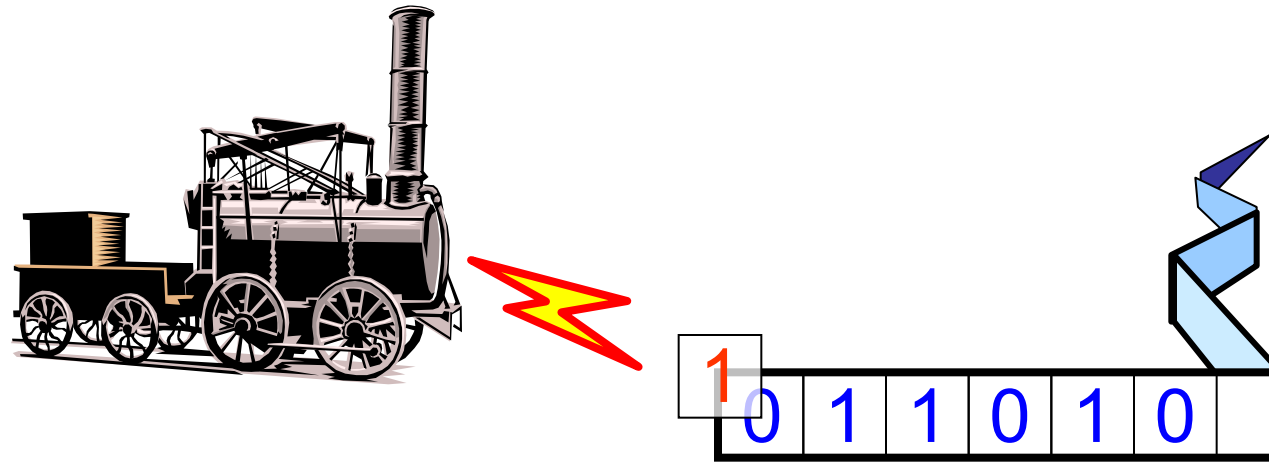
- What is the weakest form of communication that supports mutual exclusion?
- What is the weakest shared object that allows shared-memory computation?

Alan Turing



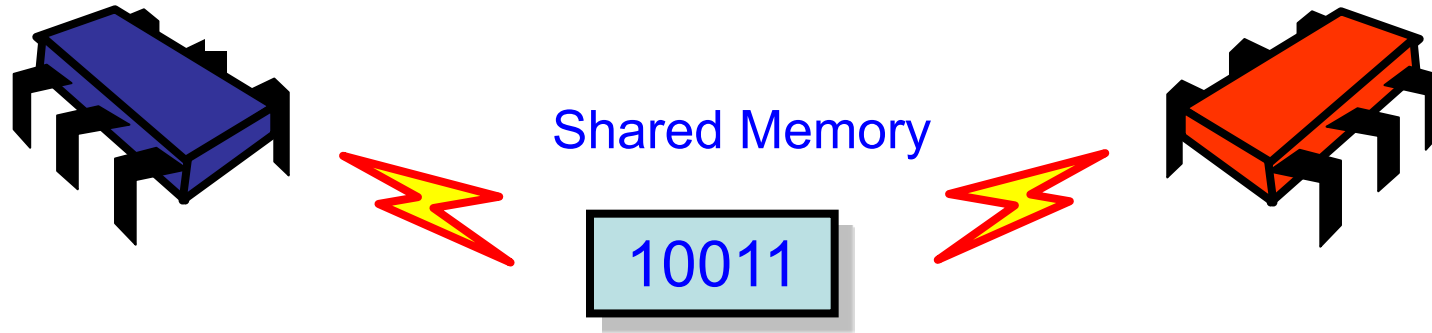
- Showed what is and is not computable on a sequential machine.
- Still best model there is.

Turing Computability



- Mathematical model of computation
- What is (and is not) computable
- Efficiency (mostly) irrelevant

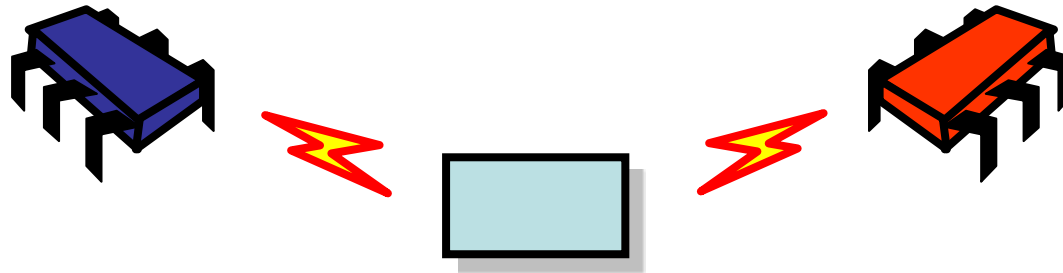
Shared-Memory Computability?



- Mathematical model of **concurrent** computation
- What is (and is not) concurrently computable
- Efficiency (mostly) irrelevant

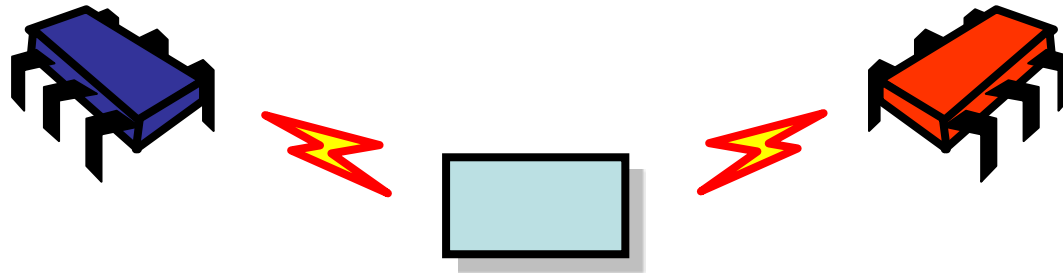
Foundations of Shared Memory

To understand modern multiprocessors we need to ask some basic questions ...



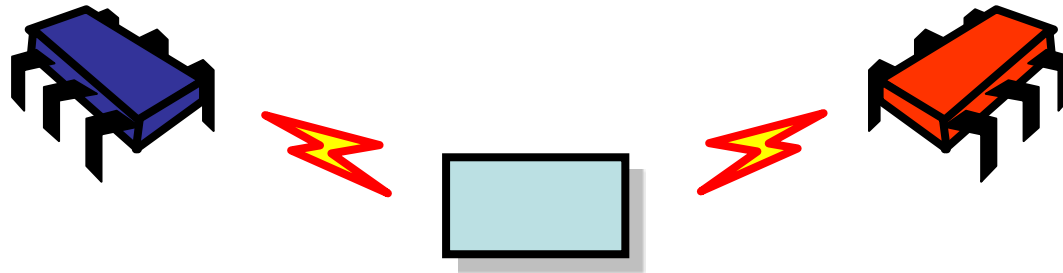
Foundations of Shared Memory

To understand modern
What is the weakest useful form of
shared memory?



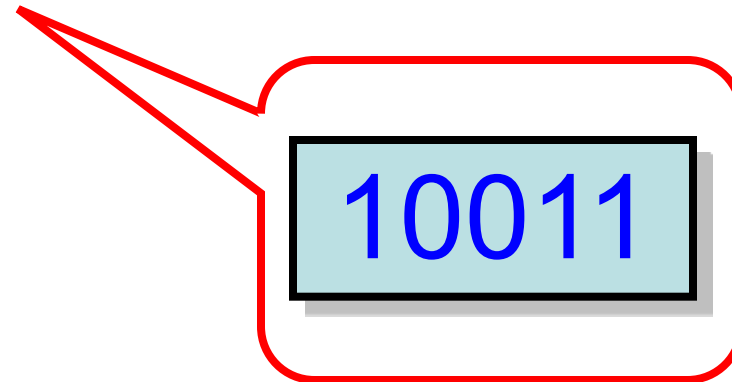
Foundations of Shared Memory

To understand modern
What is the weakest useful form of
What can it do?



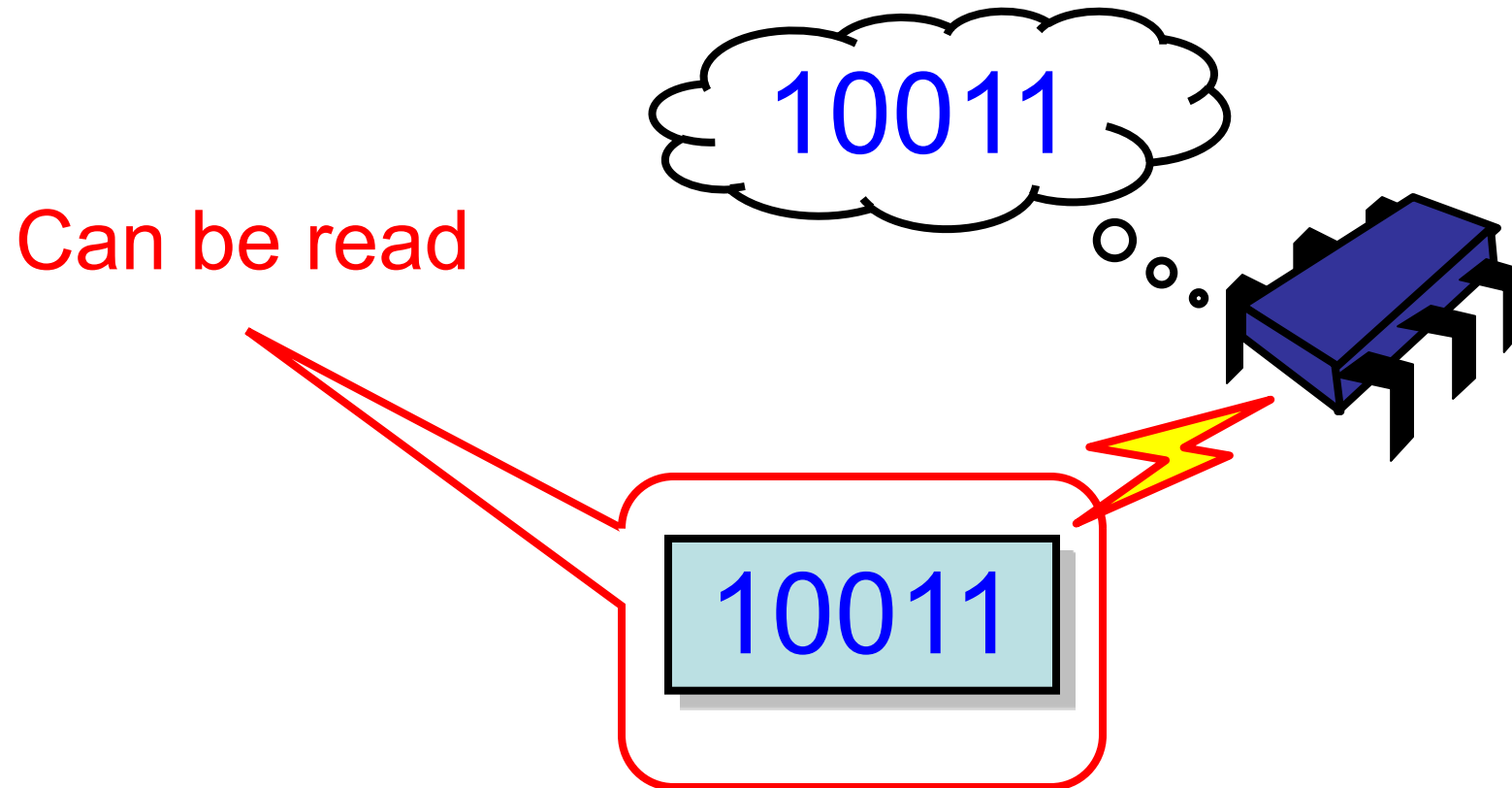
Register*

Holds a
(binary) value

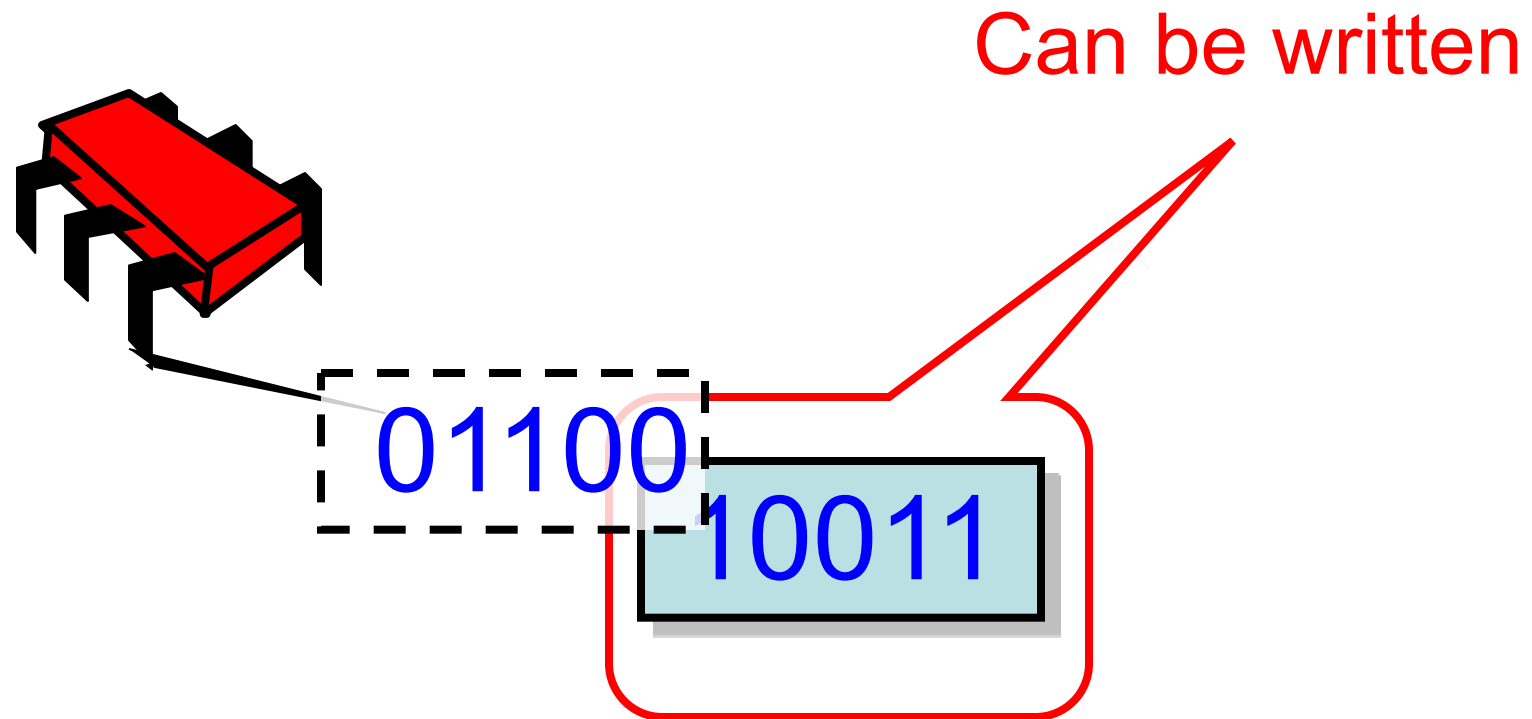


*** A memory location: name is historical**

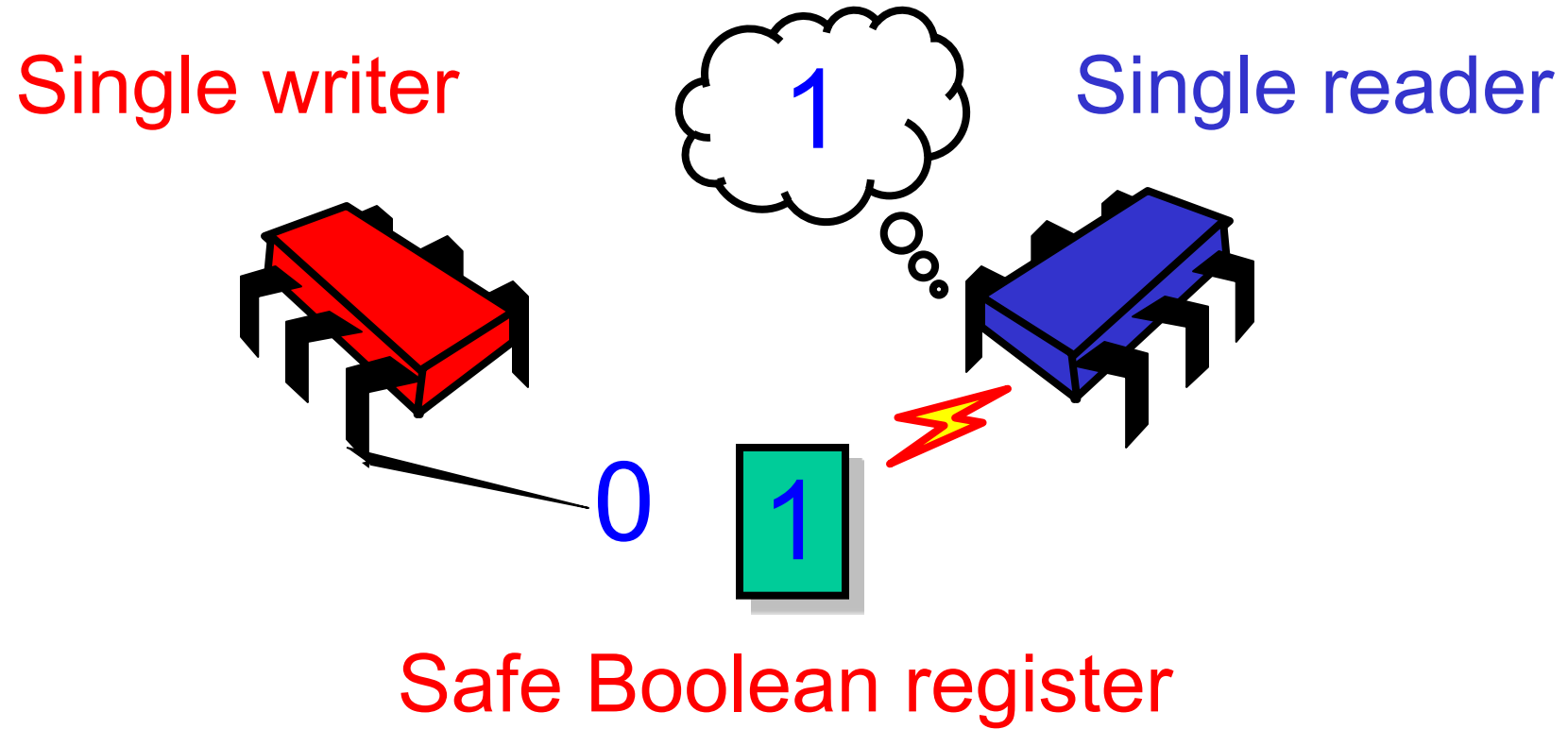
Register



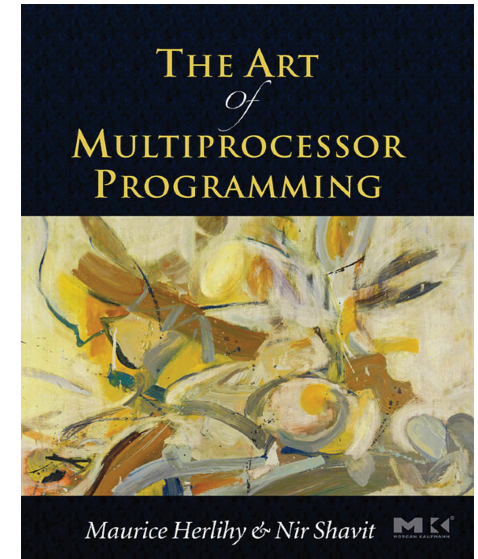
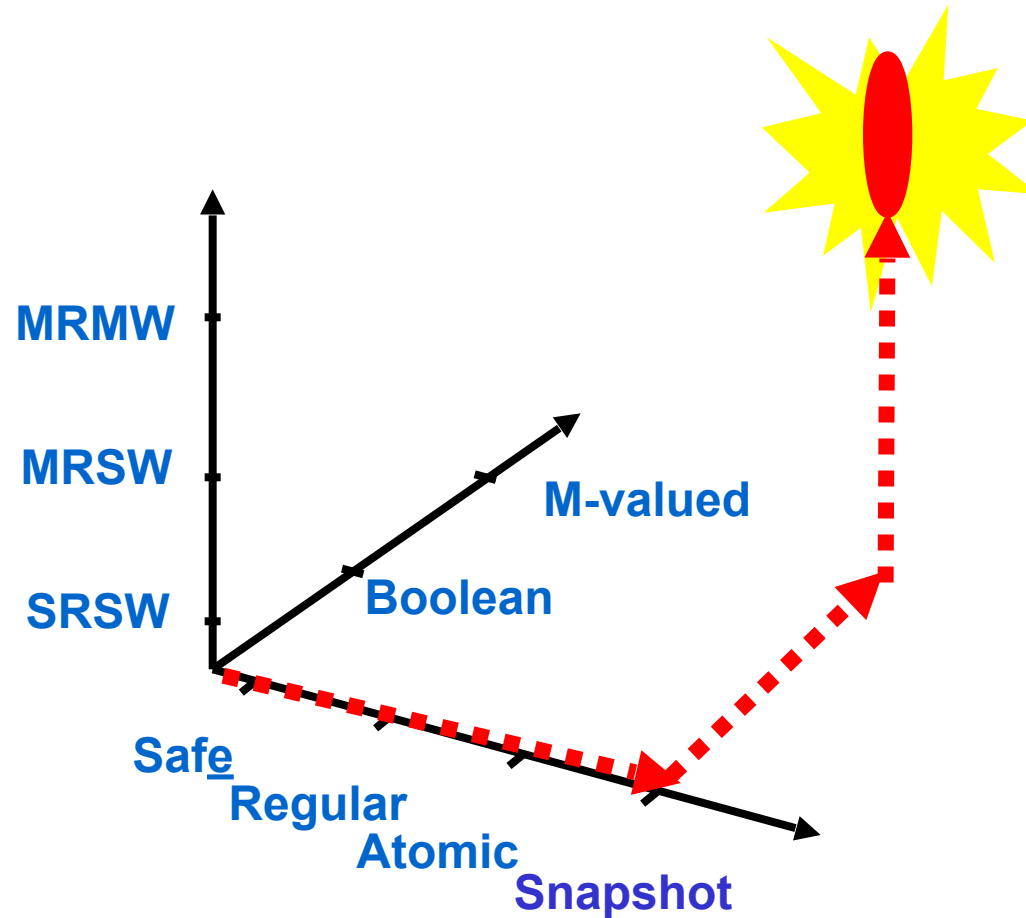
Register



From Weakest Register



All the way to a Wait-free Implementation of Atomic Snapshots



Chapter 4

Wait-Free Implementation

- Every method call completes in finite number of steps
- Implies no mutual exclusion



Rationale for wait-freedom

- We wanted atomic registers to implement mutual exclusion

Rationale for wait-freedom

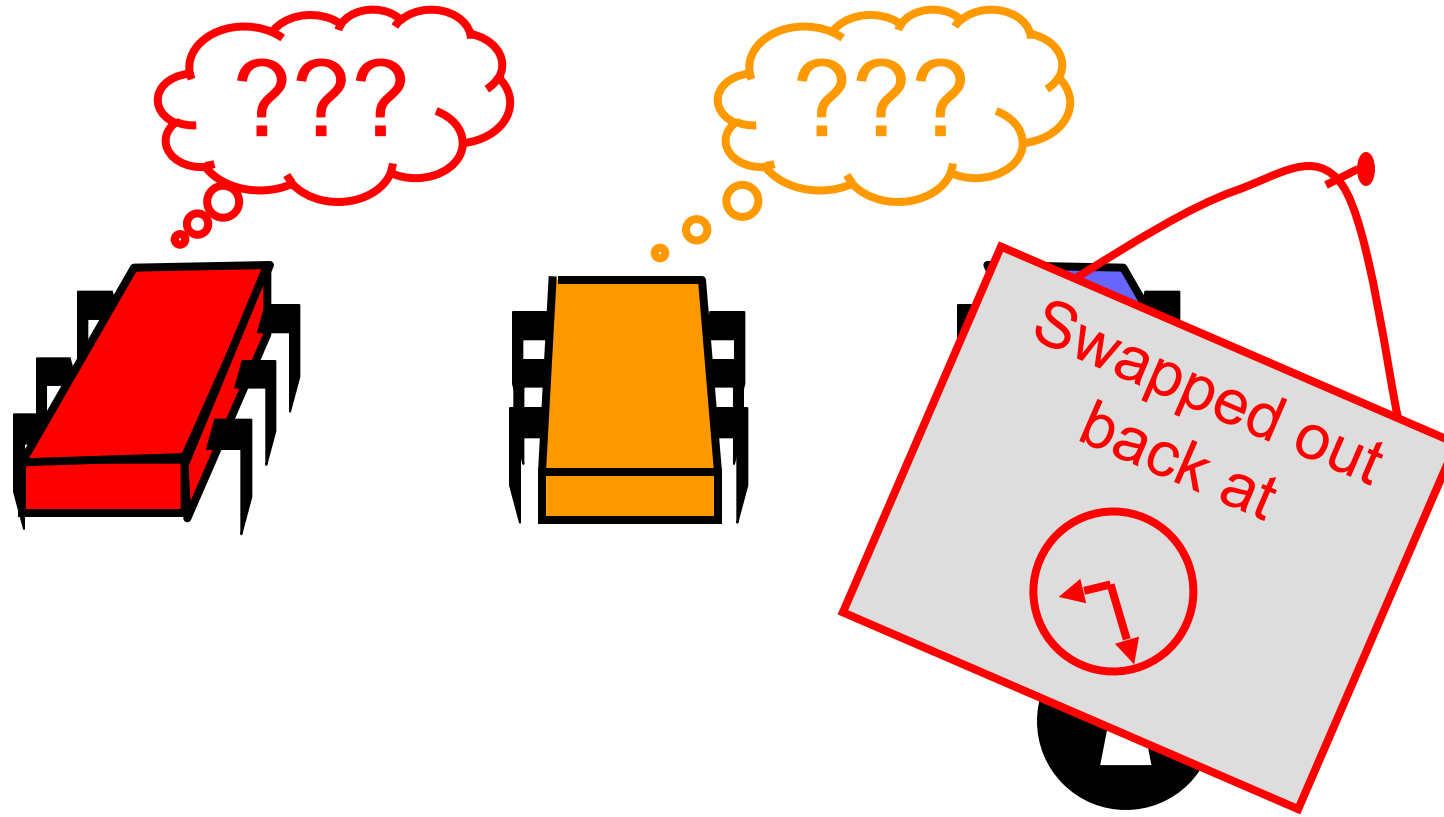
- We wanted atomic registers to implement mutual exclusion
- So we couldn't use mutual exclusion to implement atomic registers

Rationale for wait-freedom

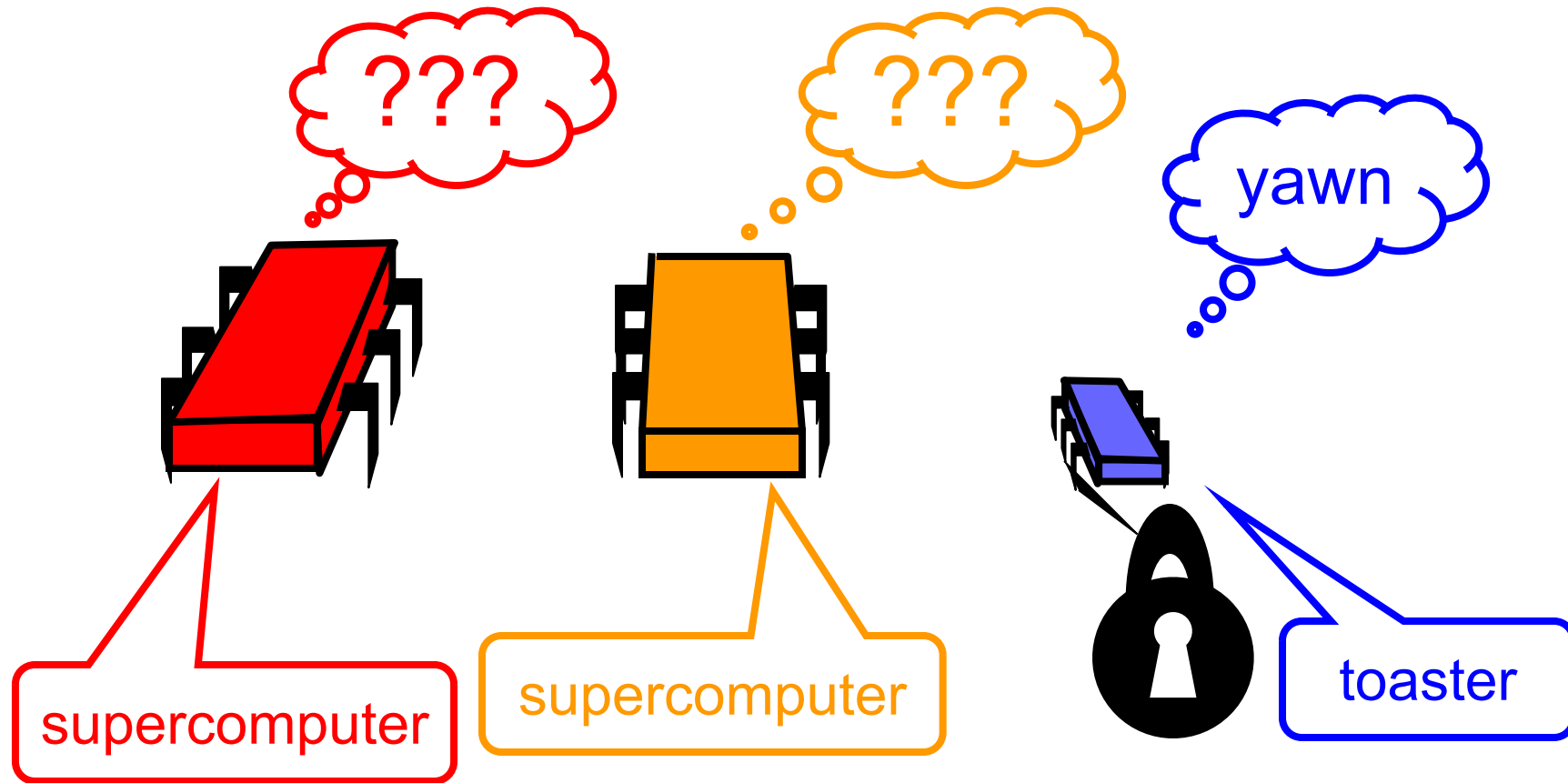
- We wanted atomic registers to implement mutual exclusion
- So we couldn't use mutual exclusion to implement atomic registers
- But wait, there's more!

What's the problem with
Mutual Exclusion?

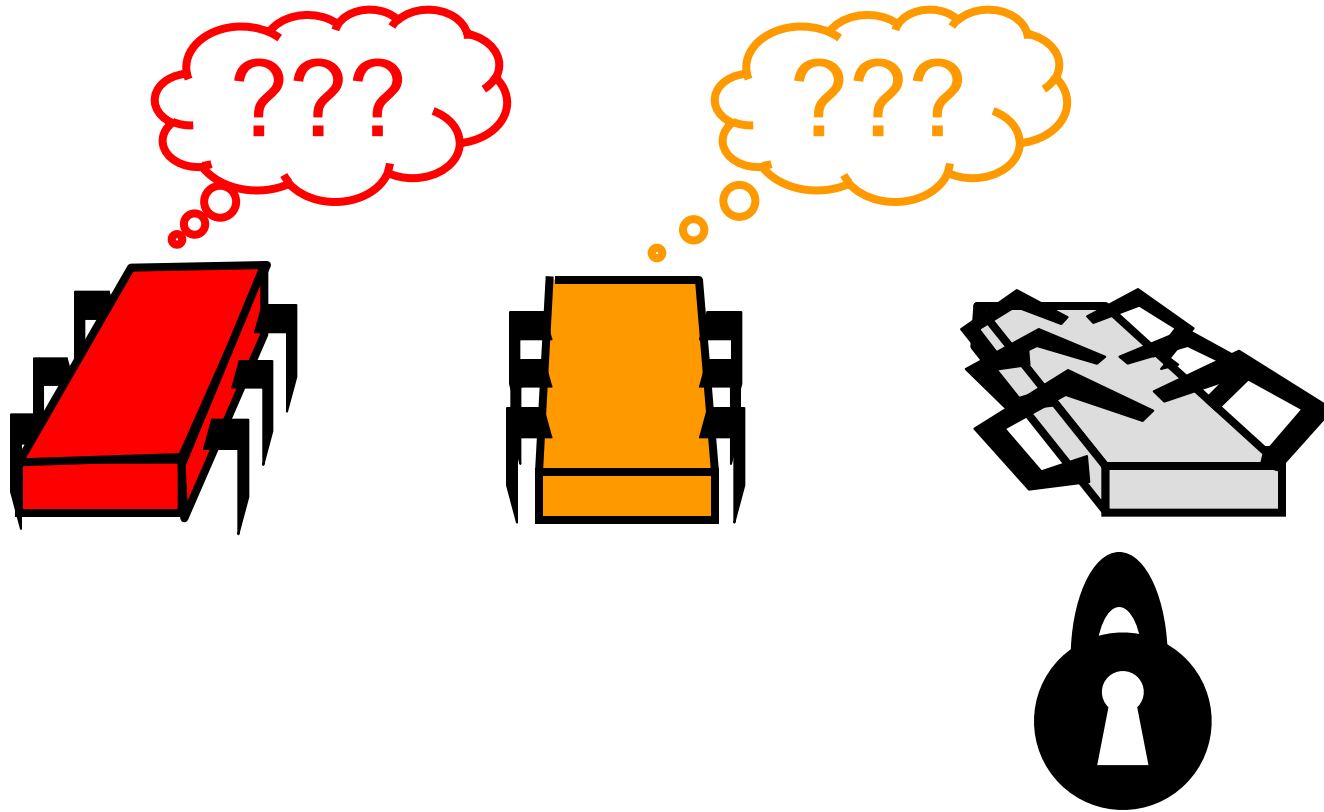
Asynchronous Interrupts



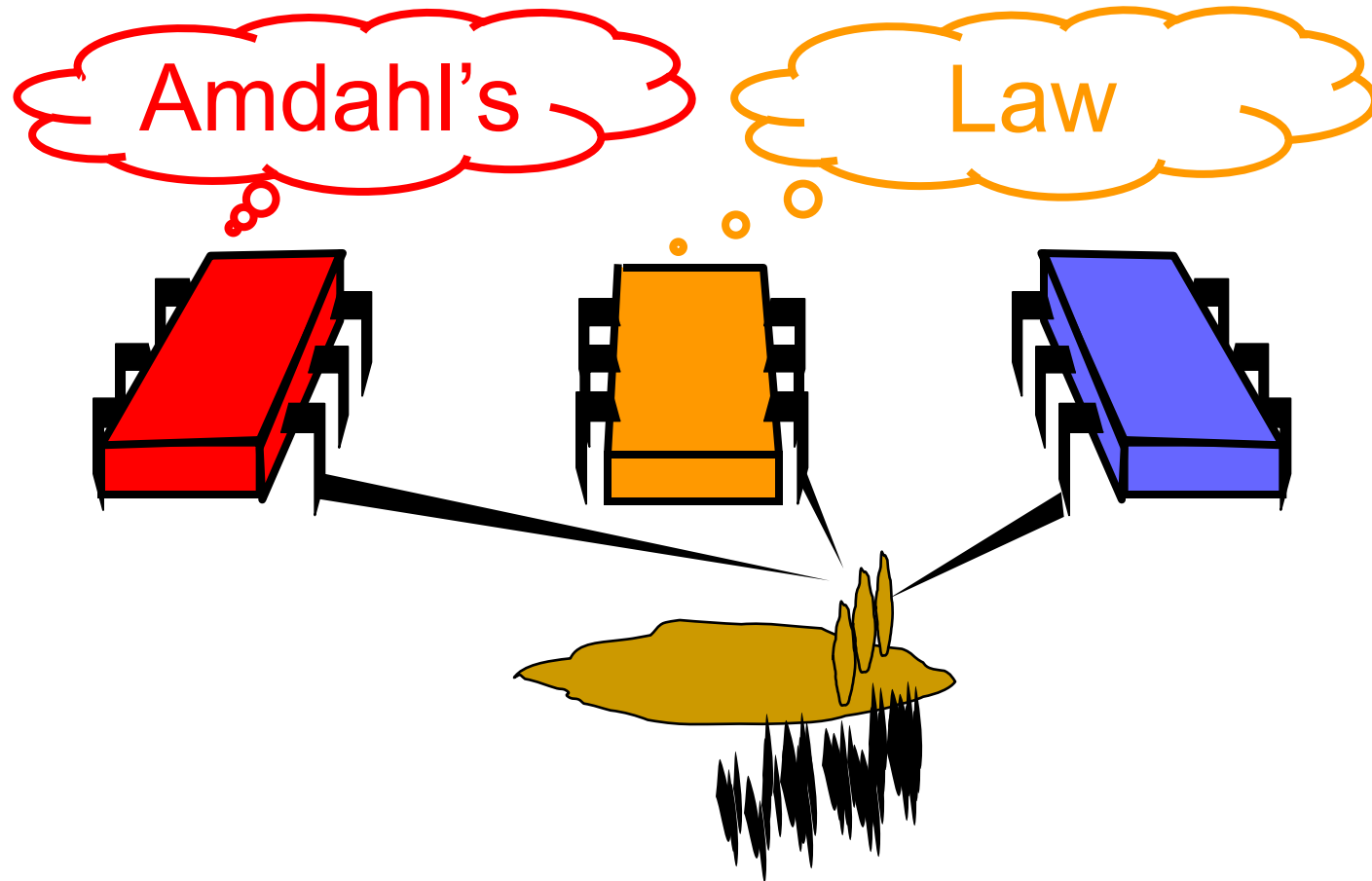
Heterogeneous Processors



Fault-tolerance



Machine Level Instruction Granularity



Basic Questions

- Wait-Free **synchronization** might be a good idea in principle

Basic Questions

- Wait-Free synchronization might be a good idea in principle
- But how do you do it ...

Basic Questions

- Wait-Free synchronization might be a good idea in principle
- But how do you do it ...
 - Systematically?

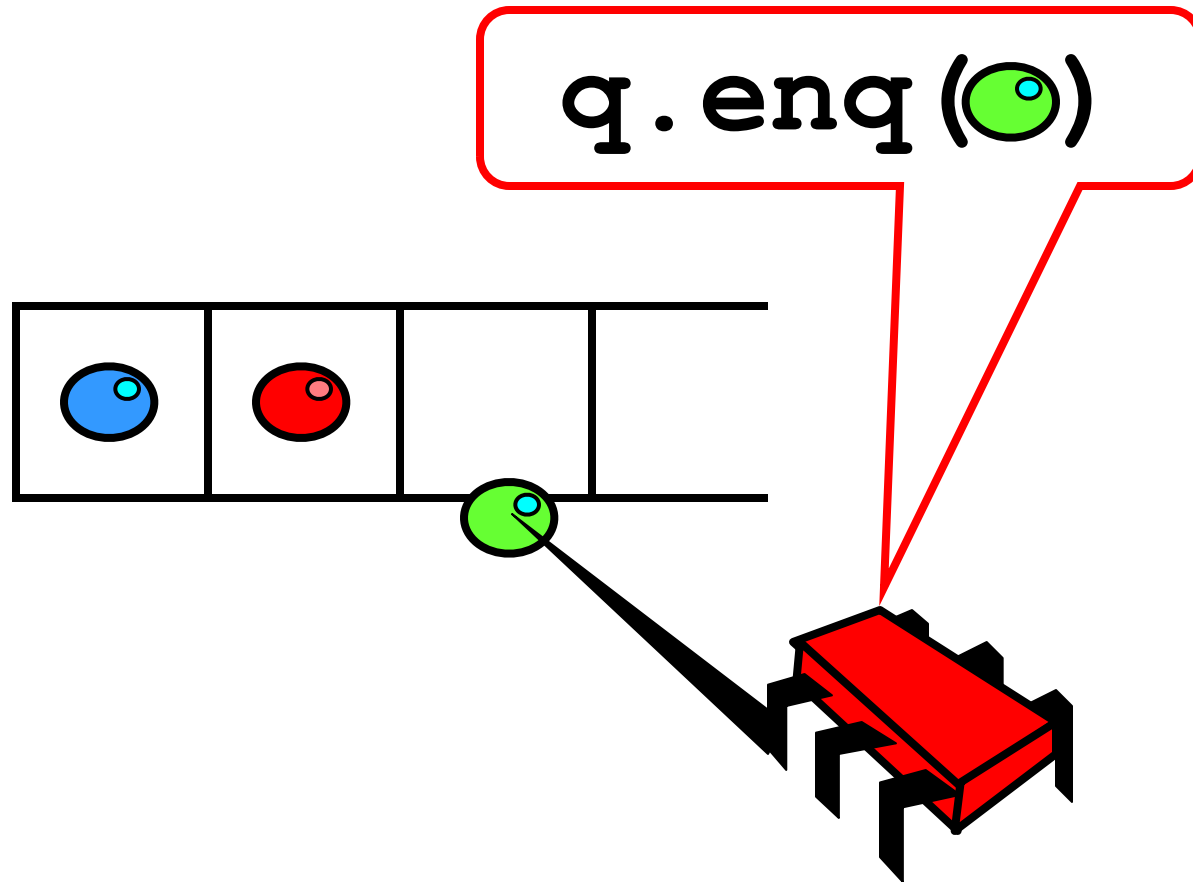
Basic Questions

- Wait-Free synchronization might be a good idea in principle
- But how do you do it ...
 - Systematically?
 - Correctly?

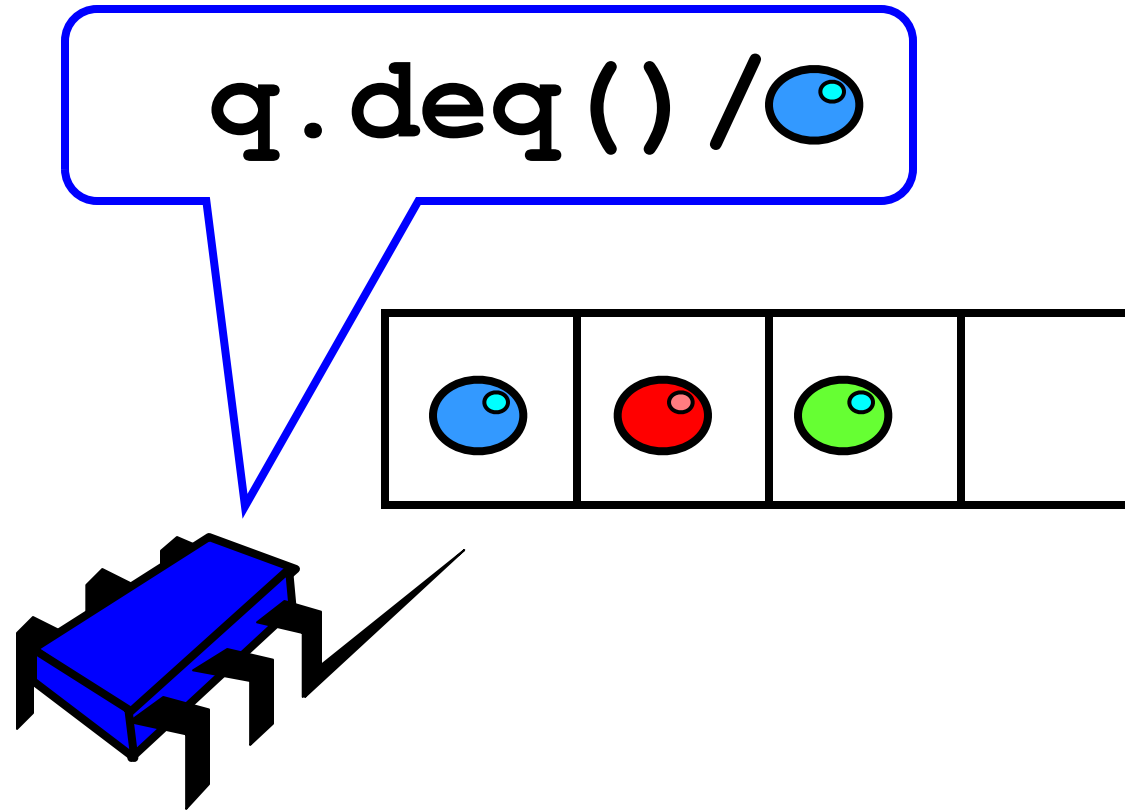
Basic Questions

- Wait-Free synchronization might be a good idea in principle
- But how do you do it ...
 - Systematically?
 - Correctly?
 - Efficiently?

FIFO Queue: Enqueue Method

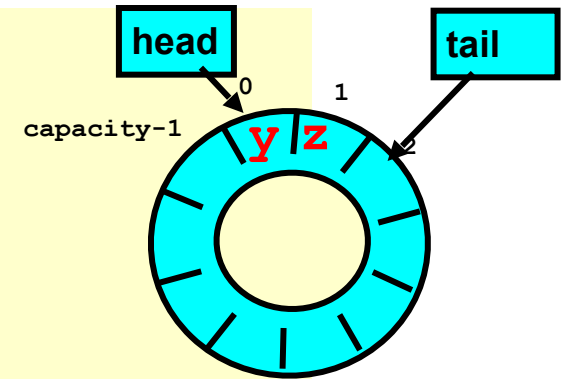


FIFO Queue: Dequeue Method

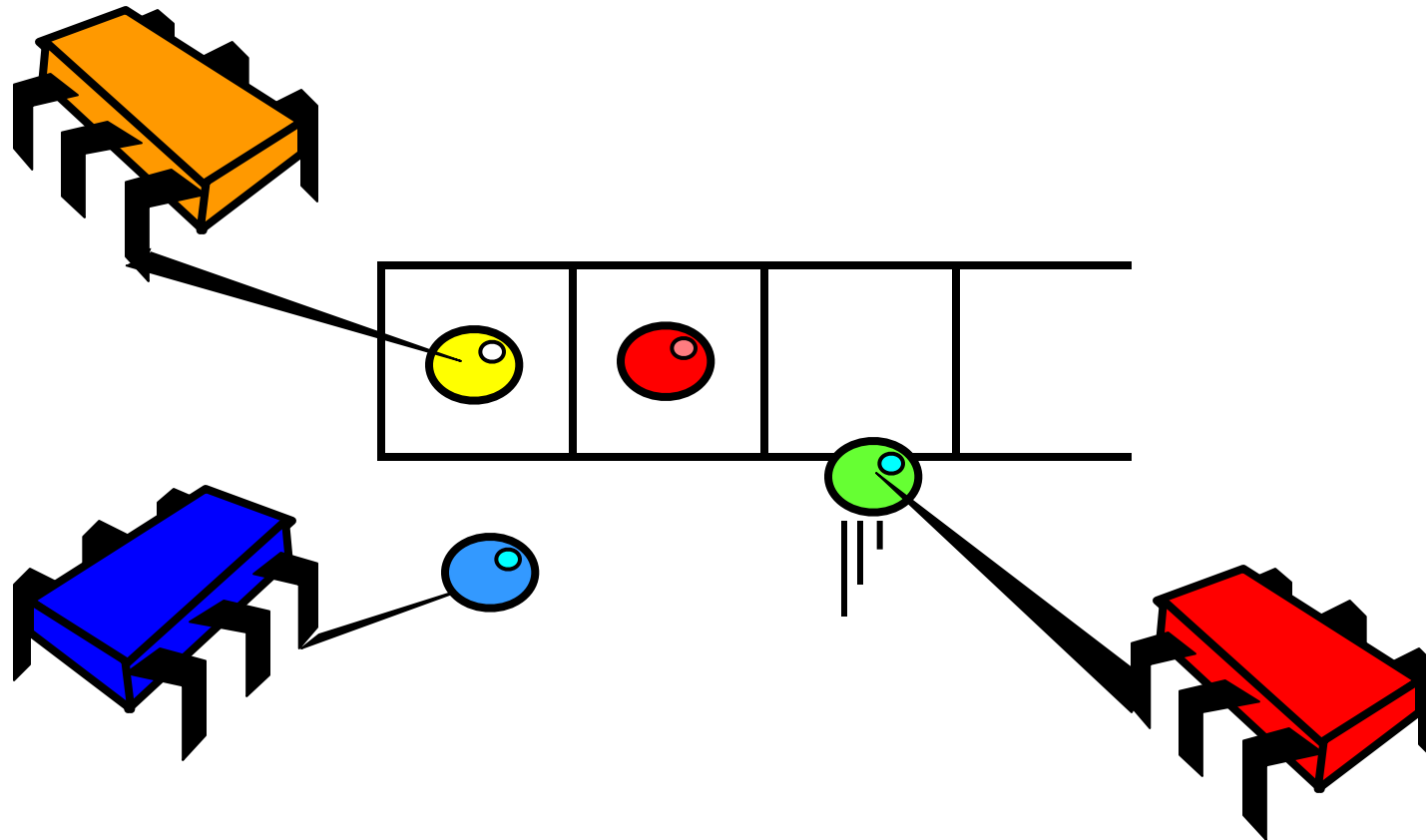


Two-Thread Wait-Free Queue

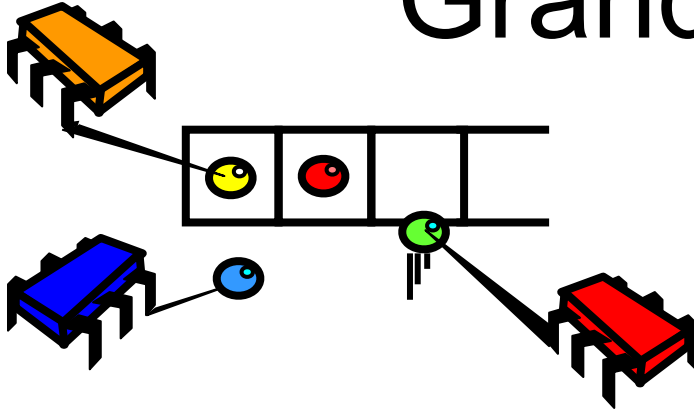
```
class LockFreeQueue[T: ClassTag](val capacity: Int) {  
  
  @volatile  
  private var head, tail: Int = 0  
  private val items = new Array[T](capacity)  
  
  def enq(x: T): Unit = {  
    if (tail - head == items.length) throw FullException  
    items(tail % items.length) = x  
    tail = tail + 1  
  }  
  
  def deq(): T = {  
    if (tail == head) throw EmptyException  
    val x = items(head % items.length)  
    head = head + 1  
    x  
  }  
}
```



What About Multiple Dequeueers?

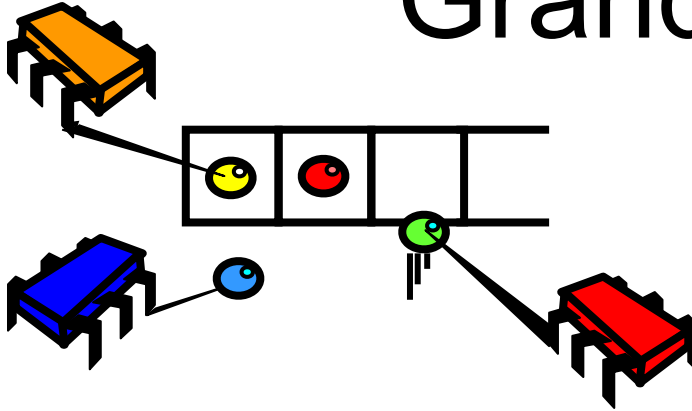


Grand Challenge



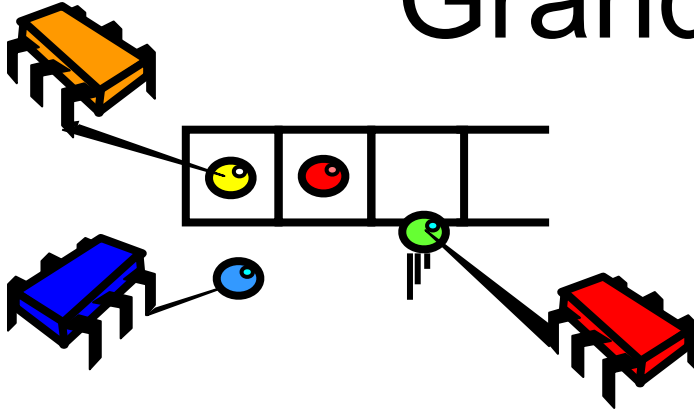
- Implement a FIFO queue

Grand Challenge



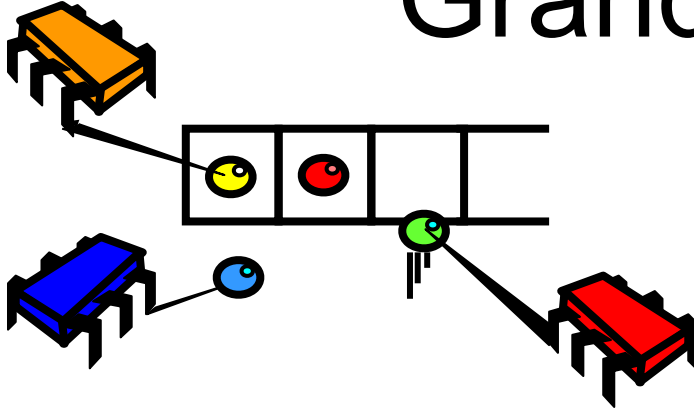
- Implement a FIFO queue
 - Wait-free

Grand Challenge



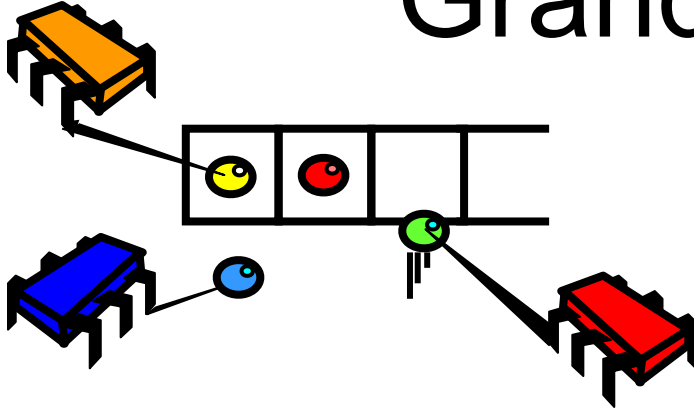
- Implement a FIFO queue
 - Wait-free
 - Linearizable

Grand Challenge



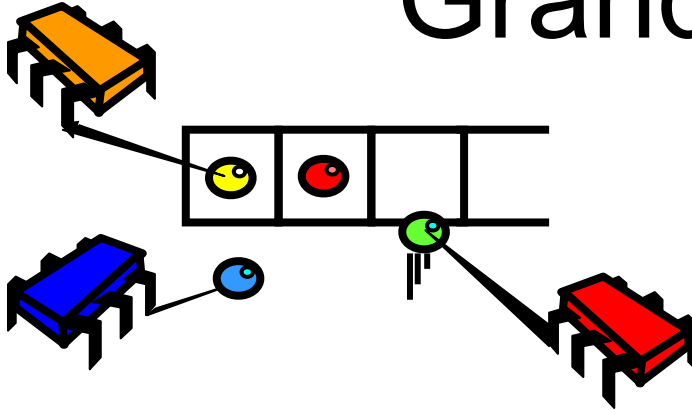
- Implement a FIFO queue
 - Wait-free
 - Linearizable
 - From atomic read-write registers

Grand Challenge



- Implement a FIFO queue
 - Wait-free
 - Linearizable
 - From atomic read-write registers
 - Multiple dequeuers

Grand Challenge



- Implement a FIFO queue

- Wait-free
- Linearizable
- From atomic read-write registers

- Multiple dequeuers

Only new
aspect

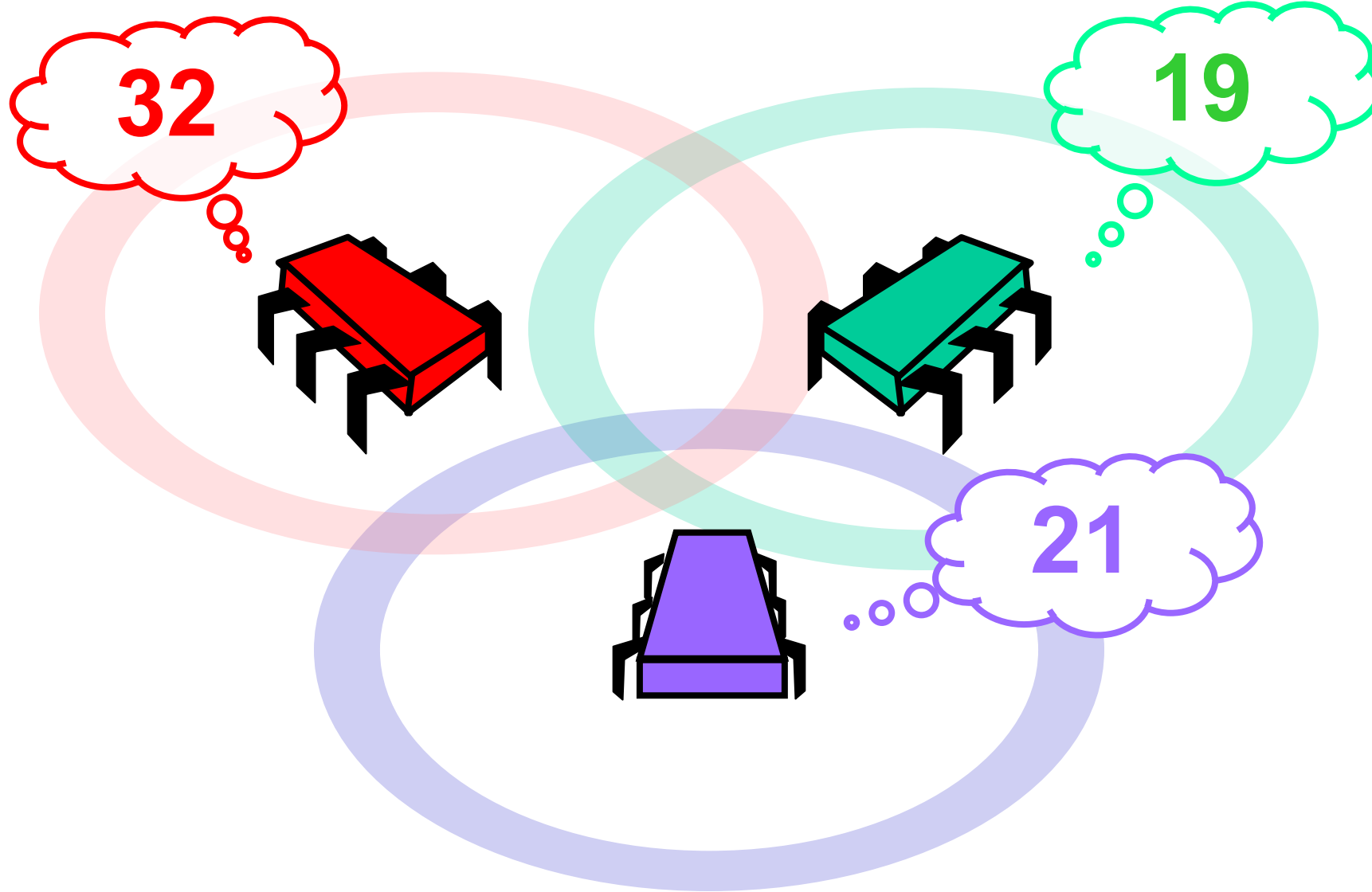
Puzzle

While you are ruminating on the grand challenge ...

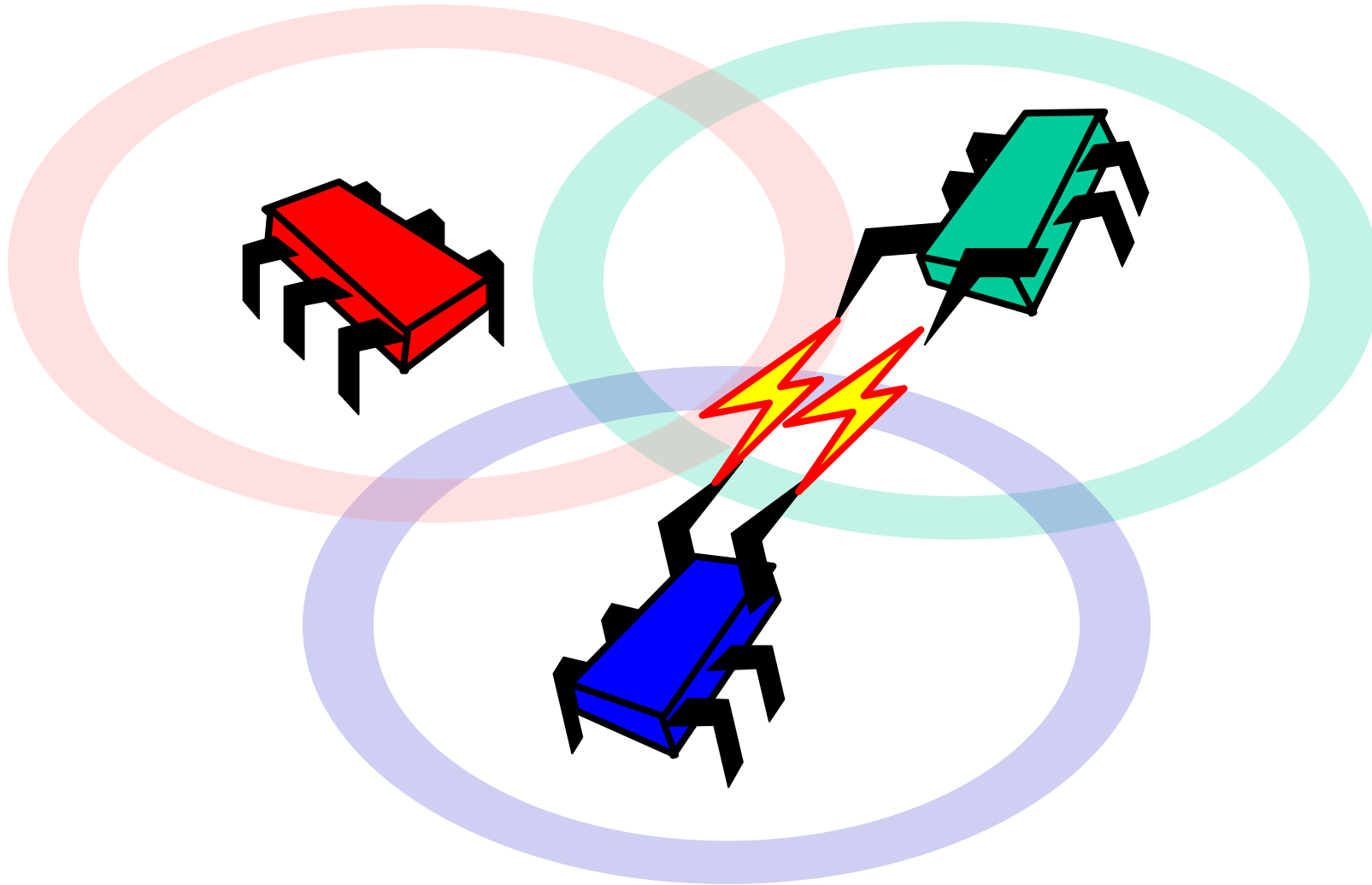
We will give you another puzzle ...

Consensus!

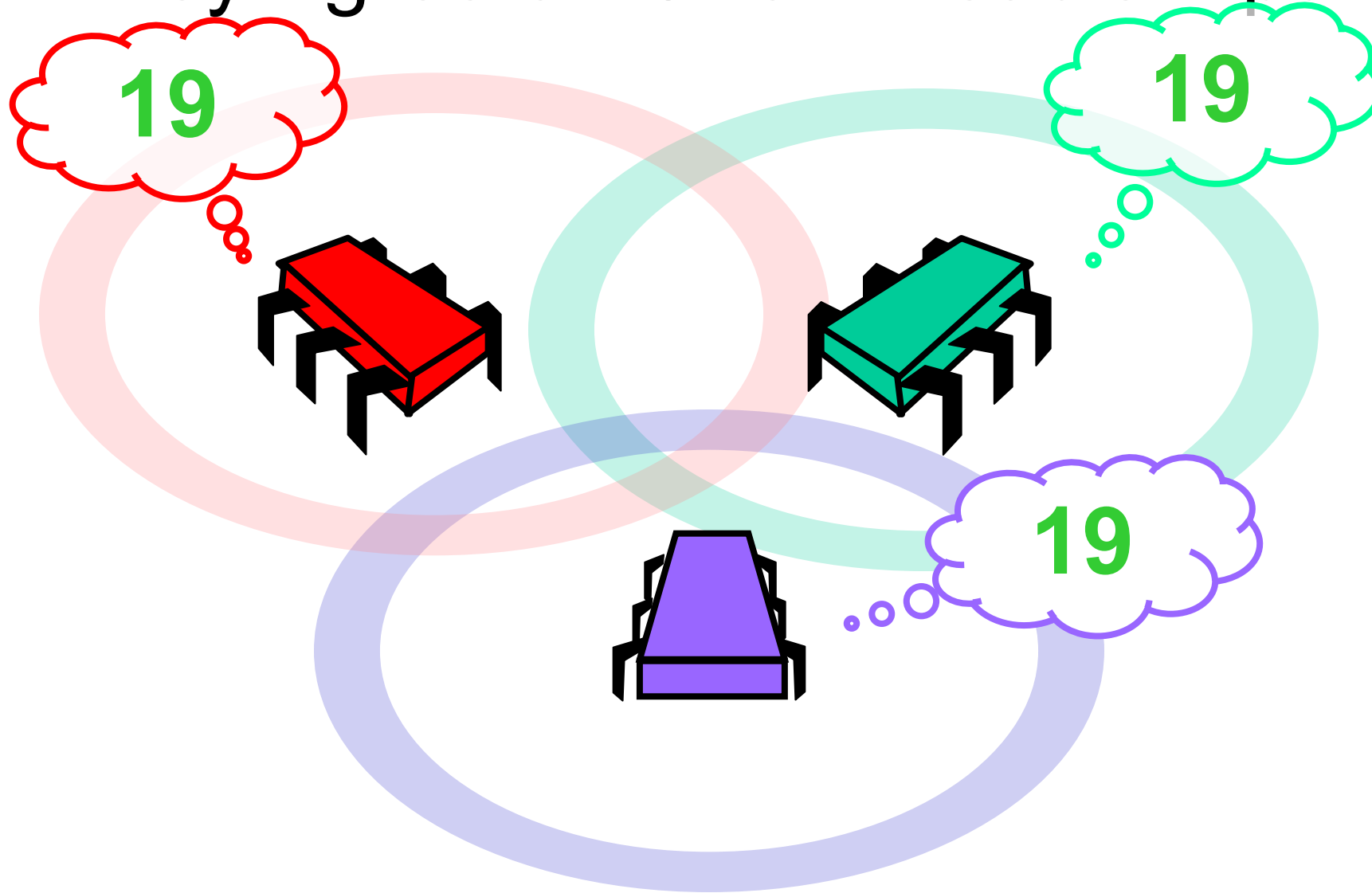
Consensus: Each Thread has a Private Input



They Communicate



They Agree on One Thread's Input



Formally: Consensus

- Consistent:
 - all threads decide the same value

Formally: Consensus

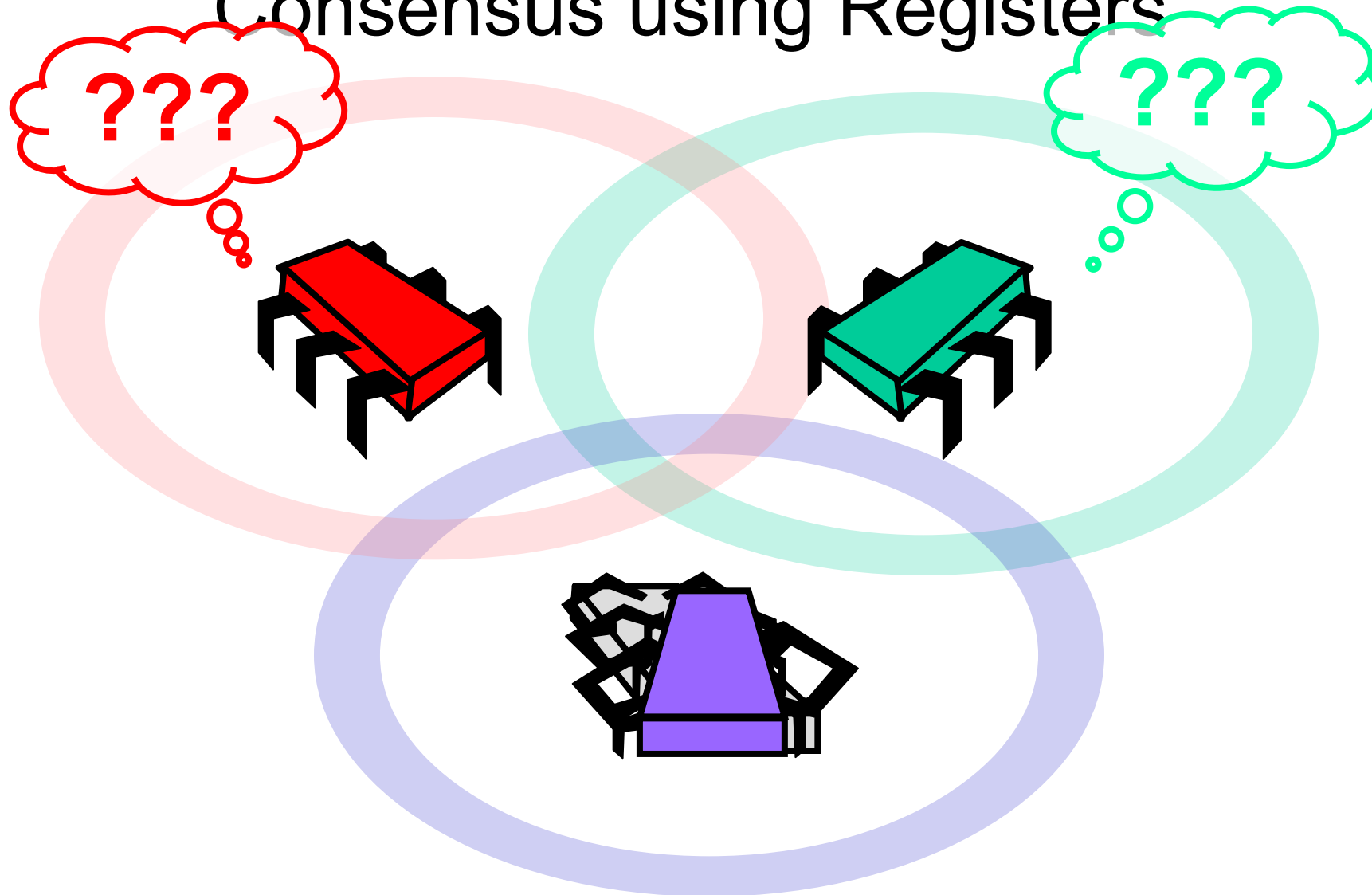
- Consistent:
 - all threads decide the same value
- Valid:
 - the common decision value is some thread's input

Let's play the consensus *Consensus Game*

- Two of you need to agree on a value, e.g., A or B
- You need to devise *a protocol to reach a consensus*
- Tell me *the maximal number of steps* for each thread (≤ 5 , please)
- We are going to communicate using the white board
- Rules: *either* reading or writing **one** register (not both)
- *No other communication,*
- **No priorities** in “thread” identities or values:
 - Either of the values can be chosen (non-triviality)
 - One of the thread's suggestions need to be chosen (validity)

Game

No Wait-Free Implementation of Consensus using Registers



Formally

- Theorem
 - There is no wait-free implementation of n -thread consensus from read-write registers
- Implication
 - Asynchronous computability different from Turing computability
- Proof
 - Using “valence trees” — see the textbook

Next lecture:
Solving n-thread Consensus

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