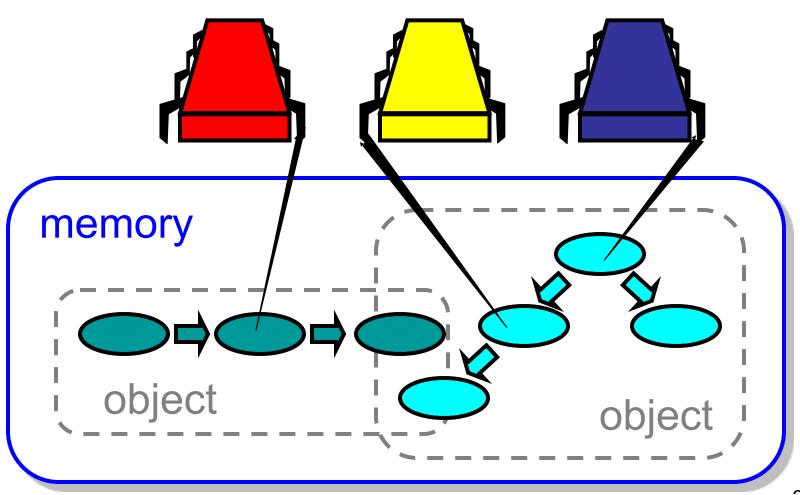
## YSC3248: Parallel, Concurrent and Distributed Programming

Concurrent Objects

#### **Concurrent Computation**



#### Objectivism

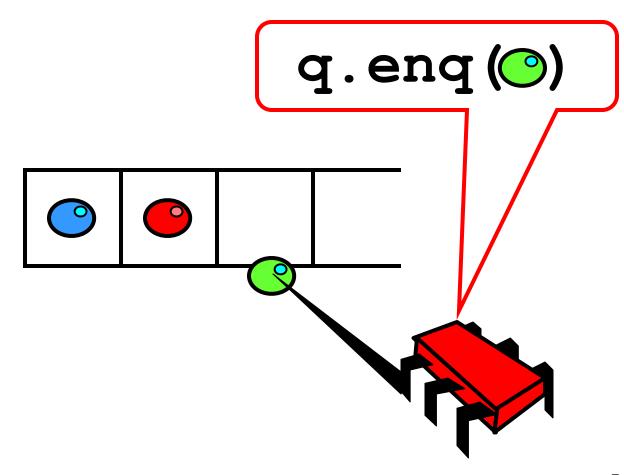
- What is a concurrent object?
  - How do we describe one?
  - How do we implement one?
  - How do we tell if we're right?

#### Objectivism

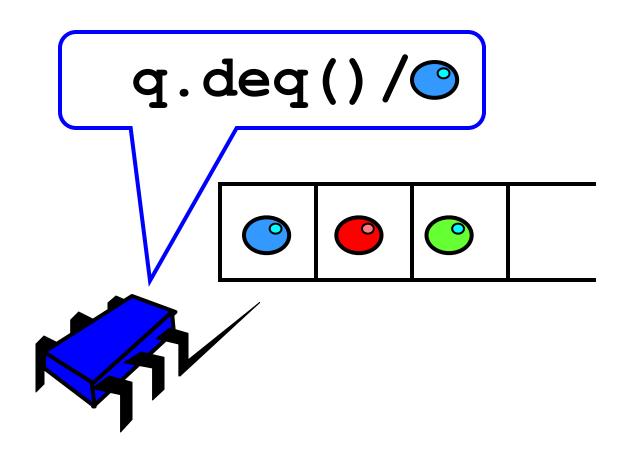
- What is a concurrent object?
  - How do we describe one?

– How do we tell if we're right?

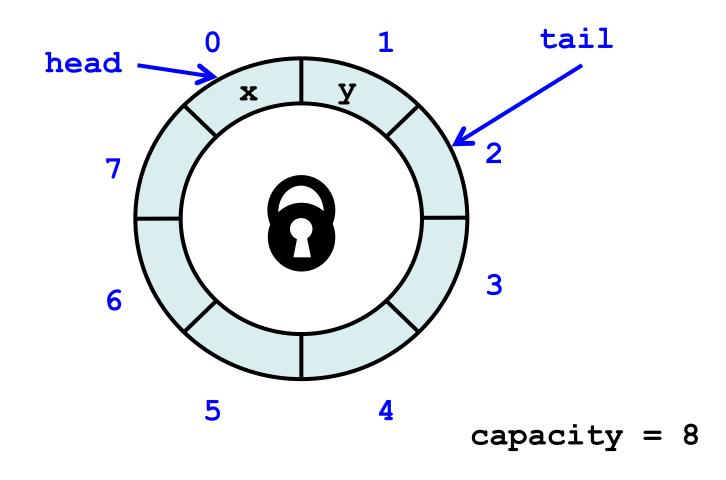
#### FIFO Queue: Enqueue Method



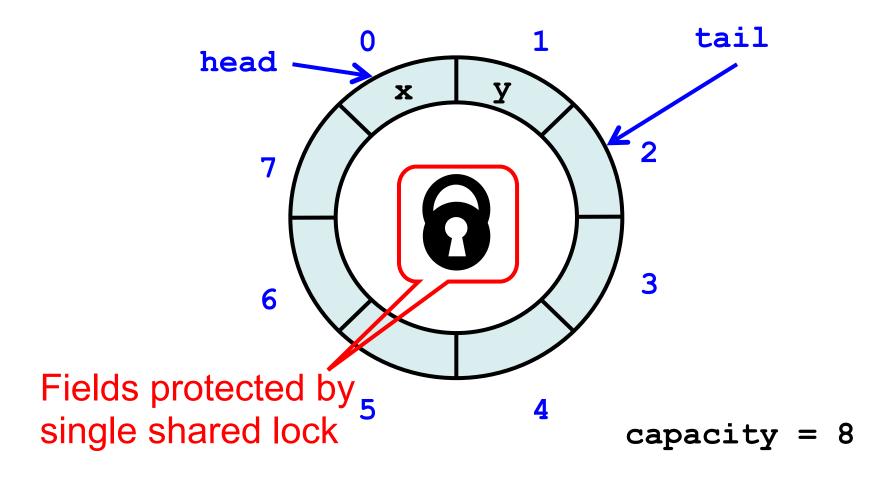
#### FIFO Queue: Dequeue Method



#### Lock-Based Queue



#### Lock-Based Queue



#### A Lock-Based Queue

```
class LockBasedQueue[T: ClassTag]
  (val capacity: Int) extends MyQueue[T] {
  private var head, tail: Int = 0
  private val items = new Array[T] (capacity)
  private val myLock = new ReentrantLock()
```

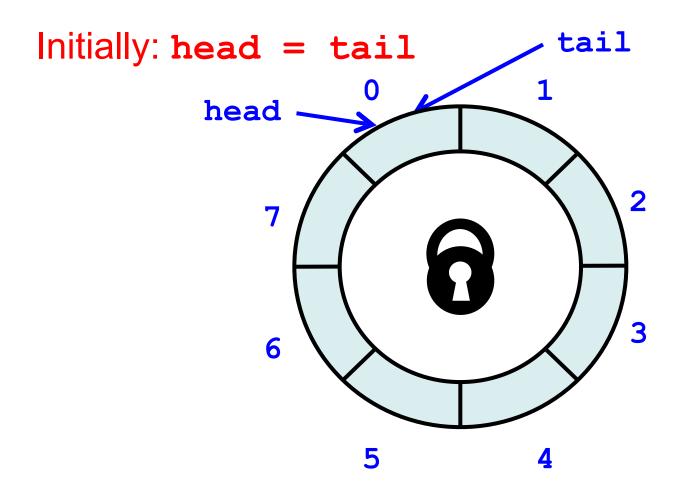
#### A Lock-Based Queue

```
class LockBasedQueue[T: ClassTag]
  (val capacity: Int) extends MyQueue[T] {

  private var head, tail: Int = 0
  private val items = new Array[T](capacity)
  private val myLock = new ReentrantLock()
```

Fields protected by single shared lock

#### Lock-Based Queue



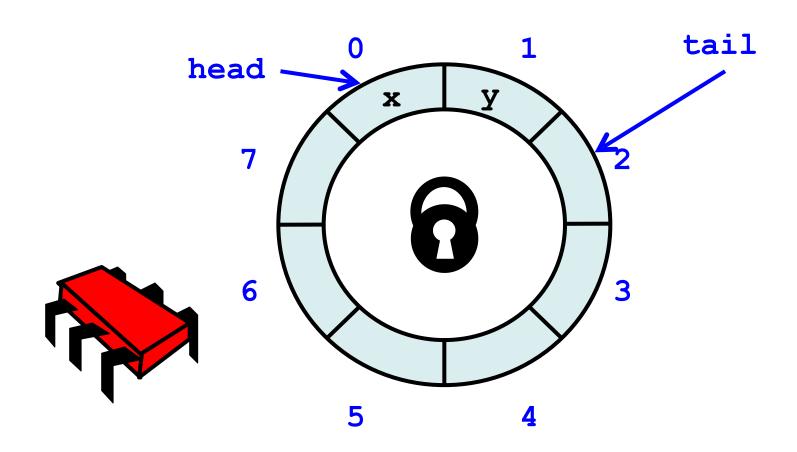
#### Lock-Based Queue

```
class LockBasedQueue[T: ClassTag]
  (val capacity: Int) extends MyQue

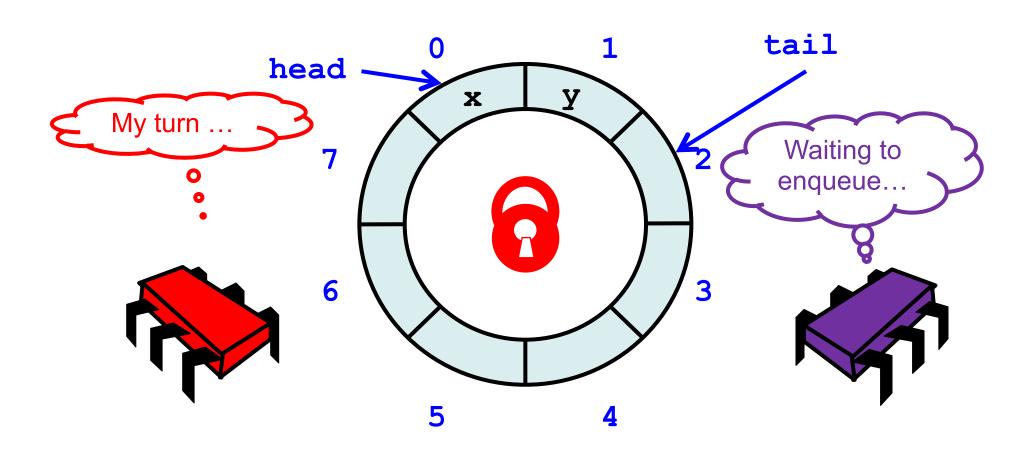
private var head, tail: Int = 0
  private val items = new Array[T] (capacity)
  private val myLock = new ReentrantLock()
```

Initially head = tail

#### Lock-Based deq()



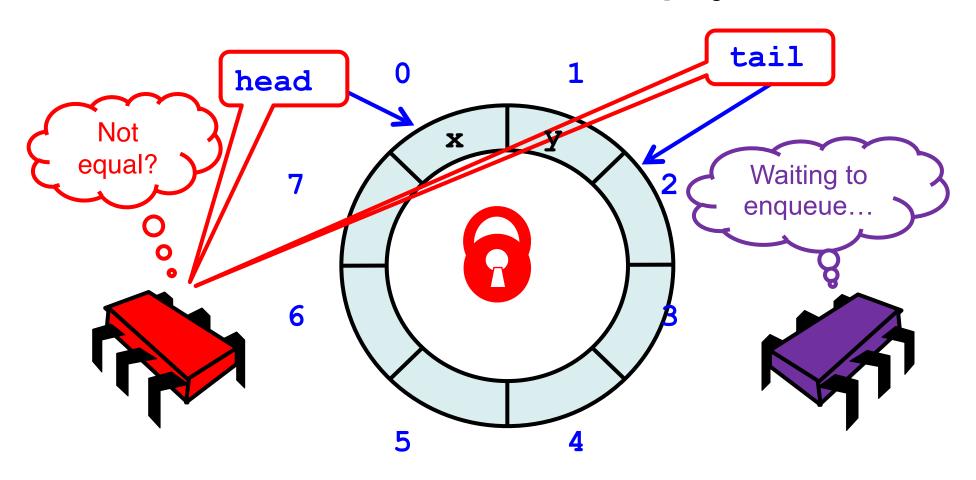
#### Acquire Lock



```
def deq() : T = {
  myLock.lock()
  try {
    if (tail == head) {
      throw EmptyException
    val x = items(head % items.length)
    head = head + 1
                                                   tail
                                        head
    X
    finally {
                                    capacity-1
    myLock.unlock()
```

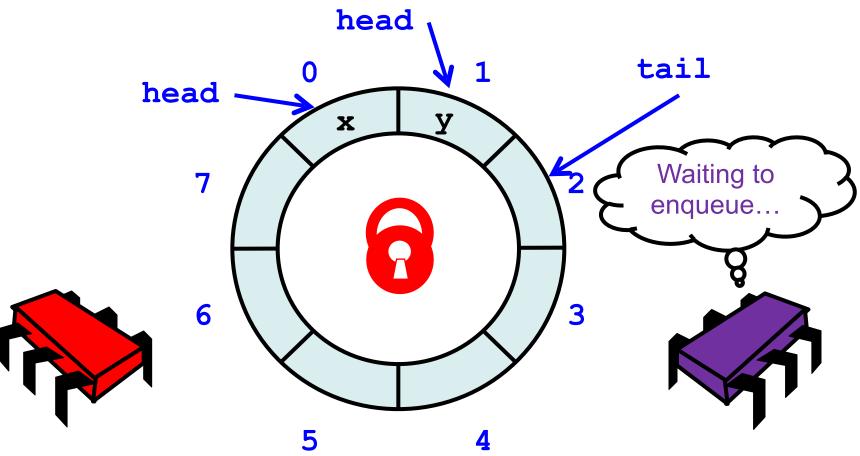
```
myLock.lock()
                                  Acquire lock at
                                   method start
  if (tail == head) {
    throw EmptyException
  val x = items(head % items.length)
  head = head + 1
                                     head
                                                tail
  finally {
                                  capacity-1
  myLock.unlock()
```

#### Check if Non-Empty



```
def deq() : T = \{
  myLock.lock()
    if (tail == head) {
      throw EmptyException
    val x = items (head % items.length)
    head = head + 1
                                                    tail
                                          head
    finally {
                                      capacity-1
    myLock.unlock
              If queue empty
             throw exception
```

### Modify the Queue head



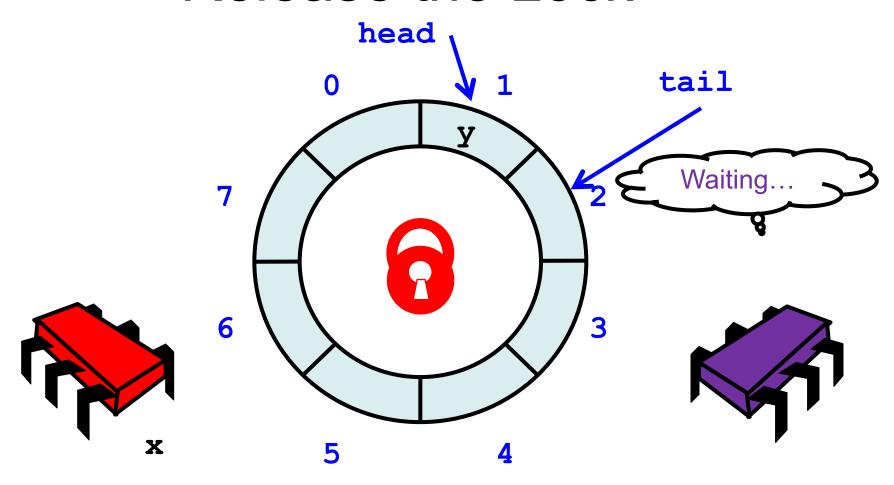
```
def deq() : T = \{
  myLock.lock()
  try {
    if (tail == head) {
      throw EmptyException
    val x = items(head % items.length)
    head = head + 1
                                           head
                                                      tail
    finally {
                                        capacity-1
    myLock.unlock()
             Queue not empty?
      Remove item and update head
```

```
def deq() : T = \{
  myLock.lock()
  try {
    if (tail == head) {
      throw EmptyException
    val x = items(head % items.length)
    head = head + 1
                                                         tail
                                              head
    finally {
                                          capacity-1
    my lock.unlock()
  Return result
```

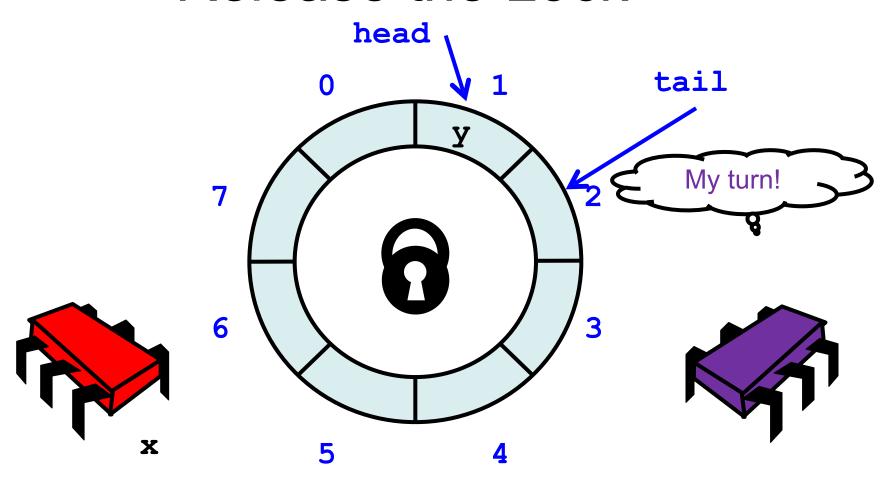
```
def deq() : T = \{
  myLock.lock()
  try {
    if (tail == head) {
      throw EmptyException
    val x = items(head % items.length)
    head = head + 1
                                                         tail
                                               head
    finally {
                                           capacity-1
    myLock.unlock()
             Release lock no
               matter what!
```

```
def deq() : T = \{
 myLock.lock()
  try {
    if (tail == head) {
      throw EmptyException
    val x = items(head % items.length)
                  modifications are mutually exclusive...
                  Should be correct because
    head = head + 1
    Χ
   finally {
    myLock.unlock()
```

#### Release the Lock



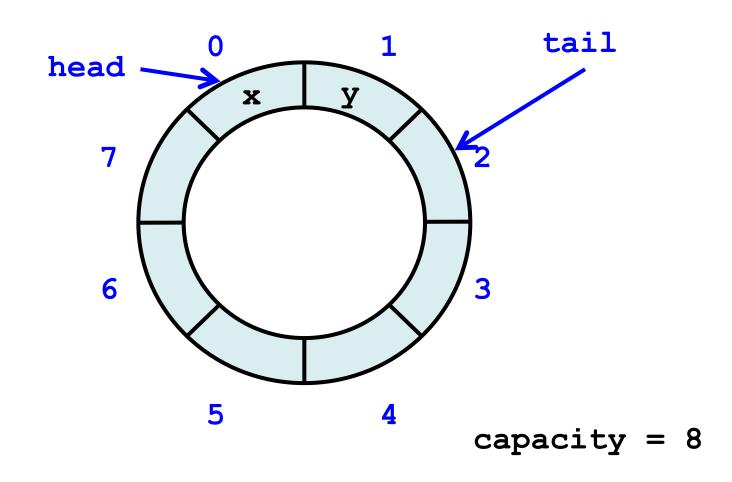
#### Release the Lock

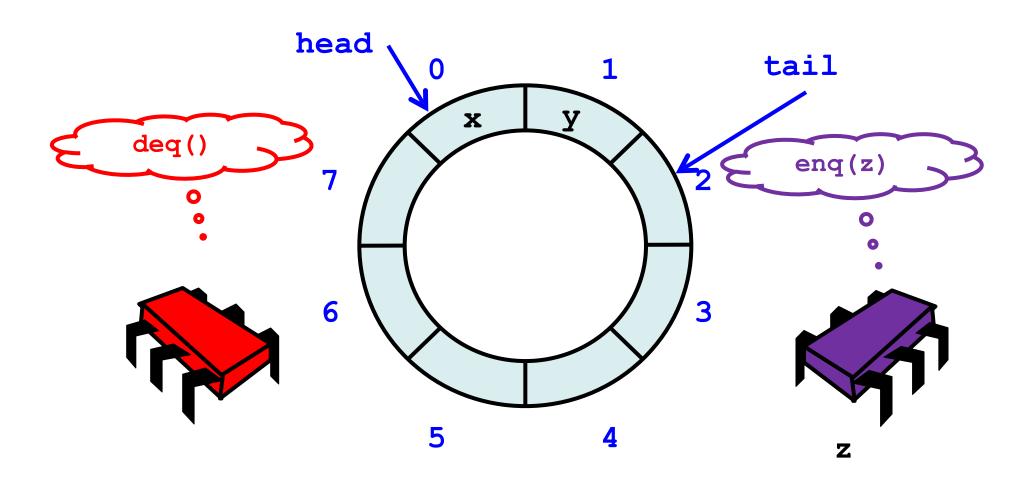


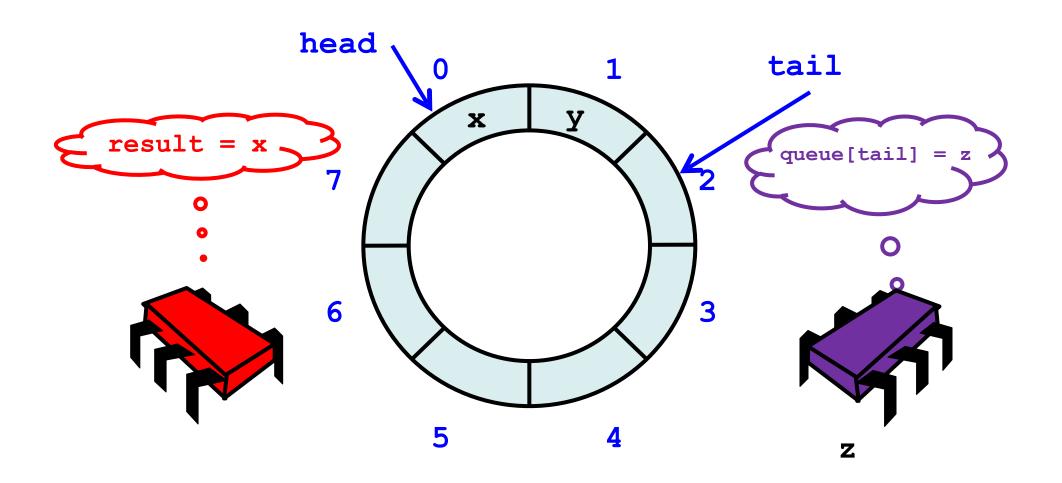
# Demo: Testing SWSR Queue

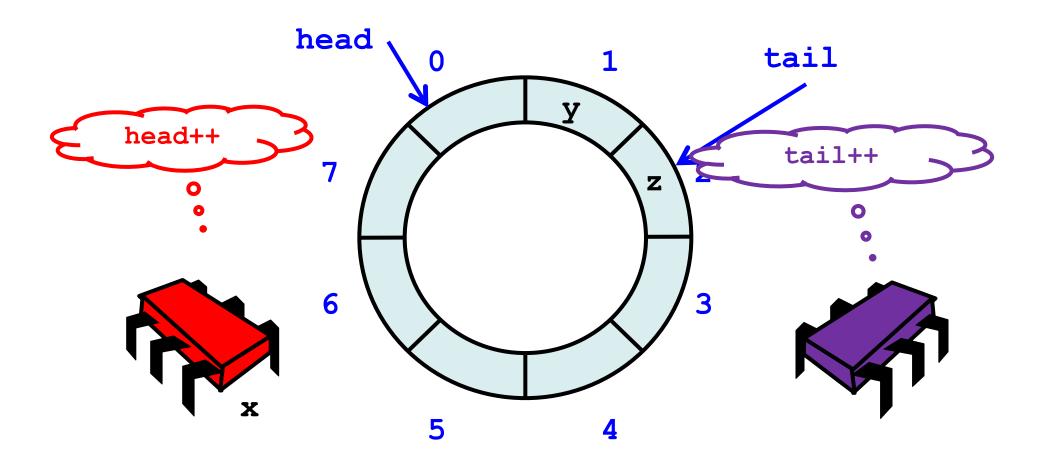
#### Now consider the following implementation

- The same thing without mutual exclusion
- For simplicity, only two threads
  - One thread enq only
  - The other deq only









```
class LockFreeQueue[T: ClassTag] (val capacity: Int) {
                                                        capacity-1
  @volatile
 private var head, tail: Int = 0
  private val items = new Array[T] (capacity)
  def enq(x: T): Unit = {
    if (tail - head == items.length) throw FullException
    items(tail % items.length) = x
    tail = tail + 1
  def deq(): T = {
    if (tail == head) throw EmptyException
    val x = items(head % items.length)
   head = head + 1
    X
```

head

```
head
                                                                        tail
class LockFreeQueue[T: ClassTag] (val capacity: Int) {
                                                        capacity-1
  @volatile
 private var head, tail: Int = 0
 private val items = new Array[T] (capacity)
  def eng(x: T): Unit = {
   if (tail - head == items.length) throw FullException
   items(tail % items.length) = x
    tail = tail + 1
                                    No lock needed!
 def deq(): T = {
   val x = items(head % items.length)
   head = head + 1
                                                           33
```

```
class LockFreeQueue[T: ClassTag] (val capacity: Int) {
 @volatile
 private var head, tail: Int = 0
 private val items = new Array[T] (capacity)
 def enq(x: T): Unit = {
   if (tail - head == items.length) throw FullException
   items(tail % items.length) = x
   tail = tail + 1
                           modifications are not mutually exclusive?
                          How do we define "correct" when
 def deq(): T = {
   if (tail == head) throw EmptyException
   val x = items(head % items.length
   head = head + 1
   Χ
```

#### What is a Concurrent Queue?

- Need a way to specify a concurrent queue object
- Need a way to prove that an algorithm implements the object's specification
- Lets talk about object specifications ...

#### Correctness and Progress

- In a concurrent setting, we need to specify both the safety and the liveness properties of an object
- Need a way to define
  - when an implementation is correct
  - the conditions under which it guarantees progress

Lets begin with correctness

## Sequential Objects

- Each object has a state
  - Usually given by a set of fields
  - Queue example: sequence of items
- Each object has a set of methods
  - Only way to manipulate state
  - Queue example: enq and deq methods

## Sequential Specifications

- If (precondition)
  - the object is in such-and-such a state
  - before you call the method,
- Then (postcondition, result)
  - the method will return a particular value
  - or throw a particular exception,
- and (postcondition, state)
  - the object will be in some other state
  - when the method returns

#### Pre and PostConditions for Dequeue

- Precondition:
  - Queue is non-empty
- Postcondition (result):
  - Returns first item in queue
- Postcondition (state):
  - Removes first item in queue

#### Pre and PostConditions for Dequeue

- Precondition:
  - Queue is empty
- Postcondition (result):
  - Throws Empty exception
- Postcondition (state):
  - Queue state unchanged

#### Why Sequential Specifications Totally Rock

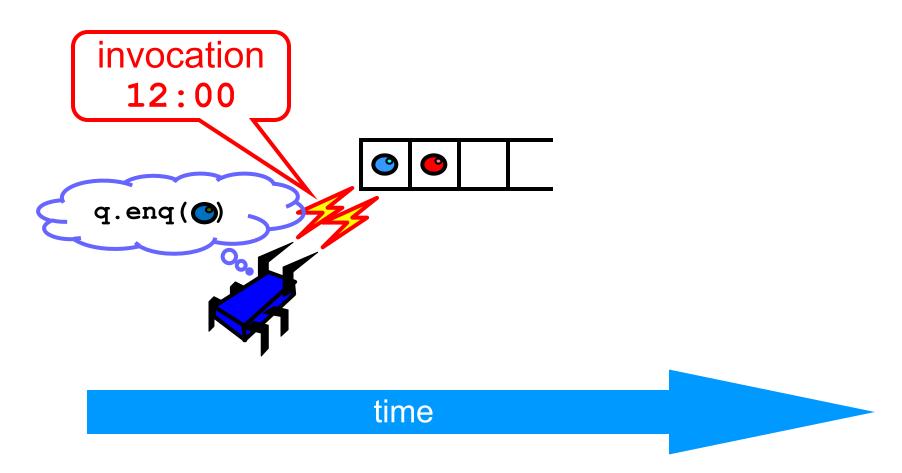
- Interactions among methods captured by side-effects on object state
  - State meaningful between method calls
- Documentation size linear in number of methods
  - Each method described in isolation
- Can add new methods
  - Without changing descriptions of old methods

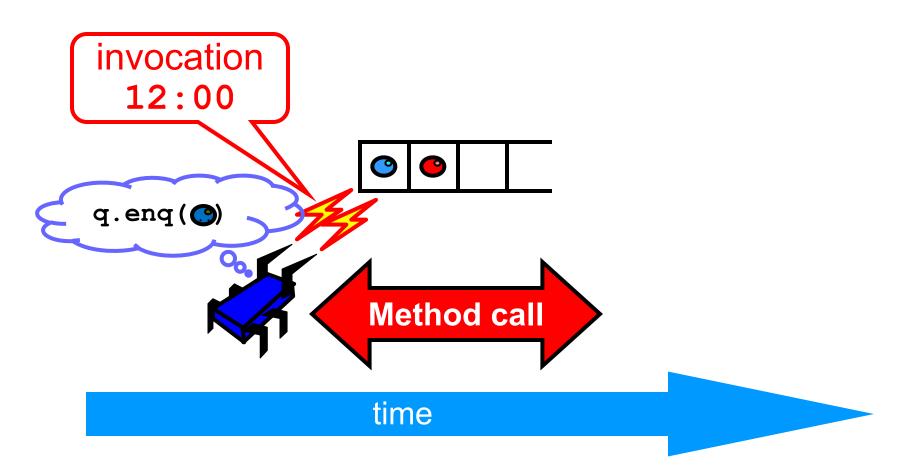
### What About Concurrent Specifications?

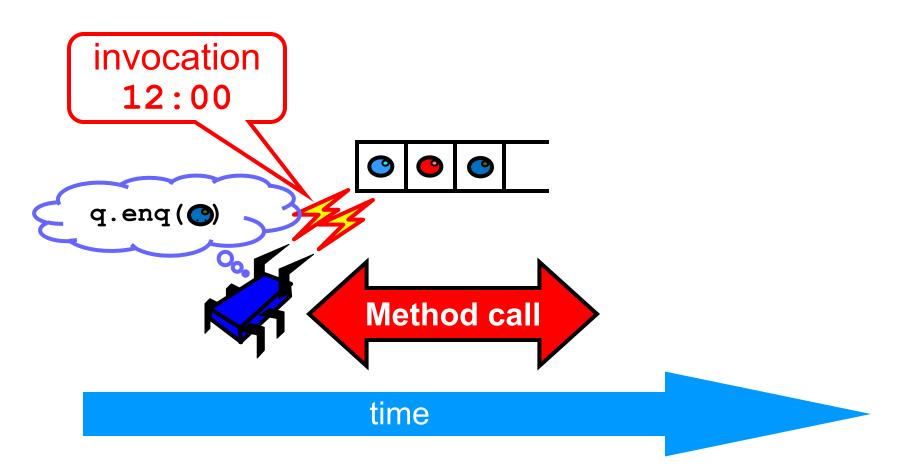
- Methods?
- Documentation?
- Adding new methods?

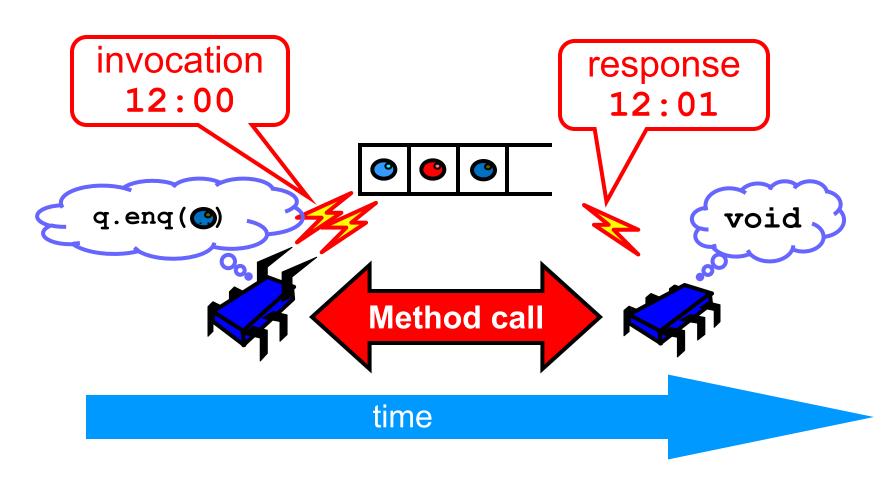


time





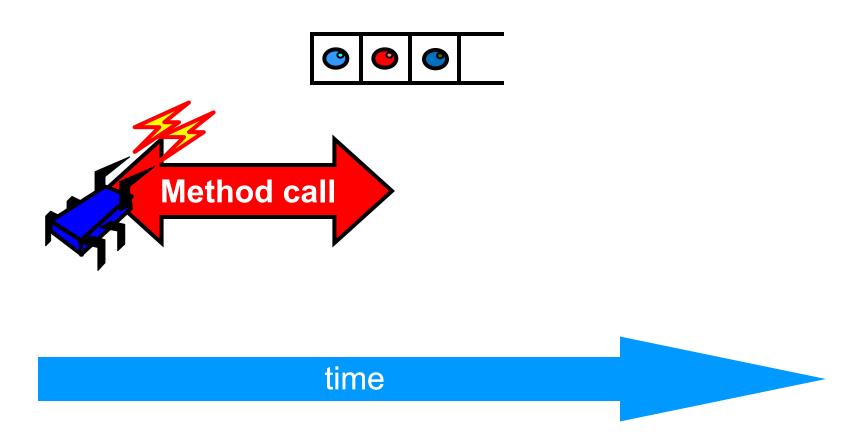


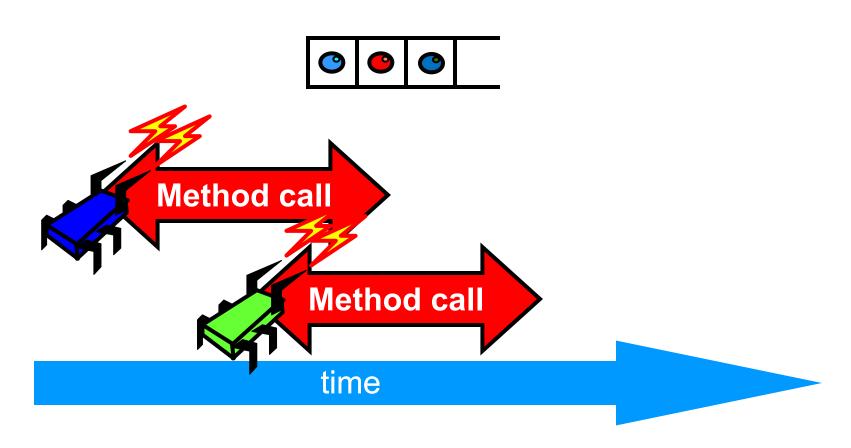


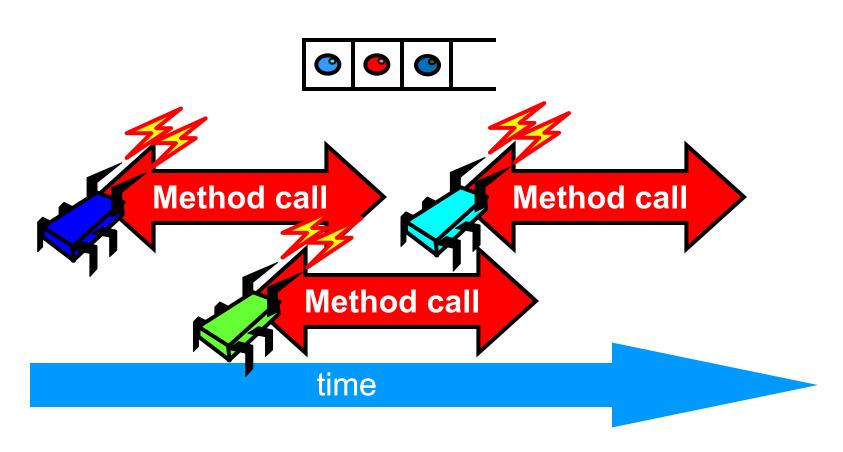
- Sequential
  - Methods take time? Who knew?
- Concurrent
  - Method call is not an event
  - Method call is a sequence of interval events.



time







- Sequential:
  - Object needs meaningful state only between method calls
- Concurrent
  - Because method calls overlap,
     object might *never* be between method calls

- Sequential:
  - Each method described in isolation
- Concurrent
  - Must characterize all possible interactions with concurrent calls
    - What if two enq() calls overlap?
    - Two deq() calls? enq() and deq()?...

- Sequential:
  - Can add new methods without affecting older methods
- Concurrent:
  - Everything can potentially interact with everything else

- Sequential:
  - Can add new methods without affecting older methods
- Concurrent:

Everything can potentially interact with everything else

## The Big Question

- What does it mean for a concurrent object to be correct?
  - What is a concurrent FIFO queue?
  - FIFO means strict temporal order
  - Concurrent means ambiguous temporal order

## Intuitively...

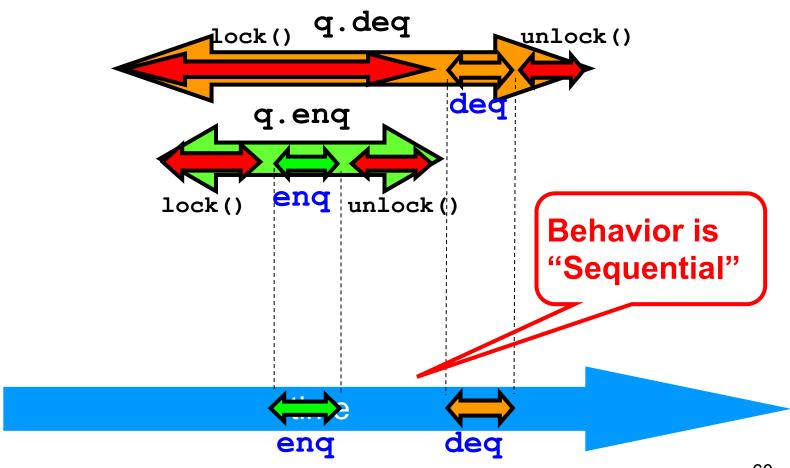
```
def deq() : T = {
 myLock.lock()
  try {
    if (tail == head) {
      throw EmptyException
    val x = items(head % items.length)
    head = head + 1
    X
  } finally {
    myLock.unlock()
```

## Intuitively...

```
def deg() : T =
 myLock.lock()
    if (tail
                head)
      throw EmptyException
    val x = items(he
                       % items.length)
    head = head + 1
                        All queue modifications
    finally
                         are mutually exclusive
   myLock.unlock()
```

#### براميرنانيواير

Lets capture the idea of describing the concurrent via the sequential



## Linearizability

- Each method should
  - "take effect"
  - Instantaneously
  - Between invocation and response events
- Object is correct if this "sequential" behavior is correct
- Any such concurrent object is
  - Linearizable™

## Is it really about the object?

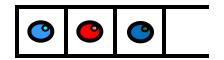
- Each method should
  - "take effect"
  - Instantaneously
  - Between invocation and response events
- Sounds like a property of an execution...
- A linearizable object: one all of whose possible executions are linearizable

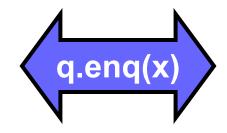
## Proving execution linearizable

- Identify "linearization points"
  - Between invocation and response events
  - Correspond to the effect of the call
  - "Justify" the whole execution
- Multiple ways to identify linearization points exist
- If none found, execution is *non-linearizable*

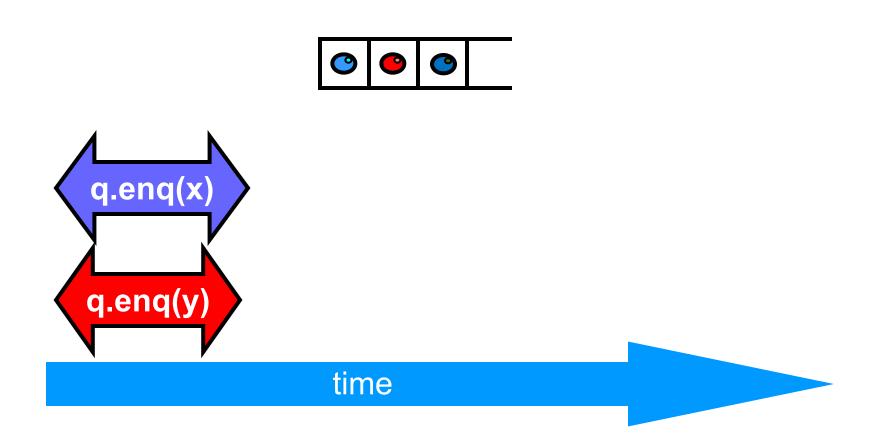


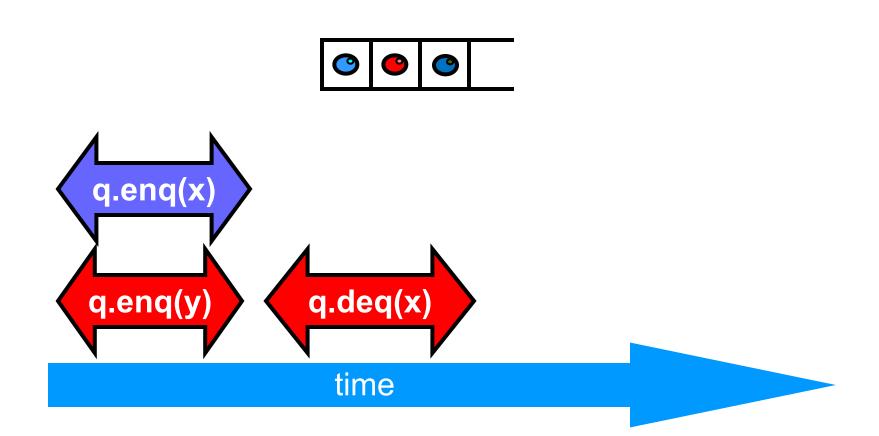
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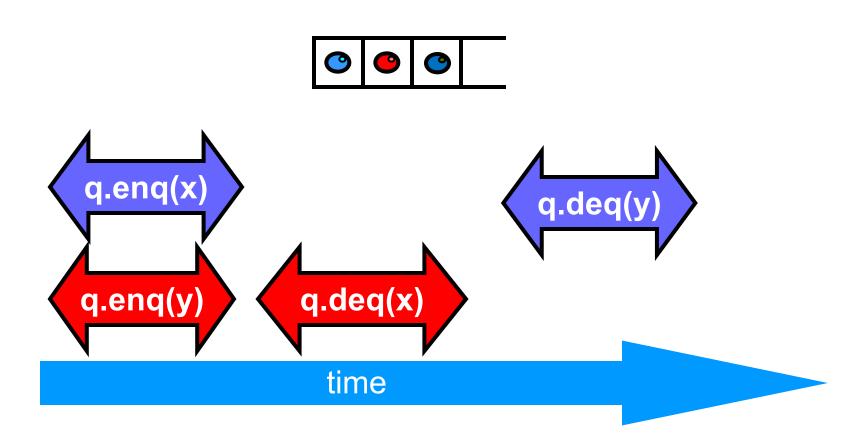


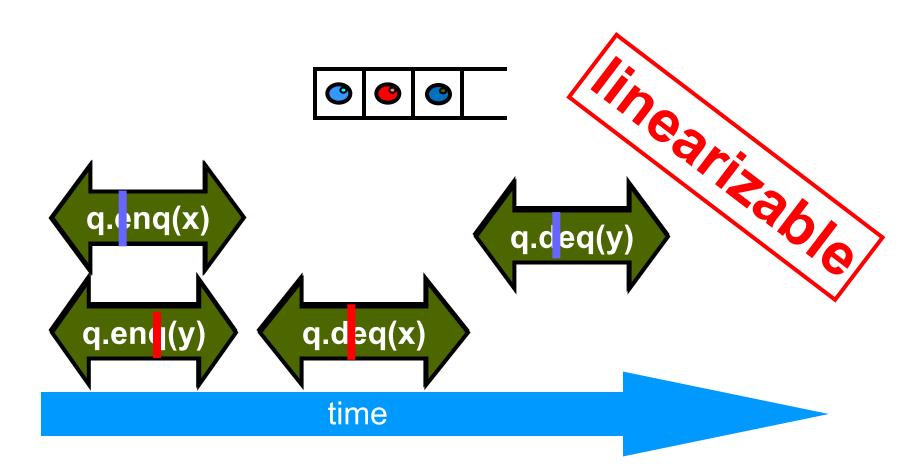
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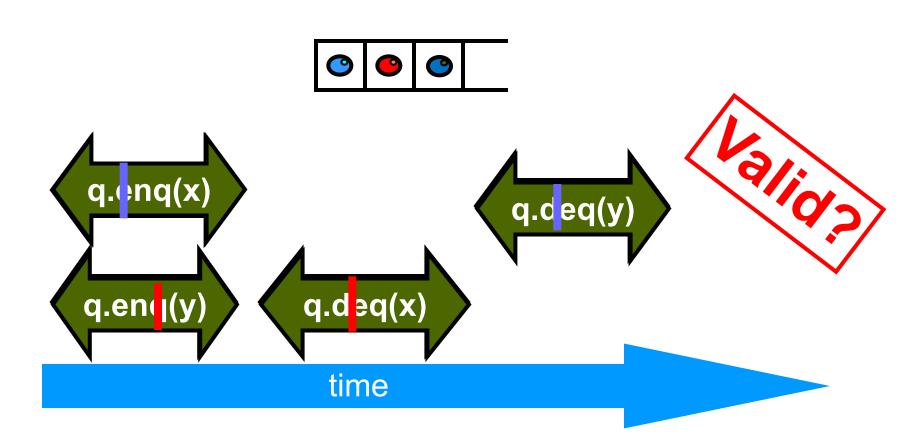






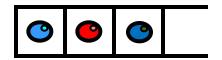


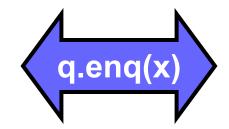




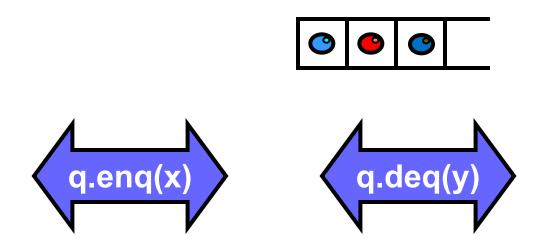


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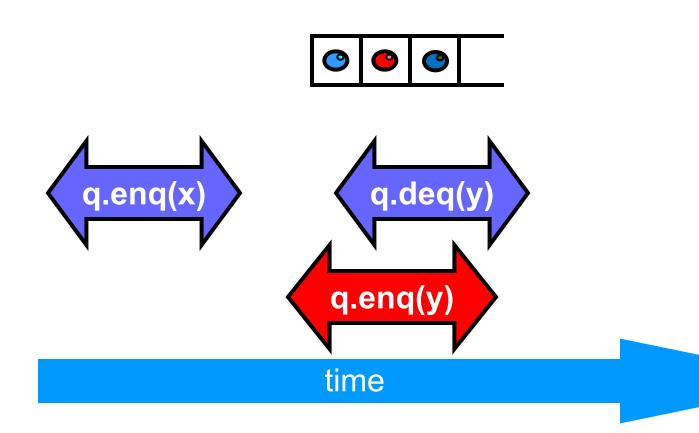


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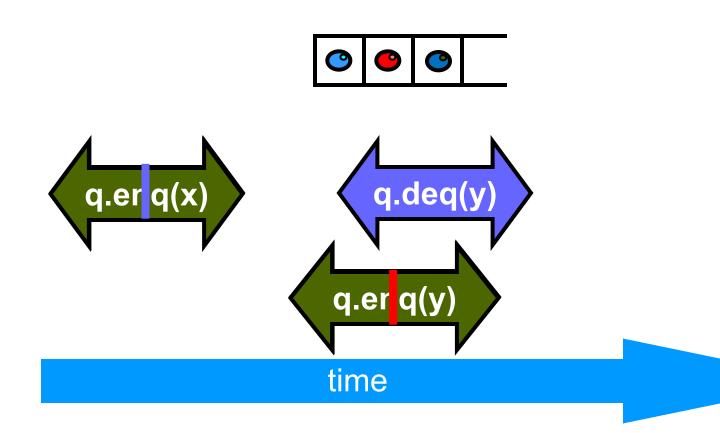


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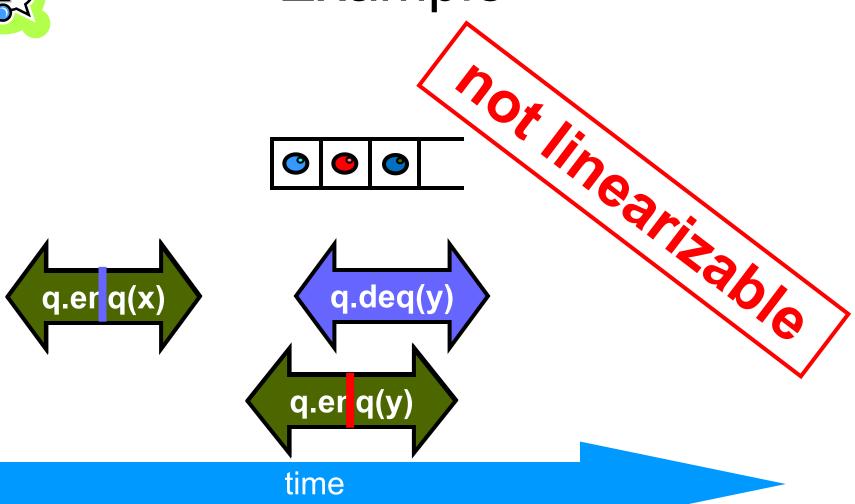






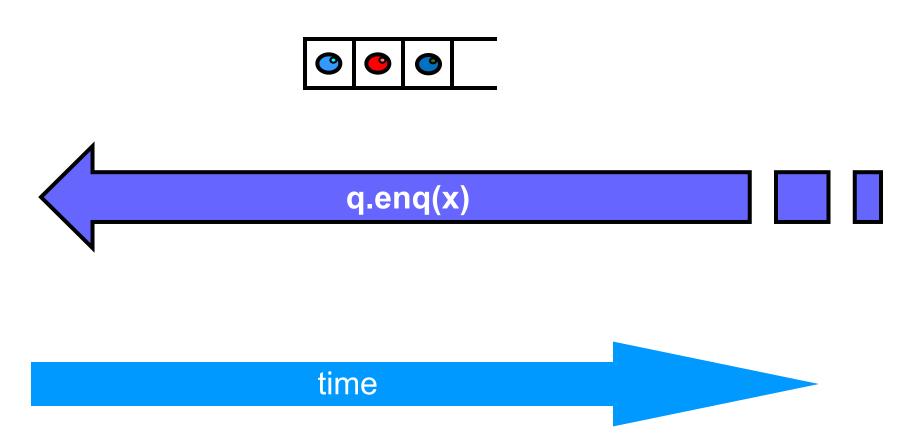




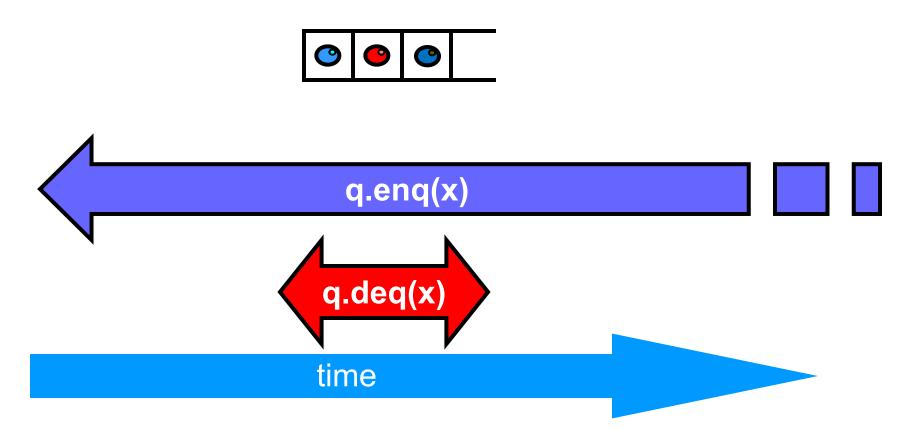




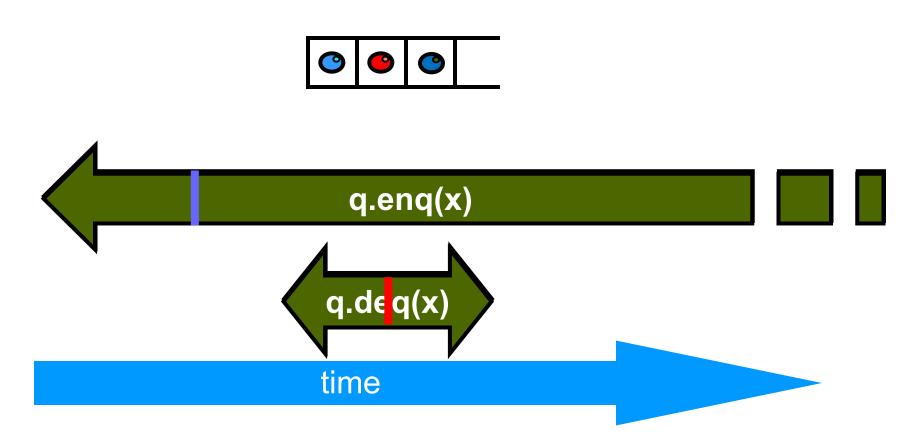
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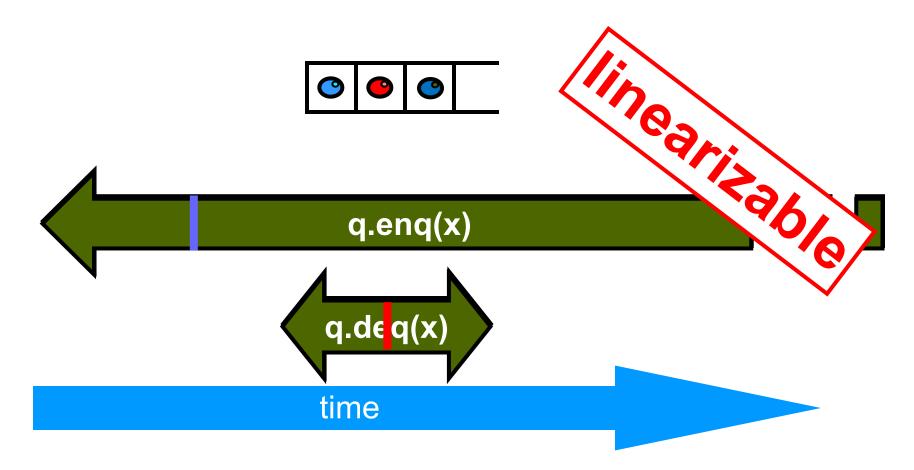


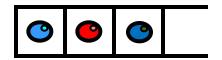


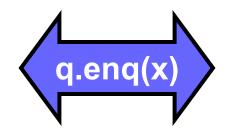




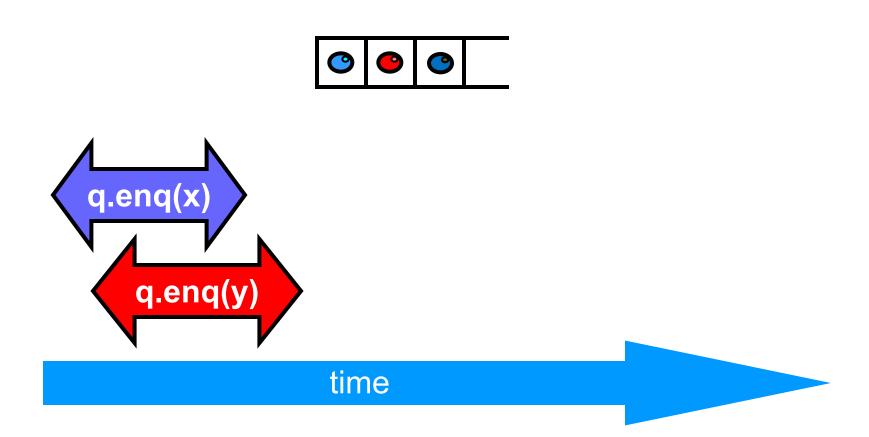


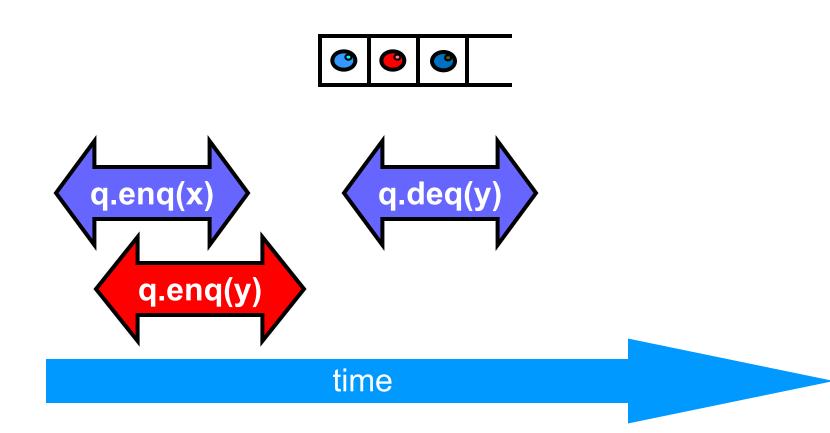




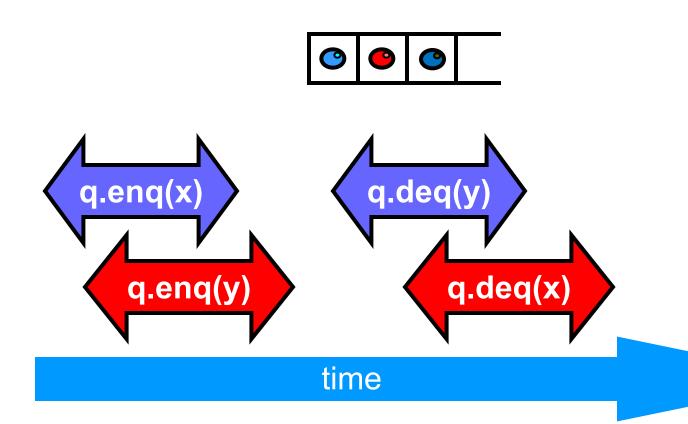


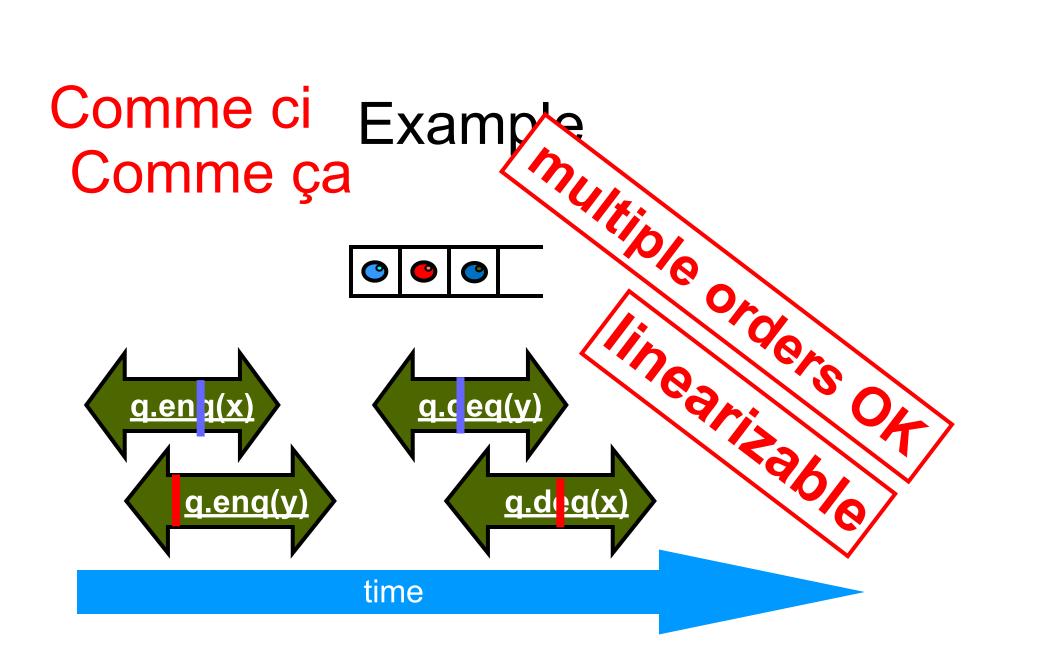
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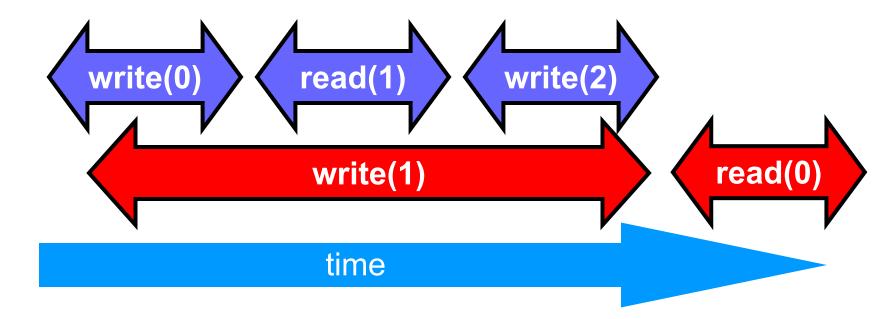


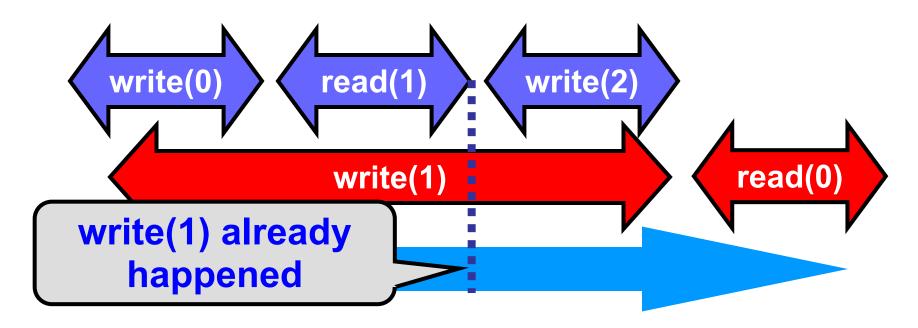


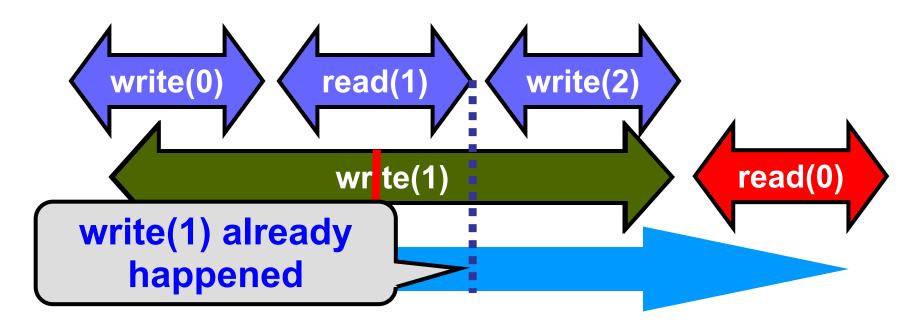


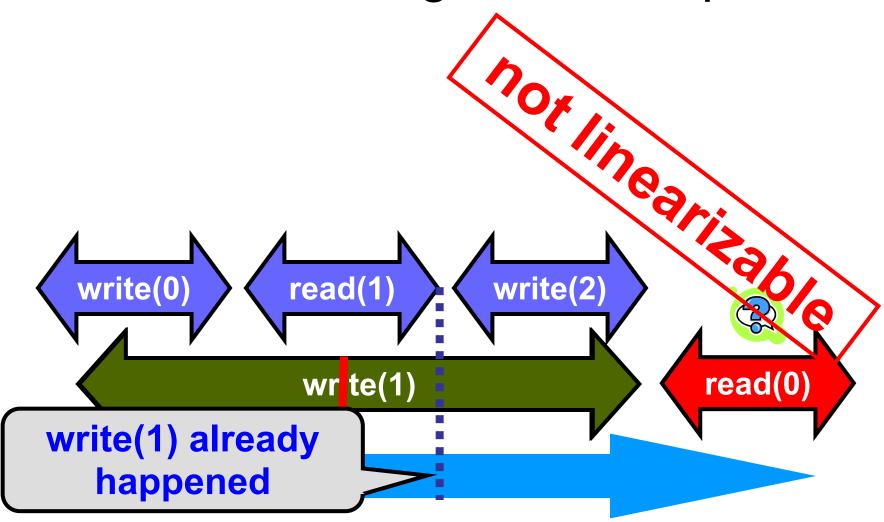


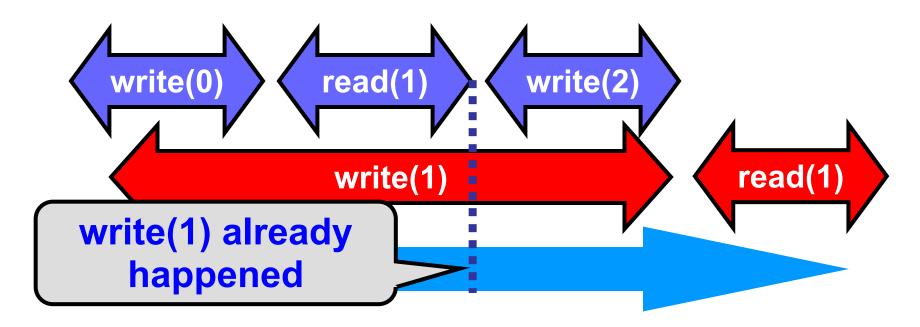


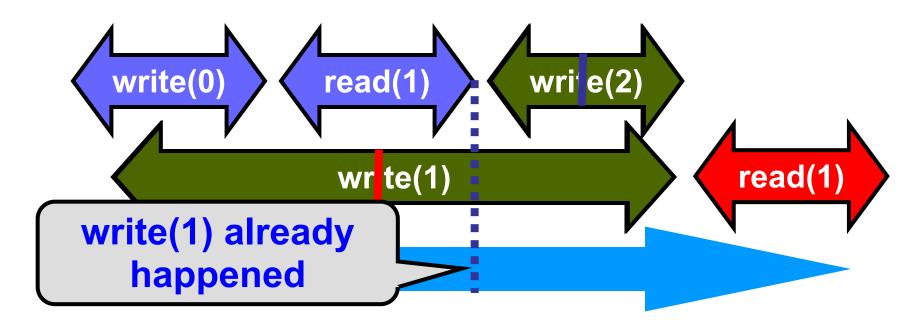


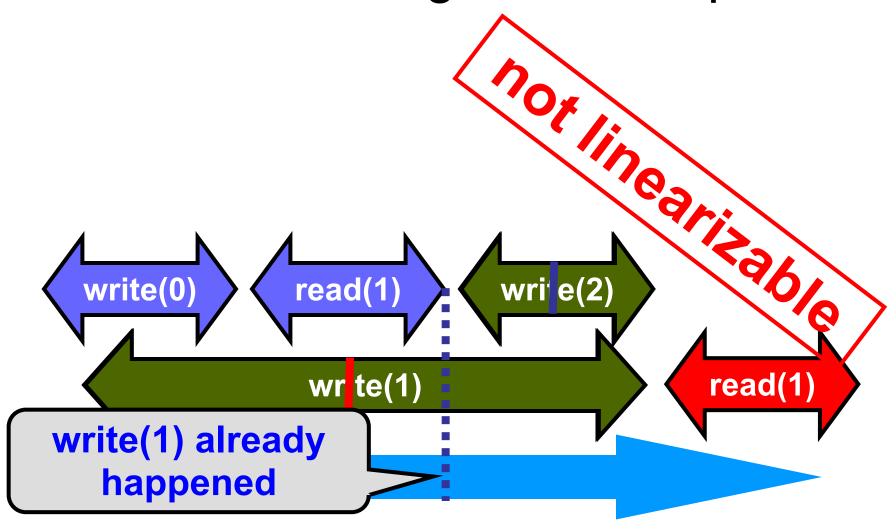


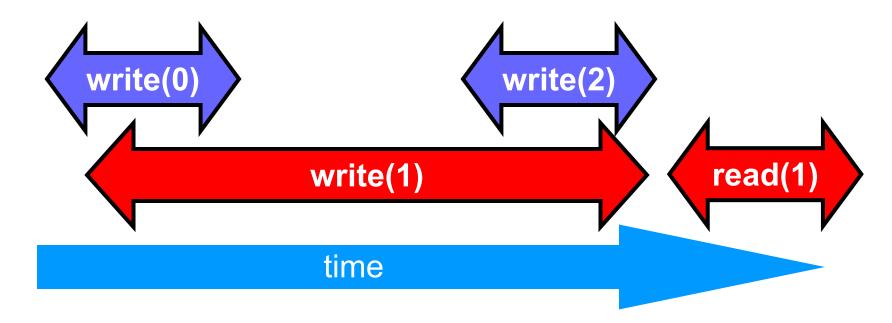


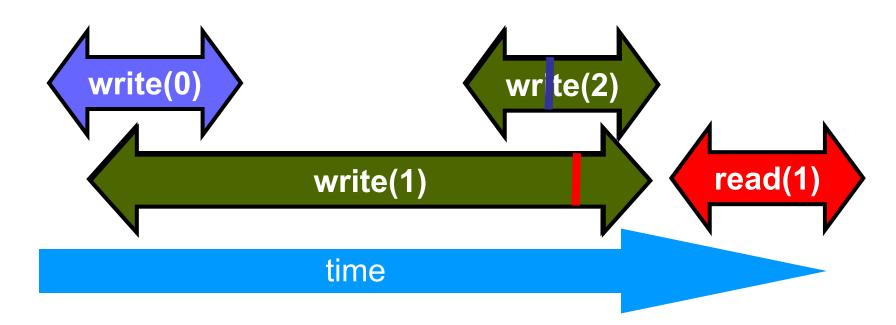


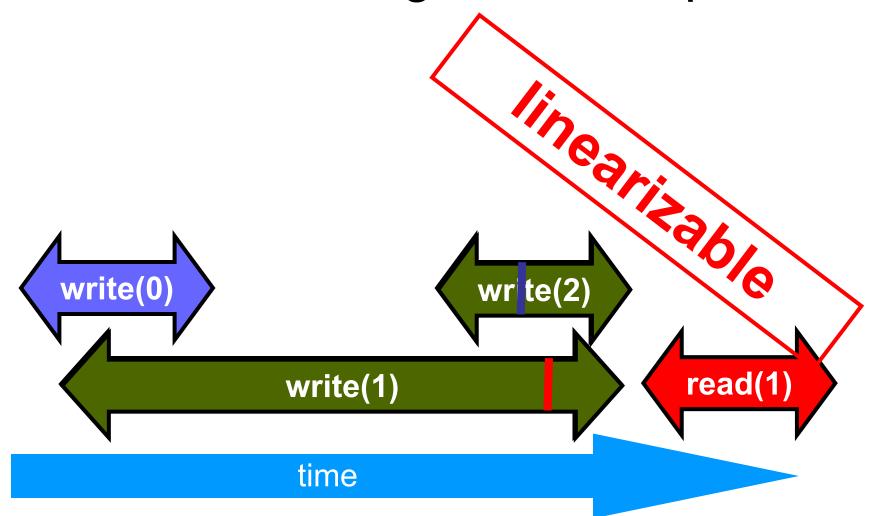












### Talking About Executions

- Why?
  - Can't we specify the linearization point of each operation without describing an execution?
- Not Always
  - In some cases, linearization point depends on the execution

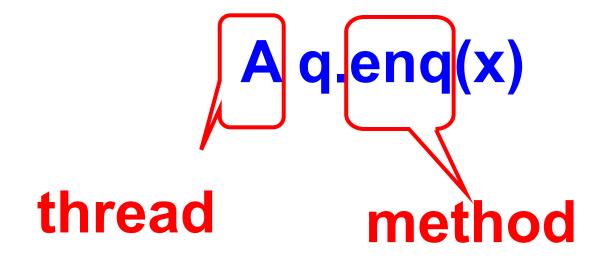
#### Formal Model of Executions

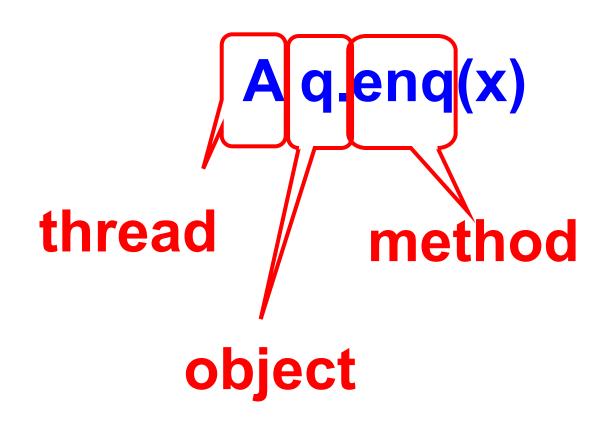
- Define precisely what we mean
  - Ambiguity is bad when intuition is weak
- Allow reasoning
  - Formal
  - But mostly informal
    - In the long run, actually more important

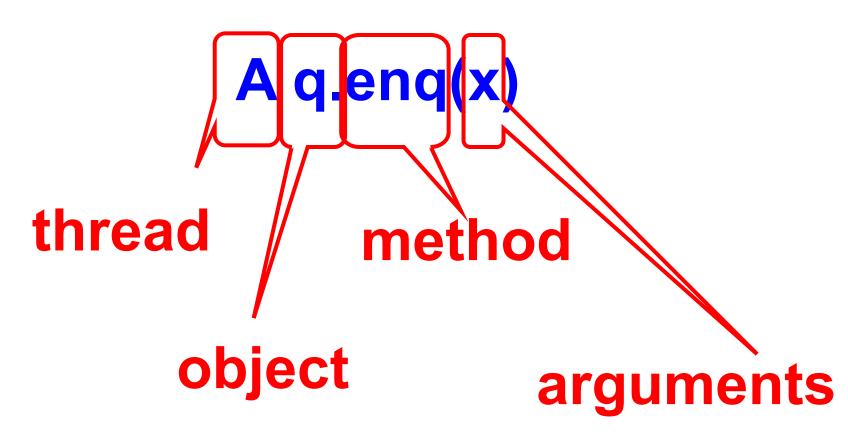
#### Split Method Calls into Two Events

- Invocation
  - method name & args
  - -q.enq(x)
- Response
  - result or exception
  - -q.enq(x) returns void
  - -q.deq() returns x
  - -q.deq() throws empty

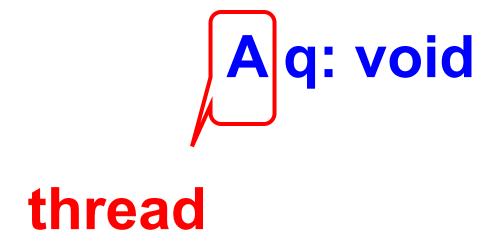
A q.enq(x)

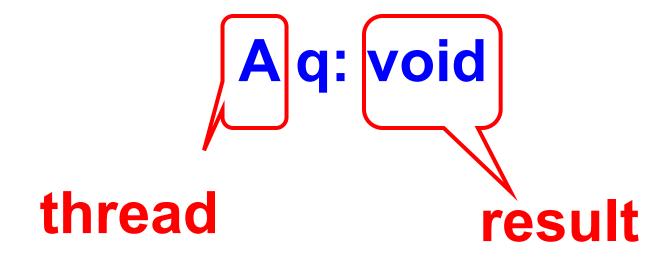


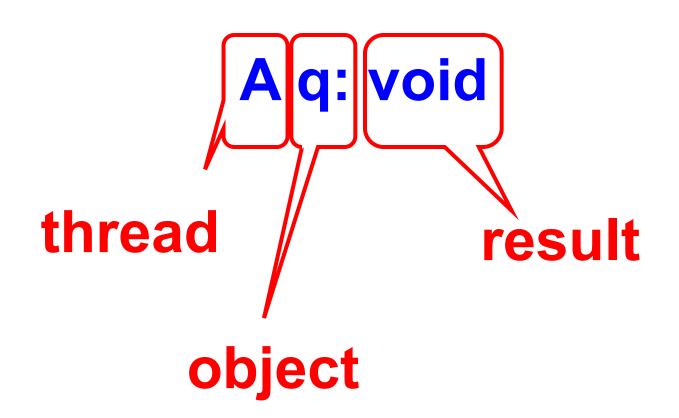




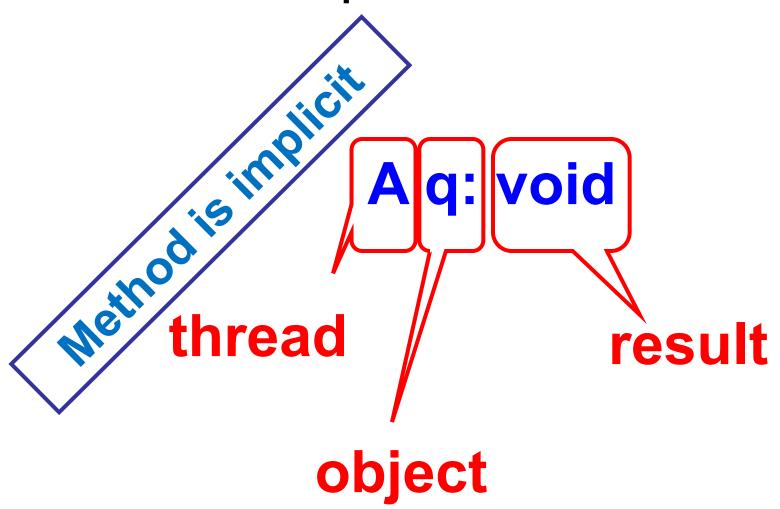
A q: void



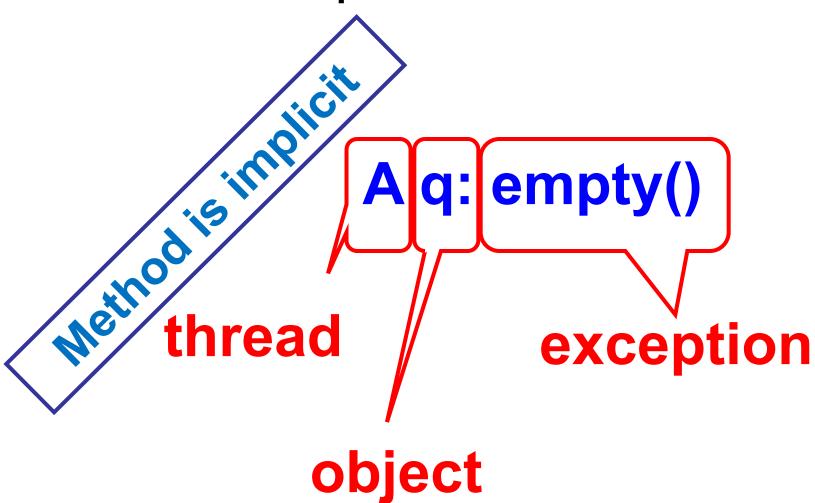




## Response Notation



## Response Notation

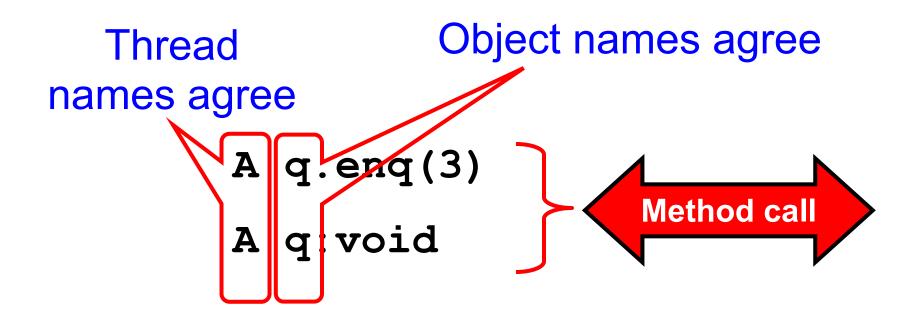


### History - Describing an Execution

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq()
B q:3
Sequence of invocations and responses
```

#### Definition

Invocation & response match if



## **Object Projections**

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

## **Object Projections**

```
A q.enq(3)
A q:void
H|q =
B q.deq()
B q:3
```

## **Thread Projections**

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

## **Thread Projections**

```
H|B = B p.enq(4)
B p:void
B q.deq()
B q:3
```

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq()
B q:3

An invocation is pending if it has no matching response
```

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq() May or may not have
B q:3 taken effect
```

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq()
B q:3 discard pending invocations
```

```
A q.enq(3)
A q:void

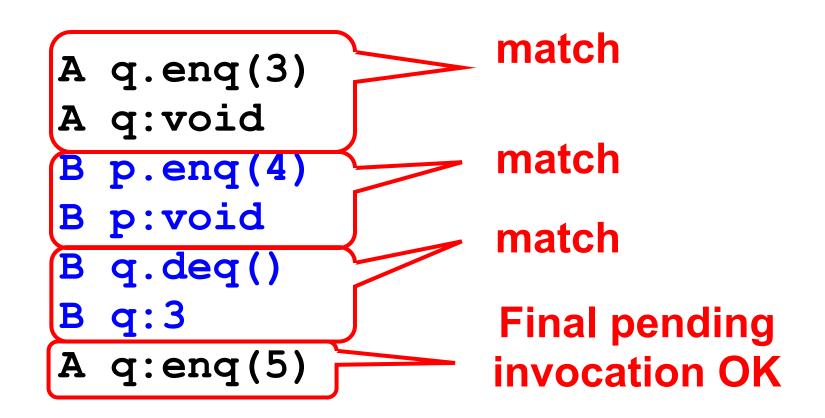
Complete(H) = B p.enq(4)
B p:void
B q.deq()
B q:3
```

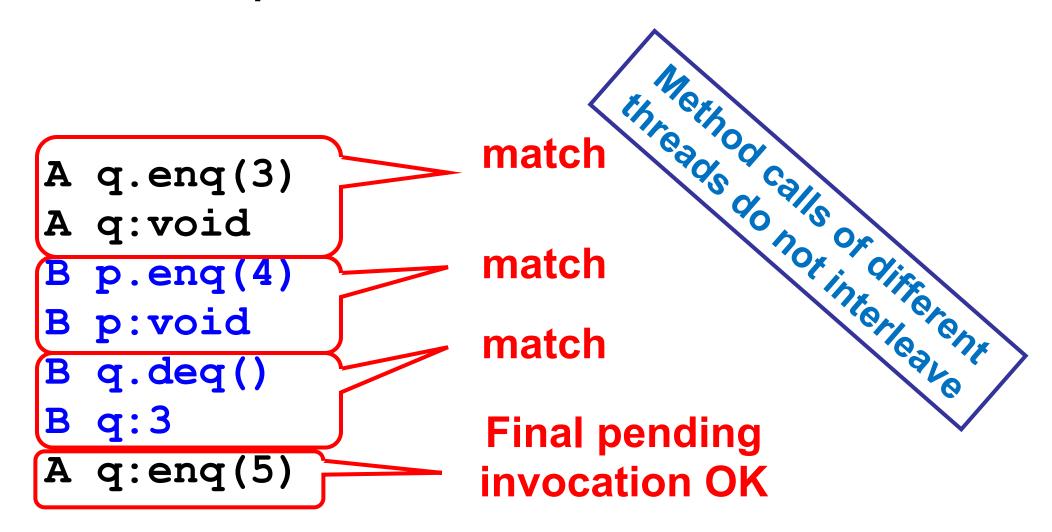
```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

```
match
A q.enq(3)
A q:void
 p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

```
match
A q.enq(3)
A q:void
                  match
 p.enq(4)
  p:void
 q.deq()
B q:3
A q:enq(5)
```

```
match
A q.enq(3)
A q:void
                   match
  p.enq(4)
  p:void
                   match
  q.deq()
  q:3
A q:enq(5)
```





#### Well-Formed Histories

```
A q.enq(3)
B p.enq(4)
B p:void
H= B q.deq()
A q:void
B q:3
```

#### Well-Formed Histories

```
Per-thread projections
sequential

A q.enq(3)
B p.enq(4)
B q.deq()
B p:void
H= B q.deq()
A q:void
B q:3
```

#### Well-Formed Histories

```
Per-thread projections
                          B p.enq(4)
     sequential
                    H|B= B p:void
                      B q.deq()
   A q.enq(3)
                          B q:3
    B p.enq(4)
    B p:void
H= B q.deq()
   A q:void
                    H|A= A q.enq(3)
A q:void
    B q:3
```

## **Equivalent Histories**

```
Threads see the same  \begin{cases} H \mid A = G \mid A \\ H \mid B = G \mid B \end{cases}  thing in both  H \mid B = G \mid B 
         A q.enq(3)
                                                                 A q.enq(3)
H= B p.enq(4)
B p:void
B q.deq()
A q:void
                                                                 A q:void
                                                                B p.enq(4)
B p:void
B q.deq()
```

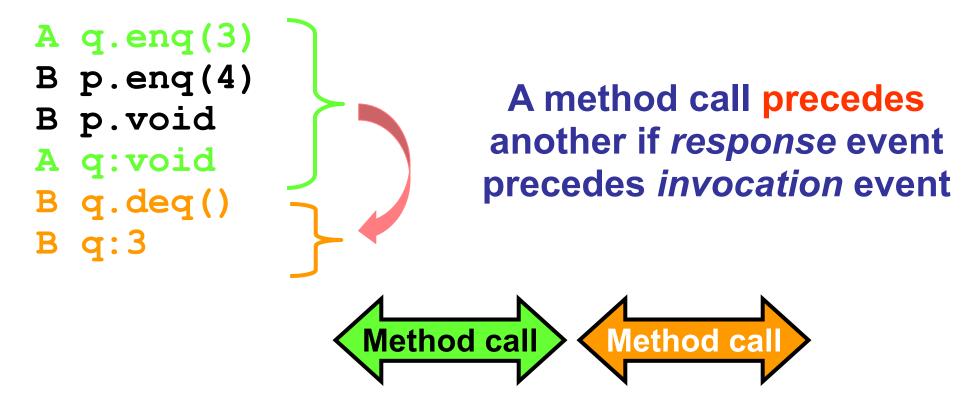
## Sequential Specifications

- A sequential specification is some way of telling whether a
  - Single-thread, single-object history
  - Is legal
- For example:
  - Pre and post-conditions
  - But plenty of other techniques exist ....

## Legal Histories

- A sequential (multi-object) history H is legal if
  - For every object x
  - H|x is in the sequential spec for x
  - Not talking about threads now!

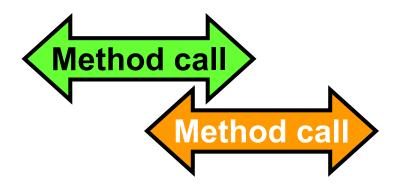
#### Precedence



#### Non-Precedence

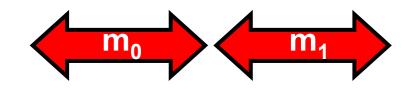
```
A q.enq(3)
B p.enq(4)
B p.void
B q.deq()
A q:void
B q:3
```

# Some method calls overlap one another



#### **Notation**

- Given
  - History H
  - method executions m₀ and m₁ in H
- We say  $m_0 \rightarrow_H m_1$ , if
  - m<sub>0</sub> precedes m<sub>1</sub>
- Relation  $m_0 \rightarrow_H m_1$  is a
  - Partial order
  - Total order if H is sequential



## Linearizability

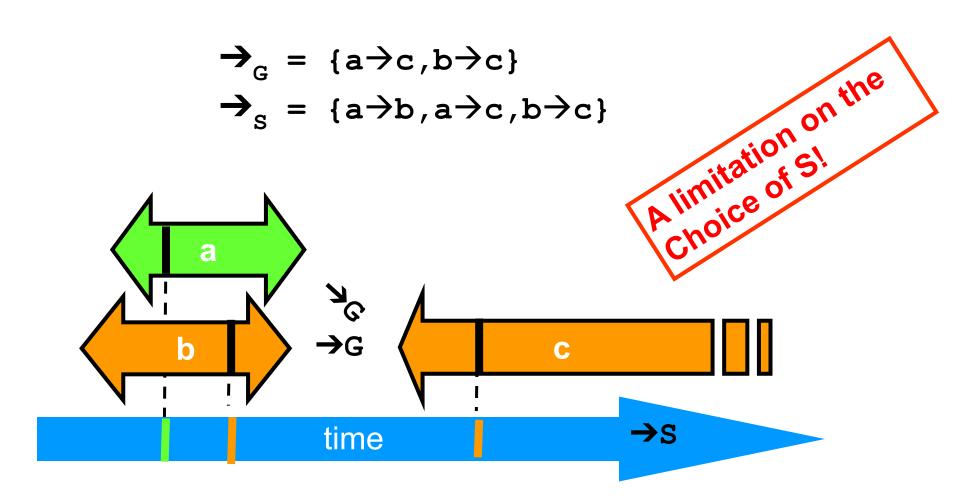
- History H is *linearizable* if it can be extended to G by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
- So that G is equivalent to
  - Legal sequential history S
  - where  $\rightarrow_{\mathsf{G}} \subset \rightarrow_{\mathsf{S}}$

(a good place for a break)

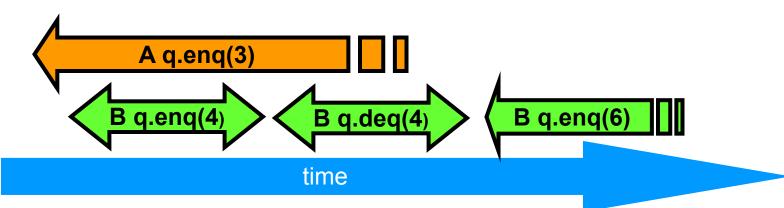
#### Remarks

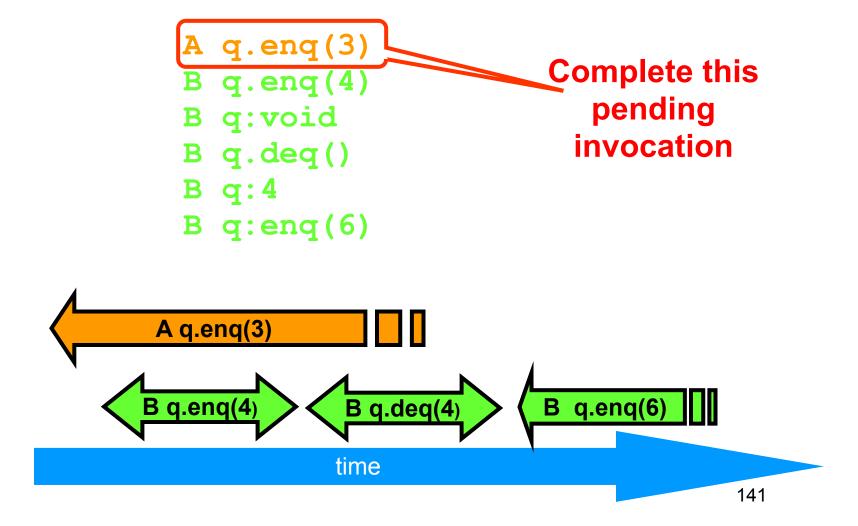
- Some pending invocations
  - Took effect, so keep them
  - Discard the rest
- Condition  $\rightarrow_G \subset \rightarrow_S$ 
  - Means that S respects "real-time order" of G

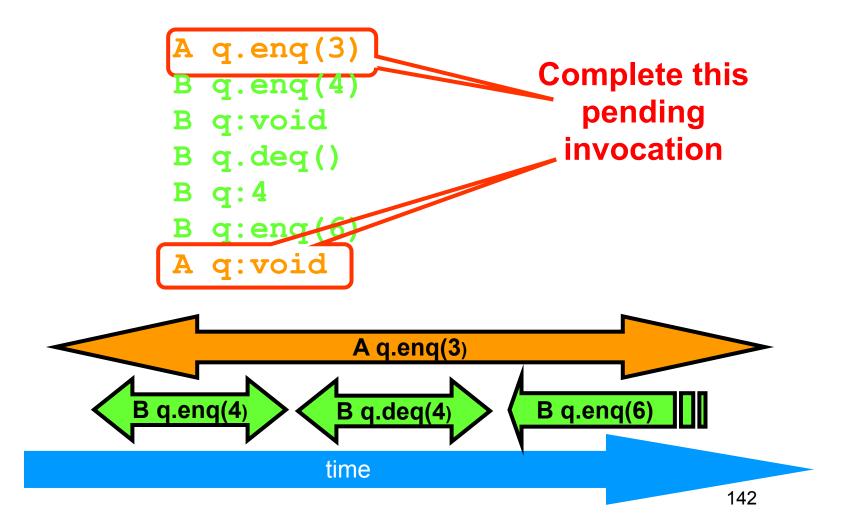
#### Ensuring $\rightarrow_{\mathbf{G}} \subset \rightarrow_{\mathbf{S}}$

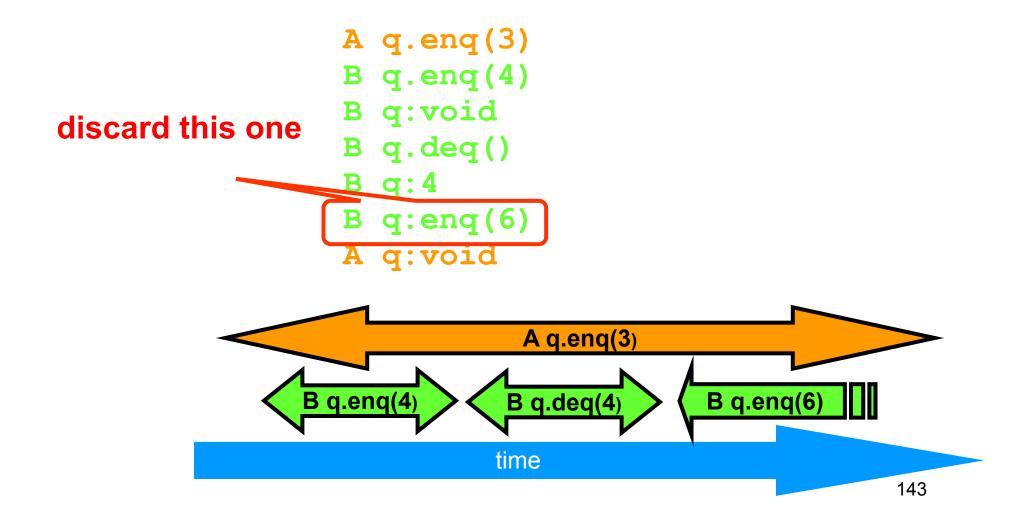


```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```





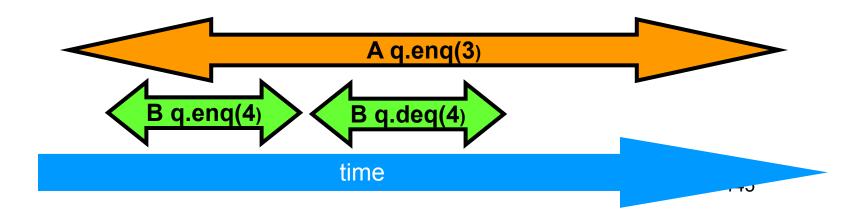




```
A q.enq(3)
  Bq.enq(4)
  B q:void
  B q.deq()
  B q:4
  A q:void
              A q.enq(3)
B q.enq(4)
             B q.deq(4)
            time
                                       144
```

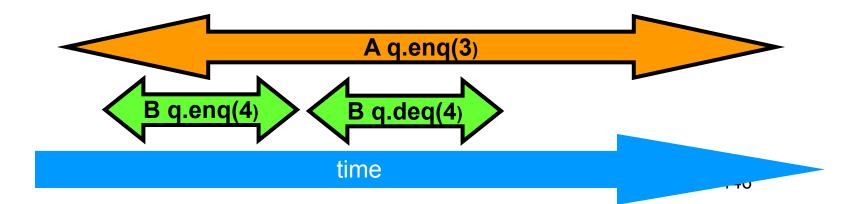
### Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```



#### Example

```
A q.enq(3)
B q.enq(4)
B q.enq(4)
B q:void
A q.enq(3)
B q.deq()
A q:void
B q:4
B q:void
B q:4
B q:4
B q:4
```

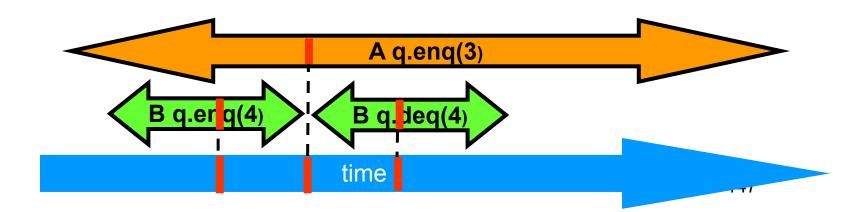


#### Example

#### **Equivalent sequential history**

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```

```
B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4
```



#### Why Does Composability Matter?

- Modularity
- Can prove linearizability of objects in isolation
- Can compose independently-implemented objects
  - A history of two linearizable objects is linearizable

# Reasoning About Linearizability: Locking

```
def deq() : T = \{
                                                  capacity-1
  myLock.lock()
  try {
    if (tail == head) {
      throw EmptyException
    val x = items(head % items.length)
    head = head + 1
    Χ
   finally {
    myLock.unlock()
```

head

# Reasoning About Linearizability: Locking

```
head
def deq() : T = \{
                                                 capacity-1
  myLock.lock()
  try {
    if (tail == head) {
      throw EmptyException
    val x = items(head % items.length)
    head = head + 1
    finally {
    myLock.unlock()
                             Linearization points
                             are when locks are
                                   released
```

#### More Reasoning: Wait-free

```
class LockFreeQueue[T: ClassTag] (val capacity: Int) {
                                                        capacity-1
  @volatile
 private var head, tail: Int = 0
  private val items = new Array[T] (capacity)
  def enq(x: T): Unit = {
    if (tail - head == items.length) throw FullException
    items(tail % items.length) = x
    tail = tail + 1
  def deq(): T = {
    if (tail == head) throw EmptyException
    val x = items(head % items.length)
   head = head + 1
    X
```

### More Reasoning: Wait-free

```
Remember that there
Is only one enqueuer
and only one dequeuer
                      , tail: Int = 0
                       = new Array[T] (capacity)
               (x: T): Uni Linearization order is order head
                            and tail fields modified
           tail = tail +
         def deq(): T
                         throw EmptyException
                         % items.length)
          head = head + 1
```

## Demo: Checking Linearizability of Wait-Free Queue

### Strategy

- Identify one atomic step where method "happens"
  - Critical section
  - Machine instruction
- Doesn't always work
  - Might need to define several different steps for a given method

### Linearizability: Summary

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being "atomic"
- Don't leave home without it



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