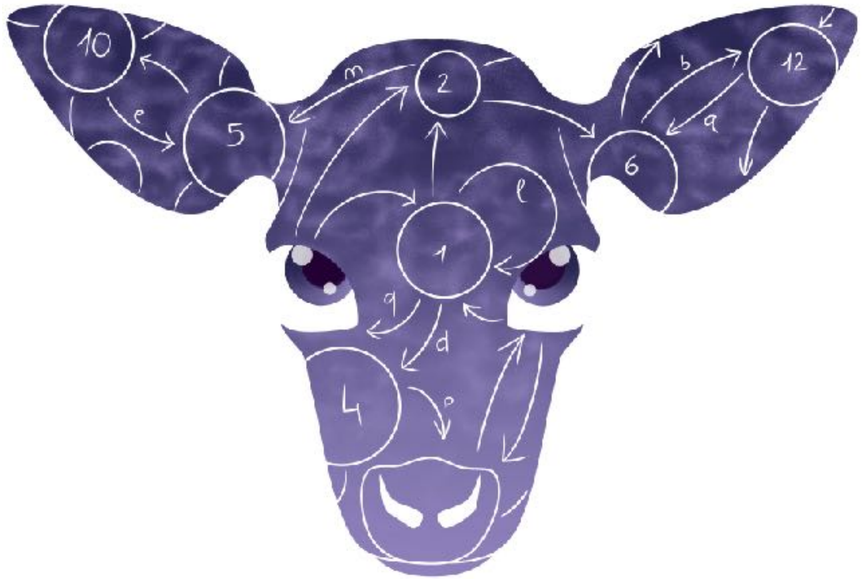


Categorical Automata Learning Framework

calf-project.org

Matteo Sammartino

S-REPLS 6

UCL ENGINEERING
Change the world

UCL

Our team

Alexandra Silva
UCL

Matteo Sammartino
UCL

Gerco van Heerdt
UCL

Joshua Moerman
Radboud University

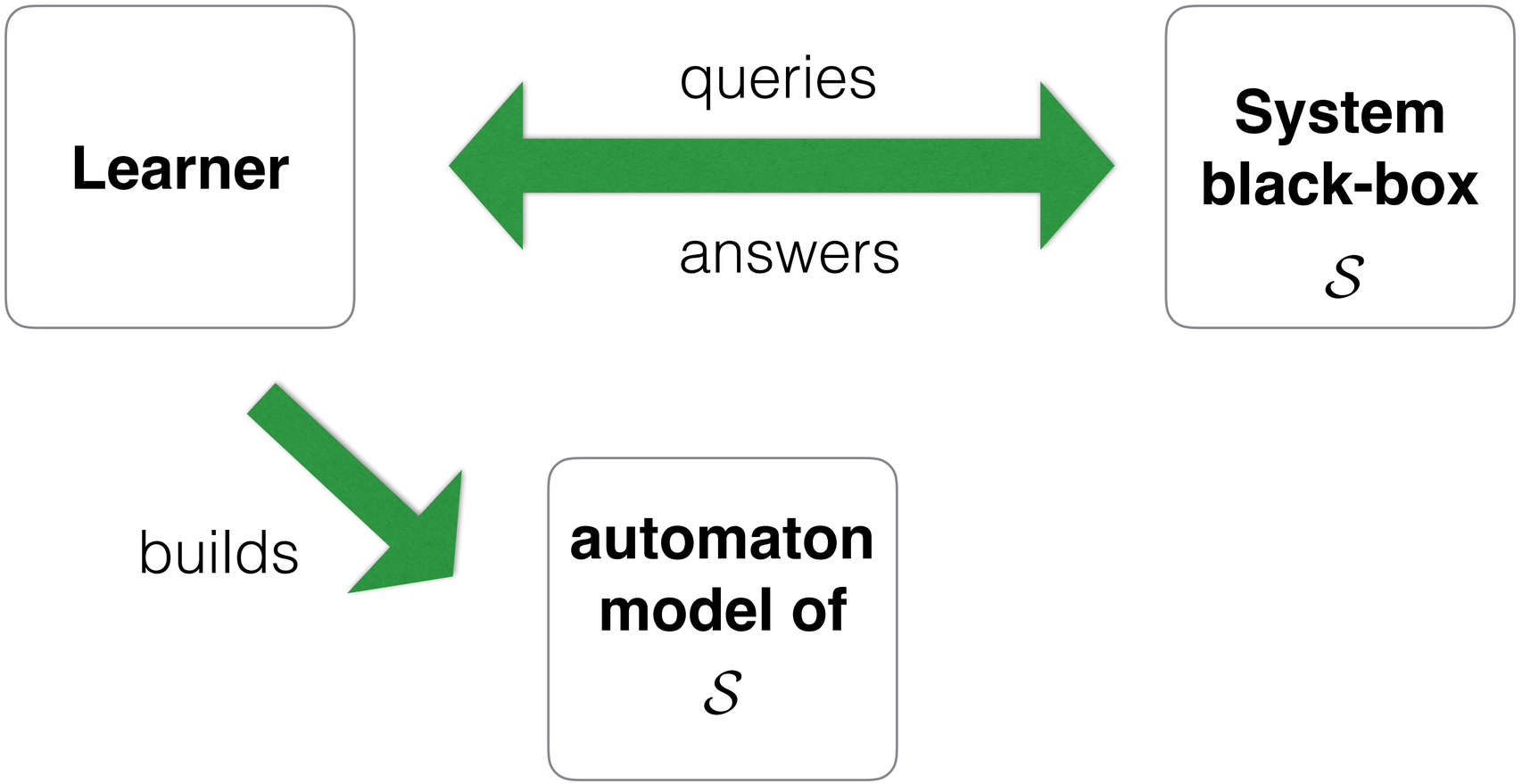
Maverick Chardet
ENS Lyon

Collaborators:

Bartek Klin
Warsaw University

Michal Szynwelski
Warsaw University

Automata learning



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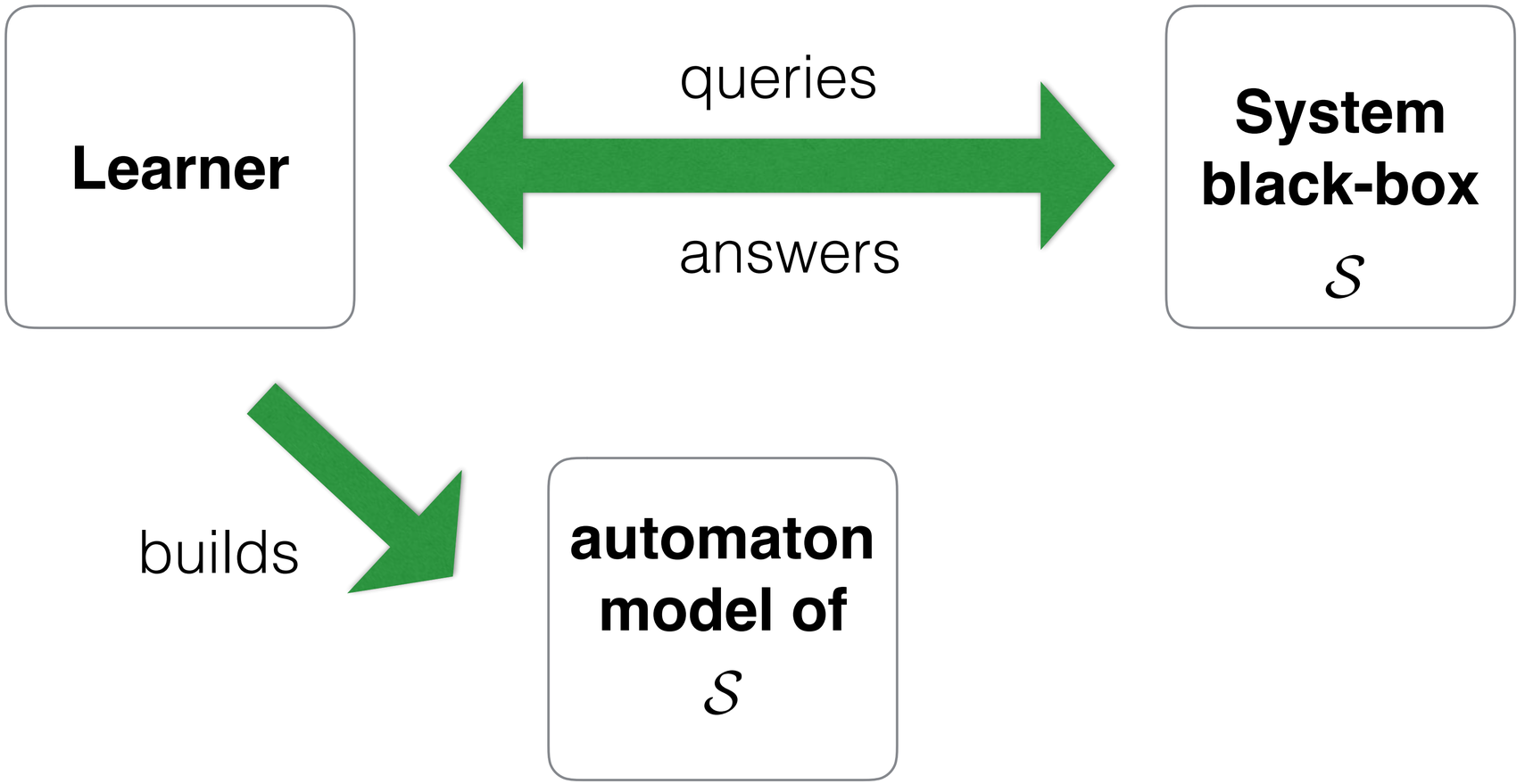

No formal specification available? **Learn it!**

L* algorithm (D.Angluin ’87)

Finite alphabet of system’s actions A

set of system behaviors is a **regular language** $\mathcal{L} \subseteq A^\star$

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Learner

Teacher

$\mathcal{L}$

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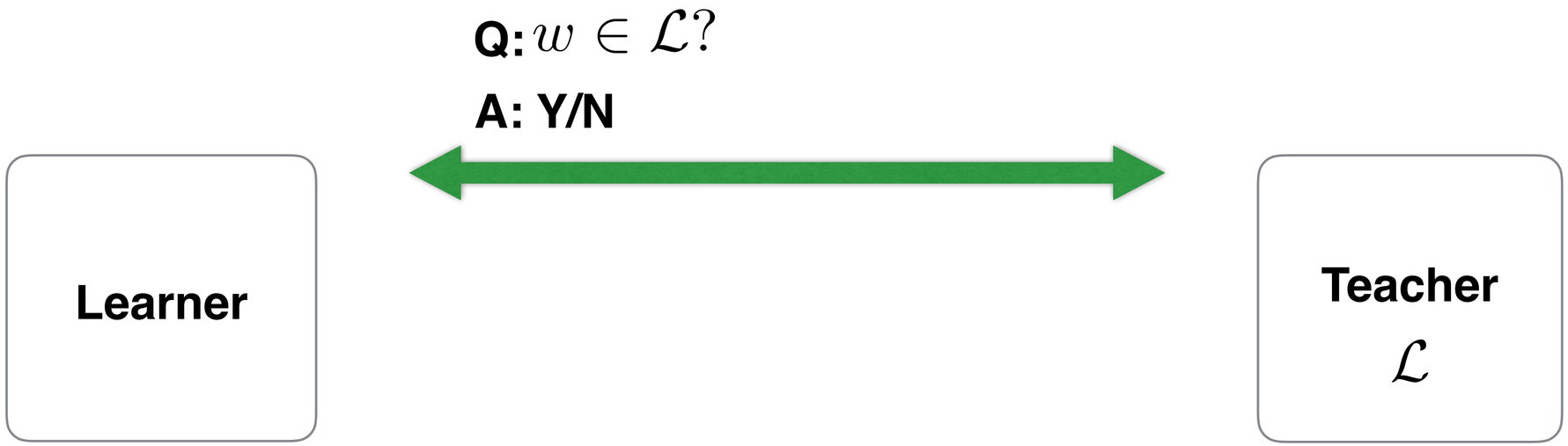

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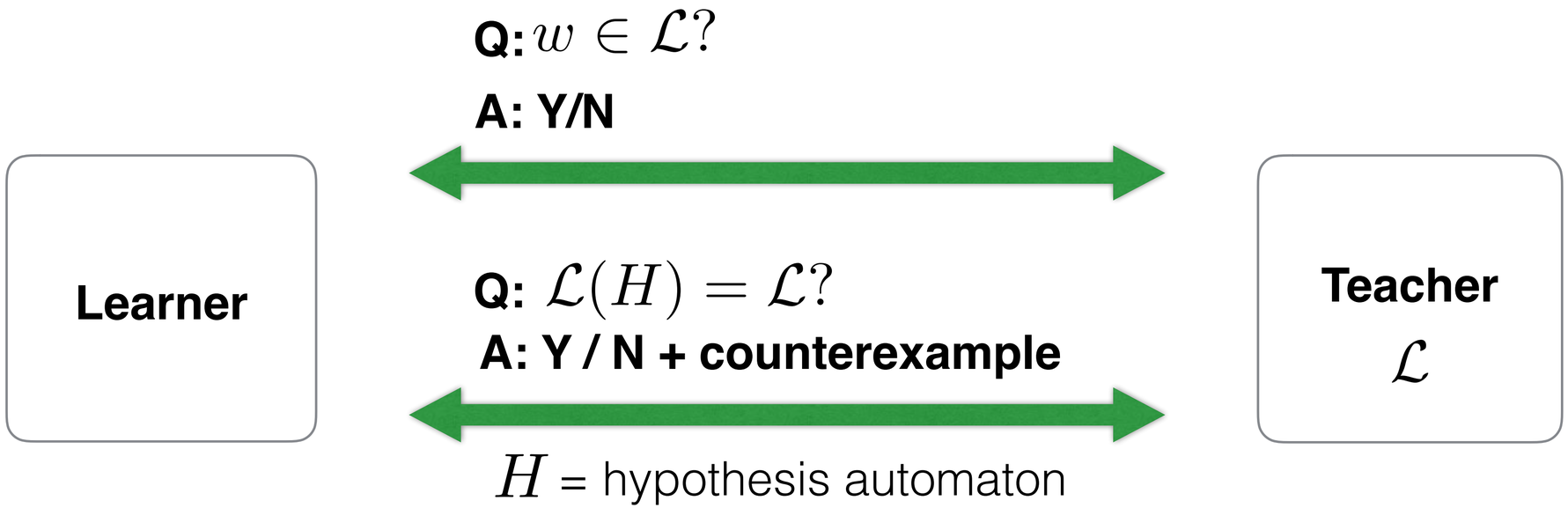

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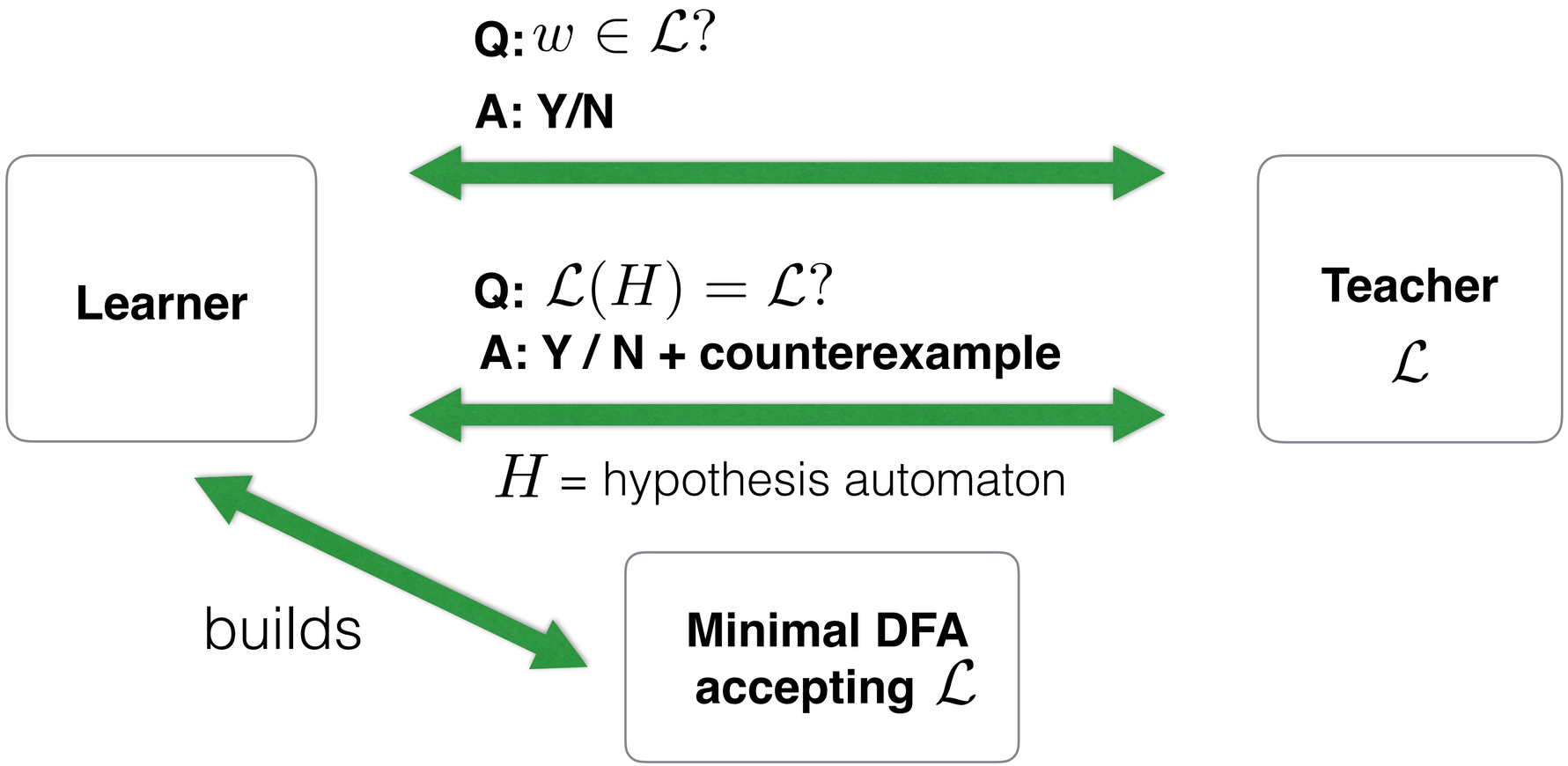
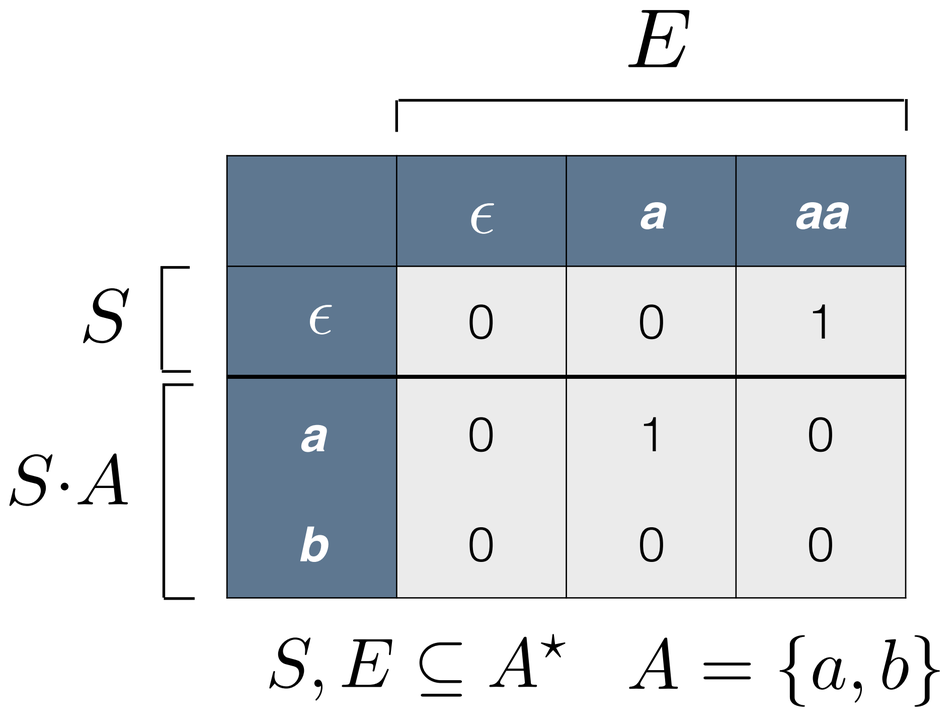


Observation table



$S, E \subseteq A^\star \quad A = \{a, b\}$

$row\colon S \cup S{\cdot}A \to 2^E$

$row(s)(e) = 1 \iff se \in \mathcal{L}$

Observation table

E

		ϵ	*a*	*aa*
S	ϵ	0	0	1
$S \cdot A$	*a*	0	1	0
	b	0	0	0

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Hypothesis automaton

$$\begin{cases} \text{states} = & \{row(s) \mid s \in S\} \\ \text{final states} = & \{row(s) \mid s \in S, row(s)(\epsilon) = 1\} \\ \text{initial state} = & row(\epsilon) \\ \text{transition function} & row(s) \xrightarrow{a} row(sa) \end{cases}$$

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Why is this correct?

Table properties

Table properties

Closed

$$\forall t \in S \cdot A \quad \exists s \in S \qquad row(t) = row(s).$$

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$$row(s) \xrightarrow{a} row(sa)$$

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$$\forall s_1, s_2 \in S \quad row(s_1) = row(s_2) \implies \forall a \in A \;\; row(s_1 a) = row(s_2 a)$$

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Fixed by extending the table

$$\forall s_1, s_2 \in S \quad row(s_1) = row(s_2) \implies \forall a \in A \;\; row(s_1 a) = row(s_2 a)$$

Pros of L* …

simple is beautiful

&

POWERFUL

Applications : Hardware verification, security/network protocols…

Generalizations : Mealy machines, I/O automata, …

A zoo of automata

Probabilistic

Weighted

Alternating

Register

Universal

Non-deterministic

Mealy Machines

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Mealy Machines

Algorithms

Correctness proofs

involved and hard to check

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Category theory comes to the rescue!

Algorithms

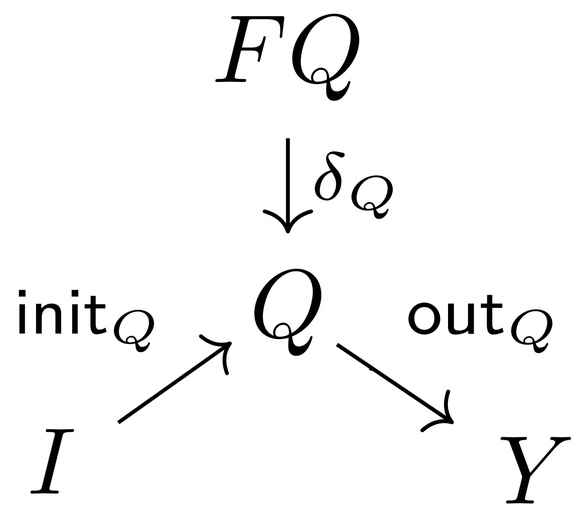
Correctness proofs

involved and hard to check

Abstract automata

Category $\mathbf{C}$ = universe of state-spaces

Endofunctor $F\colon \mathbf{C} \to \mathbf{C}$ = automaton type



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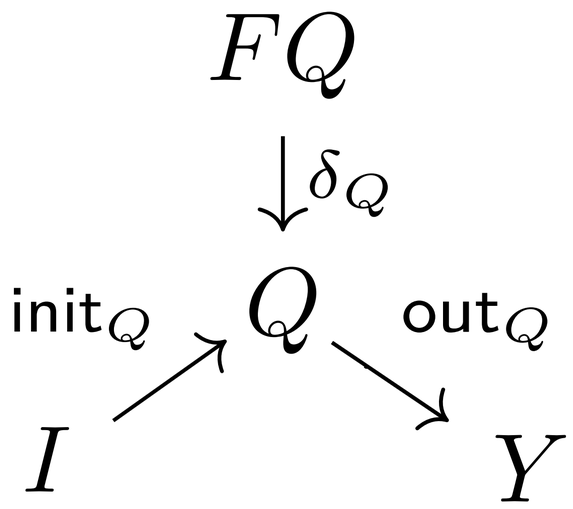
DFAs

$\mathbf{C} = \mathbf{Set}$

$F = (-) \times A$



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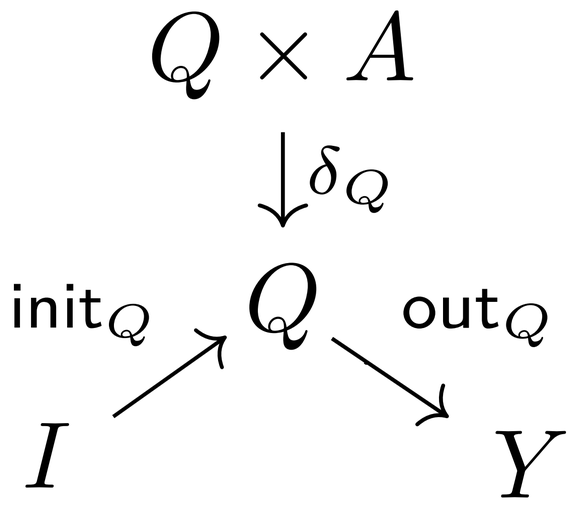

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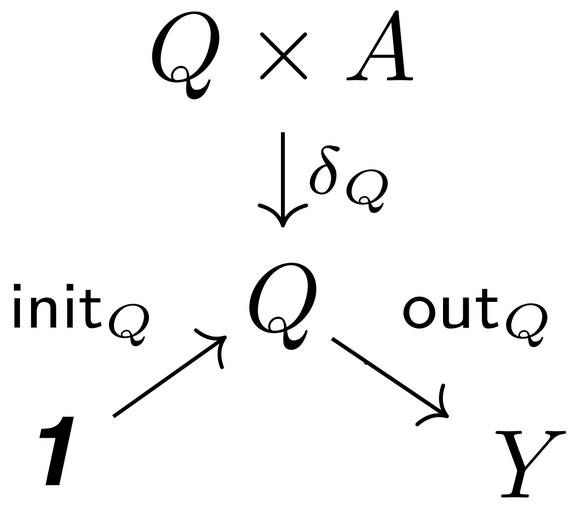

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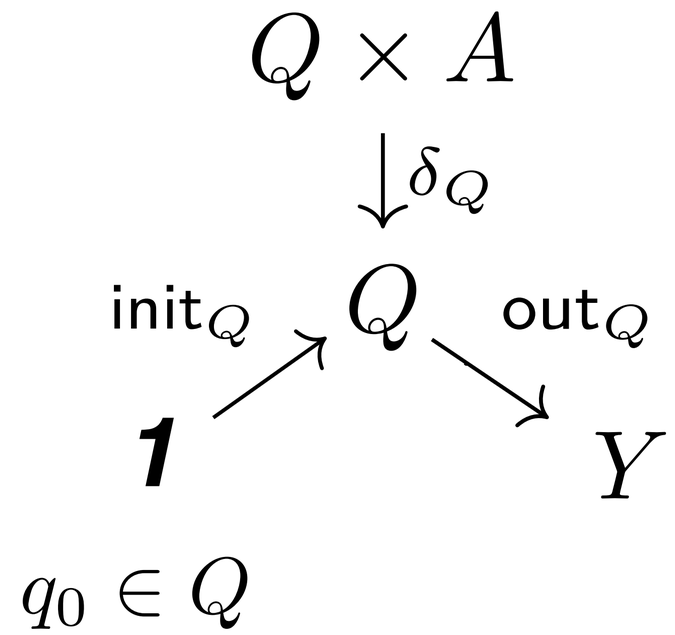

$q_0 \in Q$

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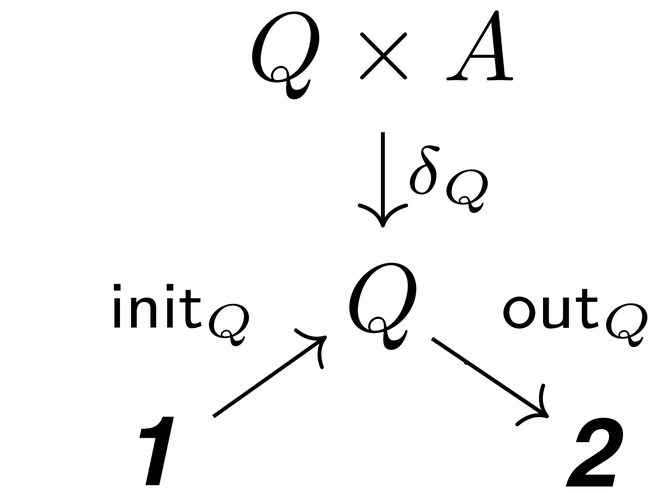

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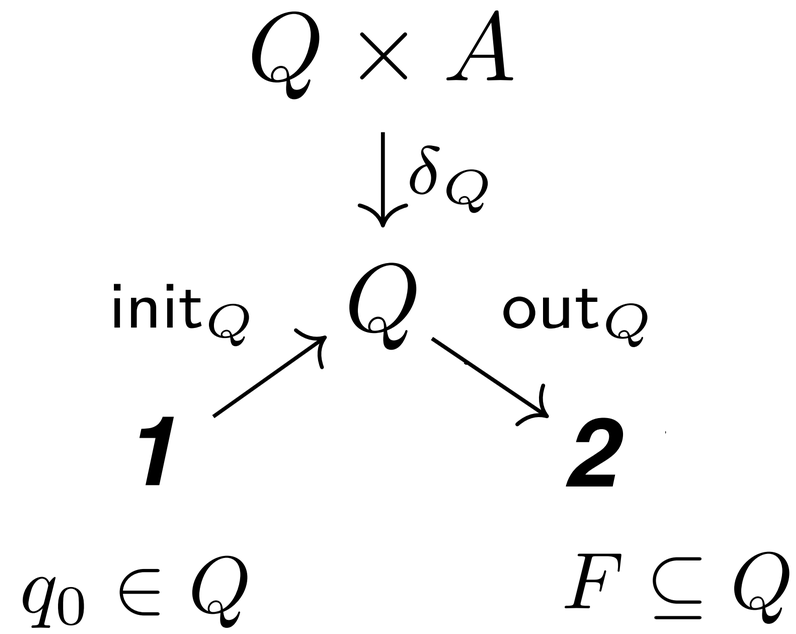


Abstract learning

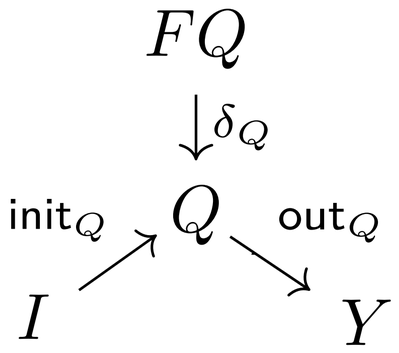
Abstract observation data structure

Abstract learning

Abstract observation data structure

approximates

Target minimal automaton



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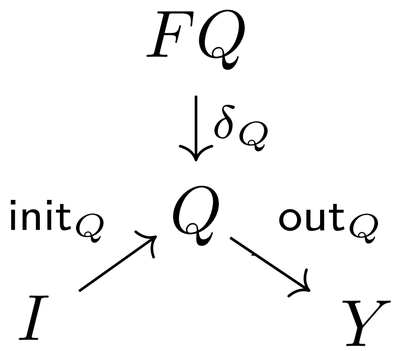
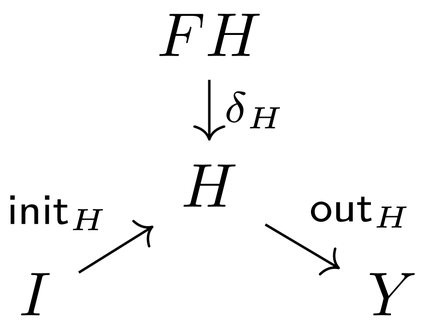


abstract closedness and consistency

Hypothesis automaton



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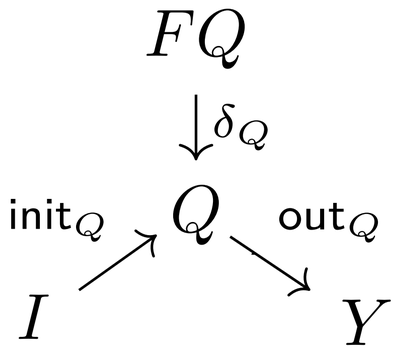

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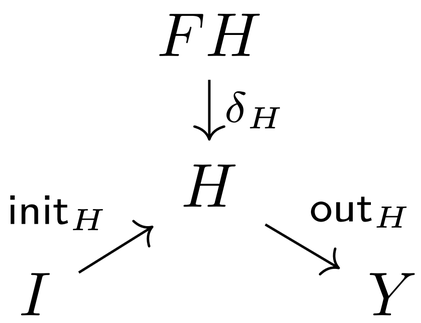

General correctness theorem

Guidelines for implementation

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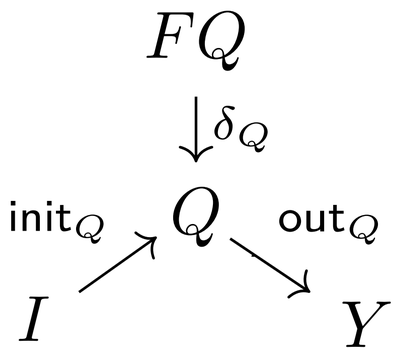

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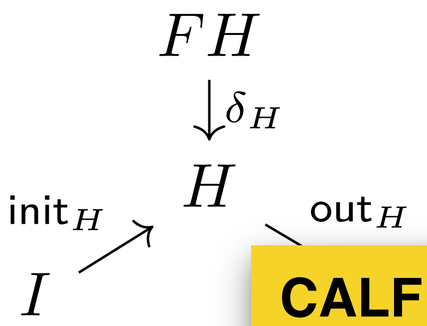

General correctness theorem

Guidelines for implementation

CALF: Categorical Automata Learning Framework (arXiv:1704.05676)

Gerco van Heerdt, Matteo Sammartino, Alexandra Silva

Degrees of freedom

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Change base category

Set	**DFAs**
Nom	**Nominal automata**
Vect	**Weighted automata**

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Side-effects (via monads)

Powerset	**NFAs**
Powerset with intersection	**Universal automata**
Maybe monad	**Partial automata**

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Discrimination trees

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Learning Nominal Automata (POPL ’17)

Joshua Moerman, Matteo Sammartino, Alexandra Silva, Bartek Klin, Michal Szynwelski

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Learning Automata with Side-effects (**arXiv:1704.08055**)

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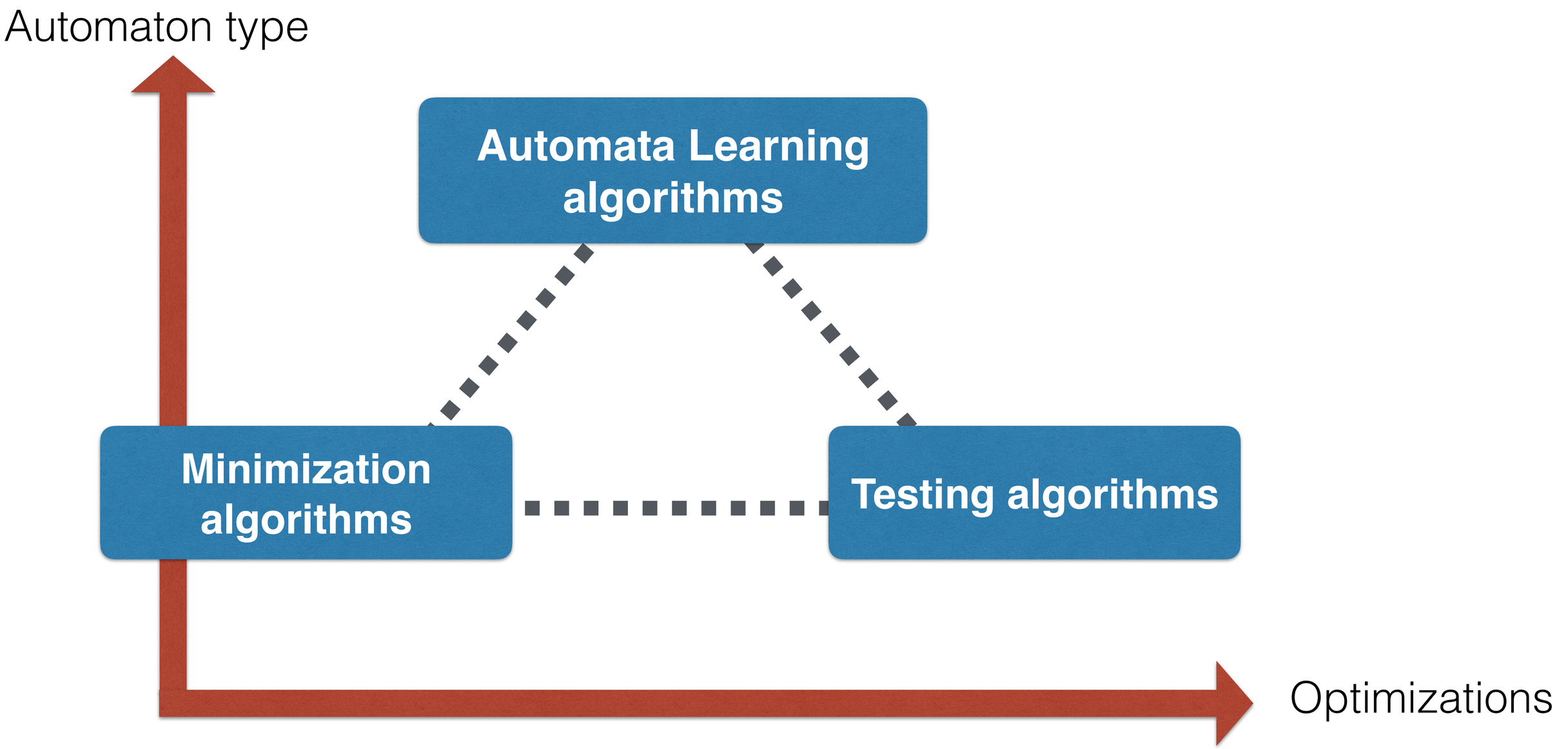
Connections with other algorithms



Abstraction helps a lot …

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but there is no free lunch!

Ongoing and future work

- **Tool** to learn control + data-flow models (as **nominal automata**)
- Applications:
  - Specification mining
  - Network verification, with amazon
  - Verification of cryptographic protocols
  - Ransomware detection