Final Work

Robotics, Kinematics, Dynamics and Control

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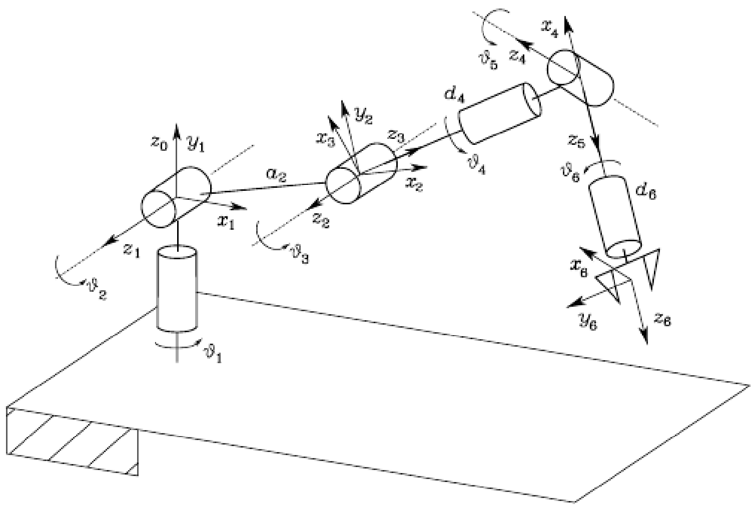
0) Introduction

Initialization of this script.

```
clear
clc
warning off
```

The following figure shows an antromphoric arm with a spherical wrist with the frames and link lenghts. Note that the base frame is the same as the frame x0,y0,z0.



1) Compute the DH parameters. Set numerical values to the link lengths and link offsets.

The following tables are showing the DH parameters of the antromphoric arm with a spherical wrist and the numerical values. The link lenghts are in meters and the angles in radiant.

$link_i$	Numerical Value
a_2	0,5
d_4	0,6
d_6	0,7

$Joint_i$	a_i	α_i	d_i	θ_i	Offset
1	0	$\frac{\pi}{2}$	0	θ_1	$\frac{\pi}{2}$
2	a_2	0	0	θ_2	0
3	0	$\frac{\pi}{2}$	0	θ_3	$\frac{\pi}{2}$
4	0	$-\frac{\pi}{2}$	d_4	θ_4	0
5	0	$\frac{\pi}{2}$	0	θ_5	$-\frac{\pi}{2}$
6	0	0	d_6	θ_6	$\frac{\pi}{2}$

```
% define link lenghts (meters)
a2 = 0.5;
d4 = 0.6;
d6 = 0.7;
% define angles (radiant)
alpha1 = pi/2;
alpha3 = pi/2;
alpha4 = -pi/2;
alpha5 = pi/2;
```

```
% DH paramters
L(1) = Revolute('alpha', alpha1, 'offset', pi/2);
L(2) = Revolute('a', a2);
L(3) = Revolute('alpha', alpha3, 'offset', pi/2);
L(4) = Revolute('d', d4,'alpha', alpha4);
L(5) = Revolute('alpha', alpha5, 'offset', -pi/2);
L(6) = Revolute('d', d6, 'offset', pi/2);
L;
```

The following code section displays the table of DH paramters with the defined offsets.

```
Robot = SerialLink(L, 'name', 'MyRobot')
```

```
Robot = 

MyRobot:: 6 axis, RRRRRR, stdDH, slowRNE, Symbolic
+---+-----------+-----------+-----------+-----------+-----------+
| j |     theta |         d |         a |     alpha |    offset |
+---+-----------+-----------+-----------+-----------+-----------+
|  1|         q1|          0|          0|     alpha1|     1.5708|
|  2|         q2|          0|         a2|          0|          0|
|  3|         q3|          0|          0|     alpha3|     1.5708|
|  4|         q4|         d4|          0|     alpha4|          0|
|  5|         q5|          0|          0|     alpha5|    -1.5708|
|  6|         q6|         d6|          0|          0|     1.5708|
```

```
+---+----------+----------+----------+----------+-----------+
```

The next code section displays the Robot itself plotted at [0,0,0,0,0,0].

```
Robot.plot([0,0,0,0,0,0])
```

```
Conversion to logical from sym is not possible.

Error in RTBPlot.plot_options (line 834)
            if reach > 0

Error in SerialLink/plot (line 213)
    opt = RTBPlot.plot_options(robot, varargin);
```

```
Robot.teach
```

2) With the aid of the Symbolic Toolbox:

(a) Implement a script/function to compute the Forward Kinematics.

First the joint varibles have to be defined.

```
%joint variables
syms qi q1 q2 q3 q4 q5 q6 as real
syms a2 d4 d6 as real
syms alpha1 alpha3 alpha4 alpha5 as real
```

The **Forward Kinematics** A matrices will be computed with the ***forward_kin()*** function which is programed in the **Appendix** chapter at end of this file.

```
% a d alpha zeta
A01 = forward_kin(0, 0, alpha1, q1);
A12 = forward_kin(a2, 0, 0, q2);
A23 = forward_kin(0, 0, alpha3, q3);
A34 = forward_kin(0, d4, alpha4, q4);
A45 = forward_kin(0, 0, alpha5, q5);
A56 = forward_kin(0, d6, 0, q6);
```

```
A06 = A01*A12*A23*A34*A45*A56;
simplify(A06)
```

(b) Implement a script/function to compute the Geometric and Analytic Jacobians.

The origin and z-axes of the reference frame of link 0 are defined.

```
z0 = [0 0 1]';
p0 = [0 0 0 1]';
```

Geometric Jacobian

The **Geometric Jacobian** will be computed with the ***geometric_jacob()*** function which is programed in the **Appendix** chapter at end of this file.

```
J = geometric_jacob(p0, z0, A01, A12, A23, A34, A45, A56)
J = simplify(J)
```

Analytical Jacobian

The **Analytical Jacobian** will be computed with the ***analytic_jacob()*** function which is programed in the **Appendix** chapter at end of this file.

```
JA = analytic_jacob(A06, q1, q2, q3, q4, q5, q6);
JA = simplify(JA)
```

(c) Implement a script/function to compute the inverse kinematics using a closed form.

The **Inverse Kinematics** A matrices will be computed with the ***inv_kin()*** function which is programed in the **Appendix** chapter at end of this file using a closed form. The function is receiving link dimensions, R30 matrix, the end-effector position pe and orientation Re as the inputs and executes the following five steps:

1. Compute the wrist position ***pw = pe - d6*ae*** (a_e is the direction vector of the end effector which is the direction of z-axis)
2. Solve inverse kinematics for Anthropomorphic arm: **q1, q2, q3**
3. Compute **R30(q1, q2, q3)**
4. Compute **R63(q4, q5, q6) = R30'*Re**
5. Solve inverse kinematics for Spherical wrist: **q4, q5, q6**

With help of the ***forward_kin()*** function used in **2a)** the R30 matrix can be computed.

```
A01 = forward_kin(0, 0, alpha1, q1);
A12 = forward_kin(a2, 0, 0, q2);
A23 = forward_kin(0, 0, alpha3, q3);
```

Applying direct kinematics.

```
A03 = A01*A02*A03;
simplify(A03);
```

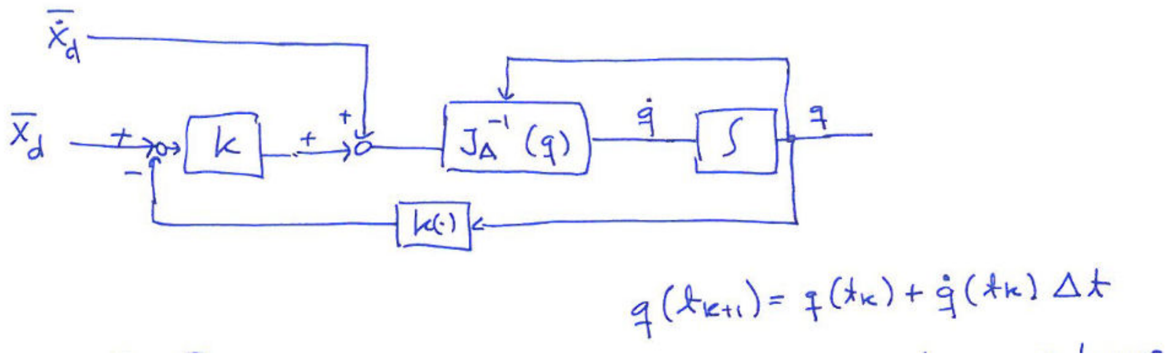
The R30 matrice is now computed.

```
R03 = A03(1:3, 1:3)
```

To solve the inverse kinematics, just call the created function i***nv_kin()*** which is in the **Appendix: Functions**.

(d) Implement a script/function to compute the inverse kinematics using an iterative form.

To compute the inverse kinematics the **Jacobian pseudo-inverse method/algorithm** will be used. The following control block diagram illustrates the procedure.



For that the function inv_kin_ps() is called and receives the loop parameters K, delta_t, initial q, desired q, A06 matrix (Direct Kinematics) and Analytical Jacobian JA matrix.

3) Using the Robotics Toolbox, model the manipulator as a SerialLink instance, and create a script file to verify the correctness of the functions implemented in the previous question.

0. Define numerical values

In order to verify the correctness of the functions implemented in the previous question, random numerical values are definied.

```
q1 = pi/5;
q2 = pi/4;
q3 = pi;
q4 = pi/5;
q5 = pi/4;
q6 = pi/3;
q = [q1, q2, q3, q4, q5, q6];
```

1. Modeling the manipulator as a SerialLink instance

```
% define link lenghts (meters)
a2 = 0.5;
d4 = 0.6;
d6 = 0.7;
% define angles (radiant)
alpha1 = pi/2;
alpha3 = pi/2;
alpha4 = -pi/2;
alpha5 = pi/2;
```

```
% DH paramters
L(1) = Revolute('alpha', alpha1, 'offset', pi/2);
L(2) = Revolute('a', a2);
L(3) = Revolute('alpha', alpha3, 'offset', pi/2);
L(4) = Revolute('d', d4,'alpha', alpha4);
L(5) = Revolute('alpha', alpha5, 'offset', -pi/2);
L(6) = Revolute('d', d6, 'offset', pi/2);
```

```
L;
```

The following code section displays the table of DH paramters with the defined offsets with help of the SerialLink.

```
Robot = SerialLink(L, 'name', 'MyRobot')
```

Plotting the robot

```
Robot.plot([0,0,0,0,0,0])
Robot.teach
```

2. Verification of the Forward Kinematics

Here the Robotics Toolbox ***.fkine()*** function is used.

```
A06_1 = Robot.fkine(q)
```

Now the previously created function will be used to compute A06_2.

```
A01 = double(forward_kin(0, 0, alpha1, q1));
A12 = double(forward_kin(a2, 0, 0, q2));
A23 = double(forward_kin(0, 0, alpha3, q3));
A34 = double(forward_kin(0, d4, alpha4, q4));
A45 = double(forward_kin(0, 0, alpha5, q5));
A56 = double(forward_kin(0, d6, 0, q6));
A06_2 = double(A01*A12*A23*A34*A45*A56)
```

Result: Both matrices **A06_1** and **A06_2** are the same, which means the function is verified.

3. Verfication of Geometric Jacobian

Here the Robotics Toolbox ***.jacob0()*** function is used.

```
J_1 = Robot.jacob0(q)
```

Now the previously created function for the geometric jacobian is used.

```
z0 = [0 0 1]';
p0 = [0 0 0 1]';
J_2 = double(geometric_jacob(p0, z0, A01, A12, A23, A34, A45, A56))
```

Result: The Geometric Jacobian matrices **J_1** and ***J_2*** are the same, which means that the function is verified.

4. Verification of Analytical Jacobian

Here the Robotics Toolbox ***.jacob0('eul')*** function is used.

```
JA_1 = double(Robot.jacob0(q,'eul'))
```

Now the previously created function for the analytical jacobian is used.

```
JA_2 = double(subs(JA,{q1 q2 q3 q4 q5 q6 a2 d4 d6 alpha1 alpha3 alpha4 alpha5},{q_1 q_2 q_3 q_4
```

Result: The Analytical Jacobians **JA_1** and **JA_2** are the same which means that the function is verified.

5. Verification of Inverse Kinematics (closed form approach)

Here the Robotics Toolbox is used.

```
Q1 = Robot.ikine(A06_1)
```

The end-effector position and orientation is obtained from the prevoius function.

```
vpe = A06_1.t;
vRe = A06_1.R;
```

Rotation matrix R03 is obtained with the substitution of the robot DH alpha parameters.

```
R03 = subs(R03,{alpha1 alpha3}, {alpha1 alpha3});
```

The function ***inv_kin()*** function is called to solve the inverse kinematics problem.

```
Q2 = inv_kin(a_2, d_4, d_6, R03, vpe, vRe)
```

The fifth solution is the same as the result from the Robotics toolbox. The configuration is called **rightarm elbow up configuration.**

Now the verification of the position has to be checked.

```
vpe'
```

```
% robot.fkine(Q_2(1,:)).transl % Equal
% robot.fkine(Q_2(3,:)).transl % Equal
% robot.fkine(Q_2(5,:)).transl % Equal
robot.fkine(Q_2(7,:)).transl
```

Now the orientation is checked.

```
vRe
```

```
% robot.fkine(Q_2(1,:)).R % Equal
% robot.fkine(Q_2(3,:)).R % Equal
% robot.fkine(Q_2(5,:)).R % Equal
robot.fkine(Q_2(7,:)).R
```

6. Verification of Inverse Kinematics (Jacobian pseudo-inverse method/algorithm)

First of all the Robotics Toolbox is used.

```
Q1_pse = robot.ikine(A06_1)
```

Now the Direct Kinematics A06 matrix and Analytical Jacobian matrix are computed, with the substitution of the robots DH alpha parameters.

```
A06 = subs(A06,{a2 d4 d6 alpha1 alpha3 alpha4 alpha5},{a_2 d_4 d_6 alpha_1 alpha_3 alpha_4 alph
JA = subs(JA,{a2 d4 d6 alpha1 alpha3 alpha4 alpha5},{a_2 d_4 d_6 alpha_1 alpha_3 alpha_4 alpha_
```

The next step is to define the loop variables. The initial q vector has been chosen close enough to the desired solution in order to speed up the convergence time.

```
q_ini = [0.5 1 3 0.5 1 1.5];
K = 20;
delta_t = 0.05;
```

Calling the previously created ***inv_kin_pse()*** function.

```
[Q_inv_2, xd_p, xd_o] = inv_kin_pse(K, delta_t, q_ini, q, A06, JA);
Q2_pse
```

Result: Q1_pse and Q2_pse are almost the same. This is the right arm elbow up configuration.

Last but not least the verification of the position and orientation is peformed.

First the position.

```
xd_p'
```

```
xd_p_test = double(subs(A06(1:3,4),{q1 q2 q3 q4 q5 q6},{q(1) q(2) q(3) q(4) q(5) q(6)}));
xd_p_test'
```

Second the orientation.

```
xd_o
```

```
xd_o_test= double(subs(A06,{q1 q2 q3 q4 q5 q6},{q_1 q_2 q_3 q_4 q_5 q_6}));
Phie = xd_o_test(1:3,1:3);
phi_test = atan2(Phie(2,3),Phie(1,3));
theta_test = atan2(sqrt(Phie(1,3)^2+Phie(2,3)^2),Phie(3,3));
psi_test = atan2(Phie(3,2),-Phie(3,1));
xd_o_test = [phi_test theta_test psi_test]
```

Appendix: Functions

2a) Forward Kinematics

The following function computes the Forward Kinematics A matrices for all joints.

```
function fwk_matrix = forward_kin(a, d, alpha, q)

% Symbolic variables
syms ai di alphai qi as real
% A matrix formula
```

```
A01i = [cos(qi) -sin(qi)*cos(alphai) sin(qi)*sin(alphai) ai*cos(qi);...
sin(qi) cos(qi)*cos(alphai) -cos(qi)*sin(alphai) ai*sin(qi);...
0 sin(alphai) cos(alphai) di;...
0 0 0 1];
fwk_matrix = subs(A01i,{ai,di,alphai,qi},{ a, d, alpha, q});

end
```

2b) Geometric Jacobian

This is the function to compute the geometric jacobian.

```
function J = geometric_jacob(p0, z0, A01, A12, A23, A34, A45, A56)

p1 = A01*p0;
p2 = A01*A12*p0;
p3 = A01*A12*A23*p0;
p4 = A01*A12*A23*A34*p0;
p5 = A01*A12*A23*A34*A45*p0;
p6 = A01*A12*A23*A34*A45*A56*p0;

z1 = A01(1:3,1:3)*z0;
z2 = A01(1:3,1:3)*A12(1:3,1:3)*z0;
z3 = A01(1:3,1:3)*A12(1:3,1:3)*A23(1:3,1:3)*z0;
z4 = A01(1:3,1:3)*A12(1:3,1:3)*A23(1:3,1:3)*A34(1:3,1:3)*z0;
z5 = A01(1:3,1:3)*A12(1:3,1:3)*A23(1:3,1:3)*A34(1:3,1:3)*A45(1:3,1:3)*z0;

% Geometric Jacobian
J = [cross(z0,p6(1:3)-p0(1:3)), cross(z1,p6(1:3)-p1(1:3)), cross(z2,p6(1:3)-p2(1:3)),...
cross(z3,p6(1:3)-p3(1:3)), cross(z4,p6(1:3)-p4(1:3)), cross(z5,p6(1:3)-p5(1:3));...
            z0  z1   z2   z3  z4  z5];
end
```

2b) Analytical Jacobian

This is the function to compute the analytical jacobian.

```
function JA = analytic_jacob(R, q1, q2, q3, q4, q5, q6)
% End-effector position & orientation
Pe = R(:,4);
Phie = R(1:3,1:3);

% ZYZ Euler angles
phi = atan2(Phie(2,3),Phie(1,3));
theta = atan2(sqrt(Phie(1,3)^2+Phie(2,3)^2),Phie(3,3));
psi = atan2(Phie(3,2),-Phie(3,1));

% Linear velocities
dPexdq1 = diff(Pe(1),q1);
dPexdq2 = diff(Pe(1),q2);
dPexdq3 = diff(Pe(1),q3);
dPexdq4 = diff(Pe(1),q4);
dPexdq5 = diff(Pe(1),q5);
dPexdq6 = diff(Pe(1),q6);
```

```matlab
dPeydq1 = diff(Pe(2),q1);
dPeydq2 = diff(Pe(2),q2);
dPeydq3 = diff(Pe(2),q3);
dPeydq4 = diff(Pe(2),q4);
dPeydq5 = diff(Pe(2),q5);
dPeydq6 = diff(Pe(2),q6);

dPezdq1 = diff(Pe(3),q1);
dPezdq2 = diff(Pe(3),q2);
dPezdq3 = diff(Pe(3),q3);
dPezdq4 = diff(Pe(3),q4);
dPezdq5 = diff(Pe(3),q5);
dPezdq6 = diff(Pe(3),q6);

% Analytical Jacobian for linear velocities
Jp = [dPexdq1 dPexdq2 dPexdq3 dPexdq4 dPexdq5 dPexdq6;...
      dPeydq1 dPeydq2 dPeydq3 dPeydq4 dPeydq5 dPeydq6;...
      dPezdq1 dPezdq2 dPezdq3 dPezdq4 dPezdq5 dPezdq6];

Jp = simplify(Jp);

% Angular velocities
dphidq1 = diff(phi,q1);
dphidq2 = diff(phi,q2);
dphidq3 = diff(phi,q3);
dphidq4 = diff(phi,q4);
dphidq5 = diff(phi,q5);
dphidq6 = diff(phi,q6);

dthetadq1 = diff(theta,q1);
dthetadq2 = diff(theta,q2);
dthetadq3 = diff(theta,q3);
dthetadq4 = diff(theta,q4);
dthetadq5 = diff(theta,q5);
dthetadq6 = diff(theta,q6);

dpsidq1 = diff(psi,q1);
dpsidq2 = diff(psi,q2);
dpsidq3 = diff(psi,q3);
dpsidq4 = diff(psi,q4);
dpsidq5 = diff(psi,q5);
dpsidq6 = diff(psi,q6);

% Analytical Jacobian for angular velocities
JPhi = [dphidq1 dphidq2 dphidq3 dphidq4 dphidq5 dphidq6;...
      dthetadq1 dthetadq2 dthetadq3 dthetadq4 dthetadq5 dthetadq6;...
      dpsidq1 dpsidq2 dpsidq3 dpsidq4 dpsidq5 dpsidq6];

JPhi = simplify(JPhi);

% Analytical Jacobian
JA = [Jp;JPhi];
```

```
end
```

2c) Inversed Kinematics (closed loop approach)

This is the function to compute the Inverse Kinematic using the closed loop approach.

```
function jointspos = inv_kin(va2, vd4, d_6, R03, vpe, vRe)

% define link lenghts
syms a2 d4 as real
% define position end effector
syms pex pey pez as real

% 1. Compute the wrist position pw = pe - d6*ae
% Anthropomorphic arm position of point W
pw = [pex, pey, pez]';
% Wrist position
vpw = vpe - d_6*vRe(:,3);

% 2. Solve inverse kinematics for Anthropomorphic arm
% Theta 3
c3 = ((pw'*pw-a2^2-d4^2)/2*a2*d4);
s3plus = sqrt(1-c3^2);
s3minus = -sqrt(1-c3^2);
theta3_I = atan2(s3plus,c3) + pi/2; % pi/2 offset added
theta3_II = atan2(s3minus,c3) + pi/2; % pi/2 offset added
% Theta 2
c2_I = ( sqrt(pw(1)^2+pw(2)^2)*(a2+d4*c3)+pw(3)*d4*s3plus )/(a2^2+d4^2+2*a2*d4*c3);
c2_II = (-sqrt(pw(1)^2+pw(2)^2)*(a2+d4*c3)+pw(3)*d4*s3plus)/(a2^2+d4^2+2*a2*d4*c3);
c2_III = ( sqrt(pw(1)^2+pw(2)^2)*(a2+d4*c3)+pw(3)*d4*s3minus )/(a2^2+d4^2+2*a2*d4*c3);
c2_IV = (-sqrt(pw(1)^2+pw(2)^2)*(a2+d4*c3)+pw(3)*d4*s3minus)/(a2^2+d4^2+2*a2*d4*c3);
s2_I = (pw(3)*(a2+d4*c3) - sqrt(pw(1)^2+pw(2)^2)*d4*s3plus)/(a2^2+d4^2+2*a2*d4*c3);
s2_II = (pw(3)*(a2+d4*c3) + sqrt(pw(1)^2+pw(2)^2)*d4*s3plus)/(a2^2+d4^2+2*a2*d4*c3);
s2_III = (pw(3)*(a2+d4*c3) - sqrt(pw(1)^2+pw(2)^2)*d4*s3minus)/(a2^2+d4^2+2*a2*d4*c3);
s2_IV = (pw(3)*(a2+d4*c3) + sqrt(pw(1)^2+pw(2)^2)*d4*s3minus)/(a2^2+d4^2+2*a2*d4*c3);
theta2_I = atan2(s2_I,c2_I);
theta2_II = atan2(s2_II,c2_II);
theta2_III = atan2(s2_III,c2_III);
theta2_IV = atan2(s2_IV,c2_IV);
% Theta 1
theta1_I = atan2(pw(2),pw(1));
theta1_II = atan2(-pw(2),-pw(1));
solspos(1,:) = [theta1_I, theta2_I, theta3_I];
solspos(2,:) = [theta1_I, theta2_III, theta3_II];
solspos(3,:) = [theta1_II, theta2_II, theta3_I];
solspos(4,:) = [theta1_II, theta2_IV, theta3_II];
% Particularize to unit link lengths
%va2 = 1.0;
%vd4 = 1.0;
vsolspos = subs(solspos,{a2 d4},{va2 vd4});
% Solution of inverse kinematics for Anthropomorphic arm
jointspos(1,1:3) = double(subs(vsolspos(1,:),{pex pey pez},{vpw(1) vpw(2) vpw(3)}));
jointspos(2,1:3) = double(subs(vsolspos(1,:),{pex pey pez},{vpw(1) vpw(2) vpw(3)}));
jointspos(3,1:3) = double(subs(vsolspos(2,:),{pex pey pez},{vpw(1) vpw(2) vpw(3)}));
```

```
jointspos(4,1:3) = double(subs(vsolspos(2,:),{pex pey pez},{vpw(1) vpw(2) vpw(3)}));
jointspos(5,1:3) = double(subs(vsolspos(3,:),{pex pey pez},{vpw(1) vpw(2) vpw(3)}));
jointspos(6,1:3) = double(subs(vsolspos(3,:),{pex pey pez},{vpw(1) vpw(2) vpw(3)}));
jointspos(7,1:3) = double(subs(vsolspos(4,:),{pex pey pez},{vpw(1) vpw(2) vpw(3)}));
jointspos(8,1:3) = double(subs(vsolspos(4,:),{pex pey pez},{vpw(1) vpw(2) vpw(3)}));

% 3.Compute R30(q1 q2 q3)
syms q1 q2 q3 as real
% Solution of inverse kinematics
q_1 = jointspos(1,:);
q_2 = jointspos(3,:);
q_3 = jointspos(5,:);
q_4 = jointspos(7,:);
% R03 matrix for each solution
vR03_1 = double(subs(R03,{q1 q2 q3}, {q_1(1) q_1(2) q_1(3)}));
vR03_2 = double(subs(R03,{q1 q2 q3}, {q_2(1) q_2(2) q_2(3)}));
vR03_3 = double(subs(R03,{q1 q2 q3}, {q_3(1) q_3(2) q_3(3)}));
vR03_4 = double(subs(R03,{q1 q2 q3}, {q_4(1) q_4(2) q_4(3)}));

% 4.Computing R63(q4 q5 q6) = R30'*Re
vR63_1 = vR03_1'*vRe;
vR63_2 = vR03_2'*vRe;
vR63_3 = vR03_3'*vRe;
vR63_4 = vR03_4'*vRe;

% 5. Solve inverse kinematics for Spherical wrist
syms nx ny nz as real
syms sx sy sz as real
syms ax ay az as real
% Rotation matrix of the spherical wrist
Re = [nx sx ax; ny sy ay; nz sz az];
n = Re(:,1);
s = Re(:,2);
a = Re(:,3);
% Theta 5
theta5_I = atan2(sqrt(a(1)^2+a(2)^2), a(3));
theta5_II = atan2(-sqrt(a(1)^2+a(2)^2), a(3));
% Theta 4
theta4_I = atan2(a(2),a(1));
theta4_II = atan2(-a(2),-a(1));
% Theta 6
theta6_I = atan2(s(3),-n(3));
theta6_II = atan2(-s(3),n(3));
% Particularized to unity lenght
vsolsori(1,:) = [theta4_I, theta5_I, theta6_I];
vsolsori(2,:) = [theta4_II, theta5_II, theta6_II];

%Final Solution
jointspos(1,4:6) = double(subs(vsolsori(1,:),{nx ny nz sx sy sz ax ay az},...
    {vR63_1(1,1) vR63_1(2,1) vR63_1(3,1) vR63_1(1,2) vR63_1(2,2) vR63_1(3,2) vR63_1(1,3) vR63_
jointspos(2,4:6) = double(subs(vsolsori(2,:),{nx ny nz sx sy sz ax ay az},...
    {vR63_1(1,1) vR63_1(2,1) vR63_1(3,1) vR63_1(1,2) vR63_1(2,2) vR63_1(3,2) vR63_1(1,3) vR63_
jointspos(3,4:6) = double(subs(vsolsori(1,:),{nx ny nz sx sy sz ax ay az},...
    {vR63_2(1,1) vR63_2(2,1) vR63_2(3,1) vR63_2(1,2) vR63_2(2,2) vR63_2(3,2) vR63_2(1,3) vR63_
```

```
jointspos(4,4:6) = double(subs(vsolsori(2,:),{nx ny nz sx sy sz ax ay az},...
    {vR63_2(1,1) vR63_2(2,1) vR63_2(3,1) vR63_2(1,2) vR63_2(2,2) vR63_2(3,2) vR63_2(1,3) vR63_
jointspos(5,4:6) = double(subs(vsolsori(1,:),{nx ny nz sx sy sz ax ay az},...
    {vR63_3(1,1) vR63_3(2,1) vR63_3(3,1) vR63_3(1,2) vR63_3(2,2) vR63_3(3,2) vR63_3(1,3) vR63_
jointspos(6,4:6) = double(subs(vsolsori(2,:),{nx ny nz sx sy sz ax ay az},...
    {vR63_3(1,1) vR63_3(2,1) vR63_3(3,1) vR63_3(1,2) vR63_3(2,2) vR63_3(3,2) vR63_3(1,3) vR63_
jointspos(7,4:6) = double(subs(vsolsori(1,:),{nx ny nz sx sy sz ax ay az},...
    {vR63_4(1,1) vR63_4(2,1) vR63_4(3,1) vR63_4(1,2) vR63_4(2,2) vR63_4(3,2) vR63_4(1,3) vR63_
jointspos(8,4:6) = double(subs(vsolsori(2,:),{nx ny nz sx sy sz ax ay az},...
    {vR63_4(1,1) vR63_4(2,1) vR63_4(3,1) vR63_4(1,2) vR63_4(2,2) vR63_4(3,2) vR63_4(1,3) vR63_
end
```

2d) Inverse Kinematics (Iterative Approach)

This is the function to compute the Inverse Kinematic using the iterative approach: **Jacobian pseudo-inverse method/algorithm**.

```
function [q_n, xd_p, xd_o_] = inv_kin_pse(K, delta_t, q_n, q, A06, JA)

% Define variables
syms q1 q2 q3 q4 q5 q6 as real

% Desired pose (position)
xd_p = double(subs(A06(1:3,4),{q1 q2 q3 q4 q5 q6},{q(1) q(2) q(3) q(4) q(5) q(6)}));

% Desired pose (orientation)
xd_o = double(subs(A06,{q1 q2 q3 q4 q5 q6},{q(1) q(2) q(3) q(4) q(5) q(6)}));
Phie = xd_o(1:3,1:3);
phi = atan2(Phie(2,3),Phie(1,3));
theta = atan2(sqrt(Phie(1,3)^2+Phie(2,3)^2),Phie(3,3));
psi = atan2(Phie(3,2),-Phie(3,1));
xd_o_ = [phi theta psi];

% While loop
error = 1;
iterations = 0;

while norm(error) > 0.01
    % Updating the Pose
    pos = double(subs(A06(1:3,4),{q1 q2 q3 q4 q5 q6},{q_n(1) q_n(2) q_n(3) q_n(4) q_n(5) q_n(6
    pos_o = double(subs(A06,{q1 q2 q3 q4 q5 q6},{q_n(1) q_n(2) q_n(3) q_n(4) q_n(5) q_n(6)}));
    Phie = pos_o(1:3,1:3);
    phi_pos = atan2(Phie(2,3),Phie(1,3));
    theta_pos = atan2(sqrt(Phie(1,3)^2+Phie(2,3)^2),Phie(3,3));
    psi_pos = atan2(Phie(3,2),-Phie(3,1));

    % Update of Error between the desired position and the updated position
    error_p = double([xd_p(1)-pos(1), xd_p(2)-pos(2), xd_p(3)-pos(3)]);
    error_o = double([phi - phi_pos, theta - theta_pos, psi - psi_pos]);
    error = [error_p error_o]';

    % Update of Analytical Jacobian JA
    JA = double(subs(JA,{q1 q2 q3 q4 q5 q6},{q_n(1) q_n(2) q_n(3) q_n(4) q_n(5) q_n(6)}));
```

```
    % Computation of the new velocity
    q_dot = double(pinv(JA)*K*error);

    % Computation of the new position
    q_n = double(q_n + q_dot'*delta_t);

    % Next iteration step
    iterations = iterations + 1;
end
end
```