# Supervised Learning: SVM Method + Model evaluation methods

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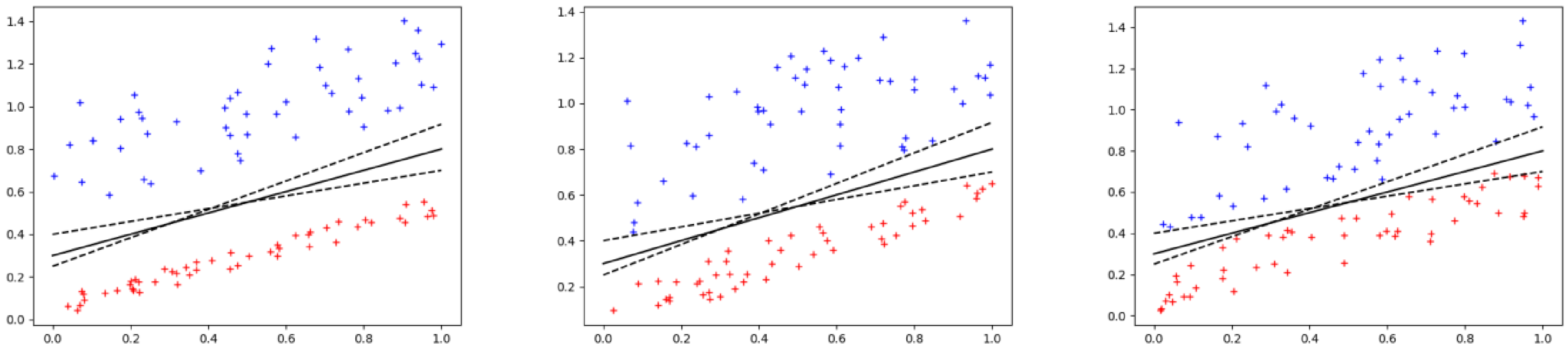
# SVM (Support Vector Machine) Method



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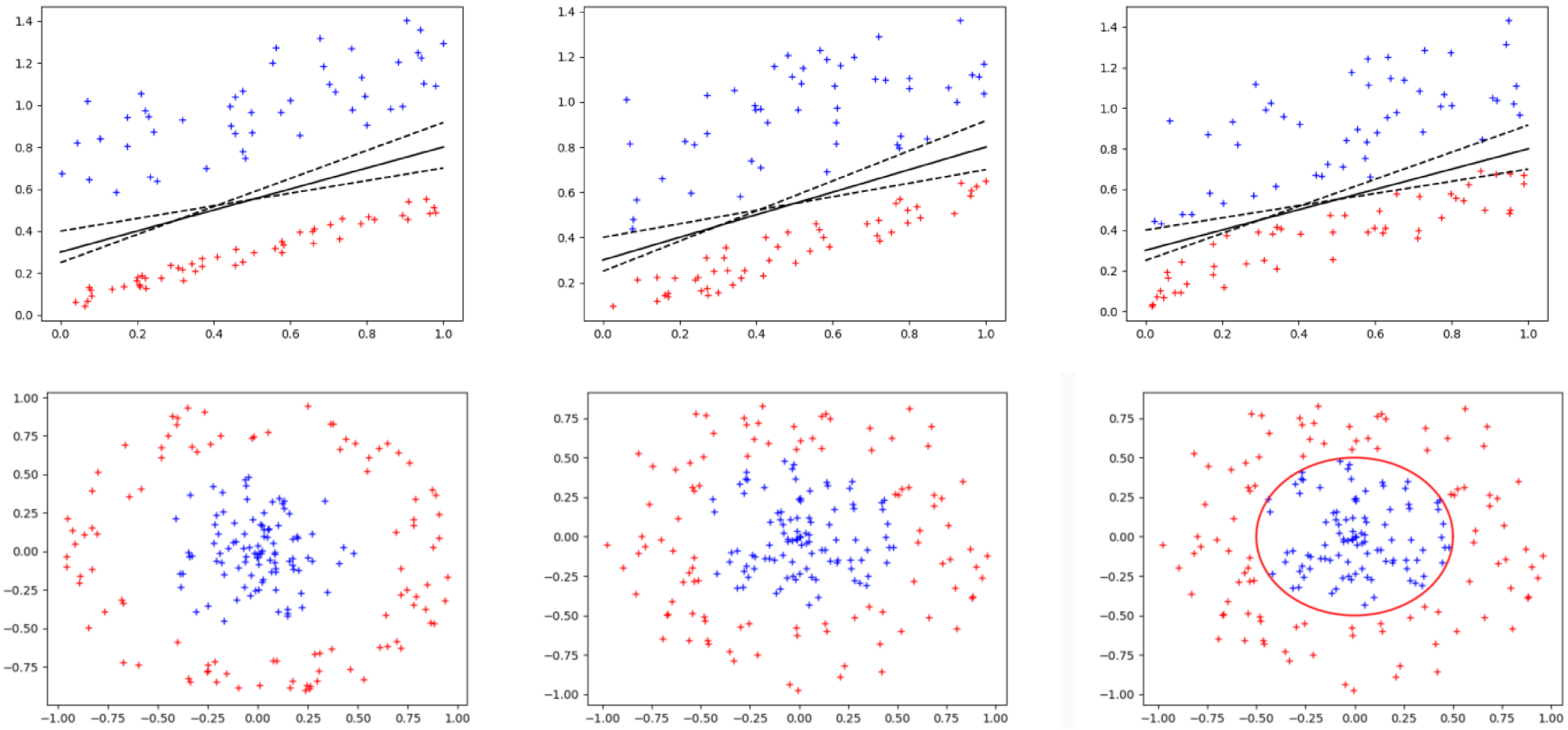
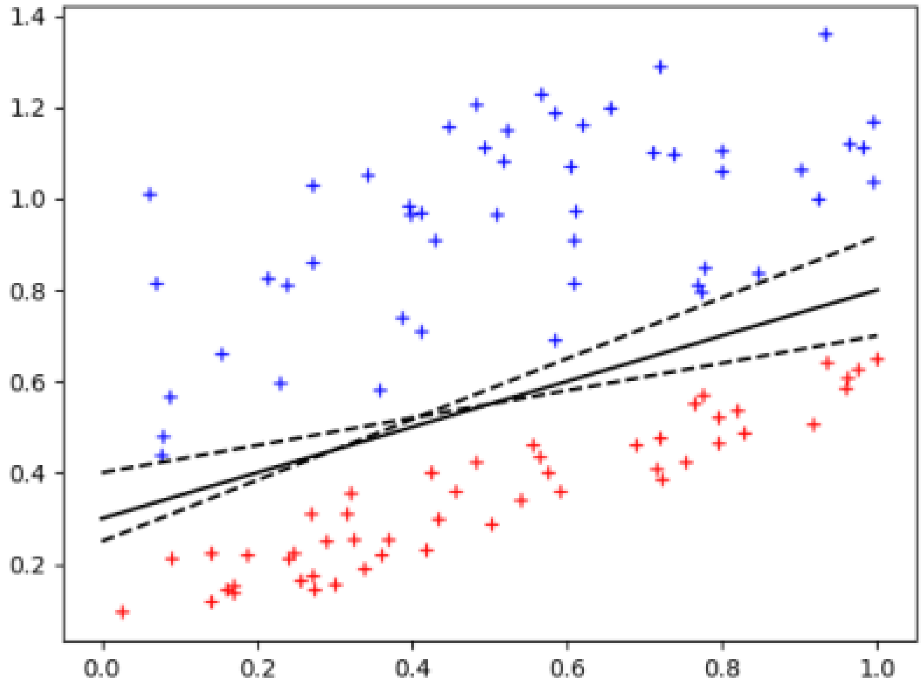


# Linear SVM (Separable classes)

- Every data-sample : $x \in \mathbb{R}^D$
- Decision Boundary : $\mathcal{H}: \; w^T x + b = 0$
- Distance Measure of Hyper plane : $d_{\mathcal{H}}(x_0) = \frac{|w^T x + b|}{||w||_2}$

- **Goal** : $w^* = argmax_w[\min_n d_{\mathcal{H}}(x_n)]$



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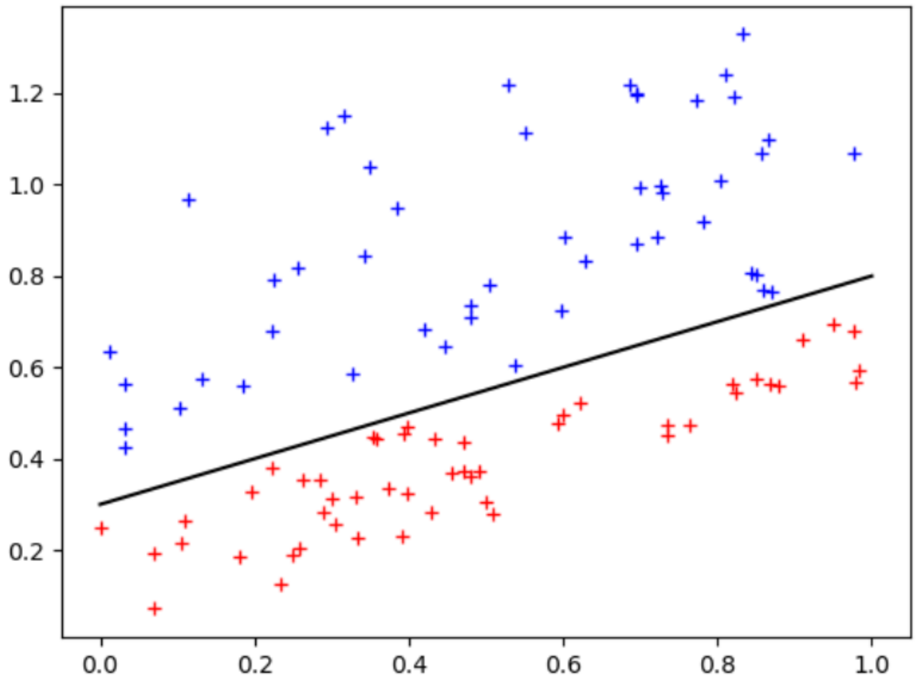
$$y_n[w^T x + b] = \begin{cases} \geq 0, class\ A \\ < 0, class\ B \end{cases}$$

$$w^* = argmax_w \frac{1}{||w||_2} [\min_n y_n [w^T x + b]]$$

**Let** $\min_n y_n [w^T x + b] = 1$

$$w^* = argmax_w \frac{1}{||w||_2}$$



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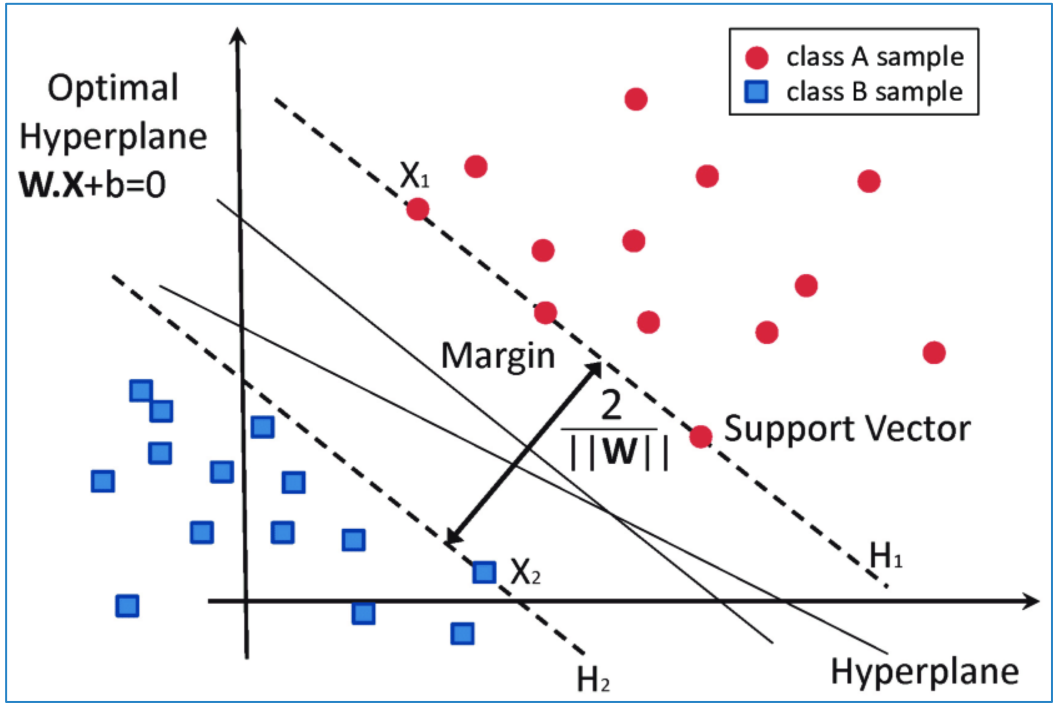
$$y_n[w^T x + b] \geq 1, \forall n$$

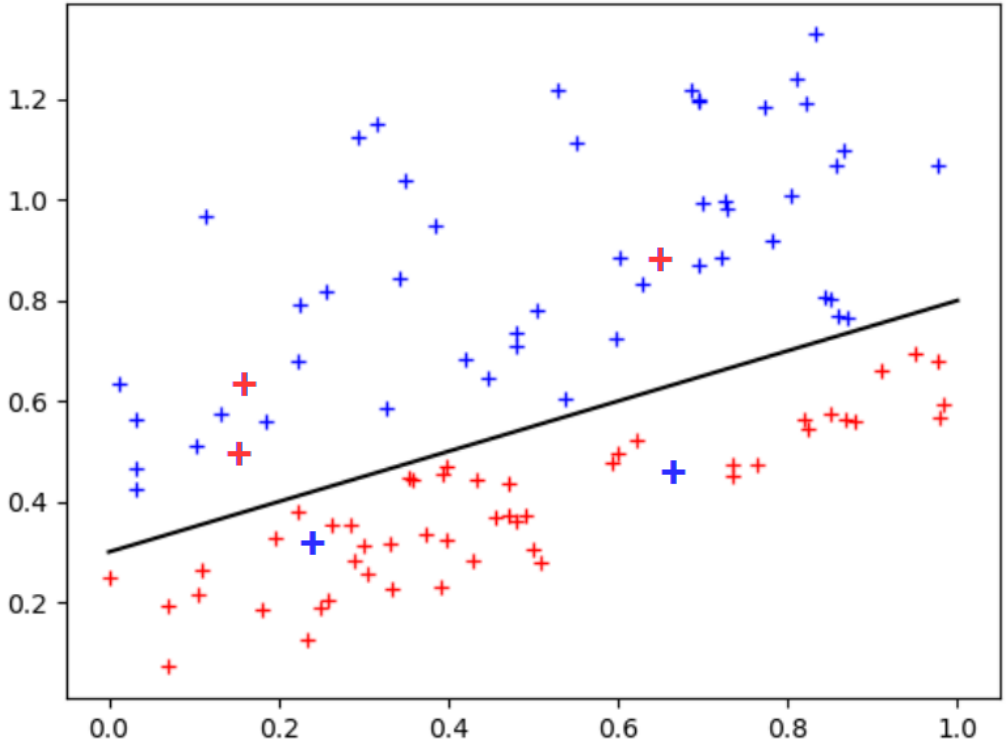
$$\min_w \frac{1}{2} ||w||_2$$



# Linear SVM (General case)

**Final form SVM optimization problem :**

$$\min_{w} \frac{1}{2} ||w||_2 + C \sum_n \xi_n$$
$$s.t.\, y_n[w^T x + b] \geq 1 - \xi_n, \forall n$$
$$\xi_n \geq 0, \forall n$$



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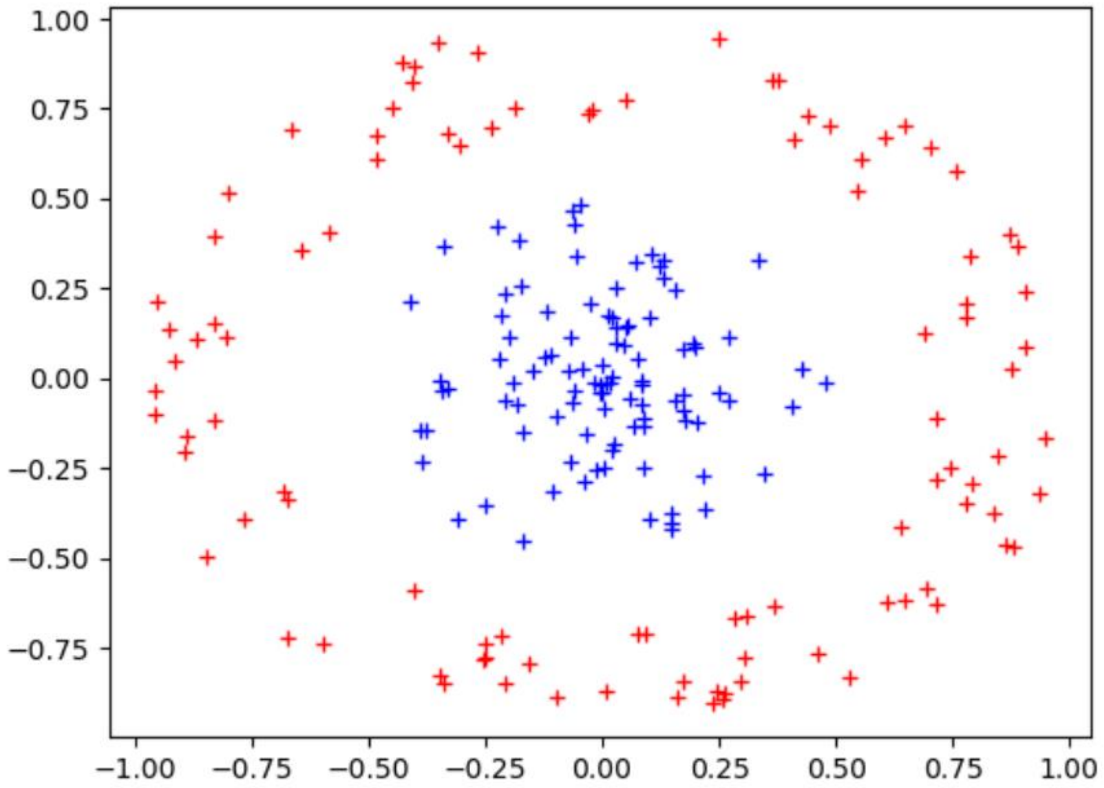

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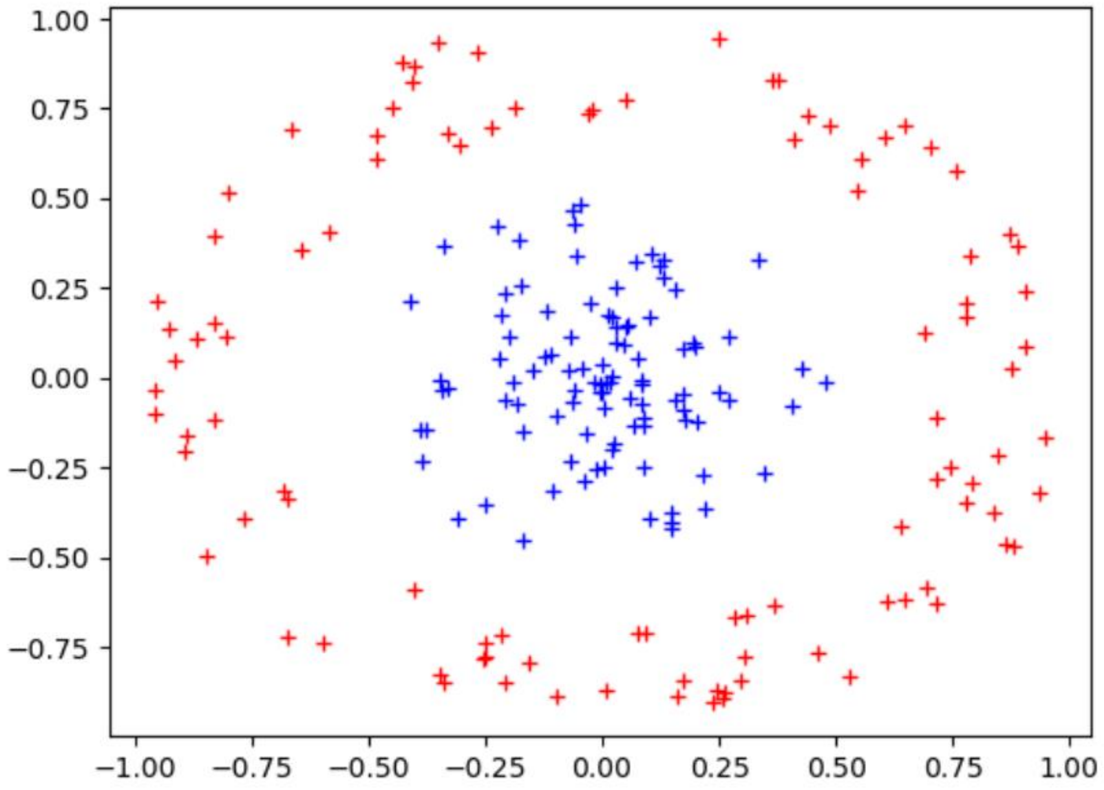
$$\boxed{\varphi(x) : \mathbb{R}^D \to \mathbb{R}^M}$$



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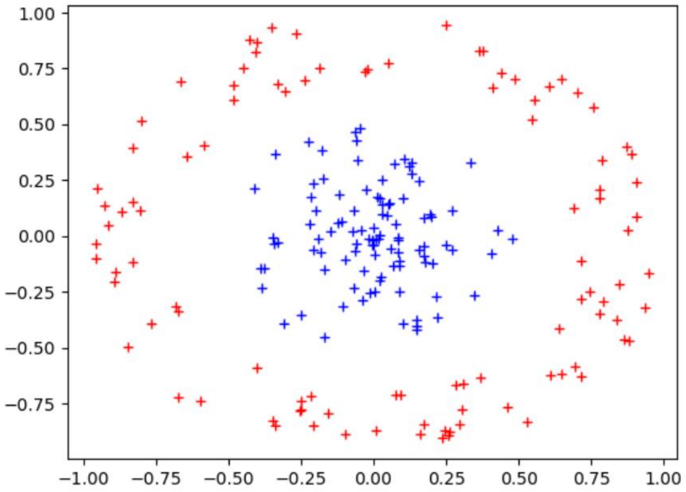
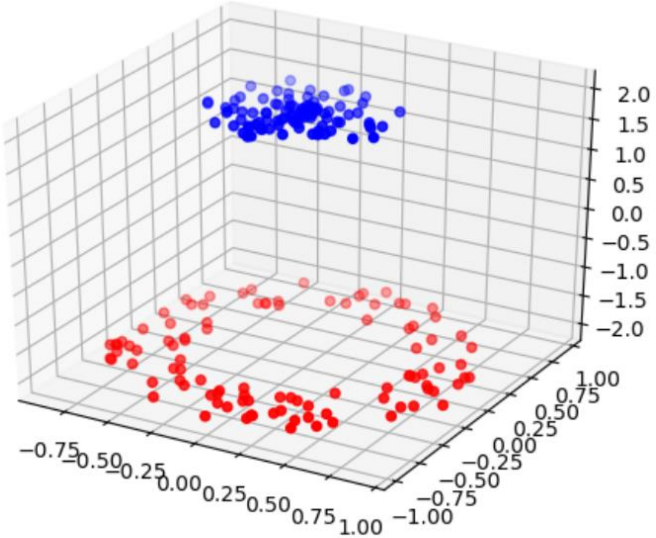
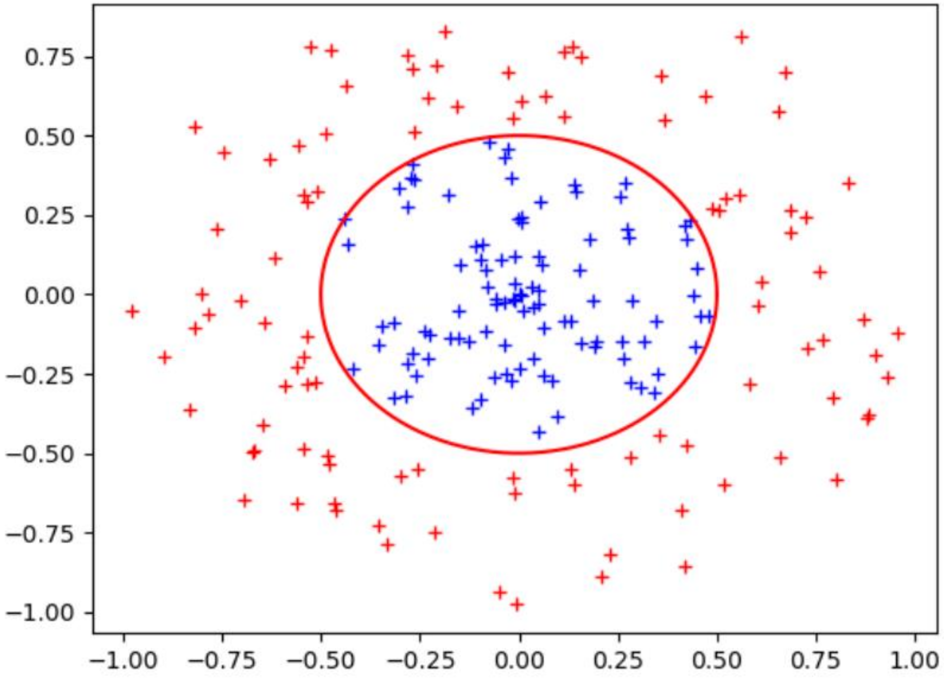


# Linear SVM (General case)

**The Most Final form SVM optimization problem :**

$$\min_{w,\{\xi_n\}} \frac{1}{2}||w||_2 + C\sum_n \xi_n$$
$$s.t.\, y_n[w^T\varphi(x) + b] \geq 1 - \xi_n, \forall n$$
$$\xi_n \geq 0, \forall n$$

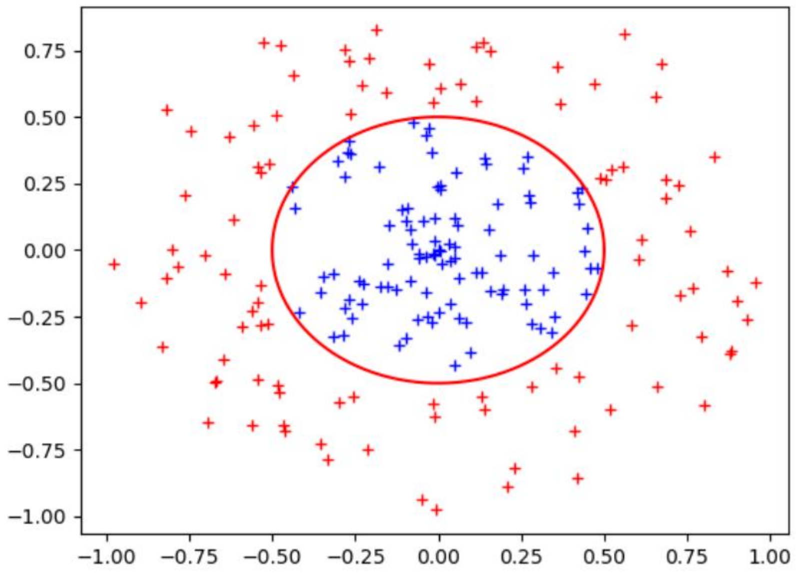


# Kernel SVM

**Problem :** Don’t know the nature of $\varphi(x)$ transformation

**Idea :** Rewrite Optimization problem without determining $\varphi(x)$

It’s possible via applying Lagrange multipliers optimization method, Dual Form of SVM, Mercer Theorem etc.



# Kernel SVM

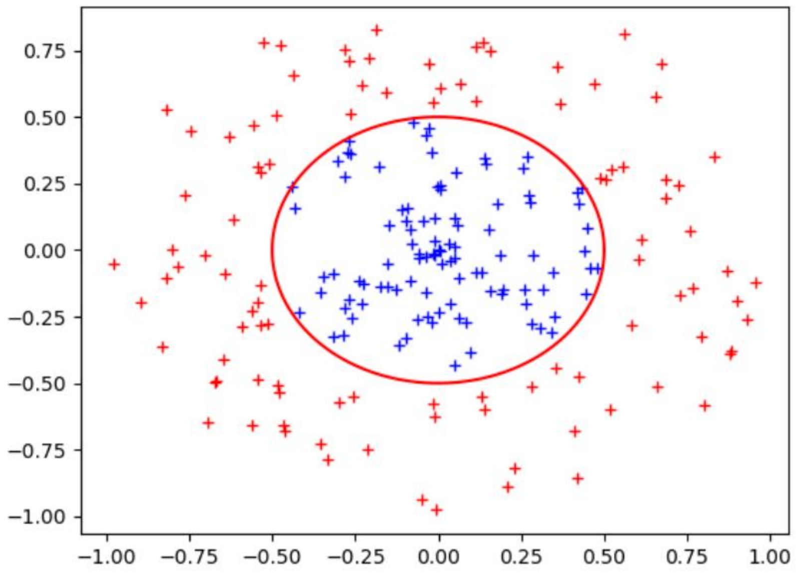
**Problem :** Don't know the nature of $\varphi(x)$ transformation

**Idea :** Rewrite Optimization problem without determining $\varphi(x)$

$$\min_{w,\{\xi_n\}} \frac{1}{2}||w||_2 + C\sum_n \xi_n$$
$$s.t.\, y_n[w^T\varphi(x)+b] \geq 1-\xi_n, \xi_n \geq 0, \forall n$$

**Dual Form :** $\varphi * \varphi^T = K(x,x')$

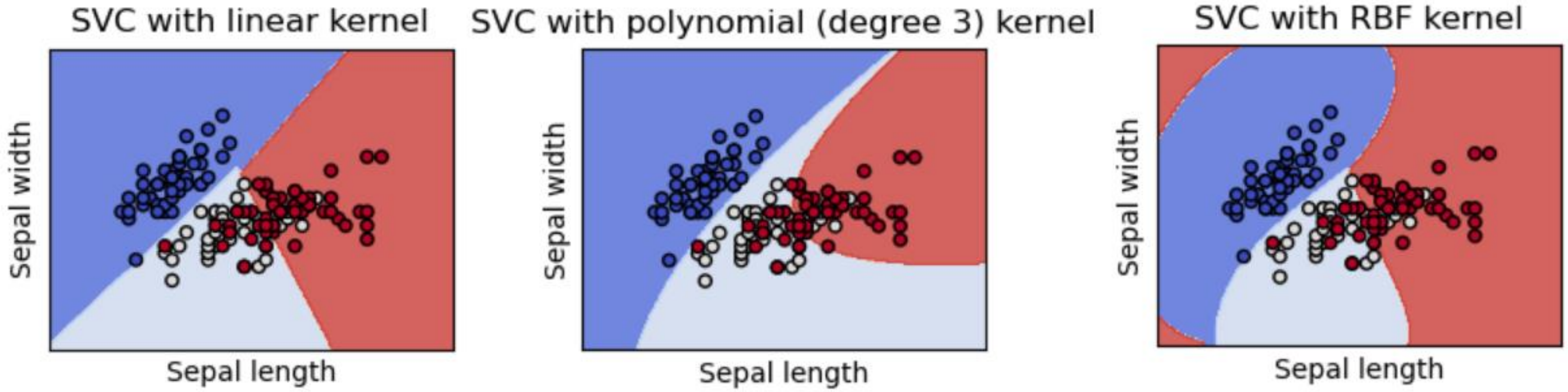
$$\begin{cases} \max L(\lambda) = \sum_{k=1}^{m} \lambda_k - \sum_{k=1}^{m}\sum_{l=1}^{m} \lambda_k\lambda_l y_k y_l \boldsymbol{K}(\boldsymbol{x_k},\boldsymbol{x_l}) \\ s.t.\, 0 \leq \lambda_k \leq C \end{cases}$$



# Kernel SVM

Kernel exemples :

- linear: $k(x, x') = x^T x'$
- polynomial:$k(x, x') = (x^T x' + 1)^d$ of different power $d = 2, 3, \dots$
- gauss-RBG: $k(x, x') = \exp(-\frac{1}{2\sigma}|x - x'|^2)$



# SVM : Advantages & Drawbacks

- Good interpretability of the model
- Effective in high dimensional spaces
- Memory efficient

- **High sensitivity to the noise** in input data
- Slow training on large dataset

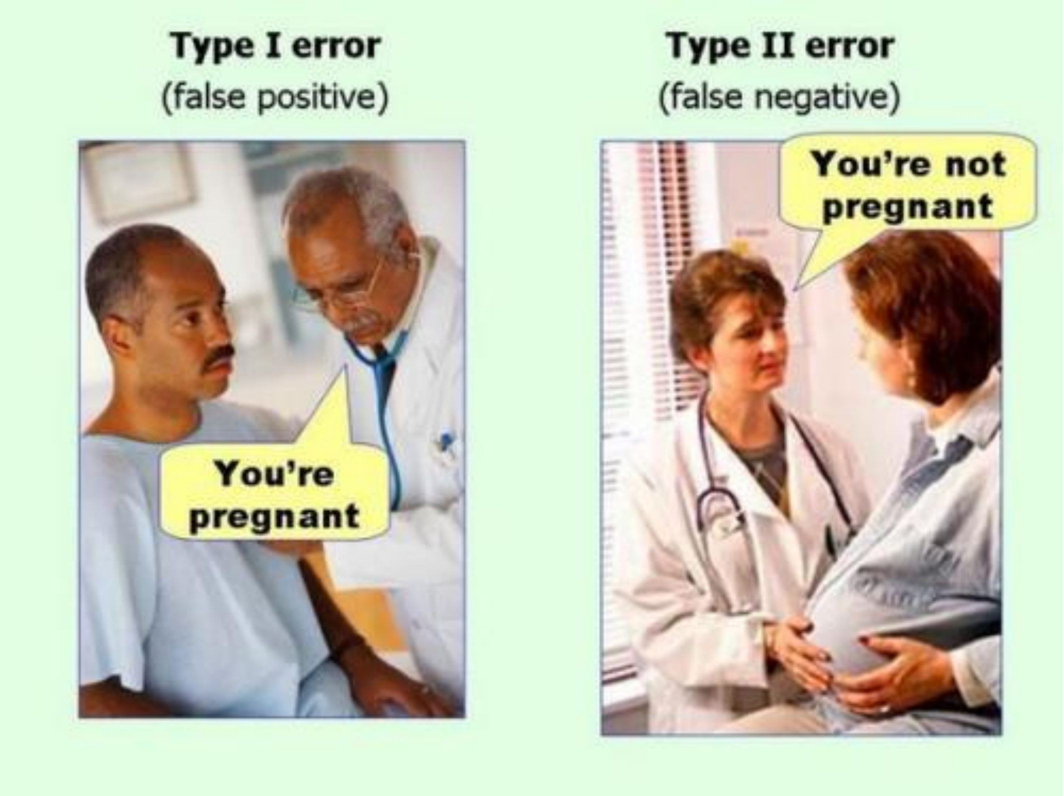
# Model evaluation methods

# Performance measures for Classification

- Simple Accuracy
- Precision
- Recall
- F-beta mesure
- ROC (and AUC)

# Confusion matrix

| | | Predicted class | |
|---|---|---|---|
| | | $P$ | $N$ |
| Actual Class | $P$ | True Positives (TP) | False Negatives (FN) |
| | $N$ | False Positives (FP) | True Negatives (TN) |



# Metrics

| Actual Class | Predicted class: $P$ | Predicted class: $N$ |
|---|---|---|
| $P$ | True Positives (TP) | False Negatives (FN) |
| $N$ | False Positives (FP) | True Negatives (TN) |

- **Accuracy** $= \frac{(\mathbf{TP}+\mathbf{TN})}{(\mathbf{TP}+\mathbf{TN}+\mathbf{FP}+\mathbf{FN})} = \frac{(\mathbf{TP}+\mathbf{TN})}{\mathbf{N}}$

  (fraction of correct predictions)

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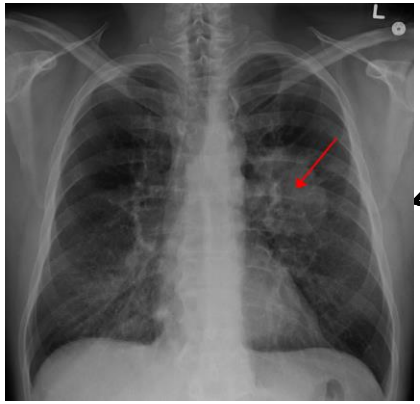

**9990** Not tumor

**10** Tumor

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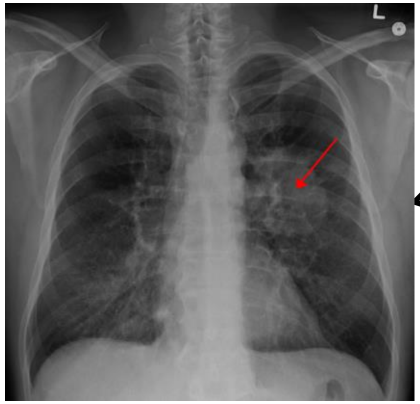

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**All is NOT tumor :**

Accuracy = 99,9%

# Metrics

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- **Precision** $= \dfrac{\mathbf{TP}}{(\mathbf{TP+FP})}$

  (fraction of correctly predicted positive values to all values predicted positive)

- **Recall** $= \dfrac{\mathbf{TP}}{\mathbf{(TP + FN)}}$

  (completeness, fraction of correctly predicted positive values to all positive values)

# Metrics

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- **Precision** $= \dfrac{\mathbf{TP}}{(\mathbf{TP+FP})}$
  (fraction of correctly predicted positive values to all values predicted positive)

  => **Precision** $= \dfrac{\mathbf{0}}{\mathbf{0}}$

- **Recall** $= \dfrac{\mathbf{TP}}{\mathbf{(TP + FN)}}$
  (completeness, fraction of correctly predicted positive values to all positive values)

  => **Recall** $= \dfrac{\mathbf{0}}{\mathbf{0+10}} = \mathbf{0}$

# Example 1&2

| | | Predicted class | |
|---|---|---|---|
| | | *P* | *N* |
| Actual Class | *P* | True Positives (TP) | False Negatives (FN) |
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**All is NOT Tumor :**

Accuracy = 99,9%

$\text{Precision} = \frac{0}{0}$

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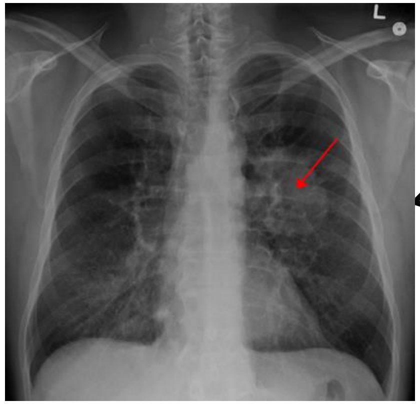

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# Example 1&2

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**All is NOT Tumor :**

Accuracy = 99,9%

$\text{Precision} = \frac{0}{0}$

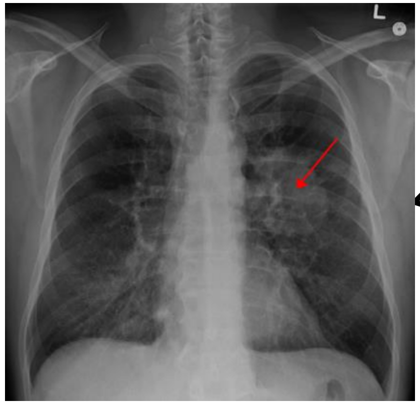
$\text{Recall} = 0$

**All is Tumor :**

Accuracy = 0,1%

Precision = 0,001

$\text{Recall} = 1$



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# Example 3

**System 1 :**

- Precision$= 70\%$
- Recall $= 60\%$

?

**System 2 :**

- Precision $= 80\%$
- Recall $= 50\%$

# Metrics

- $\mathbf{F_\beta} = \dfrac{\mathbf{1}}{\left(\boldsymbol{\beta} * \dfrac{\mathbf{1}}{\mathbf{Precision}} + (\mathbf{1} - \boldsymbol{\beta}) * \dfrac{\mathbf{1}}{\mathbf{Recall}}\right)}$ (greater $\beta$, greater importance of Precision)
- $\mathbf{F_1} = \dfrac{\mathbf{2TP}}{\mathbf{(2TP + FP + FN)}}$ (harmonic mean of precision and recall, $\beta = 0.5$)

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$F_\beta =$

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$\boldsymbol{\beta = 0.5}$

$F_\beta = 0.6461$ > $F_\beta = 0.6153$

$\boldsymbol{\beta = 0.95}$ (Pression is more important)

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$\boldsymbol{\beta = 0.5}$

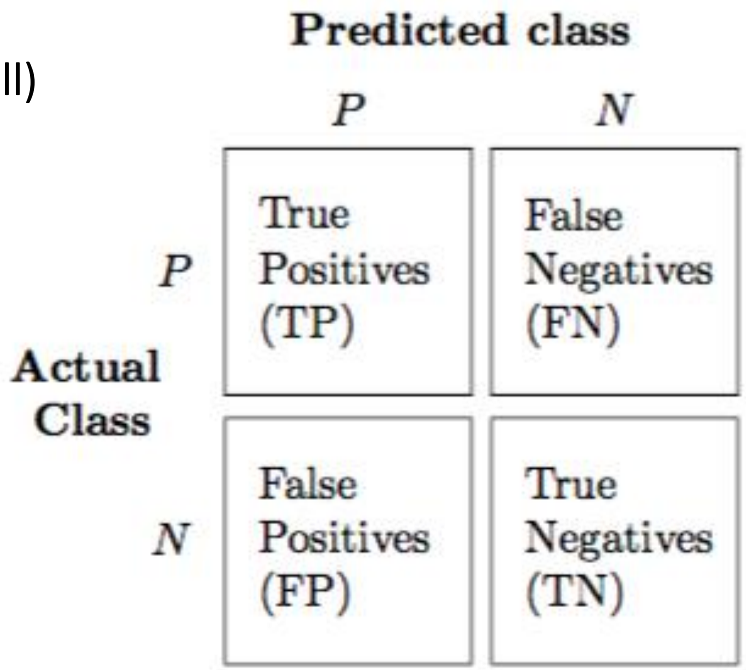
$F_\beta = 0.6461$ > $F_\beta = 0.6153$

$\boldsymbol{\beta = 0.95}$ (Pression is more important)

$F_\beta = 0.6942$ < $F_\beta = 0.7766$

# ROC (Receiver operating characteristic)

- **Sensitivity** $= \frac{\mathbf{TP}}{\mathbf{P}} = \frac{\mathbf{TP}}{\mathbf{(TP + FN)}} = \mathbf{TPR}$ (True positive rate, TPR, Recall)
- **Specificity** $= \frac{\mathbf{TN}}{\mathbf{N}} = \frac{\mathbf{TN}}{\mathbf{(FP + TN)}} = \mathbf{1 - FPR}$ (True negative rate, TNR)



# ROC (AUC)

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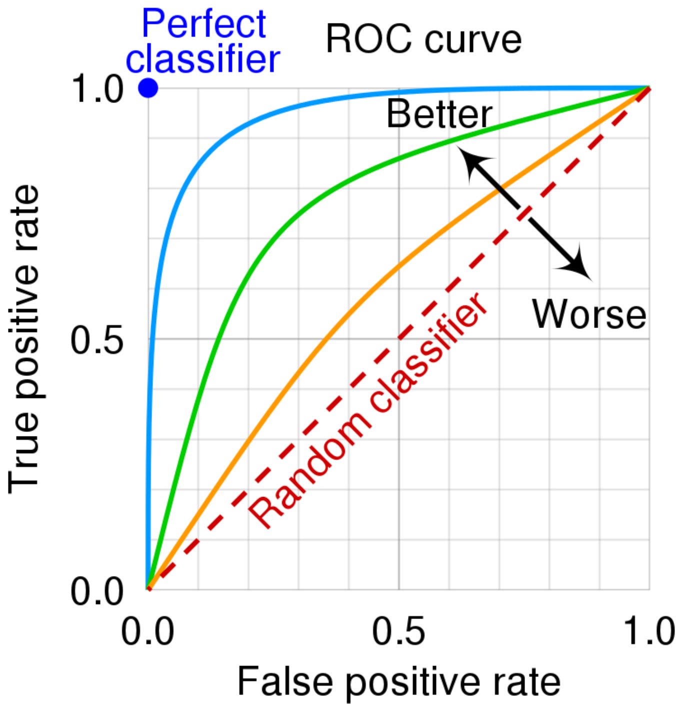

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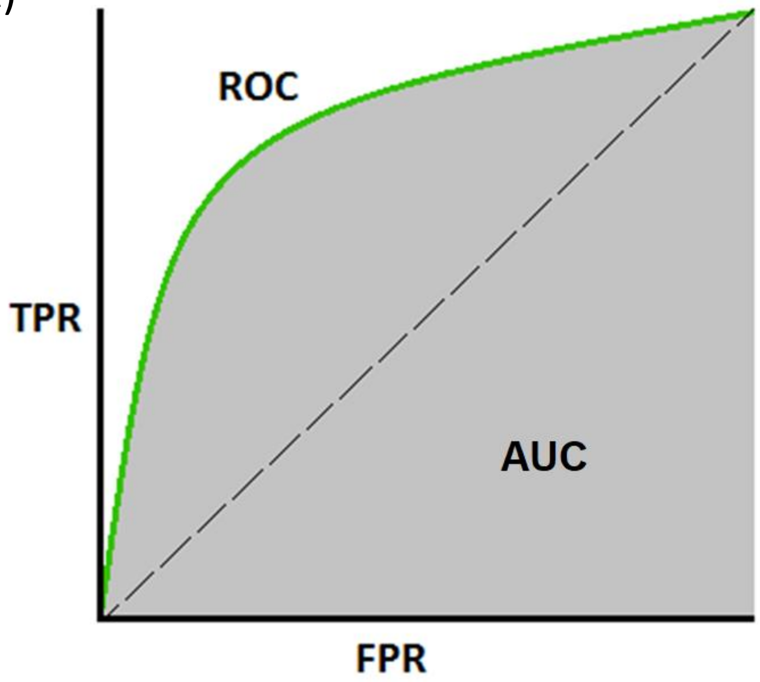

# Sources

- MIT course “Introduction to Computational Thinking and Data Science” (Prof. Eric Grimson, Prof. John Guttag)
- Open Machine Learning Course (by Yury Kashnitsky, mlcourse.ai)
- YouTube lections “Algorithms and Concepts” (by CodeEmporium)