**Experiments:**

Experiment 1

Parameters for this experiment are: . In this experiment we have 32 different variations and totally 320 generated problems.

The goal of the experiment on this data set is to compare time performance on TBased2 and Guan formulations. Results of the experiment are analyzed on the different factors, to infer what factors are most affecting each formulation. The factors (independent variables) are machines, number of jobs, due date, and process time. Tested dependent variables are solution time.

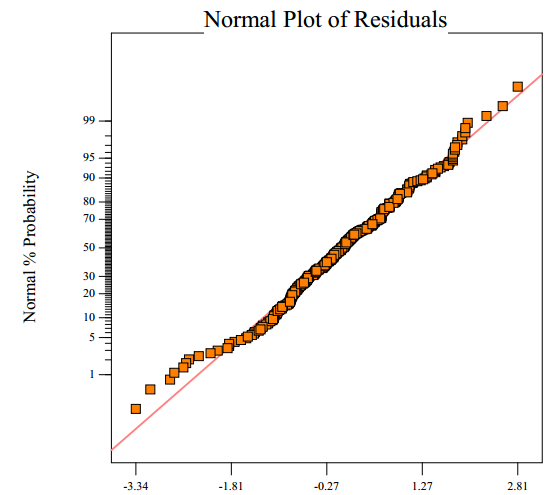
|  |  |  |
| --- | --- | --- |
|  | **Guan** | **TBased2** |
| **Total solution time** | 108585 sec. | 72190 sec. |
| **Total solution time optimal only** | 31190 sec. | 39580 sec. |
| **Total solution time mutual optimal** | 27944 sec. | 33547 sec. |
| **Median of solution time** | 6.6 sec. | 33.3 sec. |
| **Median of solution time for optimally solved problem** | 2.1 sec. | 28.5 sec. |
| **Better solution time** | 201 | 119 |
| **Optimality proved** | 277 | 300 |
| **Only one formulation proved optimality** | 14 | 37 |
| **Mutual optimality** | 263 | 263 |
| **Better solution value** | 10 | 2 |

<Table:1> Summary of solution time

Summary of solution results shown in <Table:1>, in total TBased2 solved all problems faster than Guan. Despite in total TBased2 took less time to solve all problems, in almost of the problems, 201, Guan had better solution time. Comparing only optimally solved problems, by both of the models, Guan had a better total solution time 27944 sec, vs. 33547 sec. that it took to solve with TBased2 formulation, it means that on average Guan solved each problem in 106.25 sec (SD = 261), and for TBased2 it took 127.55 sec. (SD = 308) per problem. In both formulations the standard deviations of problems that are optimally solved for both of them, are bigger than the mean, this shows that there is a big variation in solution time.

TBased2 has a little bit more optimally proved problems than Guan, 300 vs. 277, but despite that Guan formulation resulted 10 times in a better solution than TBased2, while TBased2 had only 2 results that are better than Guan. Additionally <TableId:1> shows that for each formulation there is a small set of problems that only it, was able to proof optimality, 14 for Guan, and 37 for TBased2, it indicates that the factors are affect differently on each of the formulation.

To analyze results, and understand which factors are most influencing solution time of a problem, ANOVA test was used. To test Guan solution time a log transformation was used, to better fit the ANOVA model. <Figure:1> shows a good match of the residuals to the normal distribution.

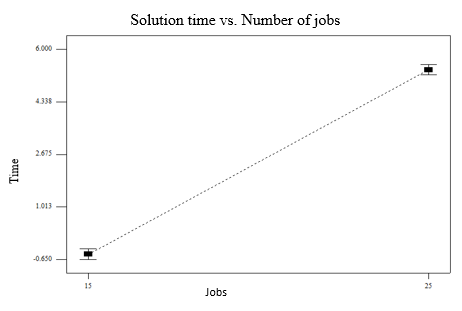


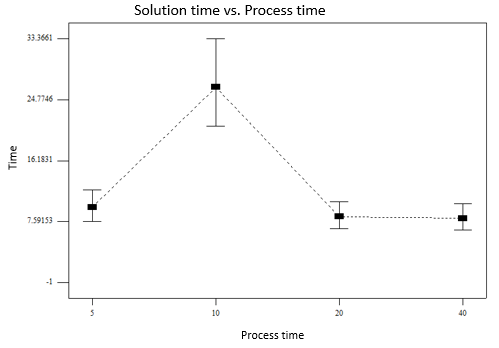
<Figure:1>. Normal plot of residuals for guan solution time.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Source** | **Sum of**  **Squares** | **df** | **Mean**  **Square** | **F**  **Value** | **p-value**  **Prob > F** |
| **Model** | 2865.464 | 18 | 159.1924565 | 73.25188724 | < 0.0001 |
| A-Machines | 7.730978 | 1 | 7.730977774 | 3.557384092 | 0.0602 |
| B-Jobs | 2727.4 | 1 | 2727.400103 | 1255.004221 | < 0.0001 |
| C-Duedate | 0.035788 | 1 | 0.035788277 | 0.016467859 | 0.8980 |
| D-ProcessTime | 77.28382 | 3 | 25.76127281 | 11.85396528 | < 0.0001 |
| AB | 0.641247 | 1 | 0.64124744 | 0.295067908 | 0.5874 |
| AC | 0.890483 | 1 | 0.890483269 | 0.409752958 | 0.5226 |
| AD | 6.352961 | 3 | 2.117653711 | 0.974431416 | 0.4051 |
| BC | 0.515116 | 1 | 0.5151165 | 0.23702917 | 0.6267 |
| BD | 18.93097 | 3 | 6.310323122 | 2.903674508 | 0.0351 |
| CD | 25.68275 | 3 | 8.560917158 | 3.93927798 | 0.0088 |
| Residual | 654.1392 | 301 | 2.173219865 |  |  |
| Lack of Fit | 41.80977 | 13 | 3.216136455 | 1.512661798 | 0.1117 |
| Pure Error | 612.3294 | 288 | 2.126143768 |  |  |

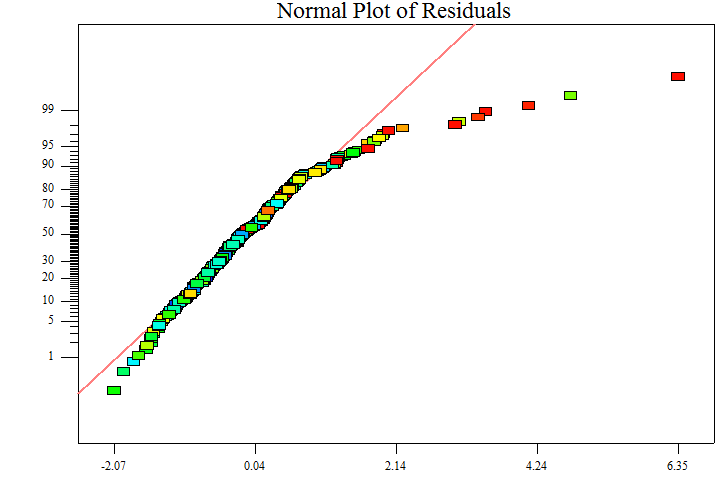
<Table:2> Natural log transformed ANOVA results for Guan formulation

<Table:2> shows received ANOVA results for testing Guan’s solution time dependent variable. There are two significant factor for guan solution time, number of jobs with , indicating that as number of jobs is growing the time it takes to solve the problem is increasing, and a second factor, process time, with . As can be seen from <Figure:12>, process time does not impact the solution time in any specific direction, it is not expected that process time will be significant factor, since it does not have a relation to the number of integer variables in Guan’s formulation. On the other hand number of jobs has a visible impact, <Figure:2>, on solution time, this result is expected since the amount of integer variables in Guan’s formulation is being affected by the number of jobs in the problem, and as the number of jobs is bigger, the number of integer variables is growing and the time for solving a problem is increasing drastically.

 <Figure:2>. Natural log transformation of time as a function of number of jobs.

<Figure:12>. Natural log transformation of time as a function of process time.

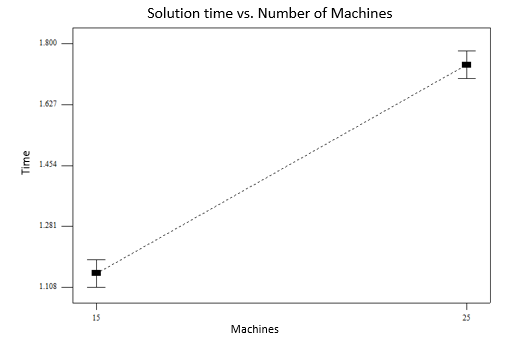
As for Guan, for the TBased2 a natural log transformation was used to better fit ANOVA model, <Figure:3> shows the residual matches, and <Table:3> summarizes the ANOVA results. All factors except due date are affecting the time that will be needed to solve the problem.

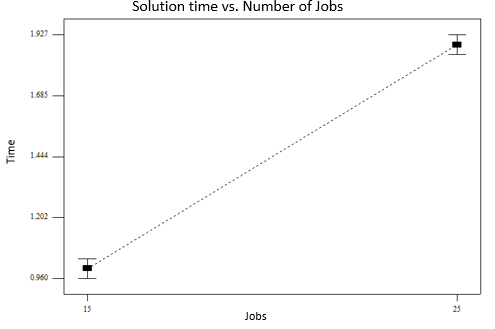
<Figure:3>. Normal plot of residuals for tbased2 solution time.

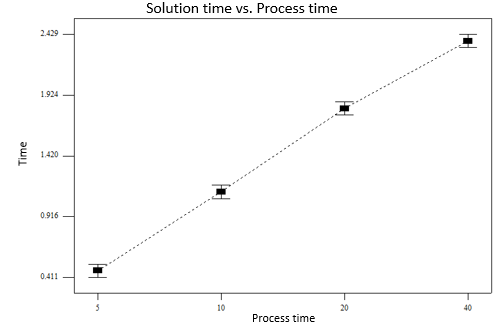
Machines factor is significant with , indicating that increase in the number of machines will cause increase in solution time. Jobs factor is significant with indicating that increase in number of jobs will cause increase in solution time. Process time factor is significant with indicating that increase in average process time for a job will increase problem solution time. <Figure:4>, <Figure:5>,<Figure:6> visualize the change in solution time, as the factor changes. It is trivial to understand why number of jobs and number of machines affect the solution time, as they directly impact the number of integer variables that a formulation will have. Process time is a little bit different, in TBased2 formulation, time horizon is the third factor, besides number of jobs and machines, that impact on the number of integer variables, in conducted tests time horizon was set as , thus increase in average process time would increase time horizon and hence increase the number of integer variables.

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| --- | --- | --- | --- | --- | --- |
| **Source** | **Sum of**  **Squares** | **df** | **Mean**  **Square** | **F**  **Value** | **p-value**  **Prob > F** |
| **Model** | 261.0214502 | 18 | 14.50119 | 115.667 | < 0.0001 |
| A-Machines | 28.19806843 | 1 | 28.19807 | 224.9184 | < 0.0001 |
| B-Jobs | 63.12257883 | 1 | 63.12258 | 503.4894 | < 0.0001 |
| C-Duedate | 0.908409631 | 1 | 0.90841 | 7.245817 | 0.0075 |
| D-ProcessTime | 164.8459918 | 3 | 54.94866 | 438.2912 | < 0.0001 |
| AB | 1.400362347 | 1 | 1.400362 | 11.16982 | 0.0009 |
| AC | 0.172973161 | 1 | 0.172973 | 1.379699 | 0.2411 |
| AD | 0.560762182 | 3 | 0.186921 | 1.49095 | 0.2170 |
| BC | 0.10721326 | 1 | 0.107213 | 0.855173 | 0.3558 |
| BD | 1.391946089 | 3 | 0.463982 | 3.700895 | 0.0122 |
| CD | 0.313144411 | 3 | 0.104381 | 0.832586 | 0.4768 |
| Residual | 37.73643569 | 301 | 0.12537 |  |  |
| Lack of Fit | 1.317130113 | 13 | 0.101318 | 0.80121 | 0.6588 |
| Pure Error | 36.41930558 | 288 | 0.126456 |  |  |

<Table:3> Natural log transformed ANOVA results for TBased2 time performance

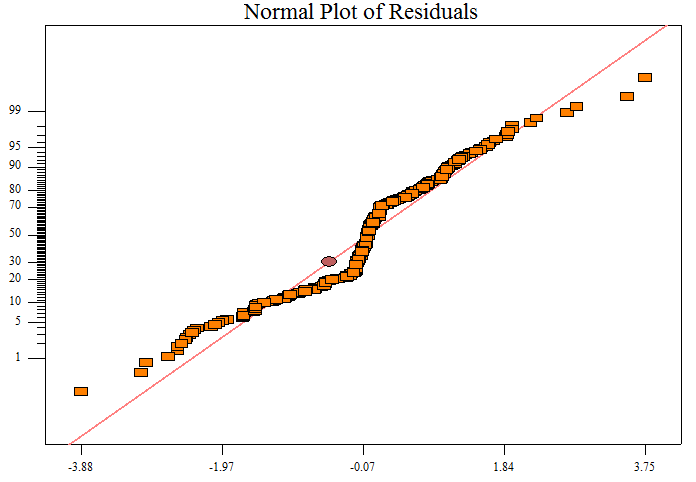
<Figure:4>. Natural log transformation of TBased2 solution time vs number of machines

<Figure:5>. Natural log transformation of TBased2 solution time vs number of jobs

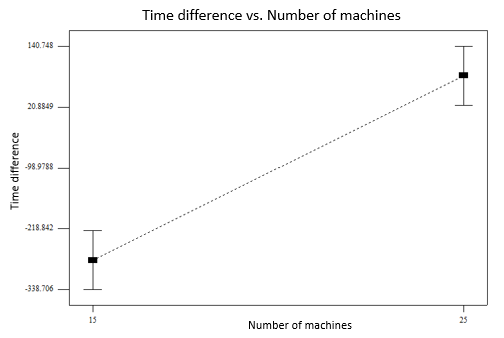
<Figure:6>. Natural log transformation of TBased2 solution time vs max job process time

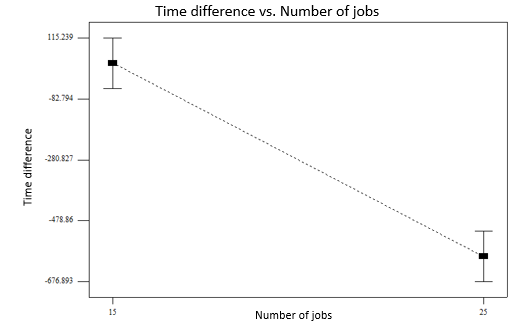
This data set is also constructed in a way that it is possible to compare what is happening to each formulation as if the time resolution changes, i.e. solving the same problem but once when a time unit is hour, and then when the time unit is half an hour. From <Figure 7>, it is seen that as the resolution is high, for example hours, TBased2 is better in solving the problems, and when moving to a more granular time units, for example quarter hours, TBased2 performs slower than Guan formulation, since Guan’s solution time is not impacted by this factor.

<Figure:7>

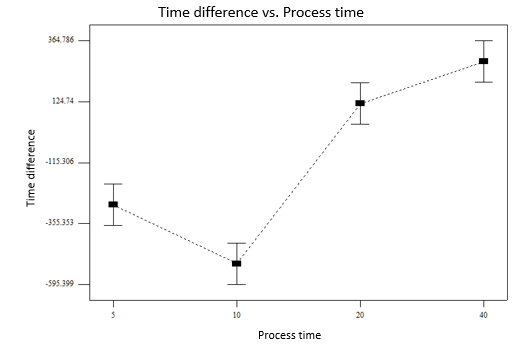
Third dependent variable that is tested is the difference of solution time between TBased2 and Guan. This time no transformation was done on the data, as there was no big benefit in doing so. Residual plot validates that we can analyses this dependent variable with ANOVA, <Figure:8>. All factors except due date, has a significant impact on the difference between the two formulations. Machines factor is significant <Figure:8>. Normal plot of residuals for solution time difference between TBased2 and Guan.

with factor , indicating that as the number of machines is growing the difference changes from negative (meaning TBased2 is faster), to positive (meaning Guan is faster), <Figure:9> visualize this.

<Figure:9>. Time difference between TBased2 and Guan formulation, as number of machines changes.

Number of jobs is significant factor with . This factor impacts the opposite way from previous one, as number of jobs is growing difference value is degrading, which means that in cases where we have many jobs, TBased2 is better performing than Guan, <Figure:10> shows this. <Figure:10>. Time difference between TBased2 and Guan formulation, as number of jobs changes.

Last significant factor is the process time, with , indicating that as the process time is getting bigger, Guan will have better solution time, <Figure:11> show the difference change. Summary of other ANOVA analysis is provided in <Table:4>. A correlation between the difference and number of machines or process time is expected, because both of the factors have a significant effect on the solution time, only in TBased2 formulation. Number of jobs has inverse correlation, which means that Guan’s performance degrades more rapidly as the number of jobs is being increased, than TBased2 formulation. So for problems with relative big number of jobs it is obvious that TBased2 has more potential of getting better result.

<Figure:11>. Time difference between TBased2 and Guan formulation, process time changes.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | Sum of  Squares | df | Mean  Square | F  Value | p-value  Prob > F |
| Model | 1.01E+08 | 28 | 3615508 | 13.06458 | < 0.0001 |
| A-Machines | 10582026 | 1 | 10582026 | 38.238 | < 0.0001 |
| B-Jobs | 7532790 | 1 | 7532790 | 27.21963 | < 0.0001 |
| C-Duedate | 148575.7 | 1 | 148575.7 | 0.536876 | 0.4643 |
| D-ProcessTime | 31822591 | 3 | 10607530 | 38.33015 | < 0.0001 |
| AB | 8272663 | 1 | 8272663 | 29.89315 | < 0.0001 |
| AC | 572309.5 | 1 | 572309.5 | 2.068032 | 0.1515 |
| AD | 8419730 | 3 | 2806577 | 10.14152 | < 0.0001 |
| BC | 300.6982 | 1 | 300.6982 | 0.001087 | 0.9737 |
| BD | 21686048 | 3 | 7228683 | 26.12074 | < 0.0001 |
| CD | 1907233 | 3 | 635744.2 | 2.297252 | 0.0778 |
| ABC | 580587.7 | 1 | 580587.7 | 2.097945 | 0.1486 |
| ABD | 6422060 | 3 | 2140687 | 7.735339 | < 0.0001 |
| ACD | 833771.7 | 3 | 277923.9 | 1.004274 | 0.3912 |
| BCD | 2453538 | 3 | 817846.1 | 2.955275 | 0.0328 |
| Residual | 80531672 | 291 | 276741.1 |  |  |
| Lack of Fit | 818208.6 | 3 | 272736.2 | 0.98538 | 0.4000 |
| Pure Error | 79713464 | 288 | 276782.9 |  |  |

<Table:4>. ANOVA results for time difference test.