Definitions:

S - number of parallel machines. is a set of n jobs, where each job has a due date size , and process time .

– Arbitrary jobs schedule with ordering, to minimize the number of tardy jobs.

– Single machine jobs schedule with ordering , and modified process time

– Completion time of schedule .

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**Lemma 1**:

**Proof:** Let be the number of machines used in time unit t. From the definition of :

So:

**Lemma 2:** All not tardy jobs in schedule will also be not tardy in schedule .

**Proof:** Lemma 1, shows that if the last job is not tardy in it will not be tardy in . Remove the last jobs from schedule , and apply the same logic again, until there are no jobs in schedule .

**Theorem:** Optimal scheduling which minimizes the number of tardy jobs on a single machine with will give a lower bound for the parallel machine problem.

**Proof:** From Lemma 2 we know that for optimal schedule , we have some schedule , which is not necessary optimal. Hence optimal schedule for single machine with jobs of will have the same or lower amount of tardy jobs.