Definitions:

total number of parallel machines.

a set of *n* jobs, where each job *i* has a due date , size , and process time .

Arbitrary jobs schedule with ordering , based on a non-decreasing order of completion times, .

Single machine jobs schedule with ordering , and modified process time

**Theorem:** an optimal solution for the single machine problem provides a lower bound on the optimal solution of the original problem.

**Proof:** due to capacity constraint, for any set of *n* jobs, , or For any solution of the original problem, the completion time of some job , is . The completion time of job in the equivalent single machine solution is then Then, for any solution *s* of the original problem, with number of tardy jobs, , there is a superior solution of the equivalent single machine problem, with tardy jobs, where . Hence, if is the optimal number of tardy jobs of the single machine problem, then, for any feasible solution *s*, providing a lower bound.

Yellow Sentence - Why we do not need to prove it?, we only proved that