**Theorem:** In number of tardy jobs problem with parallel machines, and unit job size, an upper bound on the time horizon T is .

Definitions:

– Arbitrary optimal jobs schedule with ordering, to minimize the number of tardy jobs.

- Last processing time on machine, in schedule .

– Process time of job .

– Process time of last job on machine

– Number of parallel machines

– Number of jobs

**Proof:** If in schedule there exists two machines and where , changing the schedule so the last job on will be scheduled last on will not affect the optimality criteria of the schedule. This implies that there is always optimal schedule in which

(1)

and in particular it is true for

(2)

also for any schedule

(3)

combining (2) , (3)

(4)

Which is the upper bound for time horizon.

**Upper bound analysis:**

It is interesting to compare this upper bound to the trivial . Let’s denote as

(5)

Possible values for are . Equation (4) becomes

(6)

Worst case scenario is when , so , best case is when , and it will equal to the trivial upper bound when . For example when , worst case is that proposed upper bound will be 1.5 worse, tie break is when , and the best case is that this upper bound will be twice better than the trivial upper bound.

**Adjustment for different job sizes**

If in the set of jobs, there is a job with size bigger than 1, for the upper bound heuristic it is possible to assume that all jobs are of the same size, and set them to the biggest job. In the newly created problem, adjust the number of machines to , now it is possible to apply previous result.