Thermodynamic cost to order the system and remove entropy

So, in order to order system of spins (remove disorder) we need to apply some energy TS ?

Exactly — ✅ that's *precisely* the right intuition.
Let's unpack it carefully and connect the physical, thermodynamic, and informational viewpoints.

🔹 1. Starting point: disordered spin system

Take n independent spins $s_i = \pm 1$ with no interactions ($\mu B \to 0$).
They are completely random — half up, half down on average.

- Energy of each state: $E = 0$
- Number of possible configurations: 2^n
- Entropy:

$$S = k \ln(2^n) = nk \ln 2$$

This entropy represents **disorder** — or equivalently, **missing information** about which configuration the system is in.

🔹 2. You want to "order" the system

Let's say you want to make *all spins up* — a perfectly ordered, single microstate configuration.
That means:

$$S_{\text{final}} = 0.$$

So you need to **reduce the entropy** by

$$\Delta S = -nk \ln 2.$$

3. Thermodynamic cost of removing entropy

In thermodynamics, removing entropy from a system in contact with a heat bath at temperature T mean
the bath must **absorb heat**

$$Q = T\,\Delta S_{\text{bath}} = T\,(-\Delta S_{\text{system}}) = Tnk\ln 2.$$

In words:

> You must supply at least $Q = nkT\ln 2$ joules of energy to remove the disorder from n spins at temperature T.

That energy doesn't go into the system as "useful energy" — it's dissipated as **heat into the environmer**
It's the **minimum thermodynamic price** for creating order.

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4. Relation to free energy

The initial free energy was:

$$F_i = -nkT\ln 2.$$

The final (ordered) state has

$$F_f = 0,$$

since there's only one configuration (zero entropy).

Therefore, the **change in free energy** is

$$\Delta F = F_f - F_i = nkT\ln 2.$$

That's exactly the amount of energy you need to *input* to order the system.

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5. Information-theoretic view (Landauer's principle)

Each spin carries one bit of information (up/down).
Erasing one bit — i.e., forcing it into a definite state — costs at least:

$$E_{\min} = kT\ln 2$$

of energy dissipation.

At room temperature ($T \approx 300\,\mathrm{K}$):

$$kT \ln 2 \approx 2.8 \times 10^{-21}\ \mathrm{J}.$$

So ordering $n = 10^{20}$ spins would cost about $0.28\,\mathrm{J}$.

🔹 6. Intuitive summary

Concept	Expression	Interpretation
Entropy per spin	$S = k \ln 2$	One bit of disorder per spin
Energy cost to remove disorder	$Q = TS$	Minimal energy needed to fully order the system
Free energy change	$\Delta F = nkT \ln 2$	Work required to reduce entropy
Landauer's limit	$E_{\text{bit}} = kT \ln 2$	Energy cost to erase one bit of information

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✅ **So yes:**

> To order a disordered system of spins, you must supply energy equal to TS,
> which represents the