# Project 2

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## Problem1

A.

```
last 5 rows:
          SPY      AAPL      EQIX
499 -0.011492 -0.014678 -0.006966
500 -0.012377 -0.014699 -0.008064
501 -0.004603 -0.008493  0.006512
502 -0.003422 -0.027671  0.000497
503  0.011538 -0.003445  0.015745


total standard deviation:
SPY     0.008077
AAPL    0.013483
EQIX    0.015361
dtype: float64
```

B.

```
last 5 rows:
          SPY      AAPL      EQIX
499 -0.011515 -0.014675 -0.006867
500 -0.012410 -0.014696 -0.007972
501 -0.004577 -0.008427  0.006602
502 -0.003392 -0.027930  0.000613
503  0.011494 -0.003356  0.015725


total standard deviation:
SPY     0.008078
AAPL    0.013446
EQIX    0.015270
dtype: float64
```

## Problem2

A.

portfolio value at 1/3/2025: 251862.4969

B.

To calculate Var and ES of current asset given today is 1/3/2025, I use data before 1/3/2025 to simulate, and the results:

a: Normally distributed with exponentially weighted covariance with lambda=0.97:
100 shares of SPY: VaR = $811.49, ES = $1,017.65
200 shares of AAPL: VaR = $958.43, ES = $1,201.91
150 shares of EQIX: VaR = $2,905.43, ES = $3,643.52
Portfolio: VaR = $3,829.45, ES = $4,802.28

b. T distribution using a Gaussian Copula
SPY: VaR = $726.32, ES = $1,085.74
AAPL: VaR = $988.76, ES = $1,476.83
EQIX: VaR = $3,328.70, ES = $5,060.09
Portfolio: VaR = $4,264.76, ES = $6,254.49

c. Historic simulation using the full history:
100 shares of SPY: VaR = $871.06, ES = $1,078.74
200 shares of AAPL: VaR = $1,068.56, ES = $1,438.12
150 shares of EQIX: VaR = $3,638.52, ES = $4,710.38
Portfolio: VaR = $4,570.61, ES = $6,053.84

C.

**1. Assumption of distribution**

(1) Method a uses a normal distribution, which is convenient to calculate but may underestimate extreme losses.
(2) Method b uses the T-distribution, which captures fat-tails more fully, and the results show higher risk in VaR and ES than method a, but the model and parameter estimation are more complex.
(3) Method c makes no distributional assumptions and relies directly on historical data, but is extremely sensitive to outliers.

**2. Data weight and time dependence**

(1) Method a gives higher weight to recent data through EWMA and has strong adaptability;

(2) Method b integrates multi-asset risk through Monte Carlo simulation and Copula, taking into account tail and correlation;
(3) Method c adopts the whole historical data, and if there are extreme conditions during the historical period, the results may be abnormal.

**3. Differences in actual results**

Extreme values appear in the risk estimates of individual stocks in Method c, suggesting that historical data contain abnormal events that can lead to unstable risk estimates. It is necessary to fully clean the data or adopt rolling Windows to avoid excessive influence of extreme historical events when using the historical simulation method.

# Problem3

A.

Implied volatility: 0.3351 (33.51%)

B.

$$C = S\Phi(d_1) - Xe^{-rT}\Phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$Delta(\Delta) = \frac{dc}{dS} = \Phi(d_1)$ = 0.6659

$Vega(v) = \frac{dc}{d\sigma} = S\sqrt{T}\Phi'(d_1)$ = 5.6407

$Theta(\Theta) = \frac{dc}{dt} = -\frac{S\Phi'(d_1)\sigma}{2\sqrt{T}} - rXe^{-rT}\Phi(d_2)$ = -5.5446

Change in option price when the implied volatility increased by 1% = Vega * 0.01 = $ 0.0564

Proof:

$C_1 = 3$

$when$ the implied volatility increased by 1%, $\sigma = 0.3351 + 0.01 = 0.3451$

$d_1 = \frac{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{31}{30}\right)+\left(0.1+\frac{0.3451^2}{2}\right)0.25}{0.3451\sqrt{0.25}}$ = 0.4212

$d_2 = d_1 - \sigma\sqrt{T}$ = 0.2562

$C_2 = S\Phi(d_1) - Xe^{-rT}\Phi(d_2)$ = 3.0564

$C_2 - C_1$ = $ 0.0564

# C.

$$d_1 = \frac{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = 0.4287$$

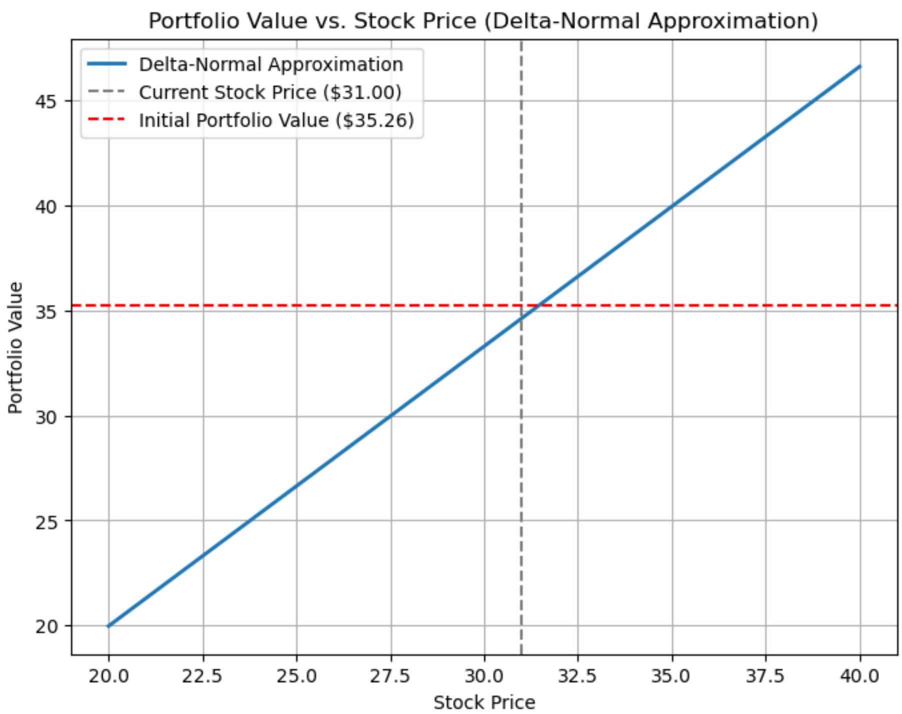
$$d_2 = d_1 - \sigma\sqrt{T} = 0.2612$$

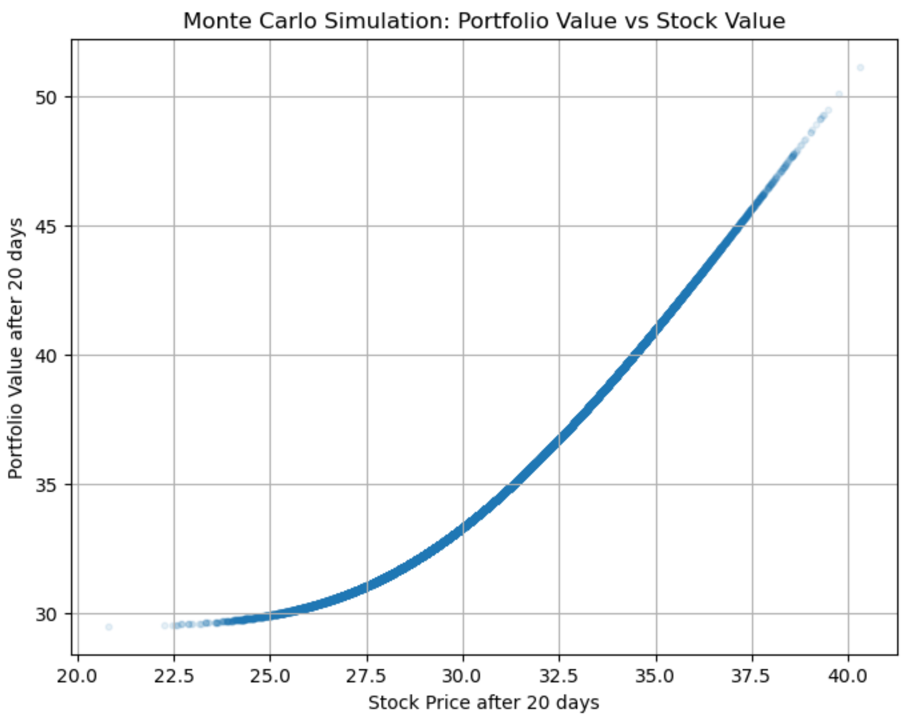
$P = Xe^{-rT}\Phi(-d_2) - S\Phi(-d_1) = 1.2593$
$C + Xe^{-rT} = 3 + 30*e^{-0.1*0.25} = 32.2593$
$P+S = 1.2593 + 31 = 32.2593$
$C + Xe^{-rT} = P+S$
Put option price is \$1.2593. Put-Call Parity holds.

# D.

**d. Delta Normal Approximation:**
VaR = \$5.3951
ES = \$6.6030
**e. Monte Carlo Simulation**
VaR = \$4.2599
ES = \$4.7265

# E.






## 1. Delta-Normal Approximation

The delta-normal method linearizes the portfolio's value change by using its Delta and Theta. It assumes that small changes in the underlying stock price translate linearly to changes in the option prices.

**(1)Assumptions:**

a. Portfolio's return is normally distributed.

b. It neglects higher-order terms like Gamma and other non-linear effects, which can be significant when the underlying moves substantially.

c. It assumes the impact of volatility (and time decay via Theta) is approximately constant over the short holding period.

**(2)Graph:**

When graphing portfolio value versus the stock price, the delta-normal approximation yields an approximately straight line. This linear relationship reflects the constant sensitivity (Delta) and a fixed time decay (Theta).

**(3)Pros and Cons:**

**Pros:** Computationally efficient, easy to implement, and works well for small moves in the underlying.

**Cons:** May underestimate risk when the portfolio exhibits significant non-linear behavior (e.g., high Gamma or when price moves are large).

## 2. Monte Carlo Simulation

Monte Carlo simulation recalculates the option prices by simulating many possible future stock price paths over the holding period and re-pricing the options at each simulated endpoint. This method fully incorporates the option's non-linear characteristics.

**(1) Assumptions**

a. Stock’s return is normally distributed over the holding period.
b. The method captures the time decay and changes in the option’s sensitivities as the underlying evolves.
c. No linearization is used. Apply non linear pricing model like Black–Scholes model.

**(2) Graph**

Typically curved. This curvature illustrates the effect of Gamma and other higher-order sensitivities that are missed by the linear delta approximation.

**(3) Pros and Cons**

**Pros:** More realistic since it captures non-linear effects and the full distribution of outcomes, making it better for assessing risk under large moves or volatile conditions.

**Cons:** Computationally more intensive and requires a larger number of simulated paths to achieve stable estimates.

## 3. Summary

The delta-normal method is simpler and faster, using a linear approximation (Delta and Theta) to predict changes in the portfolio value. However, it may underestimate risk for larger movements or when options are highly non-linear. In contrast, Monte Carlo simulation recalculates the option prices along many simulated paths, capturing non-linear payoffs and time decay more accurately. This is reflected in the graph where the delta-normal approach shows a linear relationship while the Monte Carlo simulation shows a curved, non-linear relationship between the portfolio value and the stock price.