Rejection Region

We start the hypothesis test process by determining the null and alternative hypothesis. Then we get our significance level, α, which is the probability of making a Type I error. We can determine the appropriate cut off called the critical value and find a range of values where we should reject, called the rejection region.

Critical Region

The values that separate the rejection region and non-rejection regions.

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Rejection Regions :-

The set of values for the test statistic that leads to rejection of H_0.

Acceptance Region

for a hypothesis test, a researcher collects sample data, the researcher computes a test statistic. If the statistic falls within a specified range of values, the researcher ~~computes a test statistic~~ accepts the null hypothesis. The range of values that leads ~~to~~ the researcher to accept the null hypothesis is called the region of acceptance.

example:- A researcher might hypothesize that the population mean is equal to 10. To test this null hypothesis, he/she could collect a random sample of observations and compute the sample mean. If the sample mean is close to 10 (say between 9 and 11), the researcher

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might decide to accept the hypothesis
In this region example, the region
of acceptance would be the range
of values between 9 and 11.

Central Limit Theorem

→ In probability theory, the central limit theorem (CLT) states that the distribution of a sample variable approximately a normal distribution (i.e., a 'bell curve') as the sample size becomes larger, assuming that all samples are identical in size, and regardless of the population's actual distribution ~~type~~. shape.

→ In another way, CLT is a stastical premise that given, a sufficiently large sample size from a population with a finite level of variance, the mean of all sampled variables from the same population will be approximately equal to the mean of the whole population. Furthermore, these samples approximate a normal distribution, with their variances being approximately equal to the variance of the population as the sample size gets larger, according to the law of large numbers

Key Takeaways:-

- The central limit theorem (CLT) states that the distribution of sample means approximates a normal distribution as the sample size gets larger, regardless of the population's distribution.

- Sample sizes equal to or greater than 30 are often considered sufficient for the CLT to hold.

- A key aspect of CLT is that the average of the sample means and standard deviations will equal the population mean and standard deviation.

- A sufficiently large sample can predict the characteristics of a population more accurately.

Central Limit Theorem in Finance

The CLT is useful when examining the returns of an individual stock or broader indices, because the analysis is simple, due to the relative ease of generating the necessary financial data. Consequently, investors of all types rely on the CLT to analyze stock returns, construct portfolios, and manage risk.

Example

An investor wishes to analyze the overall return for a stock index that comprises 1,000 equities. In this scenario, that investor may simply study a random sample of stocks to cultivate estimated returns of the total index. To be safe, use at least 30-50 randomly selected stocks across various sectors, should be sampled for the central limit theorem to hold. Furthermore, previously selected stocks must be swapped out with different names to help eliminate bias.

Bayes' Theorem

- Baye's' Theorem describes the probability of ~~occurrence~~ occurence of an event related to any condition. It is also considered as for the case of conditional Probability.

- Baye's Theorem is also known as the formula for the probability of "causes".

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Example:-

A bag 1 contains 4 white and 6 black balls while another bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black. Find the probability that it was drawn from Bag 1.

Let E_1 be the event of choosing bag I, E_2 is the event of choosing bag II, and A be the event of drawing a black ball.

Then, $P(E_1) = P(E_2) = \frac{1}{2}$

Also, $P(A|E_1) = P(A|E_1) = P$ (drawing a black ball from Bag I) $= \frac{6}{10} = \frac{3}{5}$

$P(A|E_2) = P$(drawing a black ball from Bag II) $= \frac{3}{7}$

By, using Bayes' Theorem, the probability of drawing a black ball from bag I out of two bags,

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) P(A|E_2)}$$

$$= \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{3}{5}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{3}{7}} = \frac{\frac{3}{10}}{\frac{42+30}{140}} = \frac{3 \times \cancel{140}^{7}}{\cancel{10} \times \cancel{72}_{\cancel{24}_{12}}}$$

$$= \frac{7}{12}$$

8. Assume that the chances of a person having a skin disease are 40%. Assuming that skin creams and drinking enough water reduces the risk of skin disease by 30% and prescription of a certain drug reduces its chance by 20%. At a time, a patient can choose any one of the two options with equal probabilities. It is given that after picking one of the options, the patient selected at random has the skin disease. Find the probability that the patient picked the option of skin screams and drinking enough water using the Bayes Theorem.

Soln:- Assume E_1: The patient uses skin creams and drinks enough water; E_2: The patient uses the drug; A: The selected patient has the skin disease.

$P(E_1) = P(E_2) = 1/2$

Using the probabilities known to us, we have

$P(A|E_1) = 0.4 \times (1-0.3) = 0.4 \times 0.7$
$= 0.28$

$P(A|E_2) = 0.4 \times (1-0.2) = 0.32$

Using Bayes Theorem, the probability that the selected patient uses skin creams and drinks enough water is given by,

$$P(E_1|A) = \frac{P(A|E_1) \cdot P(E_1)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2)}$$

$= (0.28 \times 0.5) / (0.28 \times 0.5 + 0.32 \times 0.5)$

$$= \frac{0.14}{(0.14 + 0.16)}$$

$= 0.47$

A. The probability that the patient picked the first options is 0.47

Q3. A man is known to speak the truth 3/4 times. He draws a card and reports it is king. Find the probability that is actually a king.

Let E be the event that the man reports that king is drawn from the pack of cards.

A be the event that the king is drawn.
B be the event that the king is not drawn.

Then, we have P(A) = Probability that king is drawn $= \frac{1}{4}$

P(B) = Probability that king is drawn $= \frac{3}{4}$

P(E|A) = Probability that the man says the truth that king is drawn when actually king is drawn = P(truth) $= \frac{3}{4}$

P(E|B) = Probability that the man lies that king is drawn when actually king is drawn = P(lie) = 1/4

Then, according to Bayes theorem, the probability that it is actually a king = P(A|E)

$$= \frac{P(A) \cdot P(E|A)}{P(A) \cdot P(E|A) + P(B) \cdot P(E|B)}$$

$$= \frac{\frac{1}{4} \times \frac{3}{4}}{\left[\left(\frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{3}{4}\right)\right]}$$

$$\boxed{= \frac{3}{16} \times \frac{16}{12} \quad \frac{3}{\frac{12}{16}} \quad = \frac{3}{16} \times \frac{4}{3} \\ = \frac{4}{12} \\ = 0.5}$$

$$= \frac{\frac{3}{16}}{\frac{3}{16} + \frac{3}{16}}$$

$$= \frac{\frac{\cancel{3}\,1}{16}}{\frac{\cancel{6}\,2}{16}}$$

$$= \frac{1}{2} = 0.5$$