

# Poisson and Binomial Distribution

## Poisson Distribution

Poisson distribution describes the distribution of binary data from an infinite sample. Thus it gives the probability of getting  $x$  events in a population.

## Binomial Distribution

Binomial Distribution describes the distribution of binary data from a finite sample. Thus it gives the probability of getting  $x$  events out of  $n$  trials.

Basis for  
Comparison  
meaning

Binomial  
Distribution

Binomial Distribution is one in which the probability of repeated numbers of trials are studied.

Poisson  
Distribution

Poisson Distribution gives the count of independent events occur randomly with a given period of time.

Nature  
Number of  
trials

Biparametric

Fixed

Uniparametric

Infinite



Basis for  
Comparison

Binomial  
Distribution

Poisson  
Distribution

Success

Constant Probability

Infinitesimal chance  
of Success

Outcomes

Only two possible  
outcomes, i.e.  
success or failure

Unlimited number  
of possible outcomes

Mean  
and  
Variance

Mean  $>$  Variance

Mean = Variance

Example

Coin Tossing  
Experiment

Printing mistakes  
page of a large book.

### Normal Distribution

Normal Distribution, also known as the Gaussian Distribution, is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data from the mean. In graph form, normal distribution will appear as a bell curve.



- Normal Distributions are symmetrical, but not all symmetrical distributions are normal.
- In reality, most pricing distributions are not perfectly normal.

### Properties :-

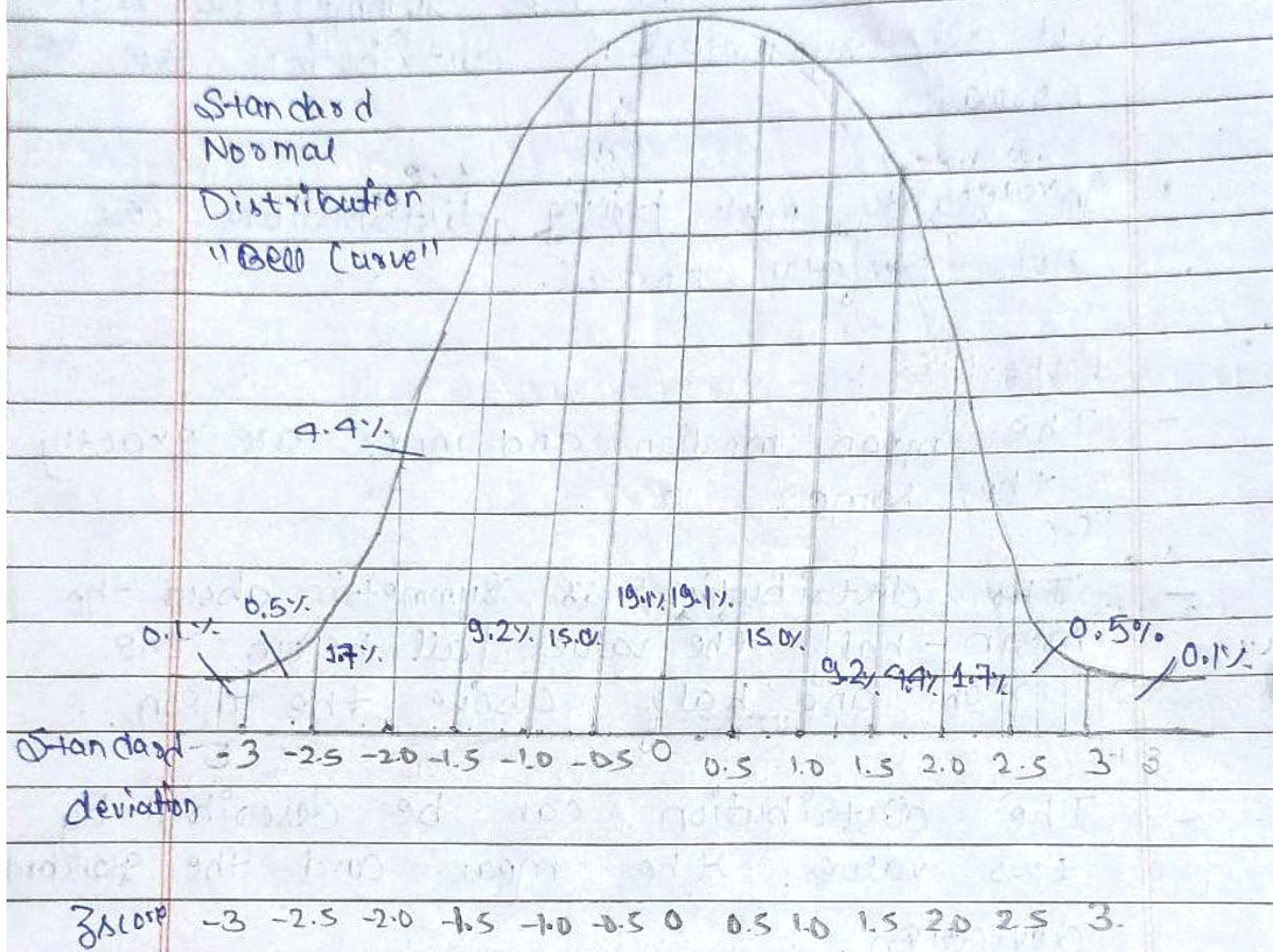
- The mean, median and mode are exactly the same.
- The distribution is symmetric about the mean - half the values fall below the mean and half above the mean.
- The distribution can be described by two values: the mean and the standard deviation.

### Standard Normal Distribution ?

- The standard normal distribution, also called the  $z$ -distribution, is a special normal distribution where the mean is 0 and the standard deviation is 1.
- Any normal distribution can be converted into the standard normal distribution by turning the individual values into  $z$ -scores.
- In a  $z$ -distribution,  $z$ -scores tell you how many standard deviation away from the mean each value lies.



Standard  
Normal  
Distribution  
"Bell Curve"



Cumulative Percent	0.1%	2.3%	15.9%	50%	84.1%	97.7%	99.9%
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## Marginal, Joint And Conditional Probabilities

### Scenario

A survey was carried out with 500 strangers in London's West End to determine people's favorite sports. The options were Football, Rugby, and the rest was grouped together in Other. The result of the test are displayed in figure 1.

	male	Female	Total
Football	120	75	195
Rugby	100	25	125
Other	50	130	180
	270	230	500

After Probability Distribution

To get Probability Distribution, we divide each number by no. of observation.

	male	female	Total
Football	0.24	0.15	0.39
Rugby	0.2	0.05	0.25
Other	0.1	0.26	0.36
	0.54	0.46	1



## Joint Probability

The joint probability is a statistical measure that is used to calculate the probability of two events occurring together at the same time -  $P(A \text{ and } B)$  or  $P(A, B)$ .

For example using figure we can see that the joint probability of someone being a male and a liking a football is 0.24.

	male	Female	Total
Football	0.24	0.15	0.39
Rugby	0.2	0.05	0.25
Other	0.1	0.26	0.36
	0.54	0.46	1

Note:-

The cells highlighted is the Joint Probability Distribution must sum to 1, because everyone in the distribution must be in one of the cells.



The Joint probability is symmetrical meaning that  $P(\text{male and Football}) = P(\text{Football and male})$  and we can also use it to find other types of distributions the marginal distribution and the conditional distribution.

### Marginal Distribution

- In probability theory and statistics, the marginal distributional of a subset of a collection of random variables is the probability distribution of the variables contained in the subset.
- It gives the probabilities of various values of the variables in the subset without reference to the values of the other variables.
- Marginal Probability is the probability of an event irrespective of the outcome of another variable.

	Male	Female	Total
Football	0.24	0.15	0.39
Rugby	0.2	0.05	0.25
Other	0.1	0.26	0.36
	0.54	0.46	1



Note:-

Whether we ignore the gender or the sport our Marginal Distributions must sum to 1.

A fun fact of marginal probability is that all the marginal probability appears in the margins.

Hence the  $P(\text{Female}) = 0.46$  which completely ignores the sport the Female prefers, and the  $P(\text{Rugby}) = 0.25$  completely ignores the gender.

### Conditional Probability

It defines the Probability of one event occurring given that another event has occurred (by assumption, presumption, assertion or evidence).

$$P(A|B) = P(A, B) / P(B)$$

If we want to calculate the probability that a person would like Rugby given that they are female, we must take the joint probability that the Person is



female and likes rugby ( $P(\text{Female and Rugby})$ )

( $P(\text{Female and Rugby})$ ) and divide it by the probability of the condition.

$$P(\text{Female, Rugby}) = 0.05$$

$$P(\text{Female}) = 0.46$$

$$P(\text{Rugby} | \text{Female}) = \frac{0.05}{0.46}$$

$$= 0.11$$