

Day-17 Python DSA

Count Inversions

<https://www.geeksforgeeks.org/problems/inversion-of-array-1587115620/1>

Bruteforce

```
def inversionCount(arr):
```

```
    n = len(arr)
```

```
    count=0
```

```
    for i in range(0,n):
```

```
        for j in range(i+1, n):
```

```
            if arr[i] > arr[j]:
```

```
                count+=1
```

```
    return count
```

```
arr= [2, 4, 1, 3, 5]
```

```
inversionCount(arr)
```

TC – $O(N^2)$

SC – $O(1)$

Optimal

```
from typing import List, Tuple
```

```
class Solution:
```

```
def mergeList(self, arr1: List[int], arr2: List[int]) -> Tuple[List[int], int]:
```

```
    n = len(arr1)
```

```
    m = len(arr2)
```

```
    i, j = 0, 0
```

```
    count = 0
```

```
    result = []
```

```
    while i < n and j < m:
```

```
        if arr1[i] <= arr2[j]:
```

```
            result.append(arr1[i])
```

```
            i += 1
```

```
        else:
```

```
            count += n - i # Count inversions
```

```
            result.append(arr2[j])
```

```
            j += 1
```

```
    while i < n:
```

```
        result.append(arr1[i])
```

```
        i += 1
```

```
    while j < m:
```

```
        result.append(arr2[j])
```

```
        j += 1
```

```
    return result, count
```

```
def mergeSort(self, lst: List[int]) -> Tuple[List[int], int]:
```

```
    if len(lst) <= 1:
```

```
        return lst, 0 # Return count as 0
```

```
    mid = len(lst) // 2
```

```
    first_half = lst[:mid]
```

```
    second_half = lst[mid:]
```

```
    fh, cnt1 = self.mergeSort(first_half)
```

```
    sh, cnt2 = self.mergeSort(second_half)
```

```
    merged, count = self.mergeList(fh, sh)
```

```
    return merged, cnt1 + cnt2 + count
```

```
def inversionCount(self, arr: List[int]) -> int:
```

```
    _, count = self.mergeSort(arr)
```

```
    return count
```

Dry Run

```
arr = [2, 4, 1, 3, 5]
```

We'll follow **merge sort** flow and keep track of:

- The current split
- Merge results
- Inversion count additions

Step 1 – Call inversionCount

`inversionCount([2, 4, 1, 3, 5])`

- Calls `mergeSort([2, 4, 1, 3, 5])`

Step 2 – First split

`mergeSort([2, 4, 1, 3, 5])`

- `mid = 2`
- `first_half = [2, 4]`
- `second_half = [1, 3, 5]`

Step 3 – Sort [2, 4]

`mergeSort([2, 4])`

- `mid = 1`
- `first_half = [2] → returns ([2], 0)`
- `second_half = [4] → returns ([4], 0)`

Merge step:

`mergeList([2], [4])`

→ Compare $2 \leq 4$ → append 2

→ Append 4

→ `result = [2, 4], count = 0`

Return: `([2, 4], 0)`

Step 4 – Sort [1, 3, 5]

mergeSort([1, 3, 5])

- mid = 1
- first_half = [1] → returns ([1], 0)
- second_half = [3, 5]

Sort [3, 5]

mergeSort([3, 5])

- mid = 1
- first_half = [3] → ([3], 0)
- second_half = [5] → ([5], 0)

Merge step:

mergeList([3], [5])

→ Compare $3 \leq 5$ → append 3

→ Append 5

→ result = [3, 5], count = 0

Return: ([3, 5], 0)

Merge [1] and [3, 5]

mergeList([1], [3, 5])

→ Compare $1 \leq 3$ → append 1

→ Append 3, 5

→ result = [1, 3, 5], count = 0

Return: ([1, 3, 5], 0)

Step 5 – Merge [2, 4] and [1, 3, 5]

`mergeList([2, 4], [1, 3, 5])`

- Compare $2 > 1 \rightarrow$ inversion count $+= (n - i) = (2 - 0) = 2$

Append 1

`result = [1], count = 2`

- Compare $2 \leq 3 \rightarrow$ append 2

`result = [1, 2]`

- Compare $4 > 3 \rightarrow$ inversion count $+= (2 - 1) = 1$

Append 3

`result = [1, 2, 3], total count so far = 3`

- Append 4

Append 5

`result = [1, 2, 3, 4, 5]`

Return: `([1, 2, 3, 4, 5], count = 3)`

Step 6 – Final count

- `cnt1 = 0` (from left part [2,4])
- `cnt2 = 0` (from right part [1,3,5])
- `count from merge = 3`

Total inversions = $0 + 0 + 3 = 3$

Output:

`inversionCount([2, 4, 1, 3, 5]) → 3`

TC – $O(N \log N)$

SC- $O(N)$