24BCON0384

Tutorial Sheet - 6

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Section - A

1. Solve $(D^2+9)y = \cos 3x$

First we will find CF (complementry function)

Auxillary eqn $m^2+9=0$

$m^2 = -9$

$m = \pm 3$

roots of eqn are imaginary & distinct.

$C.F = e^{0x}(c_1\cos 3x + c_2 \sin 3x)$

or

$[C.F = (c_1\cos 3x + c_2\sin 3x)]$

Now, we will find P.I (particular integral)

$y = \frac{\cos 3x}{D^2+9}$

replacing $D^2 \Rightarrow -a^2$

$y = \frac{\cos 3x}{-a^2+9}$

$y = \frac{\cos 3x}{-9+9}$ Not defined

$y = \frac{x\cos 3x}{f'(D)} = \frac{x\cos 3x(-2D)}{-4D^2}$ $\because D = \frac{d}{dx}$

$y = \frac{2x \cdot 3\sin 3x}{-4(-9)} = \frac{x\sin 3x}{6}$

$G.F \Rightarrow y = CI + PI \Rightarrow y = c_1\cos 3x + c_2\sin 3x + \frac{x\sin 3x}{6}$

2. Solve $(D^2-4)y = e^{2x}$

$(D^2-4)y = e^{3x}$

we find CF

Auxillary eqn $m^2-4=0$

$m^2 = 4 \Rightarrow m = \pm 2$

roots of eqn are real & distict

$[CF = c_1e^{2x} + c_2e^{-2x}]$

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Now we find PI $y = \frac{e^{2x}}{D^2 - 4}$

replacing $D \to a$

$y = \frac{e^{2x}}{4-4} = \frac{xe^{2x}}{2D} \Rightarrow \boxed{y = \frac{xe^{2x}}{4}}$

General eqn $\to y = CF + PI$

$y = c_1e^{2x} + c_2e^{-2x} + \frac{xe^{2x}}{4}$

3. Solve $(D^2 + D - 2)y = x$

= $(D^2 + D - 2)y = x$

C.F will be

$A.E \Rightarrow m^2 + m - 2 = 0$

$m^2 + 2m - m - 2 = 0$

$m(m+2) - 1(m+2) = 0 \Rightarrow m = 1, -2$

$[C.F = c_1e^x + c_2e^{-2x}]$

we find PI

$y = \frac{x}{D^2 + D - 2}$

$y = \frac{x}{-2\left[1 - \frac{D^2}{D} - \frac{D}{2}\right]} = \frac{-1}{2}\left[1 - \frac{D^2}{2} + \frac{D}{2}\right]^{-1} x$

$\because \left[(1-x)^{-1} = 1 + x + x^2 \ldots\right]$

$y = \frac{-1}{2}\left[1 + \frac{D^2}{2} + \frac{D}{2} \ldots\right] x$

$y = \frac{-1}{2}\left[1 + 0 + \frac{1}{2}\right] x \Rightarrow \left[y = \frac{-1}{2}x - \frac{1}{4}\right]$

$G.E \Rightarrow y = CF + PI$

$y = c_1e^x + c_2e^{-2x} - \frac{1}{2}x - \frac{1}{4}$

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4. Solve $(D^3+1)y = (e^x+1)^2$

$= (D^3+1)y = (e^x+1)^2$

We find C.F

$$A.E = m^3+1=0 \Rightarrow M=-1$$

$$m=-1$$

$$m+1 \overline{) m^3+1} (m^2-m+1$$

$$\underline{m^3 \pm m^2}$$

$$-m^2+1$$

$$\underline{-m^2-m}$$

$$m+1$$

$$\underline{m+1}$$

$$0$$

$$m^2-m+1=0$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow \frac{1\pm\sqrt{1-4}}{2} = \frac{1\pm\sqrt{3}i}{2}$$

$$\left[C.F = C_1e^{-x} + e^{x/2}\left(C_2\cos\frac{\sqrt{3}}{2}x + C_3\sin\frac{\sqrt{3}}{2}x\right)\right]$$

Now, we find P.I

$$F(D)y = e^{2x}+1+2e^x$$

$$\text{For } e^{2x} \Rightarrow y = \frac{e^{2x}}{D^3+1} = \frac{e^{2x}}{8+1} = \boxed{\frac{e^{2x}}{9}}$$

$$\text{For } 1 \Rightarrow y = \frac{1}{D^3+1} = [1+D^3]^{-1}$$

$$[1-D^3+D^0]\cdot 1 = [1]$$

$$\text{For } 2e^x \Rightarrow y = \frac{2e^x}{D^3+1} = \frac{2e^x}{2} = [e^x]$$

$$G.F \Rightarrow y = C.F + P.I$$

$$y = C_1e^{-x} + e^{x/2}\left[C_2\cos\frac{\sqrt{3}}{2}x + C_3\sin\frac{\sqrt{3}x}{2}\right] + \frac{e^{2x}}{9} + 1 + e^x$$

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5. Find the CF of $(D^2+1)^2 y = 24\cos x$

$(D^2+1)^2 y = 24\cos x$

C.F will be

$A.E = (m^2+1)^2 = 0 \Rightarrow m^2+1 = 0 \Rightarrow m^2 = -1$

$m = \pm 1$

roots of the equation will become twice

$m = +i, +i, -i, -i$

$C.F = e^{0x}[(c_1+c_2x).\cos x + (c_3+c_4x)\sin x]$

$C.F = (c_1+c_2x)\cos x + (c_3+c_4x)\sin x]$

Section . B.

1. Solve $(D^2+9)y = \sin^2 x$

$(D^2+9)y = \sin^2 x$

$A.E \rightarrow m^2+9 = 0 \Rightarrow m = \pm 3i$

$[C.F = e^{0x}(c_1\cos 3x + c_2\sin 3x)]$

Now, we find PI

$y = \frac{\phi(x)}{f(D)}$ $\quad y = \frac{\sin^2 x}{D^2+9}$

W.K.T $\sin^2 x = \frac{1-\cos 2x}{2}$

$y = \frac{1}{D^2+9}\left[\frac{1}{2} - \frac{\cos 2x}{2}\right]$

$y = \frac{1}{2}\left[\frac{1}{D^2+9} - \frac{\cos 2x}{D^2+9}\right]$ $\quad \{\because 1 = e^{0x}\}$

$y = \frac{1}{2}\left[\frac{e^{0x}}{D^2+9} - \frac{\cos 2x}{D^2+9}\right] = \frac{1}{2}\left[\frac{e^{0x}}{9} - \frac{\cos 2x}{-4+9}\right]$

$y = \frac{1}{2}\left[\frac{1}{9} - \frac{\cos 2x}{5}\right] = \left[y = \frac{1}{18} - \frac{\cos 2x}{10}\right]$

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$G.F \Rightarrow y = CF + PI$

$$y = e^{0x}(c_1\cos 3x + c_2\sin 3x) + \frac{1}{18} - \frac{\cos 2x}{10}$$

$n^2 = -1$

$= \pm 1$

2. Solve $(D^2-a^2)y = \cosh(ax)$

$(D^2-a^2)y = \cosh(ax)$

$A.E \rightarrow m^2 - a^2 = 0 \quad \Rightarrow m = \pm a$

$C.F = [c_1e^{ax} + c_2e^{-ax}]$

Now, we find PI

$P.I = \frac{\cosh(ax)}{D^2-a^2} = \frac{\cosh(ax)}{a^2-a^2}$ Not defined.

$= \frac{x\cosh(ax)}{2D} \times \frac{-2D}{-2D} \Rightarrow \frac{x \times -2D(\cosh(ax))}{-4D^2}$

$= \frac{2x \cdot \sinh(ax) \cdot a}{4a^2} \Rightarrow \left[y = \frac{x}{2a}\sinh(ax)\right]$

$G.F = y = C.F + PI$

$$y = c_1e^{ax} + c_2e^{-ax} + \frac{x}{2a}\sinh(ax)$$

$-x$

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3. Solve $(D^3 - 3D^2 + 2D)y = 12(x^2 - 2x + 4)$

$(D^3 - 3D^2 + 2D)y = 12(x^2 - 2x + 4)$

$A.E = m^3 - 3m^2 + 2m = 0$

$m(m^2 - 3m + 2) = 0$

$m(m^2 - 2m - m + 2) = 0$

$m(m-2)(m-1) = 0 \quad \Rightarrow m = 0, 1, 2$

$[C.F = c_1e^{0x} + c_2e^{x} + c_3e^{2x}]$

Now, we find particular integral by partial

$P.I = y = Ax^3 + Bx^2 + Cx + E$

$Dy = 3Ax^2 + 2Bx + C$

$D^2y = 6Ax + 2B$

$D^3y = 6A$

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Substituting these values in original eqn.

$$6A - 3(6Ax + 2B) + 2(3Ax^2 + 2Bx + c) = 12(x^2 - 2x + 4)$$

$$6A - 18Ax - 6B + 6Ax^2 + 4Bx + 2C = 12x^2 - 24x + 48$$

$$6A = 12 \Rightarrow \boxed{A = 2}$$

$$-18A + 4B = -24$$

$$-36 + 4B = -24 \Rightarrow 4B = 12 \Rightarrow \boxed{B = 3}$$

$$6A - 6B + 2C = 48$$

$$12 - 18 + 2C = 48 \Rightarrow 2C = 54 \Rightarrow \boxed{C = 27}$$

$$\left[P.I = 2x^2 + 3x^2 + 27x\right]$$

$$G.S \rightarrow y = C.F + P.I$$

$$y = c_1 + c_2 e^{2x} + c_3 e^{1x} + 2x^3 + 3x^2 + 27x$$

Section-C

1. Solve $(D^3 + 2D^2 + D)y = e^{-x} + \cos x + x^2$

$$(D^3 + 2D^2 + D)y = e^{-x} + \cos x + x^2$$

$$A.E \Rightarrow m^3 + 2m^2 + m = 0$$

$$m(m^2 + 2m + 1) = 0$$

$$m(m^2 + m + m + 1) = 0$$

$$m[(m+1)(m+1)] = 0$$

$$m = 0, -1, -1$$

$$\left[C.F = (c_1 + c_2 x)e^{-x} + c_3\right]$$

Now, we find solutions for p.I

i) $y = \dfrac{e^{-x}}{D^3 + 2D^2 + D}$

$$y = \frac{e^{-x}}{(-1)^3 + 2(-1)^2 + (-1)} = \frac{e^{-x}}{0} \quad \text{Not defined}$$

$$y = \frac{x e^{-x}}{3D^2 + 4D + 1} = \frac{x e^{-x}}{0} \quad \text{Not defined}$$

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$$\boxed{y = \frac{x^2 e^{-x}}{60+4} = \frac{x^2 e^{-x}}{-2}}$$

ii) $y = \frac{\cos x}{D^3+2D^2+D} = \frac{\cos x}{D \cdot D^2 + 2D^2 + D}$ $\quad D^2 \to -a^2 = -1$

$$y = \frac{\cos x}{-D-2+D} \Rightarrow \boxed{y = \frac{\cos x}{-2}}$$

iii) $y = \frac{x^2}{D^3+2D^2+D}$

$$y = \frac{x^2}{D(D^2+2D+1)}$$

$$y = \frac{1}{D}[D^2+2D+1]^{-1} x^2$$

$D^2x^2 = 2$ $\quad \{ D = \frac{d}{dx}$

$Dx^2 = 2x$

$D^3x^2 = 0$

$$y = \frac{1}{D}[1-(D^2+2D)+4D^2]x^2$$

$$y = \frac{1}{D}[x^2-2-4x+8]$$

$$y = \frac{1}{D}[x^2-4x+6]$$

$$y = \int (x^2-4x+6)\,dx$$

$$\boxed{y = \frac{x^3}{3} - 2x^2 + 6x}$$

G.S $= y = C.F + P.I$

$$y = (c_1 + c_2 x)e^{-x} + c_3 - \frac{x^2 e^{-x}}{2} - \frac{\cos x}{2} + \frac{x^3}{3} - 2x^2 + 6x$$

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2. Solve. $(D^2 - 5D + 6)y = \sin 3x$

$(D^2 - 5D + 6)y = \sin 3x$

A.E $\to m^2 - 5m + 6 = 0$

$m^2 - 3m - 2m + 6 = 0$

$m(m-3) - 2(m-3) = 0$

$(m-3)\ (m-2) = 0$

$m = 3, 2$

$\left[C.F = c_1 e^{2x} + c_2 e^{3x}\right]$

Now, we find P.I

$$y = \frac{\sin 3x}{D^2 - 5D + 6}$$

$$D^2 \to -a^2 = -9$$

$$y = \frac{\sin 3x}{-9 - 5D + 6}$$

$$y = \frac{\sin 3x}{-5D - 3}$$

$$y = \frac{-\sin 3x}{5D + 3} \times \frac{5D - 3}{5D - 3}$$

$$y = \frac{-\sin 3x\ (5D - 3)}{25D^2 - 9}$$

$$y = -\left[\frac{5D(\sin 3x) - 3\sin 3x}{25(-9) - 9}\right] = \frac{-15\cos 3x - 3\sin 3x}{-234}$$

$$y = \frac{15\cos 3x}{234} - \frac{3\sin 3x}{234}$$

$$\left[y = \frac{5\cos 3x}{78} - \frac{\sin 3x}{78}\right]$$

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G.E $\Rightarrow y = CF + PI$

$$y = c_1e^{2x} + c_2e^{3x} + \frac{5}{78}\cos 3x - \frac{\sin 3x}{78}$$

3 solve $(D^3 - D^2 - 6D)y - (1+x^2)$

$(D^3 - D^2 - 6D)y = 1 + x^2$

A.E $\rightarrow m^3 - m^2 - 6m = 0$

$m(m^2 - m - 6) = 0$

$m(m^2 - 3m + 2m - 6) = 0$

$m[m(m-3) + 2(m-3) = 0$

$m(m+2)(m-3) = 0$

$m = 0, 3, -2$

$C.F = c_1e^{0x} + c_2e^{3x} + c_3e^{-2x}$

Now, we find P.I

i) $y_1 = \frac{1}{D^3 - D^2 - 6D}$

$$y_1 = \frac{1}{-6D\left[1 + \frac{D}{6} - \frac{D^2}{6}\right]}$$

$$y_1 = \frac{-1}{6D}\left[1 + \left[\frac{-D^2}{6} + \frac{D}{6}\right]\right]^{-1}$$

$$y_1 = \frac{-1}{6D}\left[1 - \frac{D}{6} + \frac{D^2}{6}\right]$$

$$y_1 = \frac{-1}{6D}\left[1 + 0 + 0\right]$$

$$y_1 = \frac{-1}{6}\int 1\,dx$$

$$\boxed{y_1 = \frac{-x}{6}}$$

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iii) $$y_2 = \frac{x^2}{D^3 - D^2 - 6D}$$

$$y_2 = \frac{x^2}{-6D\left[1 - \frac{D^2}{6} + \frac{D}{6}\right]}$$

$$y_2 = \frac{x^2}{-6D}\left[1 + \left[\frac{D}{6} - \frac{D^2}{6}\right]\right]^{-1}$$

$$Dy = 2x, \quad D^2y = 2, \quad D^3y = 0$$

$$y_2 = \frac{-1}{6D}\left[1 - \frac{D}{6} + \frac{D^2}{6} + \frac{D^2}{36}\right]x^2$$

$$y_2 = \frac{-1}{6D}\left[x^2 - \frac{2x}{6} + \frac{2}{6} + \frac{2}{36}\right]$$

$$y_2 = \frac{-1}{6D}\left[x^2 - \frac{x}{3} + \frac{14}{36}\right]$$

$$y_2 = \frac{-1}{6}\int\left(x^2 - \frac{x}{3} + \frac{14}{36}\right)dx$$

$$y_2 = \frac{-1}{6}\left[\frac{x^3}{3} - \frac{x^2}{6} + \frac{14x}{36}\right]$$

$$\left[y_2 = \frac{-x^3}{18} + \frac{x^2}{36} - \frac{14x}{216}\right]$$

$$P.I = y = y_1 + y_2$$

$$P.I = y = \frac{-x}{6} - \frac{x^3}{} + x^2 - 14x$$

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$$P.I = y = \frac{-x^3}{18} + \frac{x^2}{36} - \frac{50x}{216}$$

$$\left[P.I = y = \frac{-x^3}{18} + \frac{x^2}{36} - \frac{25x}{108} \right]$$

$G.E \Rightarrow C.F + P.I$

$$y = C_1 e^{0x} + C_2 e^{3x} + C_3 e^{-2x} - \frac{x^3}{18} + \frac{x^2}{36} - \frac{25x}{108}$$