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PART-A

Ques 1 Define Langrage Auxaliary equation.

Ans- Lagrange Auxaliary eq^n are the system of ordinary differential equations (ODE) So, to solve first order linear differential equation (PDES) of the form

$$P(x,y,z)\frac{dz}{dx} + Q(x,y,z)\frac{dz}{dy} = R(x,y,z)$$

This is written in the form of

$$\boxed{Pp + Qq = R}$$

$$p = \frac{dz}{dx}, \; q = \frac{dz}{dy}$$

The Lagrange differential eq^n is

$$\boxed{\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}}$$

Ques 2 Solve: $xp + yq = z$

Ans- form: $\boxed{Pp + Qq = R}$, $p = \frac{dz}{dx}$, $q = \frac{dz}{dy}$

where $P = x$, $Q = y$, $z = R$

Lagrange differential eq^n

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$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

$$= \int \frac{dx}{x} = \int \frac{dy}{y} + \log c$$

$$\log x = \log y + \log c_1$$

$$\log x - \log y = \log c_1$$

$$\boxed{\frac{x}{y} = c_1}$$

$$\int \frac{dy}{y} = \int \frac{dz}{z} + \log c_2$$

$$\log y = \log z + \log c_2$$

$$\log y - \log z = \log c_2$$

$$\boxed{\frac{y}{z} = c_2}$$

So,

$$f(c_1, c_2) = 0$$

$$f\left(\frac{x}{y}, \frac{y}{z}\right) = 0 \quad \text{Ans.}$$

Ques3 Solve the PDE: $yzp + 2xq = xy$

Ans form: $Pp + Qq = R$, $p = \frac{dz}{dx}$, $q = \frac{dz}{dy}$

where,

$P = yz$, $Q = 2x$, $R = xy$

LDE:

$$\frac{dx}{yz} = \frac{dy}{2x} = \frac{dz}{xy}$$

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$$\frac{dx}{y^2} = \frac{dz}{xy} \Rightarrow \int x\,dx = \int z\,dz + c_1$$

$$\frac{x^2}{2} = \frac{z^2}{2} + c_1$$

$$\frac{x^2}{\cancel{2}} = \frac{z^2 + c_1}{\cancel{2}} \Rightarrow \boxed{x^2 - z^2 = c_1}$$

then,

$$\frac{dy}{\cancel{2x}} = \frac{dz}{\cancel{x}y}$$

$$\int y\,dy = \int 2\,dz + c_2$$

$$\frac{y^2}{2} = 2z + c_2$$

$$\frac{y^2 - 4z}{2} = c_2 \Rightarrow \boxed{y^2 - 4z = c_2}$$

So,

$$f(c_1, c_2) = 0$$

$$f(x^2 - z^2,\ y^2 - 4z) = 0 \quad \text{= Ans}$$

Ques-4. Define Charpit's Auxiliary equation.

Ans- Charpit method is a technique used to solve non-linear first order partial differential eqn of the form:

$$f(x, y, z, p, q) = 0$$

Charpit's Auxaliary equation:

$$\frac{df}{\frac{df}{dx}+p\frac{df}{dz}} = \frac{\partial q}{\frac{\partial f}{\partial y}+q\frac{\partial f}{\partial z}} = \frac{\partial z}{-p\frac{\partial f}{\partial b}-q\frac{\partial f}{\partial q}} =$$

$$\frac{\partial x}{-\frac{\partial f}{\partial p}} = \frac{\partial y}{-\frac{\partial f}{\partial q}} = \frac{\partial f}{0}$$

Ques 5 Solve $(p+q)(px+qy) = 1$

Ans eq: $p^2x + pqy + qpx + q^2y = 1$

- $\frac{\partial f}{\partial x} = p^2+pq$ • $\frac{\partial f}{\partial p} = 2p+qy+qx$
- $\frac{\partial f}{\partial y} = pq+q^2$ • $\frac{\partial f}{\partial q} = py+px+2qy$
- $\frac{\partial f}{\partial z} = 0$

then,

$$\frac{\partial f}{p^2+pq} = \frac{\partial q}{pq+q^2}$$

$$\frac{\partial p}{p\cancel{(p+q)}} = \frac{\partial q}{q\cancel{(p+q)}} = \boxed{\frac{\partial p}{p} = \frac{\partial q}{q}}$$

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$$\log p = \log q + \log a$$

$$\boxed{p = qa}$$

put the value of p in equation

$$(qa+q)(qax+qy)=1$$

$$q(a+1)q(ax+y)=1$$

$$q^2(a+1)(ax+y)=1$$

$$q^2 = \frac{1}{(a+1)(ax+y)} \Rightarrow \boxed{q = \frac{1}{\sqrt{(a+1)(ax+y)}}}$$

So,

$$p = qa$$

$$\boxed{p = \frac{a}{\sqrt{(a+1)(ax+y)}}}$$

PART-B

Ques Find the general integral of $xzp+yzq=xy$

Ans- equation: $xzp+yzq=xy$

$P=xz$, $Q=yz$, $R=xy$

Form: $Pp+Qq=R$

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$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy}$$

$$\Rightarrow \frac{dx}{xz} = \frac{dy}{yz} = \int \frac{dx}{x} = \int \frac{dy}{y} + \log c_1$$

$$= \log x = \log y + \log c_1$$

$$\log x - \log y = \log c_1$$

$$\boxed{\frac{x}{y} = c_1}$$

$$\Rightarrow \frac{dx}{xz} = \frac{dz}{xy}$$

$$\int y\,dx = \int z\,dz + c_2$$

$$xy = \frac{z^2}{2} + c_2$$

$$\boxed{xy - \frac{z^2}{2} = c_2}$$

$$f(c_1, c_2) = 0$$

$$\Rightarrow f\left(\frac{x}{y}, xy - \frac{z^2}{2}\right) = 0$$

Ans

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Ques 2 Solve $(1+y)p + (1+x)q = z$

Ans. Eqⁿ: $(1+y)p + (1+x)q = z$

form: $\underline{Pp + Qq = z}$

Here,

$$P = 1+y, \quad Q = 1+x, \quad R = z$$

LDE: $\dfrac{dx}{P} = \dfrac{dy}{Q} = \dfrac{dz}{R}$

$$\frac{dx}{1+y} = \frac{dy}{1+x} = \frac{dz}{z}$$

$$\frac{dx}{1+y} = \frac{dy}{1+x}$$

$$(1+x)\,dx = (1+y)\,dy + c_1$$

$$x + \frac{x^2}{2} = y + \frac{y^2}{2} + c_1$$

$$2x + x^2 = 2y + y^2 + c_1$$

$$\boxed{x^2 - y^2 + 2x - 2y = c_1}$$

Now put, $(1,1,0)$ in each fraction.

$$\frac{dx+dy}{1+y+1+x} = \frac{dx+dy}{2+x+y} = \frac{dz}{z}$$

$$\frac{dx+dy}{2+x+y} = \frac{dz}{z}$$

$$\frac{d(2+x+y)}{2+x+y} = \frac{dz}{z}$$

$$\log(2+x+y) = \log z + \log c_1$$

$$2+x+y = zc_2$$

$$\boxed{\frac{2+x+y}{z} = c_2}$$

$$f(c_1, c_2) = 0$$

$$f\left(x^2-y^2+2x-2y, \frac{2+x+y}{z}\right) = 0$$

Ans

Ques 3 Find the complete integral of $q = px + p^2$

Ans- eq^n: $p^2 + px = q$

let: $f = p^2 + px - q = 0$

- $\frac{\partial f}{\partial x} = p$, • $\frac{\partial f}{\partial z} = 0$, • $\frac{\partial f}{\partial q} = -1$
- $\frac{\partial f}{\partial y} = 0$, • $\frac{\partial f}{\partial p} = 2p + x$

AE:

$$\frac{\partial p}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{\partial q}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

$$\frac{\partial p}{p+p(0)} = \frac{\partial q}{0+0}$$

$$\frac{\partial p}{p} = \frac{\partial q}{0}$$

So, $dq = 0 \rightarrow \boxed{q = a}$

Put the value of q in the equation

$$\cancel{p^2 + p\cancel{x} - q = 0}$$

$$p^2 + px - a = 0$$

So, it is a quadratic equation

$$p = \frac{-x \pm \sqrt{x^2 - 4a}}{2}$$

$$dz = p\,dx + a\,dy$$

$$dz = \frac{-x + \sqrt{x^2+4a}}{2}\,dx + a\,dy$$

$$\int dz = \int -\frac{x}{2}\,dx + \int \frac{\sqrt{x^2+4a}}{2}\,dx + \int a\,dy$$

$$z = \frac{-x^2}{4}$$

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$$\therefore \int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$z = \frac{-x^2}{4} + \frac{1}{2}\left(\frac{x}{2}\sqrt{x^2+4a^2} + \frac{4a^2}{2}\log\left|x+\sqrt{x^2+4a^2}\right| + ay + c\right)$$

$$z = \frac{-x^2}{4} + \frac{1}{2}\left(\frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+4a^2}\right|\right) + ay + c$$

Ans

PART-C

Ques 1 Solve: $\cos(x+y)p + \sin(x+y)q = z$

Ans Eq: $\cos(x+y)p + \sin(x+y)q = z$

form: $\boxed{Pp + Qq = R}$

where,

$P = \cos(x+y)$, $Q = \sin(x+y)$, $R = z$

Lagrange auxiliary eq^n $\boxed{\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}}$

$$\Rightarrow \frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z}$$

$$= \frac{dx+dy}{\cos(x+y)+\sin(x+y)} = \frac{dx-dy}{\cos(x+y)-\sin(x+y)}$$

then,

$$\frac{dx+dy}{\cos(x+y)+\sin(x+y)} = \frac{dx-dy}{\cos(x+y)-\sin(x+y)}$$

$$dx-dy = \frac{\cos(x+y)-\sin(x+y)}{\cos(x+y)+\sin(x+y)}(dx+dy)$$

let $\cos(x+y)+\sin(x+y) = t$

$$-\sin(x+y)+\cos(x+y)(dx+dy) = dt$$

$$\Rightarrow dx-dy = \frac{dt}{t}$$

on integrating both sides.

$$\int dx - \int dy = \int \frac{dt}{t}$$

$$x-y = \log[\{\cos(x+y)-\sin(x+y)] - \log c_1$$

$$x-y = \log\left(\frac{\cos(x+y)+\sin(x+y)}{c_1}\right)$$

$$e^{x-y} = \frac{\cos(x+y)+\sin(x+y)}{c_1}$$

$$\Rightarrow c_1 = [\cos(x+y)+\sin(x+y)] \cdot e^{-(x+y)} \quad \text{--- (1)}$$

then,

$$\frac{dz}{z} = \frac{dx + dy}{\cos(x+y) + \sin(x+y)}$$

divide and multiply by $1/\sqrt{2}$

$$= \frac{1}{\sqrt{2}} \frac{dx + dy}{\frac{1}{\sqrt{2}}\cos(x+y) + \frac{1}{\sqrt{2}}\sin(x+y)}$$

put $\frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$ and $\sin\frac{\pi}{4}$

$$= \frac{1}{\sqrt{2}} \frac{(dx + dy)}{\sin\frac{\pi}{4}\cos(x+y) + \cos\frac{\pi}{4}\sin(x+y)}$$

$\because \sin A \cos B + \cos A \sin B = \sin(A+B)$

$$\Rightarrow \sqrt{2}\frac{dz}{z} = \frac{dx + dy}{\sin(x+y+\frac{\pi}{4})}$$

$$\sqrt{2}\int\frac{dz}{z} = \int \operatorname{cosec}\left(x+y+\frac{\pi}{4}\right)(dx+dy)$$

let $x+y+\frac{\pi}{4} = t$

$dx + dy = dt$

$$\sqrt{2}\log z = \int \operatorname{cosec} t \, dt$$

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$$\sqrt{2}\log z = \log \tan\frac{t}{2} + \log \frac{c}{2}.2$$

$$\log z^{\sqrt{2}} = \log c_2 \frac{\tan(x+y+\pi/4)}{2}$$

$$z^{\sqrt{2}} = c_2 \frac{\tan(x+y+\pi/4)}{2}$$

$$z^{\sqrt{2}} \cot\left(\frac{x+y}{2}+\frac{\pi}{8}\right) = c_2$$

So final Answer $f(c_1, c_2) = 0$

$$f\left(z^{\sqrt{2}}\cot\left(\frac{x+y}{2}+\frac{\pi}{8}\right), (\cos(x+y)+\sin(x+y)).e^{-(x-y)}\right) = 0$$

Ans.

Ques-13 Find the complete integral of $z^2 = pqxy$

Ans

$$\frac{df}{dx} = -pqy \quad , \frac{dp}{dy} = -pyx$$

$$\frac{df}{dz} = 2x \quad , \frac{df}{dp} = -qxy \quad , \frac{df}{dz} = -pxy$$

$$\frac{dx}{qxy} = \frac{dy}{pqy} = \frac{dz}{pqxy+qxyp} = \frac{dp}{-pqy+2pz}$$

$$= \frac{dq}{-pqx+2qz}$$

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$$\frac{p\,dx + x\,dp}{2xpz} = \frac{q\,dy + y\,dq}{2yqz}$$

$$\log(px) = \log qy + \log q$$

$$\boxed{p = \frac{aqy}{c_1}}$$

put ③ in eq (ii)

$$z^2 - \left(\frac{aqy}{c_1}\right) axy = 0$$

$$q = \frac{z}{y\sqrt{a}} \Rightarrow p = \frac{z\sqrt{a}}{x}$$

$$dz = p\,dx + q\,dy$$

$$dz = \frac{z\sqrt{a}}{x}\,dx + \frac{z}{y\sqrt{a}}\,dy$$

$$\int \frac{dz}{z} = \int \sqrt{a}\,\frac{dx}{x} + \int \frac{1}{\sqrt{a}}\,\frac{dy}{y}$$

$$\log z = \sqrt{a}\log x + \frac{1}{\sqrt{a}}\log y + \log c$$

$$z = x^{\sqrt{a}}\, y^{\frac{1}{\sqrt{a}}}$$

where

$$\sqrt{a} = c$$

$$\boxed{z = x^{c}\, y^{1/c}}$$

Ans