Tutorial Sheet-5

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Part - A

Q1. Define degree and order of differential equation.

Ans :• Order → It is the highest derivative in a equation.

• Degree → Highest power of the highest-order derivative. (if Polynomial)

Q2. What is the standard form of linear differential equation.

Ans : First order : $\frac{dy}{dx} \neq P(x)\, y = Q(x)$

Q3. Define exact differential equation and also discuss the condition for exactness.

Ans :- Let the differential equation be of the form

$$M(x,y)\,dx + N(x,y)\,dy = 0 \quad -①$$

eq① is said to be an exact diff. eq^n if

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \Rightarrow ②$$

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Q4. Define homogeneous differential equation.

Ans: A homogeneous differential equation is a differential equation in which all terms involve the dependent variable or its derivatives, and there is no independent terms.

$$\boxed{\frac{dy}{dx} = f\left(\frac{y}{x}\right)}$$

Q5. Solve $\frac{dy}{dx} = \sin^2(x-y+1)$

Let $x - y + 1 = t$

$$1 - \frac{dy}{dx} = \frac{dt}{dx}$$

$$1 - \frac{dt}{dx} = \sin^2(t)$$

$$\frac{dt}{dx} = \cos^2(t)$$

$$\int \sec^2(t)\,dt = \int dx$$

$$\tan(t) = x + c$$

$$\boxed{\tan(x-y+1) = x+c} \quad \text{Ans}$$

Part-B

1. Solve

Ans

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Part-B

1. Solve $3x(1-x^2)y^2 \frac{dy}{dx} + (2x^2-1)y^3 = ax^3$

Ans Let $t = y^3$

$$\frac{dt}{dx} = 3y^2 \frac{dy}{dx}$$

$$\Rightarrow x(1-x^2)\frac{dt}{dx} + (2x^2-1)t = ax^3$$

$$\Rightarrow \frac{dt}{dx} + \frac{(2x^2-1)}{x(1-x^2)} t = \frac{ax^2}{(1-x^2)}$$

$$IF = e^{\int P(dx)} = I$$

$$I = \int \frac{(2x)}{(1-x^2)} dx - \int \frac{dx}{x(1-x)(1+x)}$$

$$I = -\ln|1-x^2| + \int \frac{1}{x} + \frac{1}{2(1-x)} - \frac{1}{2(1+x)}$$

$$I = -\ln|1-x^2|$$

$$I = -\ln|x| - \frac{1}{2}\ln|1-x^2|$$

$$I = \ln\left|\frac{1}{x\sqrt{1-x^2}}\right|$$

So, $I.f = e^{I}$

$= e^{\ln\left|\frac{1}{x\sqrt{1-x^2}}\right|}$

$$I.F = \frac{1}{x\sqrt{1-x^2}}$$

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Final solution :

$$y \cdot I.F = \int IF \times Q(x)\,dx + C$$

$$t \cdot \frac{1}{x\sqrt{1-x^2}} = \int \frac{1}{x\sqrt{1-x^2}} \times \frac{ax^2}{(1-x^2)}\,dx$$

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2. Solve $\frac{dy}{dx} = \frac{x+y+1}{2x+2y+3}$

Ans: $\frac{dy}{dx} = \frac{x+y+1}{2(x+y)+3}$

Let $x+y=t$

$$1+\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 1 = \frac{t+1}{2t+3}$$

$$\frac{dt}{dx} = \frac{3t+4}{2t+3}$$

$$\int \left(\frac{2t+3}{3t+4}\right) dt = \int dx$$

$$\int \left[\left(\frac{2t}{3t+4}\right) + \left(\frac{3}{3t+4}\right)\right] dt = x+c$$

$$\frac{2}{3}\int \frac{t\,dt}{t+4/3} + 3\int \frac{dt}{3t+4} = x+c$$

$$= \frac{2}{3}\int \left[\frac{t+4/3}{t+4/3} - \frac{4/3}{t+4/3}\right] dt + \ln|3t+4| = x+c$$

$$= \frac{2}{3}\left[t - \frac{4}{3}\ln\left|t+4/3\right|\right] + \ln|3t+4| = x+c$$

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$$= \frac{2}{3}\left[x+y-\frac{4}{3}\ln\left|x+y+4/3\right| + \ln(3x+3y+4) = x\right.$$

$$= 3x+3y+4 = ce^{3(x-2y)}$$ Ans

Q3 Solve $(1+y^2) + (x-e^{\tan^{-1}y})\frac{dy}{dx} = 0$

Ans $\frac{dy}{dx} = (x-e^{-\tan^{-1}(y)}) = -dx(1+y^2)$

$$= \frac{dy}{dx} = \frac{e^{-\tan^{-1}(y)} - x}{1+y^2}$$

$$= \frac{dx}{dy} + \left(\frac{1}{1+y^2}\right)x = \frac{e^{-\tan^{-1}(y)}}{1+y^2}$$

$$= I.F = e^{\int P(y)\,dy}$$

$$= I.F = e^{\int \frac{1}{1+y^2}\,dy}$$

$$= \boxed{I.F = e^{\tan^{-1}y}}$$

Final solution

$\cancel{xe^{\tan^{-1}y} = \tan^{-1}(y)+c}$

$$xe^{\tan^{-1}y} = \int e^{-\tan^{-1}(y)} \times \frac{e^{\tan^{-1}(y)}}{1+y^2}\,dy + c$$

$$\Rightarrow xe^{\tan^{-1}(y)} = \tan^{-1}(y) + c$$ Ans

—x—

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~~Solve~~

Part-C.

1. Solve $\sin y\,dy - \cos y\,(1-x\cos y)\,dx = 0$

Sol^n Let $\cos y = t$

$-\sin y\,dy = dt$

$-dt - t(1-xt)\,dx = 0$

$-\frac{dt}{dx} = t - xt^2$

$\boxed{\frac{dt}{dx} + t = xt^2}$

$\Rightarrow\ t^{2}\frac{dt}{dx} + \frac{1}{t} = x$

Let $\frac{1}{t} = v$

$-\frac{1}{t^2}\frac{dt}{dx} = \frac{dv}{dx}$

$\Rightarrow\ -\frac{dv}{dx} + v = x$

$\boxed{\frac{dv}{dx} - v = -x}$ — ②

$I.F = e^{\int -1\,dx} = e^{-x}$

Complete sol^n is given by

$ve^{-x} = \int -xe^{-x}\,dx$

$ve^{-x} = -\left[-xe^{-x} + \int e^{-x}dx\right]$

$ve^{-x} = xe^{-x} + e^{-x} + c$

$v = (x+1) + ce^{x}$

$\frac{1}{t} = (x+1) + ce^{x}$

$\frac{1}{\cos y} = (x+1) + ce^{x}$

$\sec y = (x+1) + ce^{x}$

Ans

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C2. Solve $(3x^2y^4+2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$

Sol^n

$M = 3x^2y^4 + 2xy, \quad N = 2x^3y^3 - x^2$

$\frac{\partial M}{\partial y} = 12x^2y^3 + 2x, \quad \frac{\partial N}{\partial x} = 6x^2y^3 - 2x$

Let solve by multiplying eq① by $[x^n y^k]$

$M' = 3x^{n+2}y^{k+4} + 2x^{n+1}y^{k+1}$

$\frac{\partial M'}{\partial y} = 3(k+4)\,x^{n+2}y^{k+3} + 2(k+1)\,x^{n+1}y^k$

$N' = 2x^{n+3}y^{k+3} - x^{n+2}y^k$

$\frac{\partial N'}{\partial x} = 2(n+3)\,x^{n+2}y^{k+3} - (n+2)\,x^{n+1}y^k$

To make this eq^n reducible to exact it holds:

$$\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}$$

$3(k+4)x^{n+2}y^{k+3} + 2(k+1)x^{n+1}y^k$
$= 2(n+3)x^{n+2}y^{k+3} - (n+2)x^{n+1}y^k$

On comparing L.H.S & R.H.S we get 2 eq^{n}s

$= 3(k+4) = 2(n+3)$

$2(k+1) = -(n+2)$

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$\Rightarrow 3k - 2n = 6 - 12$

$2n - 3k = 6 \quad - (i)$

$2k + 2 = -n - 2$

$\boxed{2k + n = -4} \quad - (ii)$

$2n - 3k - 4k - 2n = 6 + 8$

$-7k = 14$

$\boxed{k = -2}$

$n = 0$

So, $If = x^0 y^{-2}$

$\boxed{If = y^{-2}}$ → multiply by main eqⁿ

$\Rightarrow (3x^2y^2 + 2xy^{-1})dx + (2x^3y - x^2y^{-2})dy = 0$

$\Rightarrow M' = 3x^2y^2 + 2xy^{-1}$

$\frac{\partial M'}{\partial y} = 6x^2y - 2xy^{-2}$

$N' = 2x^2y - x^2y^{-2}$

$\boxed{\frac{\partial N'}{\partial x} = 6x^2y - 2xy^{-2}}$

Here; $\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}}$ → Now exact d.f

$P = m'dx$

$P = \int (3x^2y^2 + 2xy^{-1})dx = \boxed{P = x^3y^2 + \frac{x^2}{y}}$

$Q = \int \left(N' - \frac{\partial P}{\partial y}\right)dy$

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$$Q = \int (2x^3y - x^2y^{-2} - 2x^3y + x^2y^{-2}).$$

$Q = 0$ $\qquad$ $P + Q = C$

Final ans = $\boxed{x^3y^2 + \frac{x^2}{y} = C}$ Ans

Solve

Q3 $(x^2y^2 + xy + 1)\,y\,dx + (x^2y^2 - xy + 1)\,x\,dy = 0$

Ans $M = (x^2y^2 + xy + 1)\,y$

$N = (x^2y^2 - xy + 1)\,x$

This above eq^n is

type $M(x,y)\,y\,dx + N(x,y)\,x\,dy = 0$

Its $I.F = \dfrac{1}{Mx - Ny}$

$$I.F = \frac{1}{[\cancel{x^2y^2} + xy + \cancel{1} - \cancel{x^2y^2} + xy - \cancel{1})xy}$$

$$I.F = \frac{1}{x^2y^2}$$

multiply I.F by main eq^n

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$$\Rightarrow \frac{1}{x^2y^2}\left[x^2y^3 + xy^2 + y\right)dx + \frac{1}{x^2y^2}(x^3y^2 - x^2y + x)dy = 0$$

$$= \underbrace{\left[y + \frac{1}{x} + \frac{1}{x^2y}\right]}_{M'} dx + \underbrace{\left[x - \frac{1}{y} + \frac{1}{xy^2}\right]}_{N'} dy = 0$$

$$\frac{\partial M'}{\partial y} = 1 - \frac{1}{x^2y^2}, \quad \frac{\partial N'}{\partial x} = 1 - \frac{1}{x^2y^2}$$

$$\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}$$

$$P = \int M' dx$$

$$P = \int\left(y + \frac{1}{x} + \frac{1}{x^2y}\right) dx$$

$$\boxed{P = xy + \ln x - \frac{1}{xy}}$$

$$Q = \int\left(N - \frac{\partial P}{\partial y}\right)dy$$

$$Q = \int -\frac{1}{y}\, dy = Q = -\ln y$$

Final sol^n

$$P + Q = C$$

$$xy + \ln x - \frac{1}{xy} - \ln y = C$$

$$\boxed{xy + \ln\left(\frac{x}{y}\right) - \frac{1}{xy} = C}$$ Ans