Tribhuvan University
Institute of Science and Technology
2073

Bachelor Level/ First Year/ First Semester/ Science — Full Marks: 80
Computer Science and Information Technology (MTH:104) — Pass Marks: 32
(Calculus and analytical Geometry) — Time: 3 hours.
(Old course)

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Attempt all questions.

Group A (10×2=20)

1. If f(x) = sin x and g(x) = -x/2. Find f(f(x)) and g(f(x)).

2. Define critical point. Find the critical point of f(x) = 2x2.

3. Evaluate $\lim_{n\to\infty} \frac{a-b^4}{n^4+a}$.

4. Find the equation of the parabola with vertex at the origin and directrix at x= 7.

5. Find a vector parallel to the line of intersection of the planes 3x + 6y − 2z = 5.

6. Evaluate $\int_{-1}^{0}\int_{-1}^{1}(x+y+1)dxdy$.

7. Find $\frac{dt}{dx}$ and $\frac{dt}{dy}$ if f(x,y) = x2 + y2

8. Evaluate $\log_{(x,y)\to(0,1)} \frac{x-xy+k}{x^2y+5xy-y^3}$

9. Show that y = ax2 + b is the solution of xy'' + y' = 0.

10. Solve $\frac{d^2y}{dx^2} - y = 0$

Group B (5×4=20)

11. Verify Rolle's theorem for f(x) = x3, x ∈ [-3,3].

12. Find the Taylor series expansion of the case at ex, at x=0.

13. Find a Cartesian equivalent of the polar equation r cos (θ-π/3) = 3.

14. Evaluate it $(x,y) \to (0,0) \frac{2y^2}{\sqrt{x^2+xy}}$

15. Obtain the general solution of $(y-z)\frac{dz}{dx} + (x-y)\frac{dz}{dy} = z - x$

Group C (5×8=40)

16. Evaluate the integrals and determine whether they converge or diverge

(a) $\int_{-1}^{\infty} \frac{dx}{x}$ (b) $\int_{-1}^{\infty} \frac{dx}{x^2}$

OR

Find the area bounded on the parabola $y = 2 - x^2$ and the line $y = -x$.

17. Find the curvature of the helix $\vec{R}(t) = (a \cos \omega t)\vec{i} + (a \sin \omega t)\vec{j} + (bt)\vec{k}$?

18. Find the volume enclosed between the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$

19. Find the extreme values of the function $F(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$

OR

Find the extreme values of $f(x,y) = xy$ subject to $g(x,y) = x^2 + y^2 - 10 = 0$.

20. Define second order partial differential equation. Define initial boundary value problem. Derive the heat equation or wave equation in one dimension.