

Tribhuvan University
Institute of Science and Technology
2075
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Bachelor Level / First Year/ First Semester/ Science
Computer Science and Information Technology (MTH. 112)
(Mathematics I)
(NEW COURSE)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt any three questions:

(3×10=30)

1. (a) A function is defined by $f(x) = |x|$, calculate $f(-3)$, $f(4)$, and sketch the graph. (5)
- (b) Prove that the $\lim_{x \rightarrow 2} \frac{|x-2|}{x-2}$ does not exist. (5)
2. (a) Find the domain and sketch the graph of the function $f(x) = x^2 - 6x$. (5)
- (b) Estimate the area between the curve $y = x^2$ and the lines $y = 1$ and $y = 2$. (5)
3. (a) Find the Maclaurin series for $\cos x$ and prove that it represents $\cos x$ for all x . (4)
- (b) Define initial value problem. Solve that initial value problem of $y' + 2y = 3$, $y(0) = 1$. (4)
- (c) Find the volume of a sphere of radius a . (2)
4. (a) If $f(x, y) = \frac{y}{x^2}$ does $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exist? Justify. (5)
- (b) Calculate $\iint_R f(x, y) dA$ for $f(x, y) = 100 - 6x^2y$ and $R: 0 \leq x \leq 2, -1 \leq y \leq 1$. (5)

Attempt any ten questions:

(10×5=50)

5. If $f(x) = \sqrt{2-x}$ and $g(x) = \sqrt{x}$, find $f \circ g$ and $f \circ f$. (5)
6. Define continuity on an interval. Show that the function $f(x) = 1 - \sqrt{1-x^2}$ is continuous on the interval $[-1, 1]$. (5)
7. Verify Mean value theorem of $f(x) = x^3 - 3x + 2$ for $[-1, 2]$. (5)
8. Starting with $x_1 = 2$, find the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$. (5)
9. Evaluate $\int_0^\infty x^3 \sqrt{1-x^4} dx$. (5)
10. Find the volume of the resulting solid which is enclosed by the curve $y = x$ and $y = x^2$ is rotated about the x -axis. (5)
11. Find the solution of $y'' + 4y' + 4 = 0$. (5)

12. Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2 + 4}$ converges or diverges. (5)

13. If $\mathbf{a} = (4, 0, 3)$ and $\mathbf{b} = (-2, 1, 5)$ find $|\mathbf{a}|$, the vector $\mathbf{a} - \mathbf{b}$ and $2\mathbf{a} + 5\mathbf{b}$. (1+2+2)

14. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined as a function of x and y by the equation $x^3 + y^3 + z^3 + 6xyz = 1$. (5)

15. Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$. (5)