Set5

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. (a) A function is defined by $f(x)=\sqrt{x-5}$ [5]
   Find domain and sketch the graph.

   (b) Prove that the limit $\lim_{x\to 0} x^2 \sin\frac{1}{x}=0$. [5]

2. (a) Sketch the curve : $f(x)=\ln(4-x^2)$. [5]

   (b) Estimate the area between the curve $y=e^{x^2}$ and the lines x = 0 and x = 1, using Simpson's rule. [5]

3. (a) Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y=2x^2-x^3$ and y = 0. [4]

   (b) Define differential equation. Solve: $x\ln x = y(1+\sqrt{3+y^2})\,y'$ if y(1) = 1. [6]

4. (a) Show that the lines and with parametric equations

   x = 1 + t y = - 2 + 3t z = 4 – t

   x = 2s y = 3 + s z = - 4 + 4s are **skew lines**; that is, they do not intersect and are not parallel (and therefore do not lie in the same plane) [6]

   (b) Find the scalar projection and vector projection of $\vec{b}=\langle 1,1,2\rangle$ onto $\vec{a}=\langle -2,3,1\rangle$ [4]

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Determine whether each of the following functions is even, odd, or neither even nor odd.

   (a) $y=x^5+x$ (b) $y=1-x^4$ (c) $y=2x-x^2$

6. Define indeterminate forms.Evaluate the following: $\lim_{x\to 0^+} x^x$.
7. Find an equation of the plane that passes through the points P(1, 3, 2), Q(3, -1, 6) and R(5, 2, 0).
8. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?
9. For what value of P, the integral $\int_1^\infty \frac{1}{x^P}dx$ is convergent?
10. The curve $y=\sqrt{4-x^2}$, $-1\le x\le 1$, is an arc of the circle $x^2+y^2=4$. Find the area of the surface obtained by rotating this arc about the x-axis.
11. Solve: $y''+y=e^x+x^3$, $y'(0)=0$ and $y(0)=2$.
12. Find the point at which the line with parametric equations,

    x = 2 + 3t y = - 4t z = 5 + t, intersects the plane

    4x + 5y – 2z = 18.
13. Show that the series $\sum_{n=1}^{\infty}\frac{1}{n(1+n)}$ converges, find its sum.
14. Find the volume of the solid that is bounded by the elliptic paraboloid $x^2+2y^2+z=16$, the planes x = 2 and y = 2 , and the three coordinate planes.
15. Find an equation of the plane through the point (2, 4, -1) with normal vector $\langle 2,3,4\rangle$. Find the intercepts and sketch the plane.

Best of luck!!!!!!

Set 5

Group-'A'

Diwash Sapkota

(Q.N. 1.a)

= solution,

$$f(x) = \sqrt{x-5}$$

f(x) exists and is real when $\sqrt{x-5} \geq 0$

$\Rightarrow (x-5) \geq 0$

$\Rightarrow x \geq 0 + 5$

$\Rightarrow x \geq 5$

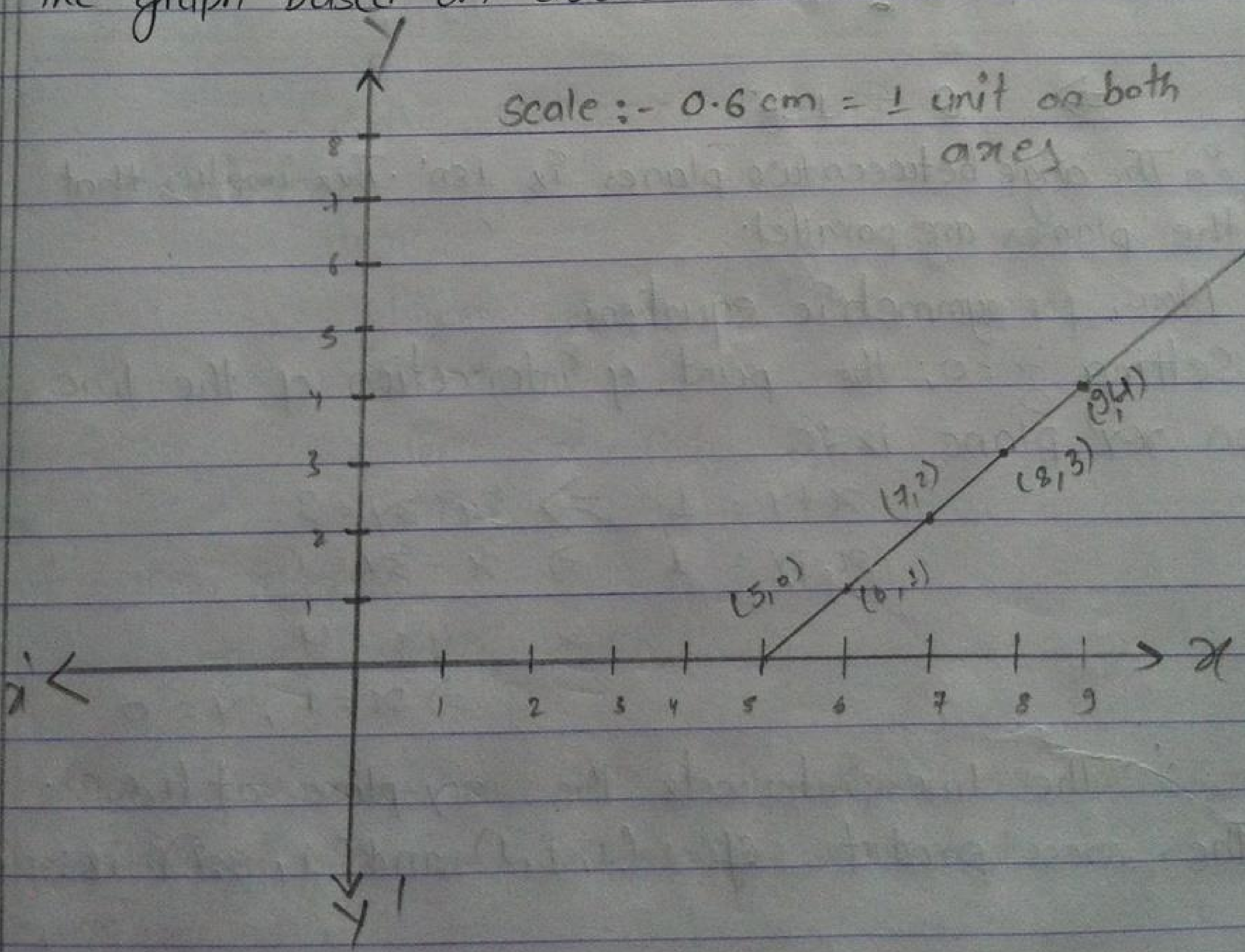
$\therefore$ Domain = $[5, \infty)$

Now, the domain & corresponding functional values are:-

~~Graph~~

x	5	6	7	8	9	10
$f(x)$	0	1	2	3	4	5

The graph based on above characteristics is :-



(Q.N. 1.b) (S-S, G-A)

= solution,

here,

$$\lim_{x \to 0} x^2 \sin 1/x$$

$$= \lim_{x \to 0} x^2 \cdot (\sin 1/x)$$

we know;

$$-1 \leq \sin(1/x) \leq 1$$

Also, $x^2 \geq 0$ for all real values of x. So, multiplying above inequality on each side by x^2;

$$-x^2 \le x^2 \sin 1/x \le x^2$$

And, Using squeeze theorem,

let $h(x) = x^2$, $g(x) = x^2 \sin(1/x)$ and $f(x) = -x^2$

so,

$$f(x) \le g(x) \le h(x)$$

And,

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} (-x^2) = 0$$

$$\lim_{x \to 0} h(x) = \lim_{x \to 0} x^2 = 0$$

since,

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} h(x);$$

$$\lim_{x \to 0} g(x) = 0$$

$$\Rightarrow \lim_{x \to 0} x^2 \sin(1/x) = 0$$

proved

(Q.N. 2.a)

= solution,

$$f(x) = \ln(4-x^2)$$

(i) Domain = $(-2, 2)$

(ii) When $x=0$, $f(x) = 1.387$. So, 1.387 is the y-intercept.

When $f(x)=0$; $x = \pm\sqrt{3}$. So, $\pm\sqrt{3}$ is the x-intercept.

(iii) $f'(x) = \frac{1}{(4-x^2)} \cdot (-2x)$

$= \frac{2x}{(x^2-4)}$

Since -2 and 2 donot lie in the domain, the only critical point is : $x=0$. When $-2<x<0$; $f'(x)>0$. And, when $0<x<2$; $f'(x)<0$. So, $f(x)$ is increasing in $(-2,0)$ and is decreasing in $(0,2)$.

(iv) $f''(x) = \frac{(x^2-4)\times 2 - 2x\times(2x)}{(x^2-4)^2}$

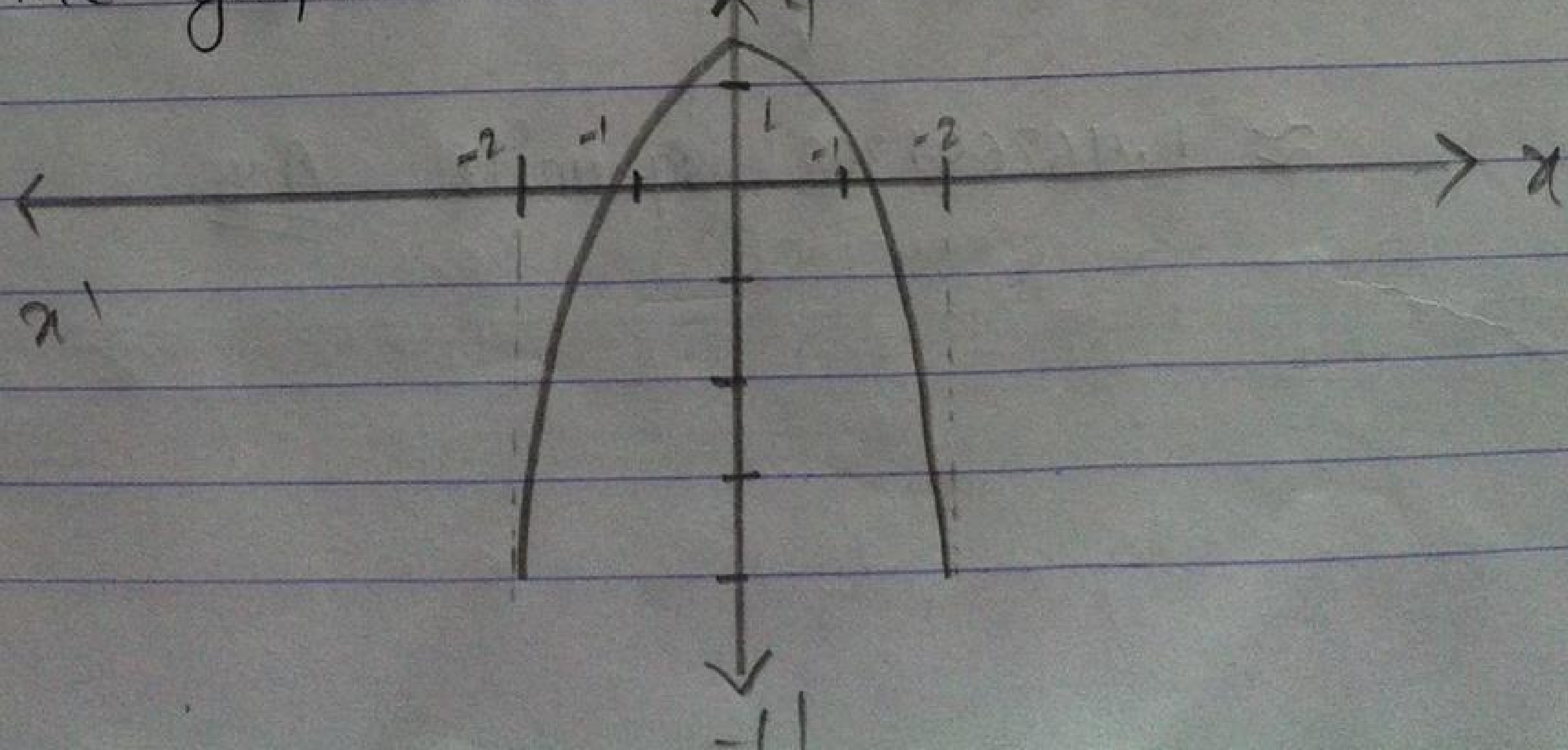
$= \frac{2x^2-8-4x^2}{(x^2-4)^2}$

$= \frac{-2x^2-8}{(x^2-4)^2}$

$= \frac{-2(x^2+4)}{(x^2-4)^2}$

There is no point of inflection. Since $(x^2+4)>0$ and $(x^2-4)^2>0$; $f''(x)<0$. So, $f(x)$ is concave downwards.

The graph based on above characteristics is:-



(Q.N.2b)

= solution,

$$y = f(x) = e^{x^2}$$

end points ⇒ $a = 0$, $b = 1$

Between 0 and 1; 10 even slices can be made.

So, $n = 10$ And,

$$\Delta x = \frac{b-a}{n} = \left(\frac{1-0}{10}\right) = 0.1$$

$\therefore x_0 = 0,\ x_1 = 0.1,\ x_2 = 0.2,\ x_3 = 0.3, \ldots x_{10} = 1$

So,

Using Simpson's rule,

$$\int_0^1 e^{x^2} \cdot dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 4f(x_9) + f(x_{10})\right]$$

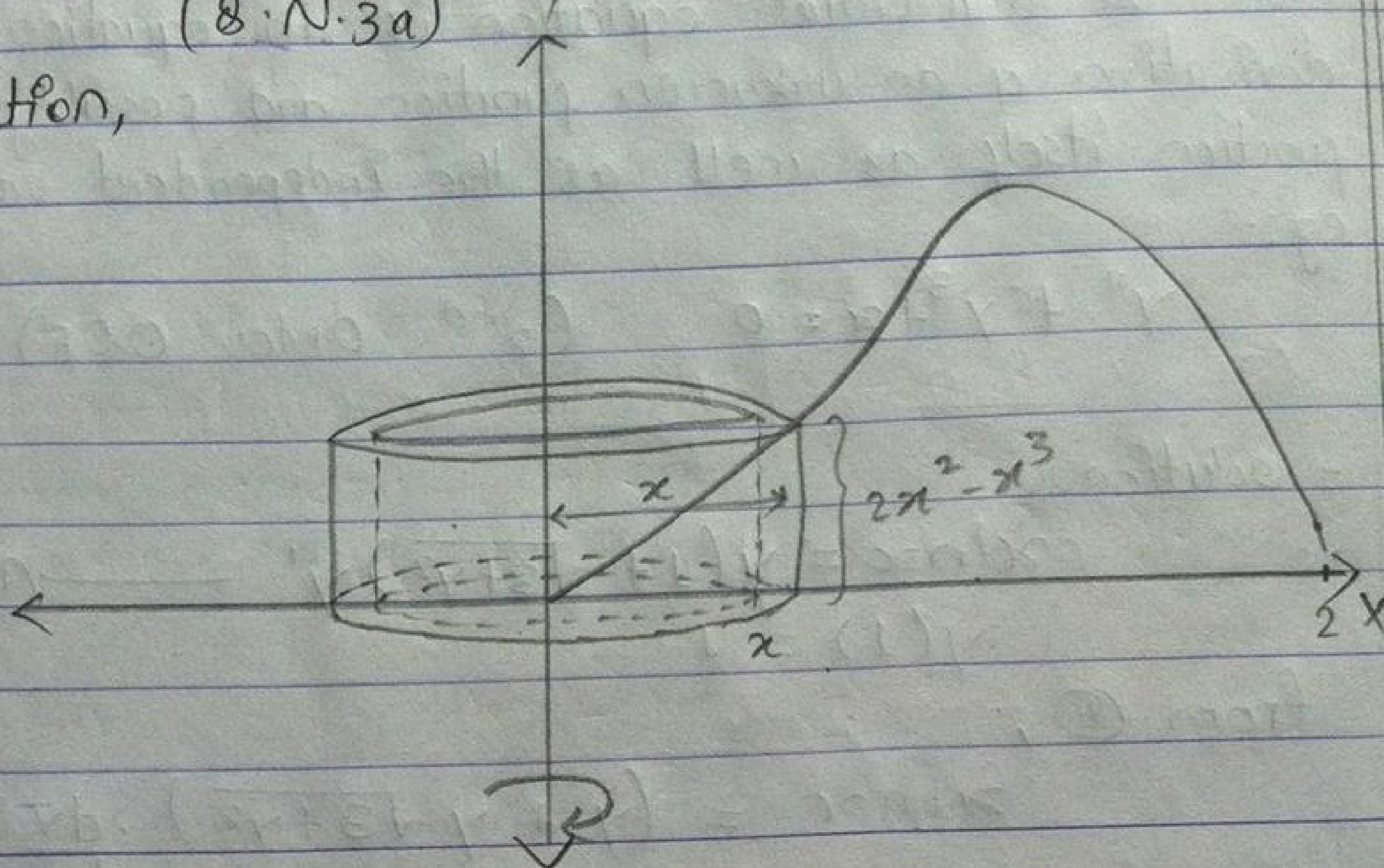
$$\approx \frac{0.1}{3}\left[e^0 + 4e^{(0.1)^2} + 2e^{(0.2)^2} + 4e^{(0.3)^2} + 2e^{(0.4)^2} + 4e^{(0.5)^2} + 2e^{(0.6)^2} + 4e^{(0.7)^2} + 2e^{(0.8)^2} + 4e^{(0.9)^2} + e^{1^2}\right]$$

$$\approx (0.033333333)\left[1 + 4.040200668 + 2.081621548 + 4.376697135 + 2.347021742 + 5.136101667 + 2.866658829 + 6.52926488 + 3.792961759 + 8.991631947 + 2.718281828\right]$$

$\approx$ 1.462681368 sq. units **Ans**

(8·N·3a)

= solution,



From the above sketch, we see a typical shell has radius x, circumference $2\pi x$, and height $f(x) = 2x^2 - x^3$.
So, by shell method,

$$\text{Volume (V)} = \int_0^2 (2\pi x)(2x^2 - x^3) \cdot dx$$

$$= 2\pi \int_0^2 (2x^3 - x^4) \cdot dx$$

$$= 2\pi \left[\frac{x^4}{2} - \frac{x^5}{5} \right]_0^2$$

$$= \frac{16}{5} \pi \text{ cubic units.}$$

Ans

(Q.N.3 b)

A differential equation is an equation involving derivatives of an unknown function and possibly the function itself as well as the independent variable.

eg:-

$$y'' + y^3 + x = 0 \quad (2^{nd} \text{ Order ODE})$$

= solution,

$$x\ln x = y(1+\sqrt{3+y^2})\,y' \quad \text{——— (1)}$$

$$y(1) = 1$$

from (1);

$$x\ln x = (y + y\sqrt{3+y^2})\cdot\frac{dy}{dx}$$

or, $x\ln x \cdot dx = (y + y\sqrt{3+y^2})\cdot dy$

Integrating both sides;

$$\int x\ln x \cdot dx = \int y\cdot dy + \frac{1}{2}\int 2y\sqrt{3+y^2}\cdot dy$$

or, $(\ln x)\cdot\frac{x^2}{2} - \int \frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{2}}}{2}\cdot dx = \frac{y^2}{2} + \frac{\cancel{2}^{1}\cdot(3+y^2)^{3/2}}{{}_{1}\cancel{2}\times 3} + P$

[∵ P = arbitrary constant]

or, $\frac{x^2}{2}\ln x - \frac{x^2}{4} = \frac{y^2}{2} + \frac{(3+y^2)^{3/2}}{3} + K$

We have; when x takes a value of 1; y also takes a value of 1. so,-

$$\frac{(1)^2}{2}\cdot\cancel{\ln 1}^{\,0} - \frac{1}{4} = \frac{1}{2} + \frac{(3+1)^{3/2}}{3} + k$$

or, $-\frac{1}{4} = \frac{1}{2} + \frac{8}{3} + K$

or, $k = -\frac{1}{4} - \frac{1}{2} - \frac{8}{3}$

or, $k = -\left(\frac{1}{4} + \frac{2}{4} + \frac{8}{3}\right)$

or, $k = -\left(\frac{3}{4} + \frac{8}{3}\right)$

or, $k = -\left(\frac{9+32}{12}\right)$

or, $k = -\left(\frac{41}{12}\right)$

So, The required general solution is :-

$$\frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) = \frac{y^2}{2} + \frac{(3+y^2)^{3/2}}{3} - \frac{41}{12}$$

Ans

(Q.N. 4.a)

= solution,

For the first line, (L_1)

$x = (1+t)$, $y = (-2+3t)$, $z = (4-t)$

For the second line, (L_2)

$x = 2s$, $y = (3+s)$, $z = (-4+4s)$

The direction vectors for L_1 and L_2 are :- $(1, 3, -1)$ and $(2, 1, 4)$ respectively. Since they are not proportional, the lines are not parallel.

If l_1 and l_2 had a point of intersection, then:

$x_1 = x_2$, $y_1 = y_2$

$\Rightarrow (1+t) = 2s$ ——— (i) $-2+3t = (3+s)$

or, $s = 3t - 5$ ——— (ii)

$z_1 = z_2$

$\Rightarrow 4 - t = 3 + 4s$

or, $4s = 7 - t$ ——— (iv)

Solving (i) and (ii);

$t = (11/5)$, $s = 8/5$

similarly,

solving (i) and (iii);

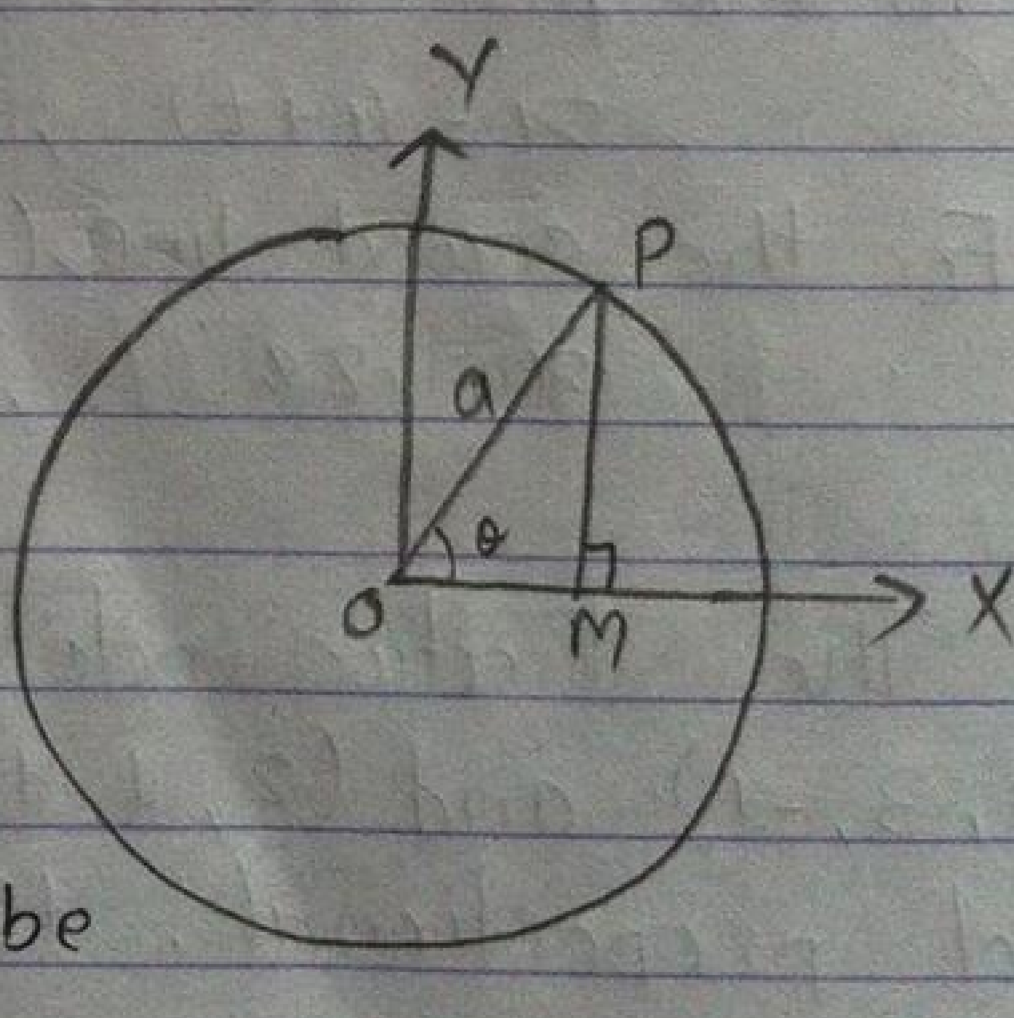
$t = (27/13)$, $s = (16/13)$

Since, the value of t and s are not same even at the same point. So, our supposition got wrong. Therefore, the given lines are skew lines.

(4.b)

= solution,

here,



Consider a circle with center at O and radius 'a' units. let OX and OY be the coordinate axes. 'P' be a point on the circumference of the circle. Then,

If 'θ' be the angle made by $\vec{OP}$ with x'; then..

$$\cos\theta = \frac{OM}{OP} = \frac{OM}{a}$$

$$\Rightarrow OM = (a\cos\theta)$$

$$\sin\theta = \frac{PM}{OP} = \frac{PM}{a}$$

$$\Rightarrow PM = (a\sin\theta)$$

Now;

$$\vec{OP} = (\vec{OM}\,\vec{i} + \vec{MP}\,\vec{j})$$

or, $\vec{r} = (a\cos\theta)\vec{i} + (a\sin\theta)\vec{j}$ ——— (1)

Next;

$$\vec{r'(t)} = (-a\sin\theta)\vec{i} + (a\cos\theta)\vec{j}$$

$$\vec{r''(t)} = (-a\cos\theta)\vec{i} + (-a\sin\theta)\vec{j}$$

And,

$$\vec{r'(t)} \times \vec{r''(t)} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a\sin\theta & a\cos\theta & 0 \\ -a\cos\theta & -a\sin\theta & 0 \end{vmatrix}$$

$$= a^2\vec{k}$$

$$\therefore |\vec{r'(t)} \times \vec{r''(t)}| = \sqrt{(a^4)} = a^2$$

Now,

$$|\vec{r'(t)}| = \sqrt{a^2\cos^2\theta + a^2\sin^2\theta} = a$$

Hence,

$$\text{Curvature } (k) = \frac{|\vec{r'(t)} \times \vec{r''(t)}|}{|\vec{r'(t)}|^3}$$

$$= \frac{a^2}{a^3}$$

$$\text{or, } k = (1/a)$$

∴ The radius of curvature of a circle with radius 'a' is (1/a).

<u>proved</u>

Group - B

(Q · N · 5)

= solution,

$$\vec{a} = (1, 1, 2)$$
$$\vec{b} = (-2, 3, 1)$$
$$|\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$$
$$|\vec{b}| = \sqrt{4+9+1} = \sqrt{14}$$
$$\cancel{|\vec{a}.\vec{b}| = \vec{b}.\vec{a} = 2} \quad \vec{a}.\vec{b} = \vec{b}.\vec{a} = -2+3+2$$
$$= 3 \text{ units.}$$

Now,

$$\text{Scalar Projection of } \vec{b} \text{ on } \vec{a} \Rightarrow \frac{\vec{a}.\vec{b}}{|\vec{a}|}$$

$$= \frac{3}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{6}$$

$$= \frac{\sqrt{6}}{2}$$

Vector projection of $\vec{b}$ on $\vec{a}$ $\Rightarrow$ $\dfrac{\vec{a} \cdot \vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{a}|}$

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \cdot \vec{a}$$

$$= \frac{3}{6} \cdot (-2\vec{i} + 3\vec{j} + \vec{k})$$

$$= \left(-\vec{i} + \frac{3}{2}\vec{j} + \frac{1}{2}\vec{k}\right)$$ Ans

---

(Q.N. 6)

The values, which can no longer be evaluated any more. are called indeterminate forms. eg:-
$\frac{0}{0}$ form, $\infty - \infty$ form, $0(\infty)$ form, 1^∞ form,

$\frac{\infty}{\infty}$ form, e.t.c.

= solution,

$$\lim_{x \to 0^+} x^x$$

$$= \lim_{x \to 0^+} e^{x \ln x}$$

$$= \exp\left(\lim_{x \to 0^+} \frac{\ln x}{(1/x)}\right) \quad \left[\because \frac{\infty}{\infty} \text{ form}\right]$$

$$= \exp\left(\lim_{x \to 0^+} \frac{1/x}{-x^{-2}}\right)$$

$= \exp\left(\lim_{x \to 0^+} -x \right)$

$= e^{-0}$

$= \underline{1}$ Ans

(Q.N.7)

solution,

P(1,3,2), Q(3,−1,6) and R(5,2,0) are the points on a plane. Then,

$\overrightarrow{PQ} = (2,-4,4) = (2\vec{i} - 4\vec{j} + 4\vec{k})$

$\overrightarrow{QR} = (2,3,-6) = (2\vec{i} + 3\vec{j} - 6\vec{k})$

Now,

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -4 & 4 \\ 2 & 3 & -6 \end{vmatrix}$$

$$= (12\vec{i} + 20\vec{j} + 14\vec{k})$$

This is a vector perpendicular to the plane of both $\overrightarrow{PQ}$ & $\overrightarrow{QR}$. So, The equation of plane is:-

$$12x + 20y + 14z + D = 0 \quad \text{——— (1)}$$

since P(1,3,2) is a point on the same plane;

$$12 \times 1 + 20 \times 3 + 14 \times 2 + D = 0$$

or, $12 + 60 + 28 + D = 0$

$\Rightarrow \quad D = (-100)$

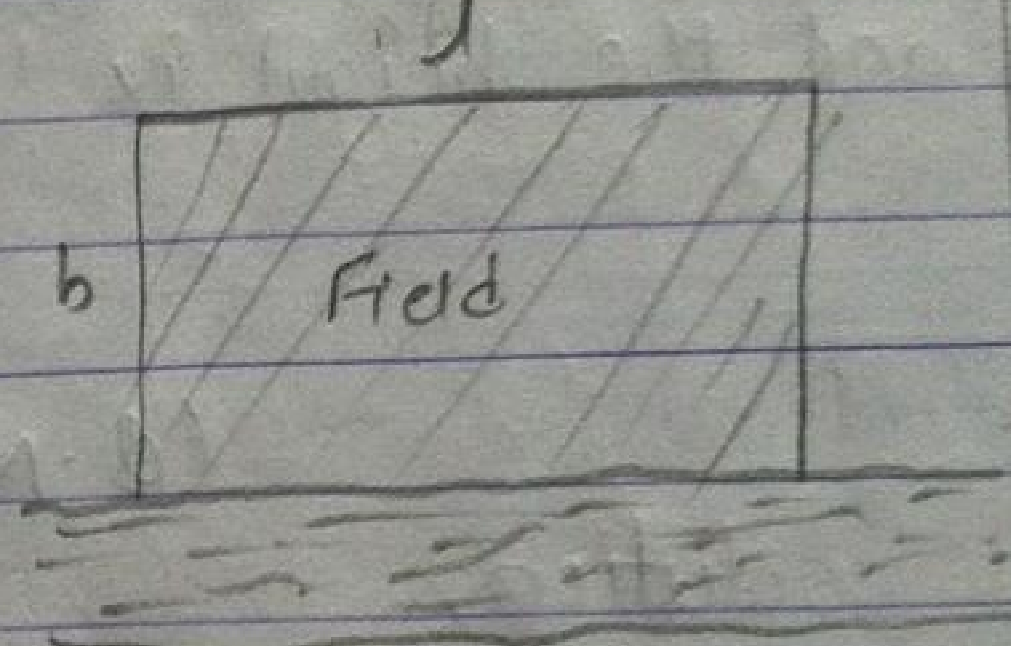
So, The equation of plane :-

$$12x + 20y + 14z - 100 = 0$$

or, $6x + 10y + 7z - 50 = 0$; which is the required equation.

(Q.N.8)

= solution,



Consider l and b as the length and the breadth of the rectangular field.

Then,

length of fencing = 2400 ft

$\Rightarrow \quad (2b + l) = 2400$

$\Rightarrow \quad \boxed{l = (2400 - 2b)}$

Now;

$$\text{Area } (A) = l \times b$$

or, $A = (2400 - 2b)\, b$

or, $A(b) = b(2400 - 2b)$

And,

$$A'(b) = (2400 - 4b)$$

For maximum area; $A'(b) = 0$

$\Rightarrow 2400 - 4b = 0$

$\Rightarrow b = 600 \text{ ft}$

$\Rightarrow l = 1200 \text{ ft}$

Also, maxima exists at the end points as well. Here end points are ~~at b>0 and b<1200~~ of breadth are (0, 1200). At $b = 0$;

$$Area = 0$$

At $b = 1200$; $Area = 0$

So,

At $b = 600$;

$$Area = 7,20,000 \ ft^2$$

$\therefore$ The area is maximum when the breadth is 600 ft and the height is 1200 ft. Ans

---

(Q.N.9)

solution,

here,

$$I = \int_1^{\infty} \frac{1}{x^p} \cdot dx$$

Firstly, Consider $p = 1$. Then;

$$I = \lim_{t \to \infty} \int_1^t \frac{1}{x} \cdot dx$$

$$= \lim_{t \to \infty} \left[\ln x \right]_1^t$$

$$= \lim_{t \to \infty} \left[\ln t - \overset{0}{\ln 1} \right]$$

$$= \lim_{t \to \infty} \left[\ln t \right]$$

$$= \infty \ (\text{Divergent}).$$

When $p \neq 1$;

$$I = \lim_{t \to \infty} \int_1^t \frac{1}{x^p} \cdot dx$$

$$= \lim_{t \to \infty} \int_1^t x^{-p} \cdot dx$$

$$= \lim_{t \to \infty} \left[\frac{x^{1-p}}{1-p} \right]_1^t$$

$$= \lim_{t \to \infty} \left[\frac{1}{(1-p) \cdot x^{p-1}} \right]_1^t$$

$$= \lim_{t \to \infty} \left[\frac{1}{(1-p)\, t^{p-1}} - \frac{1}{(1-p)} \right]$$

$$= \frac{1}{(1-p)} \cdot \lim_{t \to \infty} \left[\frac{1}{t^{p-1}} - 1 \right]$$

as $p > 1$; $(p-1) > 0$ and;

$$I = \frac{1}{(1-p)} \left(\frac{1}{\infty} - 1 \right) = \frac{1}{(p-1)}$$

(Converges)

But; if $p < 1$; $(p-1) < 0$ and

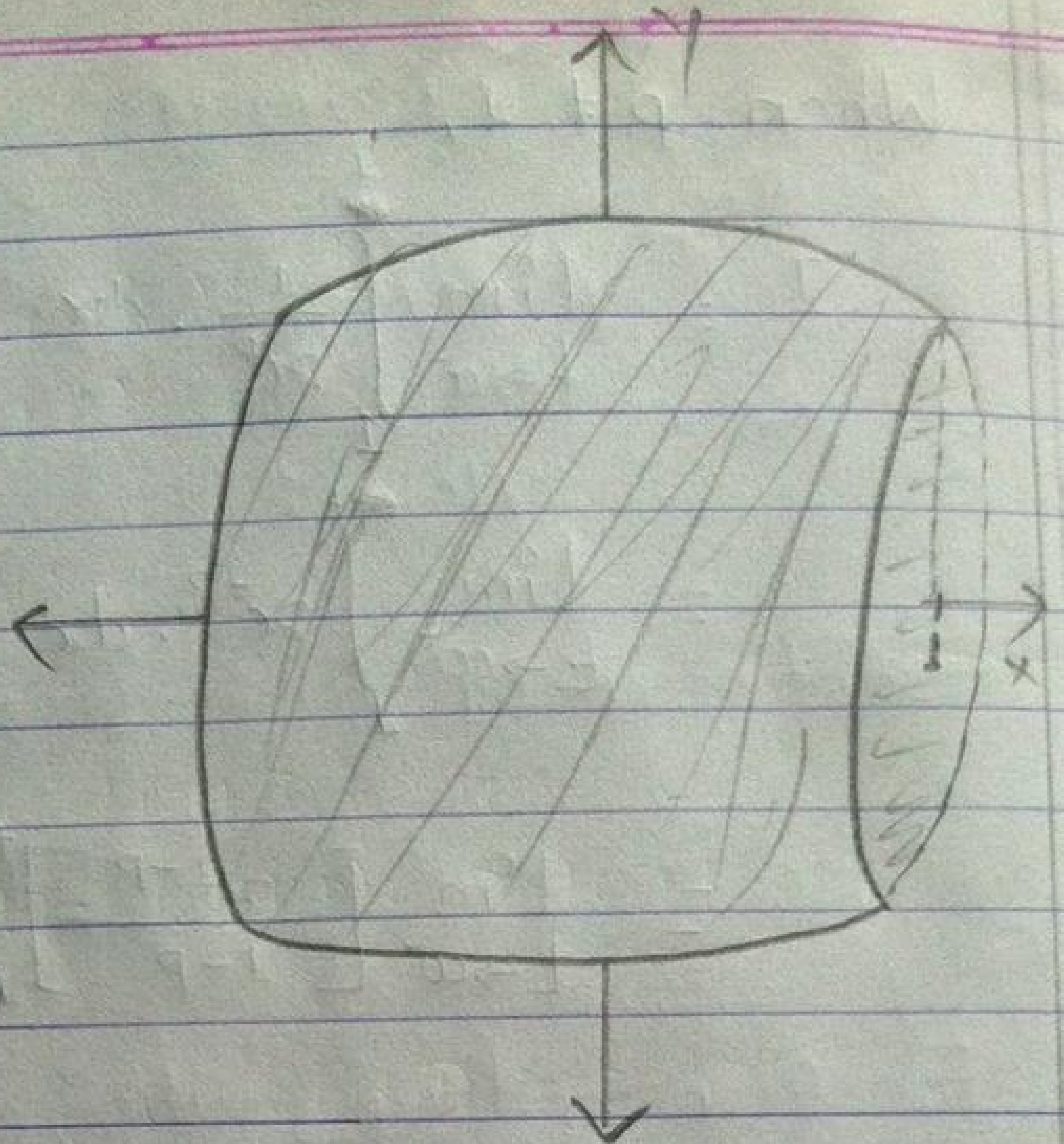
$$I = \frac{1}{(1-p)} \left(\infty - 1 \right) = \infty \text{ (Diverges)}$$

$\therefore$ The given series converges when $p > 1$.

(Q.N. 10)

= solution.

The rotating cylinder is as shown in the figure alongside. Here,



$y=\sqrt{4-x^2}$ ——— ①

$-1\le x\le 1$

so,

$$\frac{dy}{dx}=\frac{1}{2\sqrt{4-x^2}}\cdot(-2x)$$

$$\therefore \frac{dy}{dx}=\left(\frac{-x}{\sqrt{4-x^2}}\right)$$

so,

The surface area is :- $S=\int_{-1}^{1}2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

$$=2\pi\int_{-1}^{1}\sqrt{4-x^2}\cdot\sqrt{1+\frac{x^2}{4-x^2}}\cdot dx$$

$$=2\pi\int_{-1}^{1}\cancel{(\sqrt{4-x^2})}\cdot\frac{2}{\cancel{\sqrt{4-x^2}}}\cdot dx$$

$$=4\pi\left[x\right]_{-1}^{1}$$

$$=4\pi\left[1+1\right]$$

$$=8\pi \text{ sq. units}.$$

(Q.N. 11)

solution,

$$y'' + y = e^x + x^3 \quad \text{——— (1)}$$

$$y'(0) = 0$$

$$y(0) = 2$$

Solution of eqn (1) :- Complementary Function + Particular Indicator

$$= (y_c + y_p)$$

For complementary function, auxiliary equation ⇒

$$m^2 + 1 = 0$$

or, $m^2 = i^2$

⇒ $m = \pm i$

∴ Complementary Function $(y_c) = e^0 (c_1 \cos x + c_2 \sin x)$

$$= (c_1 \cos x + c_2 \sin x)$$

For particular indicators; we have:

$$y_p = G_1(x) + G_2(x) = (e^x + x^3)$$

let $y_{p1} = ke^x$

∴ $y' = ke^x$

$y'' = ke^x$

∴ eqn can be used as :-

$$ke^x + ke^x = e^x$$

or, $2ke^x = e^x$

⇒ $k = (1/2)$

$\therefore y_{p1} = \dfrac{e^x}{2}$

let $y_{p2} = (Ax^3 + Bx^2 + Cx + D)$

$\therefore y' = (3Ax^2 + 2Bx + C)$

$\therefore y'' = (6Ax + 2B)$

$\therefore$ eq^n can be used as :-

$$6Ax + 2B + Ax^3 + Bx^2 + Cx + D = x^3$$

or, $Ax^3 + Bx^2 + (6A + C)x + (2B + D) = x^3 + 0x^2 + 0x + 0$

This implies;

$A = 1$, $\quad B = 0$, $\quad 6A + C = 0$, $\quad 2B + D = 0$

$\Rightarrow C = -6A \qquad \Rightarrow D = -2B$

$\Rightarrow C = -6 \qquad \Rightarrow D = 0$

so,

$y_{p2} = (x^3 - 6x)$

$\therefore PI = \left(\dfrac{e^x}{2} + x^3 - 6x\right)$

So, The required solution is :-

$$y = (c_1 \cos x + c_2 \sin x) + \frac{e^x}{2} + x^3 - 6x \qquad \underline{\underline{\text{Ans}}}$$

(Q.N.12)

= solution,

Consider the given equation of plane as :-

$$4x + 5y - 2z = 18 \quad \text{--- (1)}$$

And,

$x = (2+3t)$, $y = -4t$, $z = (5+t)$ is another line.

When the line intersects the plane, eq^n (1) can be written as :-

$$4(2+3t) + 5(-4t) - 2(5+t) = 18$$

or, $8 + 12t - 20t - 10 - 2t = 18$

or, $-2 - 10t = 18$

or, $-10t = 20$

$\Rightarrow$ $t = -2$

At the point of intersection, $t = -2$. So the point of intersection is : $(2-6, 8, 3) = (-4, 8, 3)$.

<u>Ans</u>

(Q.N.13)

= solution,

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

$$= \sum_{n=1}^{\infty} \left[\frac{1}{n} - \frac{1}{(n+1)}\right]$$

so; $\sum_{i=1}^{n} = \left[\frac{1}{1} + \cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} + \dots \cancel{\frac{1}{n}}\right] - \left[\cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} + \dots \cancel{\frac{1}{n}} + \frac{1}{(n+1)}\right]$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n(n+1)} = \left[1-\frac{1}{n+1}\right]$$

$$= \lim_{n\to\infty}\left[1-\frac{1}{n+1}\right]$$

$\therefore S_n = 1$

So, the given series converges.

(Q. N. 14)

= solution,

Consider the given equation as:-

$$x^2+2y^2+z^2 = 16 \text{ ——— (1)}$$

$$R = [0,2]\times[0,2]$$

from (1);

$$z = (16-x^2-2y^2)$$

Now, According to fubini's theorem;

$$\text{Volume } (V) = \iint_R (16-x^2-2y^2)\cdot dA$$

$$= \int_0^2\int_0^2 (16-x^2-2y^2)\cdot dy\cdot dx$$

$$= \int_0^2 \left[16y - x^2y - \frac{2y^3}{3}\right]_0^2 dx$$

$$= \int_0^2 \left[32 - 2x^2 - \frac{16}{3}\right] dx$$

$$= \int_0^2 \left[\frac{80}{3} - 2x^2\right] . dx$$

$$= \left[\frac{80x}{3} - \frac{2x^3}{3}\right]_0^2$$

$$= \left[160/3 - 16/3\right]$$

$$= \frac{144}{3}$$

$= 48$ cubic units <u>Ans</u>

(Q.N. 15)

= solution,

normal vector $(\vec{v}) = (2\vec{i} + 3\vec{j} + 4\vec{k})$

A (2, 4, -1) is any point lying in the plane. The equation of plane with normal vector (2, 3, 4) is :-

$$2x + 3y + 4z + D = 0 \quad \text{——— (1)}$$

since the plane passes through (2, 4, -1), eqn (1) gives

$$4 + 12 - 4 + D = 0$$

$$\Rightarrow D = -12$$

$\therefore$ eqn (1) is :-

$$2x + 3y + 4z = 12$$

or, $$\frac{x}{6} + \frac{y}{4} + \frac{z}{3} = 1 \quad \text{———} \quad (\text{ii})$$

which is the required equation of plane.

From eqⁿ (ii);

x-intercept = 6

y-intercept = 4

z-intercept = 3.

The sketch of the plane is :-