

Tribhuvan University
Institute of Science and Technology
2070

Bachelor Level/ First Year/ First Semester/ Science
Computer Science and Information Technology (MTH:104)
(Calculus and analytical Geometry)
Old Course

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions.

Group A (10×2=20)

1. Define odd and even function, with example.
2. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^n 5}{4^n} = -\frac{5}{4} + \frac{5}{16} - \frac{5}{64} + \dots$ Converges to -1.
3. Test the convergence of the series $\frac{(2n!)}{n!n!}$.
4. Find the eccentricity of the curve $2x^2 + y^2 = 4$.
5. Find the angle between the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$
6. Find the velocity and acceleration of a particle whose position is
$$j(t) = (t + 1)\vec{i} + (t^2 - 1)\vec{j}, \text{ at } t = 1$$
7. Evaluate $\int_{\pi}^{2\pi} \int_{\infty}^{\pi} (\sin x + \cos y) dx dy$.
8. Find the Jacobian $j(u, v, w)$ if $x = u + v, y = 2u, z = 3w$.
9. Show that $y = x^2 + 5$ is the solution of $\frac{dy}{dx} = 2x$
10. Find $\frac{df}{dx}$ and $\frac{df}{dy}$ at (1,2) of $f(x, y) = x^2 + 2xy + 5$.

Group B (5×4=20)

11. State Rolles's theorem and verify it for the function $f(x) = \sin x$ in $[0, \pi]$.
12. Find the Taylors series and the Taylor polynomials generated by $f(x) = e^x$ at $x = 0$.
13. Find the length of the cardioids $r = 1 + \cos \theta$.
14. Find the gradient vector of $f(x, y)$ at a point $P(x_0, y_0)$. Find an equation for the tangent to the ellipse $x^2 + 4y^2 = 4$ at point $(-2, 1)$
15. Find the general solution of $y^2 z \frac{dz}{dx} - x^2 z \frac{dz}{dy} = xy^2$

Group C (5×8=40)

16. Find the area of the region bounded by $x = 2y^2, x = 0$ and $y = 3$.

Or

Investigates the convergence of the integrals

$$(a) \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

$$(b) \int_0^2 \frac{dx}{1-x}$$

17. Find the torsion, normal and curvature for the space curve

$$\vec{r}(t) = (2 \cos t)\vec{i} + (3 \sin t)\vec{j} + t\vec{k}$$

18. Evaluate $\int_1^1 \int_0^{\sqrt{1-x^2}} dy dx$.

19. Find the local maximum, minimum and saddle point of $6x^2 - 2x^3 + 3y^2 + 6xy$.

OR

Find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse

$$\frac{x^2}{8} - \frac{y^2}{2} = 1.$$

20. Define the wave equation by the modeling of vibrating string.