

Tribhuvan University
Institute of Science and Technology
2072

Bachelor Level/ First Year/ First Semester/ Science
Computer Science and Information Technology (MTH:104)
(Calculus and analytical Geometry)
Old Course

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions.

Group A (10×2=20)

1. If $f(x) = (x - 1) + x$, then prove that $f(x) \cdot f(1 - x) = 1$
2. Define critical point. Find the critical point of $f(x) = x^2$.
3. Evaluate: $\lim_{n \rightarrow \infty} \frac{3 - 5n^6}{n^6 - 3}$.
4. Find the equation of the parabola with vertex at the origin and directrix at $y = 2$
5. Find the angle between the planes $x - 2y - 2z = 5$ and $5x - 2y - z = 0$
6. Evaluate $\int_0^3 \int_0^2 (4 - y^2) dx dy$.
7. Find $\frac{dt}{dx}$ and $\frac{dt}{dy}$ if $f(x, y) = ye^2$.
8. Find the equation for the tangent plane to the surfaces $Z = f(x, y) = g - x^2 - y^2$ at the point $(1, 2, 3)$.
9. Show that $y = c_1 x e^{-2x} + c_2 e^{-2x}$ is the solution of $y'' + y' - 2y = 0$.
10. Solve $\frac{d^2 y}{dx^2} + \frac{dy}{dzx} = 0$.

Group B (5×4=20)

11. Verify Rolle's theorem for $f(x) = x^2, x \in [-1, 1]$.
12. Find the Taylor's series expression of $f(x) = \cos \theta$ at $x = 1$.
13. Find the Cartesian equation of the polar equation $r \cos \left(\theta - \frac{\pi}{3} \right) = 3$
14. Show that the function $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ is continuous at every point except the origin.
15. Solve $xz \frac{dz}{dx} + yz \frac{dz}{dy} = xy$

Group C (5×8=40)

- 16.** Find the area bounded on right by the line $y=x-2$ on the left by the parabola $x=y^2$ and below by the x-axis

Or

What is an improper integral? Evaluate

(a). $\int_2^{\infty} \frac{dx}{\sqrt{x-1}}$

(b) $\int_2^{\infty} \frac{dx}{(x-1)^2}$

- 17.** Define curvature of a curve .find that the curvature of a helix

$$\vec{R}(t) = (a \cos wt)\vec{i} + (a \sin wt)\vec{j} + (bt)\vec{k}$$

- 18.** Find the area enclosed by $r^2 = 2a^2 \cos 2\theta$

- 19.** Find the extreme values of $Z = x^3 - y^3 - 2xy + 6$.

OR

Find the extreme value of function $F(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$

- 20.** Define initial boundary values problems .Derive the heat equation or wave equation in one dimension