Exercise 5.1

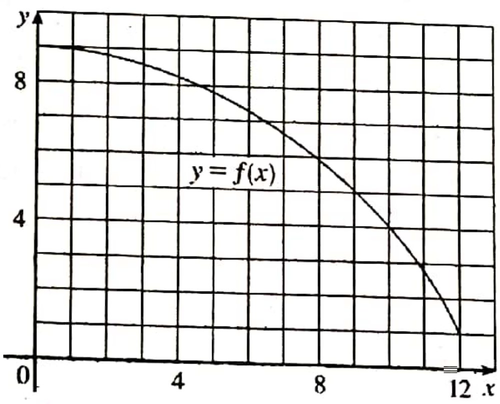
1. A particle is moving with the given data. Find the position of the particle.
   a. $V(t) = \sin t - \cos t$, $s(0) = 0$
   b. $a(t) = 10\sin t + 3\cos t$, $s(0) = 0$, $s(2\pi) = 12$
   c. $a(t) = 6t + 4$, $v(0) = -6$ cm/s, $s(0) = 9$ cm
   d. $a(t) = 12t^2 + 6t - 4$, $s(0) = 4$, and $s(1) = 1$.
2. A stone is dropped from the upper observation deck (the space deck) of the CN Tower, 450 m above the ground.
   a. Find the distance of the stone above ground level at time t.
   b. How long does it take the stone to reach the ground?
   c. With what velocity does it strike the ground?
   d. If the stone is thrown downward with a speed of 5 m/s, how long does it take to reach the ground?
3. (a) Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 12$.

   (i) L_6 (ii) R_6 (iii) M_6

   (b) Is L_6 an underestimate or overestimate of the true of area?

   (c) Is R_6 and underestimate or overestimate of the true area?

   (d) Which of the numbers L_6, R_6 or M_6 gives the best estimate? Explain.



4. (a) Estimate the area under the graph of $f(x) = \cos x$ from $x = 0$ to $x = \pi/2$ using four approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

   (b) Repeat part (a) using left endpoints.
5. (a) Estimate the area under the graph of $f(x) = \sqrt{x}$ from $x = 0$ to $x = 4$ using four approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

   (b) Repeat part (a) using left endpoints.

6. (a) Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints. Then improve your estimate by using six rectangles. Sketch the curve and the approximating rectangles.

(b) Repeat part (a) using left endpoints.

(c) Repeat part (a) using midpoints.

(d) From your sketches in parts (a) - (c), which appears to be the best estimate?

7. Evaluate the upper and lower sums for $f(x) = 1 + x^2$, $-1 \leq x \leq 1$, with $n = 3$ and 4. Illustrate with diagrams.

8. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

t(s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v(ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

9. Speedometer readings for a motorcycle at 12 second intervals are given in the table.

(a) Estimate the distance traveled by the motorcycle during this time period using the velocities at the beginning of the time intervals.

(b) Give another estimate using the velocities at the end of the time periods.

(c) Are your estimates in parts (a) and (b) upper and lower estimates? Explain.

t(s)	0	12	24	36	48	60
v(ft/s)	30	28	25	22	24	27

10. Oil leaked from a tank at a rate of r(t) liters per hour. The rate decreased as time passed and values of the rate at two hour time intervals are shown in the table. Find lower and upper estimates for the total amount of oil that leaked out.

t(h)	0	2	4	6	8	10
r(t) (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

11. Let A be the area of the region that lies under the following graphs of f(x) then, find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

(a) $f(x) = \frac{2x}{x^2+1}$, $1 \leq x \leq 3$

(b) $f(x) = x^2 + \sqrt{1+2x}$, $4 \leq x \leq 7$

(c) $f(x) = \sqrt{\sin x}$, $0 \leq x \leq \pi$

12. Determine a region whose area is equal to the given limit. Do not evaluate the limit.

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5 + \frac{2i}{n}\right)^{10}$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan\frac{i\pi}{4n}$

Answers:

1. (a) $1 - \cos t - \sin t$ (b) $-10\sin t - 3\cos t + \frac{6t}{\pi} + 3$

   (c) $t^3 + 2t^2 - 6t + 9$, (d) $t^4 + t^3 - 2t^2 - 3t + 4$
2. (a) $s(t) = 450 - 4.9t^2$, (b) 9.58s, (c) – 93.9 m/s, (d) 9.09 s
3. (a) (i) 86.6 (ii) 70.6 (iii) 79.2 (b) Overestimate (c) Underestimate d) M_6
4. (a) 0.79, underestimate (b) 1.18, overestimate
5. (a) $3 + \sqrt{2} + \sqrt{3}$ (b) $1 + \sqrt{2} + \sqrt{3}$
6. (a) 8, 6.875 (b) 5, 5.375 (c) 5.75, 5.9375 (d) C
7. Lower limit = 2.148 Upper limit = 3.407; Lower limit = 2.25 Upper limit = 3.25
8. 34.7 ft, 44.8 ft
9. (a) 1548 ft (b) 1512 ft (c) neither
10. Lower limit = 63.2, Upper limit = 70 l
11. (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{2\left(1+\frac{2i}{n}\right)}{\left(1+\frac{2i}{n}\right)^2+1}\right] \times \frac{2}{n}$ (b) $\lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\left(4+\frac{3i}{n}\right)^2 + \sqrt{9+\frac{6i}{n}}\right]$

    (c) $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\sin\frac{\pi i}{n}} \times \frac{\pi}{n}$
12. (a) $y = x^{10}$ for $5 \le x \le 7$ (b) $y = \tan x$ for $0 \le x \le \frac{\pi}{4}$

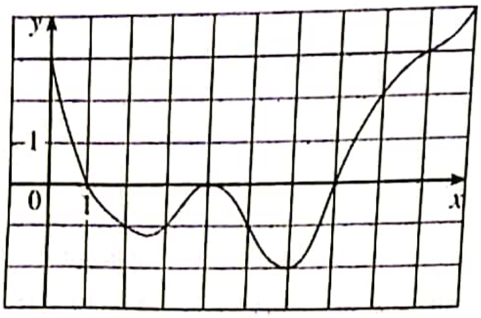
and the approximation given by the Midpoint
of the areas of the rectangles shown in figure.

Exercise 5.2

1. If $f(x) = x^2 - 2x$, $0 \le x \le 3$, evaluate the Riemann sum with $n = 6$, taking the sample points to be right endpoints. What does the Riemann sum represent?

2. If $f(x) = e^x - 2$, $0 \le x \le 2$, find the Riemann sum with $n = 4$ correct to six decimal places, taking the sample points to be midpoints. What does the Riemann sum represent?

3. The graph of a function f is given. Estimate $\int_0^{10} f(x)$ using five subintervals with (a) right endpoints, (b) left endpoints, and (c) midpoints.



4. A table of values of an increasing function f is shown. Use the table to find lower and upper estimates for $\int_{10}^{30} f(x)\,dx$.

x	10	14	18	22	26	30
f(x)	–12	–6	–2	1	3	8

5. Use the Midpoint Rule with the given value of to approximate the integral.

(a) $\int_0^8 \sin\sqrt{x}\,dx,\ n = 4$

(b) $\int_0^{\pi/2} \cos^4 x\,dx,\ n = 4$

(c) $\int_0^2 \frac{x}{x+1}\,dx,\ n = 5$

(d) $\int_1^5 x^2 e^{-x}\,dx,\ n = 4$

6. Express the limit as a definite integral on the given interval.

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \ln(1 + x_i^2)\,\Delta x,\ [2, 6]$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i}\,\Delta x,\ [\pi, 2\pi]$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} [5(x_i^*)^3 - 4x_i^*]\,\Delta x,\ [2, 7]$

(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{x_i^*}{(x_i^*)^2 + 4}\,\Delta x,\ [1, 3]$

7. Use the form of the definition of the integral. Evaluate the integral.

(a) $\int_2^5 (4 - 2x)\,dx$

(b) $\int_1^4 (x^2 - 4x + 2)\,dx$

(c) $\int_{-2}^0 (x^2 + x)\,dx$

(d) $\int_0^2 (2x - x^3)\,dx$

(e) $\int_0^1 (x^3 - 3x^2)\,dx$

8. Prove that

(a) $\int_a^b x\,dx = \frac{b^2 - a^2}{2}$

(b) $\int_a^b x^2\,dx = \frac{b^3 - a^3}{3}$

9. Evaluate the integral by interpreting it in terms of areas.

(a) $\int_{-1}^2 (1 - x)\,dx$

(b) $\int_0^9 \left(\frac{1}{3}x - 2\right)dx$

(c) $\int_{-3}^0 \left(1 + \sqrt{9 - x^2}\right)dx$

(d) $\int_{-5}^5 \left(x - \sqrt{25 - x^2}\right)dx$

(e) $\int_{-1}^2 |x|\,dx$

(f) $\int_0^{10} |x - 5|\,dx$

Answers:

1. 0.87

2. 2.32

3. (a) 6 (b) 4 (c) 2

4. $-64, 16$

5. (a) 6.18 (b) 0.589 (c) 0.9 (d) 1.6

6. (a) $\int_2^6 x \ln(1+x^2)\,dx$ (b) $\int_\pi^{2\pi} \frac{\cos x}{x}\,dx$ (c) $\int_2^7 (5x^3-4x)\,dx$ (d) $\int_1^3 \frac{x}{x^2+4}\,dx$

7. (a) -9 (b) -3 (c) $\frac{2}{3}$ (d) 0 (e) $\frac{-3}{4}$

9. (a) $\frac{3}{2}$ (b) $\frac{-9}{2}$ (c) 10.07 (d) $\frac{-25\pi}{2}$ (e) $\frac{5}{2}$ (f) 25

Exercise 5.3

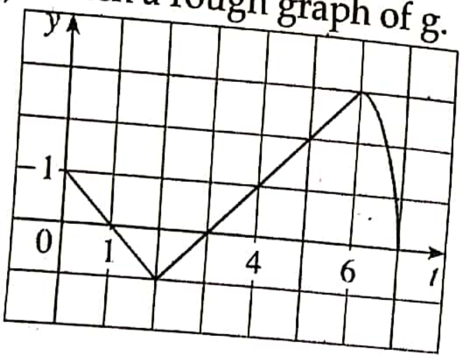
1. Let $\int_0^x f(t)\,dt$, where f is the function whose graph is shown in following figure.

(a) Evaluate g(x) for x = 0, 1, 2,3 ,4 , 5, and 6.

(b) Estimate g(7)

(c) Where does g have a maximum value and minimum value?

(d) Sketch a rough graph of g.



2. Use part 1 of the fundamental theorem of calculus to find the derivative of the function

(a) $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$

(b) $g(x) = \int_3^x e^{t^2-t}\,dt$

(c) $g(s) = \int_5^s (t-t^2)^8\,dt$

(d) $g(r) = \int_0^r \sqrt{x^2+4}\,dx$

(e) $F(x) = \int_x^{\pi} \sqrt{1+\sec t}\,dt$

(f) $G(x) = \int_x^1 \cos\sqrt{t}\,dt$

(g) $h(x) = \int_1^{e^x} \ln t\,dt$

(h) $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz$

(i) $y = \int_1^{x^4} \cos^2\theta\,d\theta$

(j) $y = \int_{\sin x}^{1} \sqrt{1+t^2}\,dt$

(k) $g(x) = \int_{1-2x}^{1+2x} t\sin t\,dt$

(*l*) $y = \int_{\cos x}^{\sin x} \ln(1+2v)\,dv$

3. Evaluate as a Riemann sum: $\lim\limits_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\sqrt{\frac{3}{n}}+\ldots+\sqrt{\frac{n}{n}}\right)$

Answers:

1. (a) $\frac{1}{2}, 0, -\frac{1}{2}, 0, \frac{3}{2}, 4$ (b) 6.2 (c) maximum at $t = 7$, minimum at $t = 3$

2. (a) $\frac{1}{x^3+1}$ (b) $e^{x^2}-x$ (c) $(s-s^2)^8$ (d) $\sqrt{r^2+4}$ (e) $-\sqrt{1+\sec x}$ (f) $-\cos\sqrt{x}$ (g) $x\,e^x$

   (h) $\frac{\sqrt{x}}{2(x^2+1)}$ (i) $4x^3(\cos x^4)^2$ (j) $-\cos x\sqrt{1+\sin^2 x}$

   (k) $(2-4x)\sin(1-2x)+(2+4x)\sin(1+2x)$ (*l*) $\sin x\ln(1+2\cos x)+\cos x\ln(1+2\sin x)$

3. $\frac{2}{3}$

Exercise 5.4

1. Evaluate the integral

(a) $\int (x^2 + x^{-2})\, dx$

(b) $\int \left(\sqrt{x^3} + \sqrt[3]{x^2}\right) dx$

(c) $\int \left(x^2 + 1 + \frac{1}{x^2 + 1}\right) dx$

(d) $\int (\sin x + \sinh x)\, dx$

(e) $\int (1 + \tan^2\alpha)\, d\alpha$

(f) $\int \frac{\sin 2x}{\sin x}\, dx$

(g) $\int_0^1 (x^{10} + 10^x)\, dx$

(h) $\int_0^{\pi/4} \frac{1 + \cos^2\theta}{\cos^2\theta}\, dx$

(i) $\int_0^{\pi/3} \frac{\sin\theta + \sin\theta \tan^2\theta}{\sec^2\theta}\, d\theta$

(j) $\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x}\, dx$

(k) $\int_1^2 \frac{(x-1)^3}{x^2}\, dx$

(l) $\int_0^2 |2x - 1|\, dx$

(m) $\int_{-1}^2 (x - 2|x|)\, dx$

2. Use a graph to estimate the x-intercepts of the curve $y = 1 - 2x - 5x^4$. Then use this information to estimate the area of the region that lies under the curve and above the x-axis.

3. The area of the region that lies to the right of the y-axis and to the left of the parabola $x = 2y - y^2$.

4. Find the area of region bounded by y-axis, $y = \sqrt[4]{x}$ and $y = 1$.

5. The velocity function (in meters per second) is given for a particle moving along a line. Find (1) the displacement and (2) the distance traveled by the particle during the given time interval.

(a) $v(t) = 3t - 5,\ 0 \le t \le 3$

(b) $v(t) = t^2 - 2t - 8,\ 1 \le t \le 6$

6. The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find (1) the velocity at time t and (2) the distance traveled during the given time interval.

(a) $a(t) = t + 4$, $v(0) = 5$, $0 \le t \le 10$ (b) $a(t) = 2t + 3$, $v(0) = -4$, $0 \le t \le 3$

7. The linear density of a rod of length 4 m is given by $\rho(x) = 9 + 2\sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.

8. The marginal cost of manufacturing x yards of a certain fabric is $C'(x) = 3 - 0.01x + 0.000006x^2$ (in dollars per yard). Find the increase in cost if the production level is raised from 2000 yards to 4000 yards.

9. A bacteria population is 4000 at time $t = 0$ and its rate of growth is 1000×2^t bacteria per hour after t hours. What is the population after one hour?

Answers:

1. (a) $\frac{x^3}{3} - \frac{1}{x} + C$ (b) $\frac{2}{5}x^{5/2} + \frac{3}{5}x^{5/3} + C$ (c) $\frac{x^3}{3} + x + \tan^{-1}x + C$ (d) $-\cos x + \cosh x + C$

(e) $\tan\alpha + C$ (f) $2\sin x + C$ (g) $\frac{1}{11} + \frac{9}{\ln 10}$ (h) $1 + \frac{\pi}{4}$ (i) $\frac{1}{2}$ (j) 40 (k) $-2 + 3\ln 2$

(l) $\frac{5}{2}$ (m) $\frac{-7}{2}$

2. 1.36
3. 4/3
4. 1/5
5. (a) (1) -1.5 (2) $\frac{41}{6}$ (b) (1) $-10/3$ (2) 98/3
6. (a) (1) $\frac{t^2}{2} + 4t + 5$ (2) $\frac{1250}{3}$ (b)(1) $t^2 + 3t - 4$ (2) 89/6
7. 140/3
8. 58000
9. 5442.7

more complicated function.

Some Formulas and Rules of Integrals

1. $\int c\,f(x)\,dx = c\int f(x)\,dx$

2. $\int k\,dx = kx + c$

3. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \; (n \neq 1)$

4. $\int e^x\,dx = e^x + C$

5. $\int \frac{1}{x}\,dx = \ln|x| + C$

6. $\int a^x\,dx = \frac{a^x}{\ln a} + C$

7. $\int \sin x\,dx = -\cos x + C$

8. $\int \cos x\,dx = \sin x + C$

9. $\int \sec^2 x\,dx = \tan x + C$

10. $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$

11. $\int \sec x \tan x\,dx = \sec x + C$

12. $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x$

13. $\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$

14. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

15. $\int \sinh x\,dx = \cosh x + C$

16. $\int \cosh x\,dx = \sinh x + C$

Note: $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$

Algebraic Substitution Rule

Exercise 5.5

1. Evaluate:

(a) $\int \ln \sqrt[3]{x}\, dx$

(b) $\int \sin^{-1}x\, dx$

(c) $\int s2^s\, ds$

(d) $\int z^3 e^z\, dz$

(e) $\int x \tan^2 x\, dx$

(f) $\int (\sin^{-1}x)^2\, dx$

(g) $\int_0^{2\pi} t^2 \sin 2t\, dt$

(h) $\int_1^2 x^4 (\ln x)^2\, dx$

2. Evaluate:

(a) $\int \sin^2 x \cos^3 x\, dx$

(b) $\int_0^{\pi} \sin^2 t \cos^4 t\, dt$

(c) $\int \cos\theta \cos^5(\sin\theta)\, d\theta$

(d) $\int \cos^2 x \tan^3 x\, dx$

(e) $\int \tan x \sec^3 x\, dx$

(f) $\int \tan^2\theta \sec^4\theta\, d\theta$

(g) $\int \tan^4 x \sec^6 x\, dx$

(h) $\int_0^{\pi/4} \tan^4 t\, dt$

(i) $\int \sin 8x \cos 5x\, dx$

(j) $\int \frac{\cos x + \sin x}{\sin 2x}\, dx$

(k) $\int_0^1 x \tan^2 x\, dx$

3. Evaluate:

(a) $\int \frac{dx}{x^2\sqrt{4 - x^2}}$

(b) $\int \frac{x^3}{\sqrt{x^2 + 4}}\, dx$

(c) $\int \frac{\sqrt{x^2 - 4}}{x}\, dx$

(d) $\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2 - 1}}\, dt$

(e) $\int \frac{t^5}{\sqrt{t^2 + 2}}\, dt$

(f) $\int_0^{0.6} \frac{x^2}{\sqrt{9 - 25x^2}}\, dx$

(g) $\int \sqrt{5 + 4x - x^2}\, dx$

(h) $\int \frac{x}{\sqrt{x^2 + x + 1}}$

4. Evaluate:

(a) $\int \frac{5x + 1}{(2x + 1)(x - 1)}\, dx$

(b) $\int_0^1 \frac{2}{2x^2 + 3x + 1}\, dx$

(c) $\int \frac{ax}{x^2 - bx}\, dx$

(d) $\int \frac{x^2 + 1}{(x - 3)(x - 2)^2}\, dx$

(e) $\int \frac{x^2 - 5x + 16}{(2x + 1)(x - 2)^2}\, dx$

(f) $\int \frac{x^2 - 2x - 1}{(x - 1)^2 (x^2 + 1)}\, dx$

(g) $\int_0^1 \frac{x^3+2x}{x^4+4x^2+3}\,dx$

Answers

1. (a) $x \ln \sqrt[3]{x} - \frac{x}{3} + C$

   (b) $x \sin^{-1}x + \sqrt{1-x^2} + C$

   (c) $\frac{s\,2^s}{\ln 2} - \frac{2^s}{(\ln 2)^2} + C$

   (d) $z^3 e^z - 3z^2 e^z + 6ze^z - 6e^z + C$

   (e) $x \tan x + \ln|\cos x| - \frac{x^2}{2} + C$

   (f) $x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}\sin^{-1}x - 2x + C$

   (g) $-2\pi^2$

   (h) $\frac{32}{5}(\ln 2)^2 - \frac{64}{25}\ln 2 + \frac{62}{125}$

2. (a) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

   (b) $\frac{\pi}{16}$

   (c) $\sin(\sin\theta) - \frac{2\sin^3(\sin\theta)}{3} + \frac{\sin^5(\sin\theta)}{5} + C$

   (d) $\frac{\cos^2 x}{2} - \ln|\cos x| + C$

   (e) $\frac{\sec^3 x}{3} + C$

   (f) $\frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} + C$

   (g) $\frac{\tan^5 x}{5} + \frac{2\tan^7 x}{7} + \frac{\tan^9 x}{9} + C$

   (h) $\frac{\pi}{4} - \frac{2}{3}$

   (i) $\frac{1}{2}\left[-\frac{\cos 3x}{3} - \frac{\cos 13x}{13}\right] + C$

   (j) $\frac{1}{2}\ln|\operatorname{cosec}x + \cot x| + \frac{1}{2}\ln|\sec x + \tan x| + C$

   (k) $x\tan x - \ln|\sec x| - \frac{x^2}{2} + C$

3. (a) $\frac{-\sqrt{4-x^2}}{4x} + C$

   (b) $\frac{(\sqrt{4+x^2})^3}{3} + 4\sqrt{1+x^2} + C$

   (c) $\sqrt{x^2-4} - 2\sec^{-1}\frac{x}{2} + C$

   (d) $\frac{1}{2}\left(\frac{\pi}{2} + \frac{\sqrt{13}}{4} - \frac{1}{2}\right)$

   (e) $\frac{\sqrt{t^2+2}}{15}(3t^4 - 8t^2 + 32) + C$

   (f) $\frac{9\pi}{500}$

   (g) $\frac{9}{2}\sin^{-1}\left(\frac{x-2}{3}\right) + \frac{x-2}{2}\sqrt{5+4x-x^2} + C$

   (h) $\sqrt{x^2+x+1} - \frac{1}{2}\ln\left|\sqrt{x^2+x+1} - \left(x+\frac{1}{2}\right)\right| + C$

4. (a) $\frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C$

   (b) $\ln\frac{9}{4}$

   (c) $a\ln|x-b| + C$

   (d) $10\ln|x-3| - 9\ln|x-2| + \frac{5}{x-2} + C$

   (e) $\frac{3}{2}\ln|2x+1| - \ln|x-2| - \frac{2}{x-2} + C$

   (f) $\ln|x-1| + \frac{1}{x-1} - \frac{1}{2}\ln(x^2+1) + \tan^{-1}x + C$

   (g) $\frac{1}{4}\ln\frac{8}{3}$

Exercise 5.6

1. Determine whether each integral is convergent or divergent. Evaluate for convergent.

(a) $\int_{3}^{\infty} \frac{1}{(x-2)^{3/2}} dx$ (b) $\int_{-\infty}^{0} \frac{1}{3-4x} dx$ (c) $\int_{2}^{\infty} e^{-5p} dp$

(d) $\int_{0}^{\infty} \frac{x^2}{\sqrt{1+x^3}} dx$ (e) $\int_{-\infty}^{\infty} xe^{-x^2} dx$ (f) $\int_{0}^{\infty} \sin^2\alpha \, d\alpha$

(g) $\int_{2}^{\infty} \frac{dv}{v^2+2v-3}$ (h) $\int_{1}^{\infty} \frac{\ln x}{x} dx$ (i) $\int_{-\infty}^{\infty} \frac{x^2}{9+x^6} dx$

(j) $\int_{0}^{\infty} \frac{x \arctan x}{(1+x^2)^2} dx$ (k) $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}}$ (*l*) $\int_{-2}^{3} \frac{1}{x^4} dx$

(m) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$ (n) $\int_{0}^{9} \frac{1}{\sqrt[3]{x-1}} dx$ (o) $\int_{\pi/2}^{\pi} \operatorname{cosec} x \, dx$

(p) $\int_{-1}^{0} \frac{e^{1/x}}{x^3} dx$ (q) $\int_{0}^{1} \ln x \, dx$

2. Use the Comparison Theorem to determine whether the integral is convergent or divergent.

(a) $\int_{0}^{\infty} \frac{x}{x^3+1} dx$ (b) $\int_{1}^{\infty} \frac{2+e^{-x}}{x} dx$ (c) $\int_{1}^{\infty} \frac{x+1}{\sqrt{x^4-x}} dx$

(d) $\int_{0}^{\infty} \frac{\tan^{-1}x}{2+e^x} dx$ (e) $\int_{0}^{1} \frac{\sec^2 x}{x\sqrt{x}} dx$ (f) $\int_{0}^{\pi} \frac{\sin^2 x}{\sqrt{x}} dx$

Answers

1. (a) converges 2 (b) diverges (c) converges $\frac{e^{-10}}{5}$ (d) diverges (e) converges 0 (f) diverges (g) converges $\frac{\ln 5}{4}$ (h) diverges (i) converges $\frac{\pi}{9}$ (j) converges $\frac{\pi}{8}$ (k) converges $\frac{32}{3}$ (*l*) diverges (m) converges $\frac{\pi}{2}$ (n) converges $\frac{9}{2}$ (o) diverges (p) converges $\frac{-2}{e}$ (q) converges -1

2. (a) converges (b) diverges (c) converges (d) converges (e) diverges (f) converges

Exercise 6.1

1. Find the area of the region bounded above by $y = e^x$, bounded below by $y = x$, and bounded on the sides by $x = 0$ and $x = 1$.
2. Find the area of the region enclosed by $x + y^2 = 0$ and $x + 3y^2 = 2$.
3. Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \pi/2$.
4. Find the area between two curves $y = \sec^2 x$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{4}$.
5. Find the area between two curves $x = \tan^2 y$ and $x = -\tan^2 y$, $\frac{-\pi}{4} \le y \le \frac{\pi}{4}$.
6. Find the area between two curves $y = \sec^2 x$ and $y = \tan^2 x$, $\frac{-\pi}{4} \le x \le \frac{\pi}{4}$.
7. Find the area of region between the curve and x–axis
   (i) $f(x) = -x^2 - 2x$, $[-3, 2]$.   (ii) $f(x) = x^2 - 6x + 8$, $[0, 3]$.
   (iii) $y = x^3 - 4x$, $-2 \le x \le 2$.
8. Find the area of region enclosed by the parabola $y = 2 - x^2$ and line $y = -x$.
9. Find the area of the region enclosed by parabola $x = y^2$ and line $x = y + 2$ in first quadrant.
10. Find the area of the region enclosed by parabola $y^2 - 4x = 4$ and line $4x - y = 16$.
11. Find the area of the region bounded by curve $x = 2y^2$, $x = 0$ and $y = 3$.
12. Find the area bounded by x–axis and curve $y = 4 - x^2$.

Answers:

1. $e - 1.5$	2. $\frac{8}{3}$ sq. unit
3. $2\sqrt{2} - 2$	4. $\frac{1}{\sqrt{2}}$
5. $4 - \pi$	6. $\frac{\pi}{2}$
7. (i) $\frac{28}{3}$ (ii) $\frac{22}{3}$ (iii) 8	8. $\frac{9}{2}$
9. $\frac{10}{3}$	10. $\frac{243}{8}$
11. 18	12. $\frac{32}{3}$

Exercise 6.2

1. The region enclosed by the x-axis and the parabola $y = f(x) = 3x - x^2$ is revolved about the vertical line $x = -1$ to generate a solid. Find the volume of solid.
2. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.
3. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.
4. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.
5. Use cylindrical shells to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.
6. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.
7. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis.
   a. $xy = 1, x = 0, y = 1, y = 3$
   b. $y = x^3, y = 8, x = 0$
   c. $x = 1 + (y - 2)^2, x = 2$

Answers:

1. $\frac{45\pi}{2}$
2. $\frac{\pi}{6}$
3. $\frac{16}{5}\pi$
4. $\frac{\pi}{6}$
5. $\frac{\pi}{2}$
6. $\frac{\pi}{2}$
7. (a) 4π (b) $768\,\pi/7$ (c) $16\,\pi/3$

Exercise 6.4

1. Find the length of cardioids $r = 1 + \cos\theta$
2. Find the length of the graph of

$$f(x) = \frac{x^3}{12} + \frac{1}{x}, \quad 1 \leq x \leq 4.$$

3. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.
4. Find the arc length function for the curve in Example 2 taking $A = (1, 13/12)$ as the starting point.
5. Find the length of the arc of the semicubical parabola $y^2 = x^3$ between the points $(1, 1)$ and $(4, 8)$.
6. Find the arc length function for the curve $y = x^2 - \frac{1}{8}\ln x$ taking $P_0(1)$ as the starting point.

Answers:

1.	8	2.	6
3.	2.27	4.	6
5.	$\frac{1}{27}(80\sqrt{10} - 13\sqrt{13})$	6.	8.1373

Exercise 6.5

1. Find the exact area of the surface obtained by rotating the curve about the x-axis.
   a. $y = x^3, 0 \leq x \leq 2$
   b. $y = \sqrt{1 + 4x}, 1 \leq x \leq 5$
   c. $y = \sin \pi x, 0 \leq x \leq 1$
   d. $x = \frac{1}{3}(y^2 + 2)^{3/2}, 1 \leq y \leq 2$

2. The following curve is rotated about the y-axis. Find the area of the resulting surface.
   a. $y = \sqrt[3]{x}, 1 \leq y \leq 2$
   b. $x = \sqrt{a^2 - y^2}, 0 \leq y \leq a/2$

Answers:

1. a. $\frac{1}{27}\pi(145\sqrt{145} - 1)$ b. $\frac{98}{3}\pi$ c. $2\sqrt{1 + \pi^2} + (2/\pi)\ln(\pi + \sqrt{1 + \pi^2})$ d. $\frac{21}{2}\pi$

2. (a) $\frac{1}{27}\pi(145\sqrt{145} - 10\sqrt{10})$ (b) πa^2