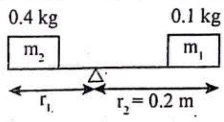
Rotational Dynamics and Oscillatory Motion

A balance scale consisting of a weightless rod has a mass of 0.1 kg on the right side 0.2 m from a pivot point. (a) How far from the pivot point on the left must 0.4 kg be placed so that balance is achieved? (b) If the 0.4 kg mass is suddenly removed, what is the instantaneous rotational acceleration of the rod? (c) What is the instantaneous tangential acceleration of the 0.1 kg mass when the 0.4 kg mass is removed?

[TU Microsyllabus 2074, W; 8.1]

Solution:

Mass of a balance scale (m) = 0.1 kg
Distance from the pivot point (r_2) = 0.2 m
Distance from the pivot point of mass 0.4 kg is r_1 = ?
Instantaneous rotational acceleration (Torque) α = ?
Instantaneous tangential acceleration (a) = ?



[Fig. 17: Masses from pivot point]

a. The torque on both sides must be equal for achieving a balance. So that, $\tau_1 = \tau_2$ (principle of moment)

$$r_1F_1 \sin 90° = r_2F_2 \sin 90°$$

$$r_1m_1g = r_2m_2g$$

$$r_1 = \frac{r_2m_2}{m_1} = \frac{0.2\times 0.1}{0.4} = 0.05\text{ m}$$

Hence, at 0.05 m distance from the mass 0.4kg, balance is achieved.

b. We know that, Torque is given by

$$\tau = I\alpha$$

or, $\alpha = \frac{\tau}{I} = \frac{r_2F_2 \sin 90°}{m_2r_2^2}$

or, $\alpha = \frac{F_2 \sin 90°}{m_2r_2}$

or, $\alpha = \frac{m_2g}{m_2r_2} = \frac{g}{r_2} = \frac{9.8}{0.2} = 49\text{ rad/sec}^2$.

Thus, rotational acceleration (Torque) $\alpha = 49\text{ rad/sec}^2$

c. We have the relation, $a = r\alpha = 0.2\times 49 = 9.8\text{ m/sec}^2$

Hence, Instantaneous tangential acceleration $a = 9.8\text{m/sec}^2$

A large wheel of radius 0.4 m and moment of inertia 1.2 kgm², pivoted at the centre is free to rotate without friction. A rope is wound it and a 2 kg weight is attached to the rope. When the weight has descended 1.5 m from its starting point (a) what is its downward velocity? (b) What is the rotational velocity of wheel? *[TU Microsyllabus 2074, W; 8.2]*

lution:

The radius of the wheel (r) = 0.4 m
Weight (m) = 2 kg
Downward velocity (V) = ?
Moment of inertia (I) = 1.2 kgm²
Height (h) = 1.5 m
Rotational velocity (ω) = ?

a. We know that, from the conservation of energy

Potential energy of weight = Kinetic energy of weight + Rotational kinetic energy of wheel.

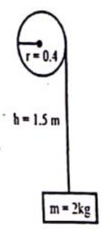
$$\therefore\ mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} \qquad \text{Where, } \omega = \frac{v}{r}$$

$$2mgh = \left(m + \frac{I}{r^2}\right)v^2$$

or, $$v = \sqrt{\frac{2mgh}{\left(m+\frac{I}{r^2}\right)}} = \sqrt{\frac{2\times 2\times 9.8\times 1.5}{\left(2+\frac{1.2}{0.4^2}\right)}}$$

$\therefore$ v = 2.5 m/sec.



[Fig. 18: Weight attached to the rope]

b. We have the relation,

$$\therefore\ \omega = \frac{v}{r} = \frac{2.5}{0.4} = 6.2\text{ rad/sec.}$$

Hence, required downward velocity and rotational velocity are 2.5m/sec and 6.2 rad/sec respectively.

4. **Suppose the body of an ice skater has a moment of inertia 4 kgm^2 and her arms have a mass 5 kg each with the centre of mass at 0.4m from her body. She starts to turn at 0.5 rev/sec on the point of her skate with her arms out stretched. She then pulls her arms inward so that their centre of mass is at the axis of her body is zero. What is the speed of rotation?** *[TU Microsyllabus 2074, W; 8.4]*

Solution:

Moment of inertia of a body (I) = 4 kgm^2

Mass of body arms (m) = 5 kg

Distance (r) = 0.4 m

Angular frequency (ω) = 0.5 rev/sec.

Speed of rotation ω_f = ?

We know that,

$$I_0 \omega_0 = I_f \omega_f$$

$$(I_{body} + I_{arms}) w_0 = I_{body} \omega_f$$

$$(I_b + 2mr^2) \omega_0 = I_b \omega_f$$

$$\omega_f = \frac{(I_b + 2mr^2)\omega_0}{I_b} = \frac{4kgm^2 + 2 \times 5 \text{ kg} \times (0.4m)^2}{4 \text{ kgm}^2} \times 0.5 \text{ rev/sec}$$

$\therefore$ $\omega_f = 0.7$ rev/sec.

Hence, required speed of rotation ω_f = 0.7rev/sec.

5. **A given spring stretches 0.1m when a force of 20N pulls on it. A 2 kg block attached to it on a frictionless surface pulled to the right 0.2 m and released.**

   a. **What is the frequency of oscillation of the block?**
   b. **What is its velocity at the midpoint?**
   c. **What is its acceleration at either end?**
   d. **What are the velocity and acceleration when x = 0.12 m on the block's first passing this point?** *[TU Microsyllabus 2074, W; 10.2]*

Solution:

The spring stretches (x) = 0.1 m

Force (F) = 20N

Mass of block (m) = 2 kg

Distance toward right (r) = 0.2m

Frequency of oscillation (f) = ?

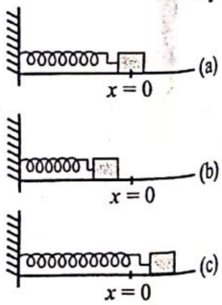
Velocity at the mid point (V_{max}) = ?

Velocity and acceleration (a_{max}) = ?

Velocity and acceleration when x = 0.12m on the blocks first passing = ?

We have, F = kx

or, $k = \frac{F}{x} = \frac{20}{0.1} = 200$ N/m



[Fig. 19: Stretched string at different positions]

a. We know that frequency (f) $= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{200}{2}} = 1.6$ Hz

b. Maximum velocity (V_{max}) = $r\omega = 0.2 \times 2\pi \times 1.6 = 2$ m/sec

c. Maximum acceleration (a_{max}) = $r\omega^2 = 0.2 \times (2\pi \times 1.6)^2 = 20.19$ m/sec

d. Velocity when x = 0.12 m passing this point,

$$V = \omega\sqrt{r^2 - x^2} = 2\pi \times 1.6 \times \sqrt{0.2^2 - 0.12^2} = 1.6 \text{ m/sec}$$

and $a = \omega^2 x = (2\pi \times 1.6)^2 \times 0.12 = 12.12$ m/sec^2.

6. **A spring mass system consists of a 2 kg mass block and spring constant 200 N/m. The block is released from the position of $x_1 = 0.2$m.**

a. What is its velocity at $x_2 = 0.1$ m?

b. What is the acceleration at this point? *[TU Microsyllabus 2074, W; 10.3]*

Solution:

Mass of the block (m) = 2 kg

Spring constant (K) = 200 N/m

Velocity of block when $x_2 = 0.1$ m is = ?

Acceleration of block (a) = ?

a. We have from the conservation of energy

$$\frac{1}{2} kx_1^2 + \frac{1}{2} m v_1^2 = \frac{1}{2} kx_2^2 + \frac{1}{2} mv_2^2$$

$$kx_1^2 + mv_1^2 = kx_2^2 + m_2v_2^2.$$

If $v_1 = 0$ i.e., start from rest position.

So, that $kx_1^2 = kx_2^2 + m_2v_2^2$

$$\therefore \quad v_2 = \sqrt{\frac{k(x_1^2 - x_2^2)}{m}} = \sqrt{\frac{200\,(0.2^2 - 0.1^2)}{2}} = 1.73\text{m/sec}$$

Thus, $v_2 = 1.73$ m/sec

b. We know that, F = ma

$kx = ma$ Since, F = kx for simple harmonic motion

$$\therefore \quad a = \frac{kx}{m} = \frac{200 \times 0.1}{2} = 10\text{m/sec}^2$$

Hence, velocity and acceleration are 1.73 m/sec and 10 m/sec² respectively.

9. A bicycle wheel of mass 2 kg and radius 0.32 m is spinning freely on its axle at 2 rev sec^{-1}. When you place your hand against the tyre, the wheel decelerates uniformly and comes to a stop in 8 sec. What was the torque of your hand against the wheel?

[TU Microsyllabus 2074, P; 8.1]

Solution

Mass of wheel (m) = 2 kg

Radius of wheel (r) = 0.32 m

Initial spinning frequency (f_0) = 2 rev sec^{-1}

Time taken (t) = 8 sec

Torque (τ) = ?

From given condition,

Initial angular frequency (ω_0) = $2\pi f_0$

Final angular frequency (ω_f) = $2\pi f_f = 0$

We know that

$$\tau = I\alpha$$

Where, I = moment of inertia,

α = angular acceleration

Then,

$$\tau = mr^2\left(\frac{\omega_f - \omega_0}{t}\right) \qquad \text{Since, } \alpha = \frac{\omega_f - \omega_0}{t}$$

$$= 2 \times (0.32)^2\left(\frac{2\pi f_f - 2\pi f_0}{t}\right)$$

$$= 2 \times (0.32)^2 \times \left(-\frac{2\pi f_0}{t}\right) \qquad \text{Since, } f_f = 0$$

$$= 2 \times (0.32)^2 \times \left(-\frac{2\pi \times 2}{8}\right)$$

$$\therefore \tau = -0.64 \text{ Nm}$$

Hence, the torque of our hand against the wheel is –0.64 Nm.

10. A grindstone with I = 240 kgm^2 rotates with a speed of 1 rev sec^{-1}. A knife blade is pressed against it, and the wheel coasts to a stop with constant declaration in 12 sec. What torque did the knife exert on the wheel? ***[TU Microsyllabus 2074, P; 8.4]***

Solution:

Here is given, moment of inertia of grindstone (I) = 240 kgm^2

Initial speed of revolution per second (f_0) = 1 rev sec^{-1}

Time taken up to stop (t) = 12 sec

Torque (τ) = ?

We know that,

$$\tau = I\alpha$$

$$= I\left(\frac{\omega_f - \omega_0}{t}\right)$$

$$= \frac{240 \times (0 - 2\pi \times 1)}{12}$$

$$\therefore \tau = 125.66 \text{ Nm}$$

Hence, the required torque exerted by knife on the wheel is 125.66 Nm.

11. Two masses, m_1 = 1kg and m_2 = 5 kg are connected by a rigid rod of negligible weight. The system is pivoted about point O. The gravitational forces act in the negative direction

a. Express the position vectors and the forces on the masses in terms of unit vectors and calculate the torque on the system.

b. What is the angular acceleration of the system at the instant shown in figure 20 below?

[TU Microsyllabus 2074, P; 8.2]

Solution:

Here is given,

a. Two masses

m_1 = 1 kg

m_2 = 5 kg

Here,

Position vector can be expressed as;

$\vec{r_1} = -2\,\hat{j}$ m

$\vec{r_2} = 4\,\hat{j}$ m

Then

$\vec{F_1} = -10\,\hat{k}$ N Since, F = mg and gravitational forces acts in the negative z-direction.

$\vec{F_2} = -50\,\hat{k}$ N

Now torque on the system

$$\vec{\tau} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2}$$

$$= -2\hat{j} \times (-10\,\hat{k}) + 4\,\hat{j} \times (-50\,\hat{k})$$

$$= 20\hat{i} - 200\,\hat{i}$$

$$= (20 - 200)\,\hat{i}\text{ Nm}$$

$\therefore \quad \vec{\tau} = -180\,\hat{i}$ Nm

b. We know that,

Torque $(\vec{\tau}) = I\vec{\alpha}$

or, $\tau = I\alpha$

or, $\alpha = \dfrac{\tau}{I}$

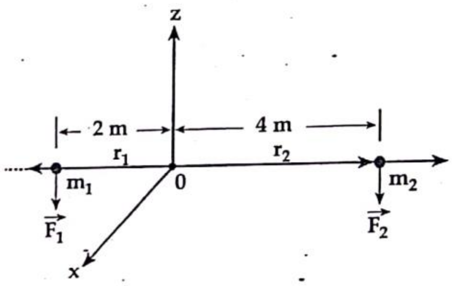
$$= \frac{\tau}{(I_1 + I_2)}$$

$$= \frac{\tau}{(m_1r_1^2 + m_2r_2^2)}$$

$$= \frac{\tau}{1 \times (-2)^2 + 5 \times 4^2}$$

$$= \frac{-180\,\hat{i}}{84}$$

$\therefore \quad \vec{\alpha} = -2.14\,\hat{i}$ rad s^{-1}

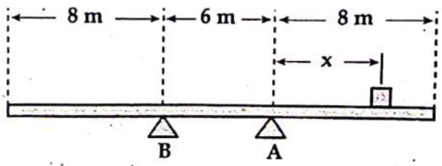


[Fig 20: Representation of two masses connected by a rigid rod of negligible weight]

Hence, required torque and angular acceleration of the system are $-180\,\hat{i}$ Nm and $-2.14\,\hat{i}$ rad s^{-1} respectively.

12. A uniform wooden board of mass 20 kg rests on two supports as shown in figure. A 30 kg steel block is placed to the light of support A. How far to the right of a can the steel the block be placed without tipping the board? *[TU Microsyllabus 2074, P; 8.7]*

Solution:



[Fig 21: A uniform wooden board rests on two rigid supports]

Here is given,

Mass of uniform wooden board

$M_T = M_w = 20$ kg

Mass of steel block $M_2 = M_s = 30$ kg

According to question distance between two supports is 6m, so centre of gravity (C.G.) of uniform wooden board lies between A and B as shown in figure 21. Let 'x' be the right of A at which steel block is placed without tipping the board. Then,

$r_1 = 3$m

$r_2 = x$ m

Now applying principle of conservation of moment

Sum of clockwise moment = Sum of anticlockwise moment

$$M_2gr_2 = M_1gr_1$$

$$30\times10\times x = 20\times10\times3$$

Therefore, $x = 2m$

Hence, steel block must be placed at 2m right from A.

13. A children's merry go-round of radius 4 m and mass 100 kg has an 80 kg man standing at the rim. The merry-go-round coasts on a frictionless bearing at 0.2 rev s⁻¹. The man walks inward 2 m toward the centre. What is the new rotational speed of the merry-go-round? What is the source of this energy? (The moment of inertia of a solid disk is $I = \frac{1}{2}mr^2$)

[TU Microsyllabus 2074, P; 8.18]

Solution:

Here is given, radius of merry go round $(r_1) = 4$ m

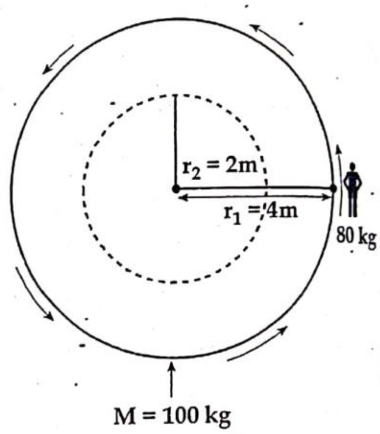
Mass of merry go round (M) = 100 kg

Mass of man (m) = 80 kg

Frequency (f) = 0.2 rev s^{-1}

Now for rotational speed (f_2), According to conservation angular momentum and energy

$$I_1\omega_1 = I_2\omega_2$$



[Fig 22: A children's merry-go-round]

According to question moment of inertia of solid disk $I = \frac{1}{2}mr^2$. Then

$$I_1 = (I + I'_1) \text{ and } I_2 = (I + I'_2)$$

or, $\left(\frac{1}{2}mr_1^2 + Mr_1^2\right) 2\pi f_1 = \left(\frac{1}{2}mr_1^2 + Mr_2^2\right) 2\pi f_2$

or, $\left(\frac{1}{2}100 \times 4^2 + 80 \times 4^2\right) \times 0.2 = \left(\frac{1}{2}100 \times 4^2 + 80 \times 2^2\right) \times f_2$

or, $416 = 1120 f_2$

$\therefore$ $f_2 = 0.371$ rev s^{-1}

Hence, required new rotational speed of the merry go round is 0.371 rev s^{-1}.

14. An oscillating block of mass 250 g takes 0.15 s to move between the end points of the motion, which are 40 cm apart. (a) What is the frequency of the motion? (b) What is the amplitude of the motion? (c) What is the force constant of the spring?

[TU Microsyllabus 2074, P; 10.5]

Solution:

Here is given, block of mass (m) = 250 gm = 250×10^{-3} kg

Time (t) = 0.15 sec

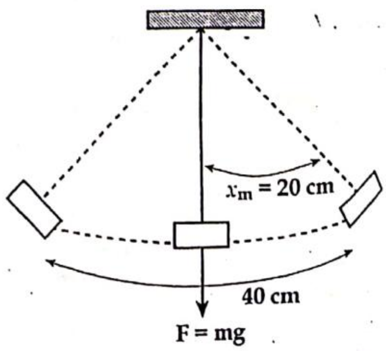
Displacement (x) = 40 cm = 40×10^{-2} m

Frequency (f) = ?

Amplitude of a motion (x_m) = ?

Force constant (k) = ?

We know that,



[Fig 23: Oscillating block between two end points]

a. $f = \frac{1}{t} = \frac{1}{0.15} = 6.67$ rev s^{-1}.

b. Amplitude $(x_m) = \frac{40 \times 10^{-2}}{2} = 20 \times 10^{-2}$m

c. Again we know that

$$2\pi f = \sqrt{\frac{k}{m}}$$

or, $4\pi^2 f^2 = \frac{k}{m}$

or, $k = 4\pi^2 f^2 m$

or, $k = 4\pi^2 (6.67)^2 \times 250 \times 10^{-3}$

$\therefore$ $k = 439.08\ Nm^{-1}$

Hence, required frequency, amplitude and force constant are 6.67 rev s^{-1}, 20×10^{-2} m and 439.08 Nm^{-1} respectively.

15. A spring (k = 200 Nm^{-1}) is compressed 10 cm between two blocks of mass m_1 = 1.5 kg and m_2 = 4.5 kg. The spring is not connected to the blocks and the table is frictionless. What are the velocities of the blocks after they are released and lose contact with the spring? Assume that the spring falls straight down to the table. *[TU Microsyllabus 2074, P; 10.18]*

Solution:

Here is given,

Force constant (k) = 200 Nm^{-1}

Displacement (x) = 10 cm = 10×10^{-2} m

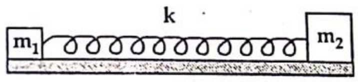
Potential energy (P.E.) = ?

Small mass (m_1) = 1.5 kg

Large mass (m_2) = 4.5 kg

Velocity of lighter mass (v_1) = ?

Velocity of heavier mass (v_2) = ?



[Fig 24: A spring connected with two blocks of having different masses]

We know that,

Potential energy of the system (P.E.) $= \frac{1}{2} kx^2$ ... (1)

According to conservation of linear momentum

$m_1 v_1 + m_2 v_2 = 0$

$m_1 v_1 = - m_2 v_2$

$v_1 = \frac{-m_2}{m_1} v_2$ ... (2)

Now kinetic energy of the system

$$(K.E.) = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2)$$

$$= \frac{1}{2}\left[m_1 \left(\frac{-m_2}{m_1} v_2\right)^2 + m_2 v_2^2\right]$$

$$= \frac{1}{2}\left[\frac{m_2^2 v_2^2}{m_1} + m_2 + v_2^2\right]$$

$$\therefore \quad K.E. = \frac{1}{2}\left[\frac{m_2^2 v_2^2}{m_1} + m_2\right] v_2^2 \quad \dots (3)$$

According to conservation of energy

Total P.E. = Total K.E.

$$\frac{1}{2} kx^2 = \frac{1}{2}\left[\frac{m_2^2 v_2^2}{m_1} + m_2\right] v_2^2$$

$$200 \times (10 \times 10^{-2})^2 = \left[\frac{4.5^2}{1.5} + 4.5\right] v_2^2$$

$$v_2^2 = \frac{200 \times (10 \times 10^{-2})^2}{18}$$

$\therefore$ $v_2 = 3.33 \times 10^{-1} \simeq 0.33\ ms^{-1}$

Then, using value of v_2 in equation (2), we get

$v_1 = -\frac{4.5}{1.5} \times 0.33$

$\therefore$ $v_1 = -1\ ms^{-1}$

Hence, velocities of masses m_1 and m_2 after they are released are $-1\ ms^{-1}$ and $0.33\ ms^{-1}$ respectively.

16. A block is oscillating with amplitude of 20 cm. The spring constant is 150 Nm^{-1}. (a) What is the energy of the system? (b) When the displacement is 5 cm, what are the kinetic energy of the block and the potential energy of the spring? *[TU Microsyllabus 2074, P; 10.13]*

Solution:

Amplitude (x_m) = 20 cm = 20×10^{-2} m

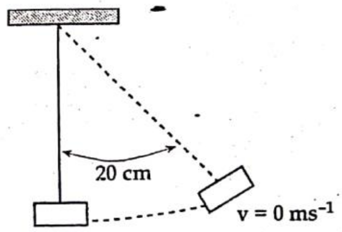
Spring constant (k) = 150 Nm^{-1}

Energy of the system (E) = ?

Displacement (x) = 5 cm = 5×10^{-2} m

Kinetic Energy (K.E.) = ?

Potential Energy (P.E.) = ?



[Fig 25: A block oscillating in the vertical plane]

a. Total energy (T.E.) = Kinetic energy (K.E.) + Potential energy (P.E.)

$= \frac{1}{2} Kx^2_m$ Since, v = 0 at maximum amplitude

Then, $= \frac{1}{2} 150 \times (20 \times 10^{-2})^2$

$= 3.0$ J

b. K.E. $= \frac{1}{2} mv^2$

$= \frac{1}{2} k(x^2_m - x^2)$

$= \frac{1}{2} 150\ [(20 \times 10^{-2})^2 - (5 \times 10^{-2})^2]$

$= 2.81$ J

c, P.E. $= \frac{1}{2} kx^2$

$= \frac{1}{2} \times 150 \times (5 \times 10^{-2})^2$

$\therefore$ P.E. = 0.19 J

Hence, required energy of the system, kinetic energy at 5 cm apart and potential energy of the spring are 3.0 J, 2.81 J and 0.19 J respectively.

Electric and magnetic field

1. A charge $q_1 = 3 \times 10^{-6}$ c is located at the origin of the x – axis. A second charge $q_2 = -5 \times 10^{-6}$ c is also on the x – axis 4 m from the origin in the positive x-direction (a) calculate the electric field at the mid-point p of the line joining the two charges (b) at what point p' on the line is the resultant field zero? *[TU Microsyllabus 2074,W; 14.1]*

Solution:

Charge located at origin $q_1 = 3 \times 10^{-6}$C

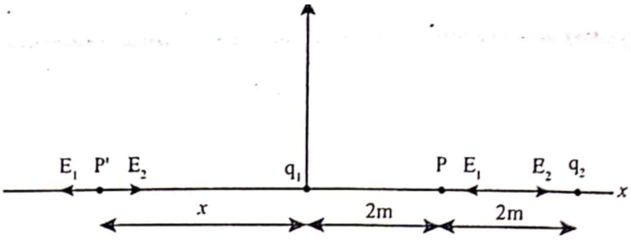
Second charge $q_2 = -5 \times 10^{-6}$C

Distance from q_1 to p point = 2m

Distance from p to q_2 = 2m

Distance from q_1 to p' = x = ?

The electric field at the mid-point p of line joining q_1 and q_2 = ?



[Fig. 25: Charges located on x-axis]

a. The electric field E_1 at a point p due the charge q_1 is

$$E_1 = \frac{q_1}{4\pi\varepsilon_0(2)^2} = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{4} = 6.75 \times 10^3 \text{ N/C toward the positive x-axis.}$$

Again, electric field E_2 at a point P due to the charge q_2 is,

$$E_2 = \frac{q_2}{4\pi\varepsilon_0(2)^2} = \frac{9 \times 10^9 \times -5 \times 10^{-6}}{4} = -11.25 \times 10^3 \text{ N/C.}$$

$\therefore$ $E_2 = 11.25 \times 10^3$ N/C which is along same direction of E_1

The positive sign means field direction along +ve x-axis.

The fields E_1 and E_2 and directed along the same direction

So, the resultant electric field at p will be

$$E = E_1 + E_2 = 6.75 \times 10^3 \text{N/C} + 11.25 \times 10^3 \text{N/C}$$

$\therefore$ $E = 18 \times 10^3$ N/C.

Hence, electric field at p due to charges q_1 q_2 is 18×10^3 N/C.

b. The electric field at a point p' due to the charge q_1 and q_2 are $E_1 = \frac{q_1}{4\pi\epsilon_o x^2}$ and $E_2 = \frac{q_2}{4\pi\epsilon_o(x+4)^2}$

The resultant field at p' will be zero. So

$\therefore$ $E = E_1 + E_2 = 0$

or, $E_1 = -E_2$

or, $|E_1| = |E_2|$

or, $\frac{q_1}{4\pi\epsilon_o x^2} = \frac{q_2}{4\pi\epsilon_o(x+4)^2}$

or, $3(x+4)^2 = 5x^2$

or, $2x^2 - 24x - 48 = 0$

or, $x^2 - 12x - 24 = 0$

or, $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4.1(-24)}}{2}$

$\therefore$ $x = 13.75$ m and $x = -1.75$m

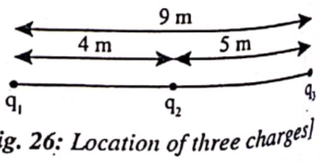
Therefore, E = 0 at x = 13.75m towards left from the origin but x = − 1.75m represents a point between the charges at which $E_1 = E_2$. However, as indicated case (a) since, between the charges, E_1 and E_2 have the same direction and consequently the resultant field is not zero.

2. Three charges $q_1 = 3 \times 10^{-6}$ c, $q_2 = -5 \times 10^{-6}$ c and $q_3 = -8 \times 10^{-6}$ c are positioned on a straight line. Find the potential energy of the charges. *[TU Microsyllabus 2074,W; 14.2]*

Solution:

We have given charges $q_1 = 3 \times 10^{-6}$ C, $q_2 = -5 \times 10^{-6}$ C and $q_3 = -8 \times 10^{-6}$ C

In figure 26 potential energy due to q_1q_2, q_1q_3 and q_2q_3 are obtained by



[Fig. 26: Location of three charges]

$$E_p = \frac{1}{4\pi\epsilon_o}\left[\frac{q_1q_2}{4} + \frac{q_1q_3}{9} + \frac{q_2q_3}{5}\right]$$

$$E_p = 9 \times 10^9 \left[\frac{(3 \times 10^{-6}) \times (-5 \times 10^{-6})}{4} + \frac{(3 \times 10^{-6})(-8 \times 10^{-6})}{9} + \frac{(-5 \times 10^{-6}) \times (-8 \times 10^{-6})}{5}\right]$$

$\therefore$ $E_p = 1.43 \times 10^{-2}$ J.

Hence, required potential energy of the charges is 1.43×10^{-2}J.

3. A potential difference of 100V is established between the two plates A and B. plate B being high potential. A proton of charge $q = 1.6 \times 10^{-19}$c is released from plate B. What will be the velocity of the proton when it reaches plate A? The mass of the proton is 1.67×10^{-27} kg. *[TU Microsyllabus 2074,W; 14.3]*

Solution:

Potential difference V = 100V

Proton charge $q = 1.6 \times 10^{-19}$ c

Mass of proton $m = 1.67 \times 10^{-27}$ kg

We know that,

$$E_k = qV$$

$$\frac{1}{2} m v_A^2 = qV$$

$$v_A = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 100}{1.67 \times 10^{-27}}} = 1.38 \times 10^5 \text{ m/sec}$$

Thus, required velocity of the proton = 1.38×10^5 m/sec

4. Suppose a copper wire carries 10A of current and has a cross-section of 10^{-6} m², each atom of copper contributes one electron that is free to move. So the electron carrier density N_n is about the same as the density of atoms which is about 7×10^{28} atom per m³. The charge on an electron is -1.6×10^{-19} (a) what is the drift velocity of the electrons? (b) How long would it take an electron to move from one terminal of a battery to the other if this wire were 1 m long?

Solution:

Current I = 10A. Cross – section area $A = 10^{-6}$m². Electron carrier density $N_n = 7 \times 10^{28}$ atom/m³.

Charge on an electron (q) = 1.6×10^{-19} c. Drift velocity V_d = ? Time taken by electron t = ?

a. We have the relation $V_d = \frac{I}{AqN_n}$

$$\therefore \quad V_d = \frac{10}{10^{-6} \times 1.6 \times 10^{-19} \times 7 \times 10^{28}} = 9 \times 10^{-4} \text{ m/sec}$$

b. Time taken $t = \frac{x}{V_d} = \frac{1}{9 \times 10^{-4}} = 1.1 \times 10^3$ sec = 11 mins.

Hence, required drift velocity and time taken by electron are 9×10^{-4} m/sec and 11 min respectively.

5. Assume that the electron in a hydrogen atom is essentially in a circular orbit of radius 0.5×10^{-10} m, and rotates about the nucleus at the rate of 10^{14} times per second. What is the magnetic moment of the Hydrogen atom due to the orbital motion of the electron?

[TU Microsyllabus 2074, W; 16.1]

Solution:

Radius of circular orbit $r = 0.5 \times 10^{-10}$ m

Frequency $f = 10^{14}$ sec⁻¹

Magnetic moment of hydrogen μ = ?

We know that $I = \frac{q}{t} = qf$. Since, $\frac{1}{t} = f$

Magnetic moment μ = IA

$$\mu = I.\pi r^2$$

$$\mu = qf\pi r^2$$

$$\mu = 1.6 \times 10^{-19} \times 10^{14} \times \pi \times (0.5 \times 10^{-10})^2$$

$$\therefore \quad \mu = 1.26 \times 10^{-25} \text{ m}^2$$

Hence, the required magnetic moment of hydrogen atom is 1.26×10^{-25} Am². It means, hydrogen atom is essentially a small bar magnet and will behave as such in a magnetic field.

6. The current of 50A is established in a slab of copper 0.5 cm thick and 2 cm wide. The slab is placed in a magnetic field B of 1.5T. The magnetic field is perpendicular to the plane of the slab and to the current. The free electron concentration in copper is 8.4 × 10²⁸ electrons/m³. What will be the magnitude of the Hall voltage across the width of the slab?

[TU Microsyllabus, 2074, W; 16.2, TU Exam 2074]

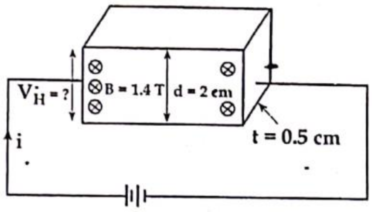
Solution:

Current (I) = 50A, thickness of copper (x) = 0.5 cm, width of copper slab (b) = 2 cm

Magnetic field (B) = 1.5T

Free electrons concentration (N) = 8.4×10^{28} electrons/m³.

Magnitude of hall voltage (V_H) = ?



[Fig. 27: Representation of Hall Effect due to charge particle in a magnetic field with in the slab of copper]

We know that $V_H = \dfrac{IB\,b}{NqA}$

$$V_H = \frac{50 \times 1.5 \times 2 \times 10^{-2}}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-4}}$$

$\therefore \quad V_H = 1.12 \times 10^{-6}$ V

Hence, required hall voltage is 1.12×10^{-6} V.

7. Four charges of equal magnitude are placed at the corners of a square as shown in figure 28. What is the electric field at the centre of the square, point O?

[TU Microsyllabus 2074, P; 14.6]

Solution:

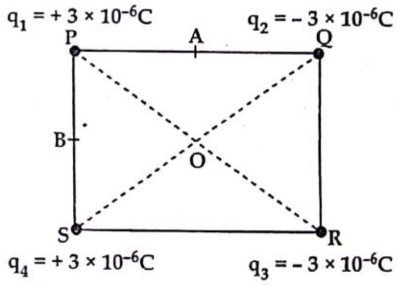
Here, four charges of equal magnitude

$q_1 = +3 \times 10^{6}$ C

$q_2 = -3 \times 10^{-6}$ C

$q_3 = -3 \times 10^{-6}$ C

$q_4 = +3 \times 10^{-6}$ C



[Fig 28: Four charges at the corners of a square having equal magnitude]

Consider PQRS be given square then PQ = PS = QR = RS = 0.23 m

From figure 28, we get

$PR = \sqrt{0.25^2 + 0.25^2} = 0.354$ m

$\therefore \quad PO = 0.176$ m = QO = SO = RO

The total electric field at the centre of the square,

$$E = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{PO^2} + \frac{q_2}{QO^2} + \frac{q_3}{RO^2} + \frac{q_4}{SO^2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3 \times 10^{6}}{0.25^2} + \frac{(-3 \times 10^{-6})}{0.25} + \frac{(-3 \times 10^{-6})}{0.25^2} + \frac{(3 \times 10^{-6})}{0.25^2}\right]$$

$E = 0\ NC^{-1}$

Hence, required the electric field at the centre of the square at point O is zero.

8. Two large parallel plates are separated by a distance of 5 cm. The plates have equal but opposite charges that create and electric field in the region between the plates. An α-particle (q = 3.2 × 10⁻¹⁹ C, m = 6.68 × 10⁻²⁷ kg) is released from the positively charged plate and it strikes the negative y charged plate 2 × 10⁻⁶ sec later. Assuming that the electric field between the plates is uniform and perpendicular to the plates, what is the strength of the electric field? ***[TU Microsyllabus 2074, P; 14.8, TU Model question 2074]***

Solution:

Here, distance between plates (s) = 5 cm = 5×10^{-2} m

Charge of α-particle (q) = 3.2×10^{-19} c

Mass of α-particle (m) = 6.68×10^{-27} kg

Time taken (t) = 2×10^{-6} sec

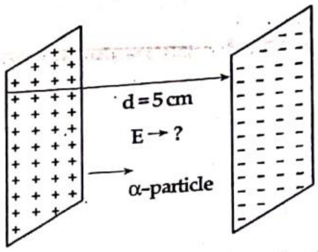
Electric field between the plates (E) = ?

We know that from equation of motion,

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}at^2 \quad [\because \text{Initially at rest, } u = 0]$$

$$a = \frac{2s}{t^2} \quad \ldots(1)$$



[Fig. 29: Two parallel plates having opposite charges at which α–particle released from +ve plate to –ve]

Again from second law of motion

$$F = ma \quad \ldots(2)$$

And electrostatic force,

$$F = qE \quad \ldots(3)$$

From equation (1), (2) and (3)

$$ma = qE$$

$$m\frac{2s}{t^2} = qE$$

$$E = \frac{2ms}{qt^2} = \frac{2 \times 6.68 \times 10^{-27} \times 5 \times 10^{-2}}{3.2 \times 10^{-19} \times (2 \times 10^{-6})^2} = 521.875$$

$\therefore \quad E \approx 522\ NC^{-1}$

Hence, strength of field between the plate is 522 NC^{-1}.

9. **An electron is placed midway between two fixed charges, $q_1 = 2.5 \times 10^{-10}$ C and $q_2 = 5 \times 10^{-10}$ C. If the charges are 1 m apart, what is the velocity of the electron when it reaches a point 10 cm from q_2?** ***[TU Microsyllabus 2074, P; 14.2]***

Solution:

Here, given two charges are

$q_1 = 2.5 \times 10^{-10}$ C

$q_2 = 5 \times 10^{-10}$ C

Distance between q_1 and q_2

$r = 1$ m

Distance between e and q_1, q_2 = $r_1 = r_2 = 0.5$ m

Distance travelled by e^- towards q_2 from initial position

$S = 0.5\text{ m} - 0.1\text{ m}$

$= 0.4$ m

Now we know that

From equation of motion

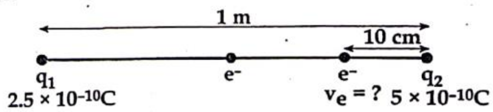
Since, $u = 0\ ms^{-1}$

$$v^2 = u^2 + 2as$$

$$v = \sqrt{2as} \quad \ldots(1)$$

Again, Electrostatic force between q_1 and e at rest condition

$$F_1 = \frac{q_1 e}{4\pi \epsilon_0 r_1^2}$$



[Fig. 30: Representation of electron in between two charges]

Electrostatic force between q_2 and e

$$F_2 = \frac{q_2 e}{4\pi \epsilon_0 r_2^2}$$

Now, Net force $F = F_2 - F_1 = \dfrac{q_1 e}{4\pi \epsilon_0 r_1^2} - \dfrac{q_2 e}{4\pi \epsilon_0 r_2^2}$

$$F = \frac{e}{4\pi\epsilon_0} = \left[\frac{q_2}{r_2^2} - \frac{q_1}{r_1^2}\right] = \frac{1.6 \times 10^{-19} \times 9 \times 10^9}{(0.5)^2} \times [5 \times 10^{-10} - 2.5 \times 10^{-10}]$$

$$= 1.44 \times 10^{-18}\ N$$

From Newton's second law of motion .

$F = ma = 1.44 \times 10^{-18}$

or, $a = \frac{1.44 \times 10^{-19}}{9.1 \times 10^{-31}}$ [$\because m = m_e = 9.1 \times 10^{-31}$ kg] .

$\therefore$ $a = 1.58 \times 10^{12}$ ms^{-2}

Substituting value of 'a' in equation (1), we get

$v = \sqrt{2 \times 1.58 \times 10^{12} \times 0.4}$

$\therefore v = 1.125 \times 10^{6}$ ms^{-1}.

Hence, required velocity of electron when it reaches a point 10 cm from q_2 is 1.125×10^{6} ms^{-1}.

10. What force is experienced by a wire of length l = 0.08 m at an angle of 20° to the magnetic field direction carrying a current of 2A in a magnitude field 1.4 T ?

[TU Microsyllabus 2074, P; 16.1]

Solution:

Given, length of wire (l) = 0.08 m

Angle between B and l is (θ) = 20°

Magnetic field (B) = 1.4 T

Current (I) = 2 A

Force experienced by a wire (F) = ?

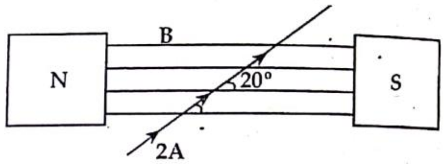
We know that,

$F = BIl \sin\theta$

$= 1.4 \times 2 \times 0.08 \times \sin 20°$

$F = 7.66 \times 10^{-2}$ N

Hence, required force experienced by wire is 7.66×10^{-2} N.



[Fig. 31: Current carrying wire in uniform magnetic field at angle 20°]

11. The earth's magnetic field at the equator is 4×10^{-5} T and is parallel to the surface of the earth in the south-north direction. A wire 2 m long of mass m = 9g is suspended by a string. The wire is also parallel to the earth's surface and carries a current of 150 A in the east-west direction. (a) What is the tension of the string? (b) What would be the tension if the current was in the west-east direction? *[TU Microsyllabus 2074, P; 16.2]*

Solution:

Here is given, Earth magnetic field at equator (B) = 4×10^{-5} T

Length of wire (ℓ) = 2m

Mass of wire (m) = 9 gm = 9×10^{-3} kg

Current carried by wire (I) = 150 A

a. Tension on the string (T_{NS}) = ?

b. Tension on the string when current was in the west-east direction (T_{WE}) = ?

For (a)

In 1st case tension on the string or wire is provided by weight and magnetic fore.

i.e.,

$T = mg + BI\ell$

$= 9 \times 10^{-3} \times 9.8 + 4 \times 10^{-5} \times 150 \times 2$

$= 0.1002$ N

$= 10.02 \times 10^{-2}$ N

For (b)

$T' = mg - BI\ell$

$= 9 \times 10^{-3} \times 9.8 - 4 \times 10^{-5} \times 150 \times 2$

$= 0.0762$

$= 7.62 \times 10^{-2}$ N. Since, g = 9.8 ms^{-2}

Hence, required tension on the string when it is at north-south direction and west-east direction are 10.02×10^{-2} N and 7.62×10^{-2} N respectively.

12. A proton is moving with a velocity $\vec{v} = (3 \times 10^5\,\hat{i} + 7 \times 10^5\,\hat{k})\ ms^{-1}$ in a region where there is a magnetic filed $\vec{B} = 0.4\hat{j}T$. What is the force experienced by the proton?

[TU Microsyllabus 2074, P; 16.12]

Solution:

Velocity of proton $(\vec{v}) = (3 \times 10^5\,\hat{i} + 7 \times 10^5\,\hat{k})\ ms^{-1}$

Magnetic field $(\vec{B}) = 0.4\hat{j}\,T$

We know, mass of proton $(m_p) = 1.67 \times 10^{-27}$ kg

Charge of proton $(q) = 1.6 \times 10^{-19}$ c

Force experienced by the proton $(\vec{F})$ = ?

We have a relation

$$\vec{F} = q(\vec{v} \times \vec{B})$$
$$= 1.6 \times 10^{-19}[(3 \times 10^5\,\hat{i} + 7 \times 10^5\,\hat{k}) \times 0.4\hat{j}]$$
$$= 1.6 \times 00^{-19}[1.2 \times 10^5\hat{k} - 2.8 \times 10^5\hat{i}]$$
$$= 1.6 \times 10^{-19}[1.2\hat{k} - 2.8\hat{i}] \times 10^5$$
$$\vec{F} = (1.92\,\hat{k} - 4.48\,\hat{i}] \times 10^{-14}\ N$$

Hence, force experienced by the proton is $(1.92\,\hat{k} - 4.48\,\hat{i}] \times 10^{-14}$ N.

13. A proton is accelerated through a potential difference of 200 V. It then enters a region where there is a magnetic filed B = 0.5 T. The magnetic field is perpendicular to the direction of motion of the proton. What is the force experienced by the proton?

[TU Microsyllabus 2074, P; 16.13]

Solution:

Here, potential difference (p.d.), V = 200 V

Magnetic field (B) = 0.5 T

Force experienced by the proton (F) = ?

We know that, at equilibrium condition, K.E. = P.E.

$$\frac{1}{2}m_p v_p^2 = qV$$
$$v_p^2 = \sqrt{\frac{2qV}{m_p}}$$
$$= \left(\frac{2 \times 1.6 \times 10^{-19} \times 200}{1.67 \times 10^{-27}}\right)^{1/2}$$

Since, mass of proton $m_p = 1.67 \times 10^{-27}$ kg, $q = 1.6 \times 10^{-19}$ C

$$= 1.957 \times 10^5\ ms^{-1}$$
$$\approx 1.96 \times 10^5\ ms^{-1}$$

Again, we know Lorentz force,

$$F = Bqv_p$$

Since, $v_p \perp^r B$

$$= 0.5 \times 1.6 \times 10^{-19} \times 1.96 \times 10^5$$
$$\therefore\ F = 1.568 \times 10^{-14}\ N$$

Hence, required force experienced by the proton is 1.568×10^{-14} N.

Fundamental of Atomic theory

1. **After being excited, the electron of a Hydrogen atom eventually falls back to the ground state. This can take place is one jump or in a series of jumps, the electron falling into lower excited states before it end up in the ground state, that is n = 3. Calculate the different photon energies that may be emitted as the atom returns to the ground state.**

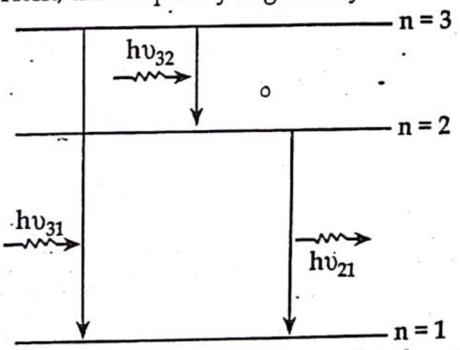
[TU Microsyllabus 2074, W;18.2]

Solution:

The possible transition lines are shown in figure 26. We know that, if an electron is initially in an allowed orbit of energy E_i and goes into another orbit of lower energy E_f. Then, the frequency is given by

$$\nu = \frac{E_i - E_f}{h}$$

$$h\nu = E_i - E_f = E_o\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$



[Fig. 26: Jump of electrons into lower excited states]

Transition from 3 to 1 states is,

$$\therefore \quad h\nu_{31} = E_3 - E_1 = E_0\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$h\nu_{31} = 13.65\ \text{eV}\left[1 - \frac{1}{9}\right] = 12.05\ \text{eV}$$

Transition from 3 to 2 states is,

$$h\nu_{32} = E_0\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 13.56\left[\frac{1}{4} - \frac{1}{9}\right] = 1.88\ \text{eV}$$

Transition from 2 to 1 state is,

$$h\nu_{21} = E_0\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 13.56\ \text{eV}\left[\frac{1}{1} - \frac{1}{4}\right] = 10.17\ \text{eV}$$

Therefore, the different photon energies that may be emitted as the atom returns to the ground state are $h\nu_{31} = 12.05$ eV, $h\nu_{32} = 1.88$ eV and $h\nu_{21} = 10.17$ eV.

2. **A beam of monochromatic neutrons is incident on a KCl crystal with lattice spacing of 3.14 Å. The first order diffraction maximum is observed when the angle θ between the incident and the atomic planes is 37°. What is the kinetic energy of the neutrons?**

[TU Microsyllabus 2074, W;19.1]

Solution:

We have lattice spacing (d) = 3.14Å

Angle of diffraction (θ) = 37°

Order of diffraction (n) = 1

Kinetic energy of the neutron (E_k) = ?

We know that, for crystal diffraction

$$2d \sin\theta = n\lambda$$

$$2d \sin\theta = \lambda \qquad \text{for } n = 1$$

$$\therefore \quad \lambda = 2 \times 3.14 \times \sin 37° = 3.78 \text{ Å}$$

From de-Broglie's hypothesis, the momentum of the neutrons is

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ Jsec}}{3.78 \times 10^{-10} \text{ m}} = 1.75 \times 10^{-24} \text{ kg m/sec.}$$

The kinetic energy of the neutrons will be

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{(1.75 \times 10^{-24})^2}{2 \times 1.67 \times 10^{-27}} = 9.21 \times 10^{-22} \text{J}$$

Thus, the required kinetic energy $E_k = 9.21 \times 10^{-22}$ J.

3. **What are the shortest and longest wavelengths of the layman series? Where, R = 1.097 × 10⁷/m.**

Solution:

Let, n_1 and n_2 are the number states. We know that an expression of wavelength $\left(\frac{1}{\lambda}\right) = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For layman series (n_1) = 1. The longest wavelength corresponds to the smallest value of n_2 which is Lyman series is 2 thus,

$$\frac{1}{\lambda_{longest}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.097 \times 10^7 \left(\frac{3}{4}\right)$$

$$\text{or,} \quad \frac{1}{\lambda_L} = 0.8226 \times 10^7 \text{ m}^{-1}.$$

$$\therefore \quad \lambda_L = 1.215 \times 10^{-7} \text{ m} = 1215 \text{ Å}$$

The shortest wavelength corresponds to the largest value of n_1 that is ∞ which is

$$\frac{1}{\lambda_{shortest}} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$\text{or,} \quad \frac{1}{\lambda_s} = 1.0968 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$\text{or,} \quad \frac{1}{\lambda_s} = 1.0968 \times 10^7 \text{ m}^{-1}$$

Therefore $\lambda_{shortest} = 0.9117 \times 10^{-7}\text{m} \approx 912$ Å.

Therefore, the longest and shortest wave lengths of layman series are 1215 Å and 912 Å respectively.

4. **Find the energy of the neutron in unit eV whose de-Broglie's wavelength is 1Å, given mass of the neutron is 1.67 × 10⁻²⁷ kg, h = 6.62 × 10⁻³⁴ J-sec.**

Solution:

We have, mass of neutron (m) = 1.67×10^{-27} kg

Planck's constant (h) = 6.62×10^{-34} J sec.

de-Broglie's wavelength (λ) = 1 Å

$$\text{We know that } \lambda = \frac{\lambda}{\sqrt{2mE_k}} \Rightarrow E_k = \frac{h^2}{2m\lambda^2}$$

$$\text{or,} \quad E_k = \frac{(6.62 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (1 \times 10^{-10})^2} = 13.01 \times 10^{-21} \text{ J}$$

$$\therefore \quad E_k = \frac{13.01 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV}$$

This is required energy of neutron in unit eV.

5. **Calculate the uncertainty in the measurement of velocity of electron in H-atom of radius 1Å.**

Solution:

The radius of H–atom (r) = 1 Å = 1×10^{-10} m

Uncertainty distance (Δx) = 2r = 2×10^{-10} m

We have, the uncertainty principle

$$\Delta x \,.\, \Delta p = \hbar \Rightarrow \Delta p = \frac{\hbar}{\Delta x}$$

We know that, $\Delta p = m\Delta v \Rightarrow m\Delta v = \frac{\hbar}{\Delta x}$

$$\therefore \quad \Delta v = \frac{\hbar}{m\Delta x} = \frac{h}{2\pi m \Delta x} \qquad \text{Since, } \hbar = \frac{h}{2\pi}$$

$$\Delta v = \frac{6.62 \times 10^{-34}}{2\pi \times 9.1 \times 10^{-31} \times 2 \times 10^{-11}} = 5.76 \times 10^5 \text{ m/sec.}$$

Thus, uncertainty in the measurement of velocity of the electron is 5.76×10^5 m/sec.

6. **Calculate the shortest and the longest wavelength of the Balmer series of Hydrogen.**

[TU Microsyllabus 2074, P; 18.1]

Solution:

The longest wavelength corresponding to the smallest value of n_j, which for the Balmer series is $n_j = 3$ and $n_k = 2$.

Therefore, we know that,

$$\frac{1}{\lambda_{longest}} = R\left[\frac{1}{n_k^2} - \frac{1}{n_j^2}\right]$$

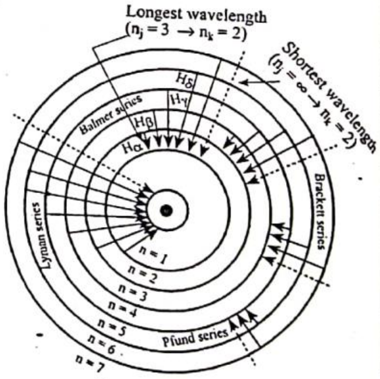
$$= R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= 1.0968 \times 10^7 \text{ m}^{-1} [0.25 - 0.11$$

$$\frac{1}{\lambda_{longest}} = 1.523 \times 10^6 \text{ m}^{-1}$$

$$\text{Hence, } \lambda_{longest} = \frac{1}{1.523 \times 10^6 \text{ m}^{-1}} = 6.54 \times 10^{-7} \text{ m}$$

$$\therefore \quad \lambda_{longest} = 6565 \text{ Å}.$$



[Fig. 27: Shortest and longest wavelength of H- atom in Balmer series]

The shortest wavelength corresponds to the largest value of n_j that is ∞.

$$\frac{1}{\lambda_{shortest}} = R\left[\frac{1}{n_k^2} - \frac{1}{n_j^2}\right]$$

$$= 10.968 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{\infty^2}\right]$$

$$= 1.0968 \times 10^7 [0.25 - 0]$$

$$\frac{1}{\lambda_{shortest}} = 2742000 \text{ m}^{-1}$$

$$\text{Hence, } \lambda_{shortest} = \frac{1}{2742000 \text{ m}^{-1}}$$

$$= 3.6469 \times 10^{-7}$$

$$\therefore \quad \lambda_{shortest} \simeq 3646 \text{ Å}.$$

Hence, required shortest and longest wavelength of the Balmer series of Hydrogen are 3646 Å and 6565 Å respectively.

7. What are (a) the energy, (b) the momentum, and (c) the wavelength of photon that is emitted when a Hydrogen atom undergoes a transition from the state n = 3 to n = 1? (The momentum of the photon is given by $\frac{h\upsilon}{C}$). *[TU Microsyllabus 2074, P; 18.2, TU Exam 2074]*

Solution:

Given energy state of Hydrogen atom are $n_1 = 1$, $n_2 = 3$

Energy (E) = ?

Momentum (p) = ?

Wavelength (λ) = ?

We know that,

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where, R = Rydberg constant i.e. $R = 1.09 \times 10^7\ m^{-1}$. Then,

$$\frac{1}{\lambda} = 1.09 \times 10^7 \left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$\therefore\quad \lambda = 1.03 \times 10^{-7}$ m

Again, energy (E) = $h\frac{c}{\lambda}$

$$= 6.62 \times 10^{-34} \times \frac{3 \times 10^8}{1.03 \times 10^{-7}}$$

$$= 1.93 \times 10^{-18}\ J$$

$$\approx 12.06\ eV$$

Momentum (p) = $\frac{h}{\lambda} = \frac{6.62 \times 10^{-34}}{1.03 \times 10^{-7}} = 6.42 \times 10^{-27}\ kg\ ms^{-1}$

Hence, required energy, momentum and wavelength are 12.06 eV, $6.42 \times 10^{-27}\ kg\ ms^{-1}$ and 1.03×10^{-7} m respectively.

8. The shortest wavelength of the Paschen series from Hydrogen is 8204 Å. From this fact, calculate the Rydberg constant. *[TU Microsyllabus 2074, P; 18.3]*

Solution:

Here is given, shortest wavelength of Paschen series $\lambda_{shortest} = 8204$ Å $= 8204 \times 10^{-10}$ m

Rydberg constant (R) = ?

We know that,

$$\frac{1}{\lambda} = R\left[\frac{1}{n_k^2} - \frac{1}{n_j^2}\right]$$

Here, shortest wavelength correspond to largest value of n_j i.e. $n_j = \infty$ and for Paschen series $n_k = 3$.

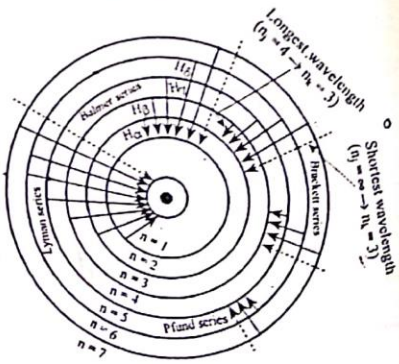
Then,

or, $\frac{1}{\lambda_{shortest}} = \left[\frac{1}{3^2} - \frac{1}{\infty^2}\right]$

or, $\frac{1}{8204 \times 10^{-10}} = R \times \frac{1}{9}$

$\therefore\quad R = 10970258 = 1.097 \times 10^7\ m^{-1}$

Hence, required Rydberg constant $1.097 \times 10^7\ m^{-1}$.



[Fig 28: Shortest and longest wavelength of H- atom in Paschen series]

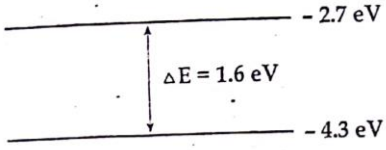
9. The ground state and the first excited-state energies of potassium atoms are – 4.3 eV and –2.7 eV respectively. If we use potassium vapour in the Franck-Hertz experience at what voltages would we see drops in the plot of current versus voltage? *[TU Microsyllabus 2074, P; 18.19]*

Solution:

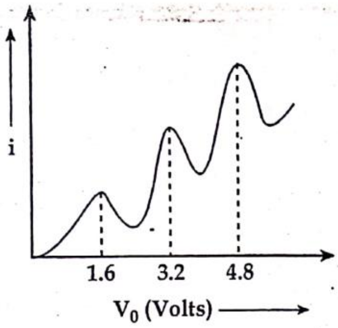
Here is given, energies corresponding to ground state and the first excited states of potassium are – 4.3eV and –2.7 eV respectively.

Here, the energy difference between the ground state and the first excited state is

$V_0 = -2.7\text{ eV} - (-4.3\text{ eV}) = 1.6\text{ eV}$



[Fig 29: Energy gap between states]



[Fig 30: Representation of dependency of plate current i on the accelerating voltage Vo]

As V_0 increases beyond the 1.6 eV, the current begins to increase again because, although the electrons can and do collide inelastically and lose 1.6 eV of energy, they still have enough energy remaining to overcome the small retarding voltage. When $V_0 = 2 \times 1.6\text{ V} = 3.2\text{ V}$ or $3 \times 1.6\text{ V} = 4.8\text{ V}$ and so on, dips in the current occur again because now electron can undergo, two, three, or more inelastic collisions with the potassium atom in each collision they lose 1.6 eV energy.

10. The de-Broglie wavelength of proton is 10^{-13} m. (a) what is the speed of the proton? (b) Through what potential difference must the proton be accelerated to acquire such a speed? ***[TU Microsyllabus 2074, P; 19.2]***

Solution:

Here is given, de-Broglie wavelength of proton (λ) = 10^{-13} m

Speed of proton (v) = ?

Potential difference (V) = ?

Mass of proton (m_p) = 1.67×10^{-27} kg

We know that,

$$\text{de-Broglie wavelength } (\lambda) = \frac{h}{m_p v}$$

$$\text{or,} \quad 10^{-13} = \frac{h}{1.67 \times 10^{-27} \times v}$$

$$\text{or,} \quad v = \frac{6.626 \times 10^{-34}}{10^{-13} \times 1.67 \times 10^{-27}}$$

$$\therefore \quad v = 3.967 \times 10^{6}\text{ ms}^{-1}$$

Again, we have a relation,

Kinetic Energy (E_k) = eV

Where, $e = 1.6 \times 10^{-19}$ C

Then,

$$\frac{1}{2} m_p v^2 = eV$$

$$V = \frac{1}{2}\frac{m_p v^2}{e}$$

$$= \frac{1}{2}\frac{(1.67 \times 10^{-27}) \times (3.967 \times 10^{6})^2}{1.6 \times 10^{-19}} = 8.2015$$

$$\therefore \quad V = 8.20 \times 10^{4}\text{ Volt}$$

Hence, required speed of proton and accelerating p.d. are $3.967 \times 10^{6}\text{ ms}^{-1}$ and 8.20×10^{4} volt respectively.

11. An α-particle is emitted from a nucleus with an energy of 5 MeV (5×10^6 eV). Calculate the wavelength of an α-particle with such energy and compare it with the size of the emitting nucleus that has a radius of 8×10^{-15} m. *[TU Microsyllabus 2074, P; 19.7]*

Solution:

Here is given, energy of α-particle (E_k) = 5 MeV = $5 \times 10^6 \times 1.6 \times 10^{-19}$J

Wavelength of α-particle (λ_α) = ?

Radius of the nucleus (r) = 8×10^{-15} m

We know that, from de-Broglie hypothesis

$$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha E_k}}$$

$$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 5 \times 10^6 \times 1.6 \times 10^{-19}}}$$ Since, α-particle ($_2He^4$) consist of 2n and 2p $\simeq$ 4 m_p

$\therefore \quad \lambda_\alpha \doteq 6.409 \times 10^{-15}$ m

Hence, wavelength of an α-particle i.e. 6.409×10^{-15} m is less than that of radius i.e. 8×10^{-15} m.

12. A neutron spectroscopy a beam of mono energetic neutrons is obtained by reflecting reactor neutrons from a beryllium crystal. If separation between the atomic planes of the beryllium crystal is 0.732 Å, what is the angle between the incident neutron beam and the atomic planes that will yield a monochromatic beam of neutrons of wavelength 0.1 Å.

[TU Microsyllabus 2074, P; 19.11]

Solution:

Here is given, Separation between the atomic plane (d) = 0.732 Å = 0.732×10^{-10} m

Wavelength of monochromatic beam of neutrons (λ) = 0.1 Å = 0.1×10^{-10}m

Angle between the incident neutron beam and the atomic planes (θ) = ?

From Brogg's law,

$$2d \sin\theta = n\lambda$$

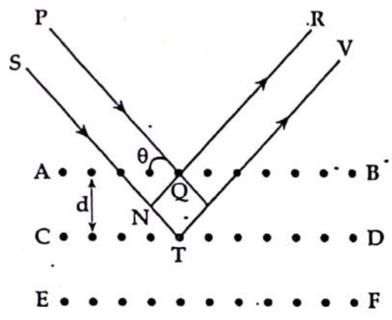
or, $\sin\theta = \frac{n\lambda}{2d}$ [Since, n = 1, for first order]

or, $\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right)$

or, $\theta = \sin^{-1}\left(\frac{0.1 \times 10^{-10}}{2 \times 0.732 \times 10^{-10}}\right)$

$\therefore \quad \theta = 3.92°$

Hence, required angle between the incident neutron bean and the atomic plane is 3.92°.



[Fig. 31: Bragg's diffraction]

13. A small particle of mass 10^{-6} g moves along the x-axis; its speed is uncertain by 10^{-6} ms^{-1}.

a. What is the uncertainty in the x-coordinate of the particle?

b. Repeat the calculation for an electron assuming that the uncertainty in its velocity is also 10^{-6} ms^{-1}. *[TU Microsyllabus 2074, P; 19.16]*

Solution:

Here, given mass of small particle (m) = 10^{-6} g = $10^{-6} \times 10^{-3}$ kg

Speed of particle (v) = 10^{-6} ms^{-1}

Uncertainty in the x-coordinate of the particle (Δx) = ?

Uncertainty in velocity (Δv) = 10^{-6} ms^{-1}

We know that, from Heisenberg's uncertainty principle,

$$\Delta x \, \Delta p \geq \hbar$$

or, $\Delta x \, \Delta p = \frac{h}{2\pi}$

a. Here, $\Delta p = m_{particle} \Delta v$

$$\therefore \Delta x = \frac{h}{2\pi\Delta p}$$

$$= \frac{h}{2\pi m\Delta v}$$

$$= \frac{6.626 \times 10^{-34}}{2\pi \times 10^{-9} \times 10^{-6}}$$

$$= 1.05 \times 10^{-19} \text{ m}$$

$$\therefore \Delta x = 1.05 \times 10^{-19} \text{ m}$$

b. Mass of electron $m_e = 9.1 \times 10^{-31}$kg

Then, uncertainty in x-coordinate or position of electron

$$\Delta x = \frac{h}{2\pi\Delta p}$$

$$= \frac{h}{2\pi m\Delta v}$$

$$= \frac{6.626 \times 10^{-34}}{2\pi \times 9.1 \times 10^{-31} \times 10^{-6}} = 115.768$$

$$\therefore \Delta x = 115.768 \approx 116 \text{ m.}$$

Hence, uncertainty in x-coordinate for a particle and electron are 1.05×10^{-19} m and 116 m respectively.

14. The uncertainty in the position of a particle is equal to the de-Broglie wavelength of the particle. Calculate the uncertainty in the velocity of the particle in terms of the velocity of the de-Broglie wave associated with the particle.

[TU Microsyllabus 2074, P;19.19, TU Exam 2074]

Solution:

Here is given, uncertainty in the position of a particle say Δx = de-Broglie wavelength of the particle (λ)

Uncertainty in the velocity (Δv) = ?

We know that, from uncertainty principle,

$$\Delta x \, \Delta p \geq \frac{h}{2\pi}$$

$$\text{or,} \quad \Delta x = \frac{h}{2\pi\Delta p}$$

$$= \frac{h}{2\pi m\Delta v} \quad \text{... (1)}$$

Again, from de-Broglie hypothesis,

$$\lambda = \frac{h}{mv} \quad \text{... (2)}$$

According to question, from equation (1) and (2), we get

$$\frac{h}{2\pi m\Delta v} = \frac{h}{mv}$$

$$\therefore \Delta v = \frac{v_{wave}}{2\pi}$$

Hence, uncertainty in velocity is equal to the $\frac{1}{2\pi}$ times velocity of the de-Broglie wave.

Method of Quantum mechanics

1. **Consider the particle in the ground state is represented by a wave function $\Psi(x)$ = B Si $\left(\frac{\pi x}{a}\right)$ within $0 < x < a$. where $B = \sqrt{\frac{2}{a}}$. What is (a) the average position (b) the averag momentum (c) the average energy of such particle?** *[TU Microsyllabus 2074, W; 20.2*

Solution:

We have the given wave function $\Psi(x) = B \operatorname{Sin}\left(\frac{\pi x}{a}\right) = \sqrt{\frac{2}{a}} \operatorname{Sin}\left(\frac{\pi x}{a}\right)$

a. The average position means the expectation value of positions $\bar{x}$ or $< x > = \int_0^a \Psi^*(x)\, x\, \Psi(x)\, dx$

$$\text{or,} \quad <x> = \frac{2}{a}\int_0^a \sin^2\left(\frac{\pi x}{a}\right) x\, dx$$

$$\text{or,} \quad <x> = \frac{2}{a}\frac{a^2}{\pi^2}\int_0^a \sin^2\left(\frac{\pi x}{a}\right)\left(\frac{\pi x}{a}\right)\left(\frac{\pi}{a}\right) dx$$

$$= \frac{2a}{\pi^2}\left[\frac{1}{4}\left(\frac{\pi x}{a}\right)^2 - \frac{\pi x \operatorname{Sin}\left(\frac{2\pi x}{a}\right)}{4a} - \frac{\cos\left(\frac{2\pi x}{a}\right)}{8}\right]_0^a$$

$$= \frac{2a}{\pi^2}\left\{\left(\frac{\pi^2}{4} - 0 - \frac{1}{8}\right) - \left(0 - 0 - \frac{1}{8}\right)\right\}$$

$$\therefore \quad <x> = \frac{a}{2}$$

b. The average value of momentum can be found by

$$<p> = \bar{P} = \int_0^a \Psi^*(x)\, P\, \Psi(x) dx$$

$$\text{or,} \quad <P> = \frac{2}{a}\int_0^a \operatorname{Sin}\left(\frac{\pi x}{a}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)\sin\left(\frac{\pi x}{a}\right) dx, \qquad \text{Since, } P = -i\hbar\frac{\partial}{\partial x}.$$

$$\text{or,} \quad <P> = \frac{2}{a}\int_0^a \operatorname{Sin}\left(\frac{\pi x}{a}\right)\left(-i\hbar\frac{\pi}{a}\cos\frac{\pi x}{a}\right) dx$$

$$= \frac{2}{a}(-i\hbar)\int_0^a \left(\operatorname{Sin}\frac{\pi x}{a}\right)\left(\cos\frac{\pi x}{a}\right)\left(\frac{\pi}{a}\right) dx.$$

$$= \frac{2}{a}(-i\hbar)\left[\frac{\sin^2\left(\frac{\pi x}{a}\right)}{2}\right]_0^a$$

$$\therefore \quad <P> = 0.$$

This result makes physical sense. The particle is moving back and forth between the walls of the well. The probability of finding the particle moving toward right is the same as probability of finding it moving forward the left. Thus, the average value of the momentum has to be zero.

c. The average energy $\bar{E}$ or <E> can be calculated as

$$<E> = \int_0^a \Psi^*\left(i\hbar\frac{\partial}{\partial t}\right)\Psi\, dx. \qquad \text{Since, } E = i\hbar\frac{\partial}{\partial t}.$$

$$\text{or,} \quad <E> = \int_0^a \Psi^*\left(i\hbar\frac{\partial}{\partial t}\right)\Psi_0\, e^{\frac{-E_0 t}{\hbar}}\, dx$$

Where, to describe the particle in the infinite potential well is described by $\Psi = Ae^{\frac{-iEt}{\hbar}}$

i.e., $\Psi = A \sin \omega t = Ae^{-i\omega t} = Ae^{\frac{-iEt}{\hbar}}$ for $E = \hbar\omega$.

$$\therefore \quad \langle E \rangle = \int_0^a \Psi^* (i\hbar) \left(\frac{-iE_0}{\hbar}\right) \Psi_0 e^{\frac{-iE_0 t}{\hbar}} dx$$

$$\text{or,} \quad \langle E \rangle = \int_0^a E_0 \Psi^* \Psi \, dx$$

$$\therefore \quad \langle E \rangle = \langle E_0 \rangle$$

Thus, when the particle is in the ground state, the Eigen function associated with the particle was given by $\Psi = A\, e^{\frac{-iE_0 t}{\hbar}}$. In this case the particle has a well defined energy $E = E_0$. We expect that the average value will be the actual value.

2. **Calculate the normal Zeeman splitting of the calcium 4226 Å line when the atoms are placed in a magnetic field of 1.2 Tesla.** ***[TU Microsyllabus 2074, W; 21.1]***

Solution:

The wave length (λ) = 4226 Å = 4226×10^{-10}m.
Magnetic field (B) = 1.2T
Normal Zeeman splitting ($d\lambda$) = ?

We know that, $|dE| = \Delta E = \frac{|e|}{2m} B\hbar$

$$|dE| = \frac{1.6 \times 10^{-19} \times 1.2 \times 1.05 \times 10^{-34}}{2 \times 9.1 \times 10^{-31}} \qquad \text{Where, } \hbar = \frac{h}{2\pi}$$

$$dE = 1.11 \times 10^{-23}\,\text{J} = 6.92 \times 10^{-5}\,\text{eV}.$$

We know that, $E_{photon} = h\upsilon = \frac{hc}{\lambda}$

$$dE = -\frac{hc}{\lambda^2} d\lambda$$

$$|dE| = \frac{hc}{\lambda^2} |d\lambda|$$

$$\text{So,} \quad |d\lambda| = \frac{\lambda^2}{hc} |dE|$$

$$|d\lambda| = \frac{4.226 \times 10^{-7} \times 1.11 \times 10^{-23}}{6.63 \times 10^{-34} \times 3 \times 10^{8}} = 9.96 \times 10^{-12}\,\text{m} = 0.0996\text{Å}$$

Thus, The normal Zeeman splitting $|d\lambda|$ = 0.0996Å.

3. **A beam of Hydrogen atom is used in Stern-Gerlach type experiment. The atom emerges from the oven with a velocity 10^4 m/sec. They enter a region 20 cm long where there is a magnetic field gradient 3×10^4T/m. The field gradient is perpendicular to the incident velocity of the atoms. The mass of the Hydrogen atom is 1.67×10^{-27} kg. What is the separation of the two components of the beam as they emerge from the magnet?**

[TU Microsyllabus 2074, W; 21.2]

Solution:

Velocity of atom (v) = 10^4m/sec

Magnetic field gradient $\left(\frac{dB}{dz}\right) = 3 \times 10^4$ J/m.

Mass of Hydrogen atom (m) = 1.67×10^{-27}kg.

Separation of the two components of beam = ?

In the ground state, Hydrogen atom has no net orbital magnetic dipole moment. The only dipole moment is the one associated with the spin of the electron in the 1s state, that is $(\mu_s) = -\frac{|e|}{m}\vec{s}$

So, from Stern-Gerlach experiment

$\therefore\ F_z = \mu_s \frac{dB}{dz} = -\frac{|e|}{m} S_z \frac{dB}{dz} = \pm\frac{1}{2}\frac{|e|}{m}\hbar\frac{dB}{dz}$ Since, m = mass of element

Using Newton's Second law, $F_z = a_z m_{atom}$

or, $a_z = \frac{F_z}{m_{atom}} = \frac{|e|\hbar\frac{dB}{dz}}{2m\ m_{atom}}$

$\therefore\ a_z = \frac{1.60 \times 10^{-19} \times 1.05 \times 10^{-34} \times 3 \times 10^4}{2 \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-27}} = 1.65 \times 10^8\ m/sec^2$

The deflection of each component in the direction of the force (z -axis) will be

$\Delta z = \frac{1}{2} a_z t^2$ Where, t is the time that the atom spend in the magnet.

This time can be found by dividing the length of the magnet by the incident velocity of the atoms.

So, $t = \frac{0.20\ m}{10^4 m/sec} = 2 \times 10^{-5}$ sec.

Therefore, $\Delta z = \frac{1}{2} \times 1.65 \times 10^{10} \times 4 \times 10^{-10}$

$\Delta z = 3.3 \times 10^{-2}\ m = 3.3$ cm.

The two possible values for m_s. Some atoms will be deflected upward and some downward. Therefore, the separation between the two components of the beam will be $2\Delta z$. So, 2×3.3 cm = 6.6 cm.

4. **Show by direct substitution into the time dependent Schrodinger equation for the free particle, that $\psi(x, t) = A \cos(kx - \omega t)$ is not a solution.** *[TU Microsyllabus 2074, P;20.1]*

Solution:

Here is given, Wave function $\psi(x, t) = A \cos(kx - \omega t)$ ... (1)

We know that, time dependent Schrödinger wave equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x, t)}{\partial x^2} = i\hbar\frac{\partial\psi(x, t)}{\partial t} \quad ...(2)$$

From equation (1) and (2), we get

$$\frac{-\hbar^2}{2m}\frac{\partial^2[A\cos(kx - \omega t)]}{\partial x^2} = i\hbar\frac{\partial[A\cos(kx - \omega t)]}{\partial t}$$

$$\frac{\hbar^2 k^2}{2m} A\cos(kx - \omega t) = i\hbar(\omega) A\sin(kx - \omega t)$$

$$\frac{\hbar^2 k^2}{2m}\psi(x, t) = i\omega\hbar A\sin(kx - \omega t) \quad \textbf{(not satisfied)}$$

Hence, $\psi(x, t) = A\cos(kx - \omega t)$ is not a solution of time dependent Schrodinger equation for the free particle.

5. **For a free Quantum particle show that the wave function $\Psi(x, t) = A \cos kx\ e^{-i\omega t}$ satisfies the time dependent Schrodinger equation.** *[TU Microsyllabus 2074, P; 20.2 and TU Model 2074]*

Solution

Here,wave function $\Psi(x, t) = A\cos kx\ e^{-i\omega t}$ ...(1)

We know that, time dependent Schrodinger wave equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = i\hbar\frac{\partial\Psi(x, t)}{\partial t} \quad ...(2)$$

From equation (1) and (2), we get

$$-\frac{\hbar^2}{2m}\frac{\partial^2(A\cos kx\ e^{-i\omega t})}{\partial x^2} = i\hbar\frac{\partial(A\cos kx\ e^{-i\omega t})}{\partial t}$$

or, $\frac{\hbar^2 k^2}{2m}\Psi(x, t) = \hbar\omega\,\Psi(x, t)$

or, $\frac{p^2}{2m}\Psi(x, t) = E\Psi(x, t)$

...(3) Since, $p = \hbar k$ and $E = \hbar\omega$

Again, time dependent Schrodinger wave equation can be written as,

$$E\Psi(x, t) = H\Psi(x, t) \quad \ldots(4)$$

Hence, equation (3) and (4) are same so the given wave function satisfies the time dependent Schrodinger equation.

6. Explain, why the following Eigen function are not acceptable solution of the Schrodinger equation. *[TU Microsyllabus 2074, P; 20.3]*

a. $\chi(x) = 0$ for $x \le 0$

$\chi(x) = A\cos kx$ for $x \ge 0$

b. $\chi(x) = A\dfrac{e^{iKx}}{x}$

c. $\chi = A\,\ell nkx$

Solution:

Here is given, wave functions are as following;

a. $\chi(x) = 0$ for $x \le 0$

$\chi(x) = A\cos kx$ for $x \ge 0$

b. $\chi(x) = A\dfrac{e^{iKx}}{x}$

c. $\chi = A\,\ell nkx$

We know that, time independent Schrodinger wave equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\chi}{\partial x^2} = E\chi \qquad \text{For potential energy } E_p = 0$$

or, $$\frac{\partial^2\chi}{\partial x^2} + \frac{2m}{\hbar^2}(E - E_p)\chi = 0 \qquad \ldots (1) \text{ For certain potential } E_p$$

Then, those solutions are acceptable which satisfied equation (1).

For (a):

$\chi(x) = 0$ for $x \le 0$. It is not a solution of S.W.E because S.W.E. has finite wave function as a solution not zero and there is no meaning.

For (a) $\chi(x) = A\cos kx$ for $x \ge 0$

Equation (1), will be $$\frac{d^2(A\cos kx)}{dx^2} + \frac{2m(E - E_p)\chi}{\hbar^2} = 0$$

$$-k^2A\cos kx + \frac{2m(E - E_1)\chi}{\hbar^2} = 0$$

$$-k^2\chi + \frac{2m}{\hbar^2}.(E - E_p)\chi = 0$$

Again, for (b) $\chi = \dfrac{A\,e^{ikx}}{x}$

Here, $$\frac{d\chi}{dx} = \frac{A\,ike^{ikx}}{x} + A\,e^{ikx}\left(\frac{-1}{x^2}\right)$$

$$\frac{d^2\chi}{dx} = \frac{A\,(ik)^2\,e^{ikx}}{x} + A\,ik\,e^{ikx}\left(\frac{-1}{x^2}\right) - \frac{A\,ike^{ikx}}{x^2} + A\,e^{ikx}(-)(-2)\frac{1}{x^3}$$

$$= \frac{-Ak^2\,e^{ikx}}{x} - \frac{2A\,ik}{x^2}e^{ikx} + \frac{2Ae^{ikx}}{x^3}$$

$$= -k^2\chi - \frac{2\,ik\,\chi}{x} + \frac{2\chi}{x^2}$$

The Schrodinger wave equation,

$$\frac{d^2\chi}{dx^2} + \frac{2m}{\hbar^2}(E - E_p)\chi = 0$$

or, $$-k^2\chi - \frac{2ik\chi}{x} + \frac{2\chi}{x^2} + \frac{2m}{\hbar^2}(E - E_p)\chi = 0$$

i.e., $-k^2 - \dfrac{2ik}{x} + \dfrac{2}{x^2} + \dfrac{2m}{\hbar^2}(E - E_p) = 0$ has no solution and meaningless.

c. For $\chi = A\,\ell nkx$. Then,

$$\frac{d\chi}{dx} = \frac{d(A\,\ell nkx)}{dx} = \frac{Ad\,\ell nkx}{d(kx)}\,\frac{d(kx)}{dx}$$

$$= Ak\frac{1}{kx}$$

$$= \frac{A}{x} \text{ and } \frac{d^2\chi}{dx^2} = -\frac{A}{x^2}$$

Now, Schrodinger wave equation,

$$\frac{d^2\chi}{dx^2} + \frac{2m}{\hbar^2}(E - E_p)\chi = 0$$

$$-\frac{A}{x^2} + \frac{2m}{\hbar^2}(E - E_p)\chi = 0$$

Hence, given Eigen functions are not acceptable solution of the Schrodinger equation because they does not satisfied in the equation which are discussed above.

7. What is the probability of finding a particle in a well of width 'a' at a position $\frac{a}{4}$ from the wall if n = 1, if n = 2, if n = 3. Use the normalized wave function, $\psi(x, t) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}\,e^{\frac{-iEt}{\hbar}}$. *[TU Microsyllabus 2074, P; 20.12]*

Solution:

Here is given, width of well = a

Position $(x) = \dfrac{a}{4}$

Normalized wave function $\psi(x, t) = \sqrt{\dfrac{2}{a}}\sin\dfrac{n\pi x}{a}\,e^{\frac{-iEt}{\hbar}}$

We know that, probability of finding a particle, $P = \psi^*\psi$

Then, at $x = \dfrac{a}{4}$

$$P = \left(\sqrt{\frac{2}{a}}\sin\frac{n\pi}{a}\cdot\frac{a}{4}e^{\frac{+iEt}{\hbar}}\right)\left(\sqrt{\frac{2}{a}}\sin\frac{n\pi}{a}\cdot\frac{a}{4}e^{\frac{-iEt}{\hbar}}\right)$$

$$P = \frac{2}{a}\sin^2\frac{n\pi}{4} \qquad \ldots (1)$$

If $n = 1$, $P_1 = \dfrac{2}{a}\sin^2\dfrac{\pi}{4} = \dfrac{2}{a}\left(\dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{1}{a}$

For $n = 2$, $P_2 = \dfrac{2}{a}\sin^2\dfrac{2\pi}{4} = \dfrac{2}{a}$

And for $n = 3$, $P_3 = \dfrac{2}{a}\sin^2\dfrac{32\pi}{4} = \dfrac{2}{a}\sin^2 135° = \dfrac{2}{a}\cdot\left(\dfrac{1}{\sqrt{2}}\right)^2$

$\therefore \quad P_3 = \dfrac{1}{a}$

Hence, the probability of finding a particle in a well of width of a position $x = \frac{a}{4}$ form the wall for n = 1, n = 2 and n = 3 are $\frac{1}{a}$, $\frac{2}{a}$ and $\frac{1}{a}$ respectively.

8. In the Bohr Model of hydrogen atom, the electron is assumed to move in circular orbits around the proton, that is motion takes place in a plane that we call the any plane. Use the uncertainty principle in z-direction, i.e. $\Delta P_z \geq \hbar$ and the fact that $\overline{P}_z^2 \geq (\Delta P_z)^2$ to show that the motion of the electron cannot be planar motion. *(TU Microsyllabus 2074, P. 21.3)*

Solution:

According to uncertainity principle position of a particle is more accurately (i.e., smaller Δx), the momentum is less accurately (i.e., larger Δp) and vice-varsa. If the particle is fully move in xy-plane the uncertainty in z is zero. i.e., $\Delta z = 0$ but the uncertainity principle suggest that,

$$\Delta z \, \Delta P_z \geq \hbar$$

It will be violated if Δz is zero and ΔP_z is finite. Therefore Δz should be greater than zero it means the motion of the particle cannot be planar.

9. (a) How many atomic states are there in Hydrogen with n = 3?

(b) How are they distributed among the sub shells? Label each state with appropriate set of Quantum numbers n, ℓ, m_ℓ, m_s.

(c) Show that the number of states in a shell, that is, states having the same n, is given by $2n^2$. *[TU Microsyllabus 2074, P; 21.6 and TU Exam 2074]*

Solution:

Here is given,

a. Principle Quantum number (n) = 3

Number of atomic states $= \frac{n(n+1)}{2} = \frac{3(3+1)}{2} = 6$

They are

1s

2s 2p

3s 3p 3d

b.

SN	State	n	ℓ	m_ℓ	m_s	No. of states in shell
1	For 1s state	1	0	0	$\frac{1}{2}$	1
2	For 2s state	2	0	0	$\frac{1}{2}$	1
3	For 2p state	2	1	0, ±1	$\frac{1}{2}$	3
4	For 3s state	3	0	0	$\frac{1}{2}$	1
5	For 3p state	3	1	0, ±1	$\frac{1}{2}$	3
6	For 3d state	3	2	0, ±1, ±2	$\frac{1}{2}$	5

c. Now, number of states in shell having Quantum number $1s^2$, $2s^2$, $2p^6$, $3s^2$ $3p^6$ i.e., 18.

Fundamental of solid-state physics

1. **Consider a copper wire of cross-section area 1 mm^2 carrying a current 1A. What is the drift velocity of the electron? The density and molecular weight of Cu are 9 gm/cm^3 and 64 g/mole respectively.** *[TU Microsyllabus 2074, W; 23.1]*

Solution:

Area of Copper wire (A) = 1 $mm^2 = 10^{-6} m^2$ | Current (I) = 1A

Density of copper (ρ) = 9 gm/cm^3 | Molecular weight = 60 gm/mol.

Drift velocity of electron (V_d) = ?

We know that,

$$V_d = \frac{J}{Nq}$$

The current density $J = \frac{I}{A} = \frac{1}{10^{-6}} = 10^6 \text{ Amp}/m^2$.

For copper (monovalent), the number of free electrons per unit volume N is equal to the number of atoms Per unit volume N_{atoms}. So that,

$$N_{atoms} = \text{Number of moles}/cm^3 \times \text{number of atoms/mole}$$

Where, the number of atoms per mole is given by Avogadro's number $N_A = 6.02 \times 10^{23}$ atoms/mole. thus,

$$N = N_{atoms} = \frac{9 \text{ gm}/cm^3}{64 \text{ gm/mole}} \times (6.02 \times 10^{23} \text{ atoms/mole})$$

$$= 8.4 \times 10^{22} \text{ atoms}/cm^3 = 8.4 \times 10^{28} \text{ atoms}/m^3$$

$$\therefore V_d = \frac{10^6}{8.4 \times 10^{28} \times 1.6 \times 10^{-19}} = 7 \times 10^{-5} \text{ m/sec.}$$

Thus, required drift velocity (V_d) = 7×10^{-5} m/sec.

2. **Consider a current carrying copper wire, the number of free electrons in copper is 8.4×10^{28} electron/m^3. (a) Calculate the Fermi energy for Cu (b) At what temperature, T_f will the average thermal energy K_BT_F of a gas be equal to that energy?** *[TU Microsyllabus 2074, W; 23.2]*

Solution:

Number of free electron in copper (N) = 8.4×10^{28} electron/m^3

Mass of electron (m) = 9.1×10^{-31} kg

a. We know that, the expression of Fermi energy at T = 0K is

$$E_f(0) = \frac{\hbar^2}{2m}(3N\pi^2)^{2/3}$$

$$E_f(0) = \frac{1.05 \times 10^{-34}}{2 \times 9.1 \times 10^{-31}}(3 \times 8.4 \times 10^{28} \times \pi^2)^{2/3}$$

$$E_f(0) = 11.1 \times 10^{-19} J = 6.95 \text{ eV}$$

b. We know that $E_f(0) = K_BT_f$. Where, K_B = Boltzmann constant = 1.38×10^{-23} J/k and T_f = Fermi Temperature. Thus,

$$T_f = \frac{E_f(0)}{K_B} = \frac{11.1 \times 10^{-19} J}{1.38 \times 10^{-23} J/k} = 80{,}500 \text{ K}$$

Hence, the required Fermi energy and Fermi temperature are 6.95 eV and 80,500 K respectively.

3. **The electrical conductivity of Cu at room temperature is $5.9 \times 10^7 \Omega^{-1} m^{-1}$. The Fermi energy for copper is 6.95 eV and carrier density 8.4×10^{28} electrons/m^3. Calculate the mean free path of the electrons.**

Solution:

Electrical conductivity (σ) = $5.9 \times 10^7 \Omega^{-1} m^{-1}$ | Fermi energy (E_f) = 6.95 eV

Carrier density (N) = 8.4×10^{28} electons/m^3 | Mean free path (l) = ?

Fermi velocity can be obtained by $(E_f) = \frac{1}{2}mv_f^2$

$$v_f = \left(\frac{2E_f}{m}\right)^{\frac{1}{2}} = \left(\frac{2 \times 6.95 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}\right)^{\frac{1}{2}} = 1.56 \times 10^6 \text{ m/sec}$$

Again, we have the relation of electric conductivity

$$\sigma = \frac{Nq^2\tau}{m}$$

$$\text{or, } \tau = \frac{\sigma m}{Nq^2} = \frac{5.9 \times 10^7 \times 9.1 \times 10^{-31}}{8.4 \times 10^{28} \times (1.6 \times 10^{-19})^2}$$

$$\therefore \tau = 2.50 \times 10^{-14} \text{ sec}$$

We know that, the mean free path

$$l = v_f\tau = 1.56 \times 10^6 \times 2.50 \times 10^{-14}$$

$$\therefore l = 3.90 \times 10^{-8} \text{ m} = 390 \text{Å}$$

Hence, required mean free path of electrons is 390 Å.

4. **The electrical conductivity of copper at 300 K is $5.9 \times 10^7 \Omega^{-1}m^{-1}$ (a) What is the thermal conductivity at room temperature (b) What is the thermal conductivity at 1000 K?**

Solution:

The electrical conductivity at temperature 300 K is (σ) = $5.9 \times 10^7 \Omega^{-1}m^{-1}$

Thermal conductivity at room temperature (K_1) = ?

a. We know that Wiedemann Frantz law, $\frac{K}{\sigma T} \propto L = 2.45 \times 10^{-8} W\Omega/deg^2$

$$\frac{K_1}{\sigma T} = \frac{\pi^2}{3}\left(\frac{K_B}{e}\right)^2 = 2.45 \times 10^{-8} w\Omega/deg^2 = L \text{ (say)}$$

$$K_1 = \sigma_1T_1L = 5.9 \times 10^7 \times 300 \times 2.45 \times 10^{-8} = 407.1 \text{ W/mk.}$$

b. We know that,

$$\frac{\sigma_1}{\sigma_2} = \frac{T_2}{T_1} \qquad \text{Since, } \sigma \propto \frac{1}{T}$$

$$\frac{\sigma_{300}}{\sigma_{1000}} = \frac{1000}{300}$$

$$\text{or, } \sigma_{1000} = \frac{300 \times \sigma_{300}}{1000}$$

$$\therefore \sigma_{1000} = \frac{300 \times 5.9 \times 10^7}{1000} = 1.77 \times 10^7 \Omega^{-1}m^{-1}$$

Thus, $K_2 = \sigma_2 T_2 L = 1.77 \times 10^7 \times 1000 \times 2.45 \times 10^{-8}$

$\therefore K_2 = 41.92$ w/mk

Hence, required thermal conductivity at 300 K and 1000 K are 407.1 W/mK and 41.92 W/mK respectively.

5. **Copper has a face-centered cubic structure with a one-atom basis. The density of copper is 8.96 gm cm^{-3} and its atomic weight is 63.5 g $mole^{-1}$. What is the length of the unit cube of the structure?** *[TU Microsyllabus 2074 P; 21.1 TU Exam 2074]*

Solution:

Here is given, density of copper (ρ) = 8.96 g cm^{-3}

Number of atom per unit cell (N) = 4 | Since, Fcc structure

Avargado number (N_A) = 6.023×10^{23} atoms $mole^{-1}$

Length of the unit cube (a) = ?

Atomic weight of copper (m) = 63.5 g $mole^{-1}$

We know that,

$$\frac{N}{V} = \frac{\rho N_A}{m}$$

$$\text{or, } \frac{N}{a^3} = \frac{\rho N_A}{m} \qquad \text{Since, volume of unit cell} = a^3$$

$$\text{or, } a^3 = \frac{Nm}{\rho N_A} = \frac{4 \times 63.5}{8.96 \times 6.023 \times 10^{23}}$$

$$\text{or, } a^3 = 4.71 \times 10^{-23}$$

$$\text{or, } a = 3.61 \times 10^{-7} \text{ cm}$$

$$\therefore a = 3.61 \text{ Å} \qquad \text{Since, } 10^{-8} \text{ cm} = 1 \text{ Å}$$

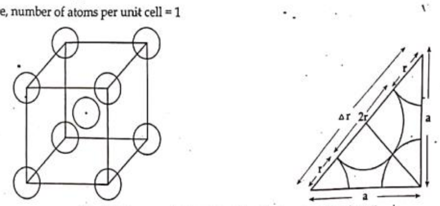
Hence, the required length of the unit cubic of the structure is 3.61 Å.

6. **Assuming that atoms in a crystal structure are arranged in close-packed spheres, what is the ratio of the volume of the atoms to the volume available for the simple cubic structure? Assume a one-atom basis.** *[TU Microsyllabus 2074, P; 22.3]*

Solution:

Each corner atom in a cubic unit cell is shared by a total number of eight unit cells so that each corner atom contributes only $\frac{1}{8}$ of its effective part to a unit cell. Since, there are in all 8 corner atoms their total contribution is equal to $\frac{8}{8} = 1$

Therefore, number of atoms per unit cell = 1



[Fig. 66: Representation of simple cubic structure of unit cell]

From figure 66,

$a = 2r$

or, $r = \frac{a}{2}$

Therefore, Volume occupied by the atom in the unit cell $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$

Volume of unit cell $= a^3$

Thus, Packing fraction $= \frac{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}{a^3} = \frac{\pi}{6} = 0.52 = 52\%$

Therefore, 52% space of unit cell is occupied by the atoms. Here the atoms are loosely packed. Only Polonium at a certain temperature is known to exhibit such a structure. For example, KCl which has alternate ions of K and Cl also behaves like a simple lattice as regards scattering of X-rays because the two ions are almost identical.

7. Assuming that atoms in a crystal structure are arranged in close-packet spheres, what is the ratio of the volume of the atoms to the volume available for the body-centered cubic structure? Assume a one-atom basis. *[TU Microsyllabus 2074, P; 22.4]*

Solution:

For a body cantered unit cell, the atomic radius can be calculated from figure 67 as follows. From figure,

$AH = 4r$ and $DH = a$

From the triangle AHD,

$AH^2 = AD^2 + DH^2$ ...(1)

From the triangle ABD

$AD^2 = AB^2 + BD^2$

$AD^2 = a^2 + a^2$ ... (2)

Substituting equation (2) in (1), we get

$AH^2 = 2a^2 + a^2$

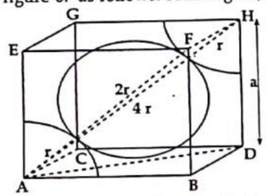
or, $AH^2 = 3a^2$

or, $(4r)^2 = 3a^2$

or, $16r^2 = 3a^2$

or, $r^2 = \frac{3a^2}{16}$

or, $r = \frac{\sqrt{3}}{4}a$



[Fig. 67: Representation of body centred cubic structure]

Packing Fraction $= \frac{\text{Number of atoms present per unit cell} \times \text{Volume of atom}}{\text{Volume of the Unit Cell}}$

Number of atoms per unit cell = 2

Volume of 2 atoms, $v = 2 \times \frac{4}{3}\pi r^3$

Side of the unit cell, $a = \frac{4r}{\sqrt{3}}$ Since, atomic radius, $r = \frac{\sqrt{3}}{4}a$

Volume of the unit cell, $V = a^3$

Packing fraction (P.F.) $= \frac{v}{V} \times 100\%$

Substituting for v and V, we have

$\therefore$ P.F. $= \frac{2 \times \frac{4}{3}\pi r^3}{a^3} \times 100\%$

$= \frac{2 \times \frac{4}{3}\pi \times \left(\frac{\sqrt{3}}{4}\right)^3 a^3}{a^3} \times 100\%$

$= 68\%$

Hence, we can say that 68% volume of the unit cell of Bcc is occupied by atoms and remaining 32% volume is vacant. Thus the Packing Density is 68%. Since the packing density is greater than simple than cubic, it has tightly packed structure, when compared to Sc.

8. Assuming that atoms in a crystal structure are arranged in close-packet spheres, what is the ratio of the volume of the atoms to the volume available for the face centered cubic structure? Assume a one-atom basis. *[TU Microsyllabus 2074, P; 22.5]*

Solution:

For a body centred cubic unit cell, the atomic radius can be calculated from figure 68 as follows.

Consider the triangle ABC,

$AC^2 = AB^2 + BC^2$

or, $(4r)^2 = a^2 + a^2$

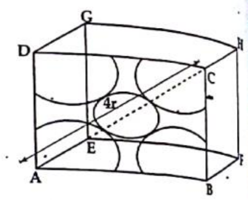
or, $16r^2 = 2a^2$

or, $r^2 = \frac{2a^2}{16}$

Taking square root on both sides, we have

$\sqrt{r^2} = \frac{\sqrt{2a^2}}{\sqrt{16}}$

$\therefore$ $r = \frac{a\sqrt{2}}{4}$



[Fig. 68: Representation of face centred cubic structure]

Volume of 4 atoms (v) $= 4 \times \frac{4}{3}\pi r^3$

Side of the unit cell (a) $= \frac{4r}{\sqrt{2}}$ Since, atomic radius, $r = \frac{\sqrt{2}}{4}a$

Volume of the unit cell (V) $= a^3 = \left(\frac{4r}{\sqrt{2}}\right)^3$

Packing fraction (P.F.) $= \frac{v}{V} \times 100\%$

Substituting for v and V, we have

P.F. $= \frac{4 \times \frac{4}{3}\pi r^3}{\left(\frac{4r}{\sqrt{2}}\right)^3} \times 100\%$

or, P.F. $= \frac{16}{3}\pi r^3 \times \frac{2\sqrt{2}}{64r^3} \times 100\%$

$\therefore$ °P.F. $= \frac{\pi\sqrt{2}}{6} = 74\%$

Hence, 74% of the volume of an Fcc unit cell is occupied by atoms and the remaining 26% volume of the unit cell is Vacant. Thus the packing density is 74%. Since the packing density is very high, the Fcc structure has closely (or) tightly packed structure.

9. The dissociation energy of KF molecule is 15.12 eV. The ionization energy of K is 4.34 eV, and the electron affinity of F is 4.07 eV. What is the equilibrium separation constant for the KF molecule? *[TU Microsyllabus 2074, P; 22.9]*

Solution:

Here is given, dissociation energy of KF molecule (D.E.) = 5.12 eV

Ionization energy of K (I.E.) = 434 eV

Electron affinity of F (E.A.) = 4.07 eV

Equilibrium separation constant (r) = ?

Here, to ionize K-atom, energy of 4.34 eV is provided, i.e.,

$K + 4.34\text{ eV} \rightarrow K^+ + e^-$

On the other hand, when F captures one electron, the energy released is, 4.07 eV

$F + e^- \rightarrow F^- + 4.07\text{ eV}$

Here, net energy released (dissociation energy)

= E.A. – Coulomb attraction P.E. – I.E.

5.12 eV = 4.07 eV – Coulomb attraction P.E. – 4.34 eV

Coulomb attraction P.E. = – (5.12 eV + 4.34 eV – 4.07 eV)

$$= -5.39 \text{ eV}$$

$$= -\frac{e^2}{4\pi\epsilon_0 r}$$

or, $$-5.39 \text{ eV} = -\frac{9\times10^9\times(1.6\times10^{-19})^2}{r}$$

or, $$5.39\times1.6\times10^{-19} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{r}$$

∴ $r = 2.675\times10^{-10}$ m = 2.675 Å

Hence, the equilibrium separation constant for the KF molecule is 2.675 Å.

10. The energy gaps of some alkali halides are KCl = 7.6 eV, KBr = 6.3 eV, KI = 5.6 eV. Which of these are transparent to visible light? At what wave length does each become opaque?

[TU Microsyllabus 2074, P; 24.6]

Solution:

Given, energy gaps of some alkali halides are;

Energy gap of KCl, $(E_g)_{KCl}$ = 7.6 eV

Energy gap of KBr, $(E_g)_{KBr}$ = 6.3 eV

and energy gap of KI, $(E_g)_{KI}$ = 5.6 eV

We know that, the corresponding emission wavelengths are

$$\lambda = \frac{hc}{E_g}$$

$$\lambda_{KCl} = \frac{6.62\times10^{-34}\times3\times10^8}{7.6\times1.6\times10^{-19}}$$

$$= 1.633\times10^{-7} \text{ m} = 163.3 \text{ nm}$$

Again,

$$\lambda_{KBr} = \frac{6.62\times10^{-34}\times3\times10^8}{6.3\times1.6\times10^{-19}} = 193.0 \text{ nm}$$

And, $$\lambda_{KI} = \frac{6.62\times10^{-34}\times3\times10^8}{5.6\times1.6\times10^{-19}} = 221.7 \text{ nm}$$

Hence, $\lambda_{visible} > \lambda_{KCl}, \lambda_{KBr}$ and λ_{KI} i.e., E_{gap} of all alkali halides is very much greater than $E_{visible\ light}$ i.e., (2.7 eV to 1.6 eV)

Therefore, all color photons are transmitted through them with no absorption hence they are transparent to visible light.

If λ_{KCl}, λ_{KBr} and λ_{KI} equivalent to (380 – 750) nm they gets absorbed and each become opaque.

Note:

Visible light	
Color	Wavelength (λ) 'nm'
V = I	380 – 450
B	450 – 495
G	495 – 570
Y	570 – 590
O	590 – 620
R	620 – 750

11. The density of aluminum is 2.70 gmcm^{-3} and its molecular weight is 26.98 gmole^{-1} (a) Calculate the Fermi Energy (b) If the experimental value of E_F is 12 eV, what is the electron effective mass in aluminum? Aluminum is trivalent. *[TU Microsyllabus 2074, P; 24.8]*

Solution:

Here is given,

Density of aluminum (ρ) = 2.70 gm cm^{-3}

Molecular weight of Aluminum (m) = 26.98 gm mole^{-1}

a. Fermi energy (E_F) = ?

b. Fermi energy (E_F') = 12 eV = $12\times1.6\times10^{-19}$ J

Effective mass (m_e^*) = ?

We know that,

Number of free electrons per unit volume

$$N = \frac{v\rho N_A}{m}$$

Where,

v = valency of atom

N_A = Avogardro's constant

i.e. $$N = \frac{3\times2.70\times(6.02\times10^{23})}{26.98} = 1.807\times10^{23} \text{ electrons cm}^{-3}$$

$$= 1.807\times10^{29} \text{ electrons m}^{-3}$$

Now, we have relation

$$E_F = \frac{\hbar^2}{2m_e}(3N\pi^2)^{2/3}$$

$$= \frac{\left(\frac{6.626\times10^{-34}}{2\pi}\right)^2}{2\times9.1\times10^{-31}}(3\times1.807\times10^{29}\times\pi^2)^{2/3}$$

$$= 1.867\times10^{-18} \text{ J}$$

$$= \frac{1.867\times10^{-18}}{1.6\times10^{-19}} \text{ eV}$$

∴ E_F = 11.66 eV

b. Again, for m_e^*

$$E_F' = \frac{\hbar^2}{2m_e}(3N\pi^2)^{2/3}$$

$$m_e^* = \frac{\hbar^2}{2E_F'}(3N\pi^2)^{2/3}$$

$$= \frac{\left(\frac{6.62\times10^{-34}}{2\pi}\right)^2}{2\times12\times1.6\times10^{-19}}(3\times1.807\times10^{29}\times\pi^2)^{2/3}$$

$$m_e^* = 8.847\times10^{-31} \text{ kg}$$

∴ $m_e^* = 0.97\, m_e$

Hence, required Fermi energy and electron effective mass in Aluminum are 11.66 eV and 0.97 m_e respectively.

1. The energy gap in Silicon is 1.1 eV. The average effective mass is 0.31m_e. Calculate the electron concentration in the conduction band of Silicon at room temperature 300k. Assume, $E_f = \frac{E_g}{2}$. *[TU Microsyllabus 2074, W; 25.1]*

Solution:

The energy gap in Silicon (E_g) = 1.1 eV

The average effective mass (m^*) = 0.31m_e

Electron concentration (n) = ?

We know that, the electron concentration of Silicon Semiconductor

$$n = N_c e^{\frac{E_f - E_g}{K_B T}} \text{ or } N_c e^{\frac{-(E_g - E_f)}{K_B T}} \qquad \text{Where, } N_c = 2\left(\frac{m_e K_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$$

$$\text{or, } n = 2\left(\frac{m^* K_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} e^{\frac{E_f - E_g}{K_B T}} \qquad \text{Since, } E_c = E_g \text{ when } E_v = 0$$

$$n = 2\left\{\frac{0.31 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{2\pi \times (1.05 \times 10^{-34})^2}\right\}^{\frac{3}{2}} \times e^{\frac{-0.55 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-34} \times 300}} \qquad \text{Where, } m^* = 0.31\, m_e \text{ and } E_f = \frac{E_g}{2}.$$

$\therefore \quad n = 2.6 \times 10^{15}$ per m^3.

Hence, required electron concentration in the conduction band at room temperature is 2.6×10^{15} per m^3.

Since, electron concentration of typical metal $\approx 10^{28}$ electron $/m^3$.

2. **A sample of Si is dipped with Phosphorous. The donor impurity level lies 0.045 eV below the bottom of the conduction band. At T = 300K, E_f is 0.010 eV above the donor level. Calculate (a) the impurity concentration (b) the number of ionized impurities (c) the free electron concentration and (d) hole concentration (For Si, E_g = 1.100 eV, m_e^* = 0.31 m_e, m_h^* = 0.38m_e)**

[TU Microsyllabus 2074, W; 25.2]

Solution:

Here, we assumed energy of valence band (E_v) = 0. So, $E_g = E_c$. Since, $E_g = E_c - E_v$.

Then, Fermi energy (E_f) = 1.1 eV – 0.045 eV + 0.010 eV

$\therefore$ E_f = 1.065 eV.

a. For N_d calculation, we know that the relation

$$N_c e^{\frac{E_f - E_c}{K_B T}} = N_v^{\frac{E_v - E_f}{K_B T}} + N_d \left[1 - \frac{1}{e^{\frac{E_d - E_f}{K_B T}} + 1}\right]$$

$$\text{or,}\quad N_c e^{\frac{E_f - E_c}{K_B T}} = N_v e^{\frac{-E_f}{K_B T}} + N_d \left[1 - \frac{1}{e^{\frac{E_d - Ef}{K_B T}} + 1}\right] \qquad \text{... (1)}$$

To find, $N_c = 2\left(\frac{m_e^* K_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$

$$N_c = 2\left(\frac{2\pi\, m_e^* K_B T}{h^2}\right)^{\frac{3}{2}}$$

$$N_c = 2\left\{\frac{2 \times 3.14 \times 0.31 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.62 \times 10^{-34})^2}\right\}^{\frac{3}{2}}$$

$\therefore$ $N_c = 4.39 \times 10^{24}\,m^{-3}$. Again $N_v = 2\left(\frac{m_h^* K_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$

$$N_v = 2\left(\frac{2\pi\, m_h^* K_B T}{h^2}\right)^{\frac{3}{2}} = 2\left\{\frac{2 \times 3.14 \times 0.38 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.62 \times 10^{-34})^2}\right\}^{3/2}$$

$\therefore$ $N_v = 5.95 \times 10^{24}\,m^{-3}$.

Substituting for N_c, N_v, E_f and E_d in equation (1), we get

$$4.39 \times 10^{24}\, e^{\frac{1.065 - 1.100}{0.026}} = 5.95 \times 10^{24}\, e^{\frac{-1.065}{0.026}} + N_d \left[1 - \frac{1}{e^{\frac{-0.010}{0.026}} + 1}\right]$$

Where, $K_B T = 1.38 \times 10^{-23} \times 300 = 0.026$ eV

or, $1.13 \times 10^{24} = 9.5 \times 10^{6} + 0.41\,N_d$

$\therefore$ $N_d = 2.79 \times 10^{24}\,m^{-3}$

b. The number of ionized impurities are given by

$$N_d^+ = N_d \left[1 - \frac{1}{e^{\frac{E_d - E_f}{K_B T}} + 1}\right] = 2.7 \times 10^{24}\left[1 - \frac{1}{e^{\frac{-0.010}{0.025}} + 1}\right]$$

$\therefore$ $N_d = 1.08 \times 10^{24}\,m^{-3}$

c. The free electron concentration

$$n = N_c e^{\frac{E_f - E_c}{K_B T}} = 4.39 \times 10^{24}\, e^{\frac{(1.065 - 1.100)}{0.026}} = 1.08 \times 10^{24}\,m^{-3}$$

d. The hole concentration $P = N_v e^{\frac{E_v - E_f}{K_B T}} = 5.95 \times 10^{24}\, e^{\frac{-1.065}{0.026}} = 1.88 \times 10^{6}\,m^{-3}$

Hence, required impurity concentration, number of ionized impurities, free electron concentration and hole concentration are $2.79 \times 10^{24} m^{-3}$, $1.08 \times 10^{24}\,m^{-3}$, $1.08 \times 10^{24}\,m^{-3}$ and $1.88 \times 10^{6}\,m^{-3}$ respectively.

3. The band gap in pure germanium is E_g = 0.67 eV.

a. Calculate the number of electrons per unit volume in the conduction band at 250 K, 300 K, and at 350 K.

b. Do the same for Silicon assuming E_g = 1.1 eV. The effective mass of the electrons in germanium is 0.12 m and in Silicon 0.31 m, where m is the free electron mass.

[TU Microsyllabus 2074, P; 25.1]

Solution:

Here is given, band gap in pure germanium $(E_g)_{Ge} = 0.67$ eV

Fermi energy in pure germanium $(F_F)_{Ge} = \frac{E_g}{2} = \frac{0.67}{2} = 0.33$ eV

a. Number of electrons per unit volume (N_e) = ?

$T_1 = 250$ K

$T_2 = 300$ K

$T_3 = 350$ K

We know that,

$$N_e = N_c e^{-\frac{(E_c - E_F)}{k_B T}}$$

$$= \frac{1}{4}\left(\frac{2m_e^* k_B T}{\hbar^2 \pi}\right)^{\frac{3}{2}} e^{-\frac{(E_c - E_F)}{k_B T}}$$

For germanium

$m_e^* = 0.12\, m_e$

$$(N_e)_{Ge} = \frac{1}{4}\left(\frac{2\times 9.1\times 10^{-31}\times 0.12\times 1.38\times 10^{-23}\,T}{(1.05\times 10^{-34})^2\times \pi}\right)^{\frac{3}{2}} e^{-\frac{(0.67-0.335)\times e}{1.38\times 10^{-23}T}}$$

$$= \frac{1}{4}\left(\frac{2\times 9.1\times 10^{-31}\times 0.12\times 1.38\times 10^{-23}\,T}{(1.05\times 10^{-34})^2\times \pi}\right)^{\frac{3}{2}} e^{-\frac{(0.67-0.335)\times 1.6\times 10^{-19}}{1.38\times 10^{-23}T}}$$

$$= 2.03\times 10^{20}\, T^{\frac{3}{2}}\, e^{-\frac{3.887\times 10^3}{T}}$$

Now for $T_1 = 250$ K

$(N_e)_{Ge}$ at 250 K

$$= 2.03\times 10^{20}\,(250)^{\frac{3}{2}}\, e^{-\frac{3.887\times 10^3}{250}}$$

$\therefore$ $(N_e)_{Ge} = 1.518\times 10^{17}$ electrons m^{-3}

Again at 300 K

$$(N_e)_{Ge} = 2.03\times 10^{20}\,(300)^{\frac{3}{2}}\, e^{-\frac{3.887\times 10^3}{300}}$$

$(N_e)_{Ge}$ at 300 K $= 2.63\times 10^{18}$ electrons m^{-3}

And, $(N_e)_{Ge}$ at 350 K

$$(N_e)_{Ge} = 2.03\times 10^{20}\,(350)^{\frac{3}{2}}\, e^{-\frac{3.887\times 10^3}{350}}$$

$$= 2.03\times 10^{20}\times (350)^{\frac{3}{2}}\, e^{-11.057} \qquad \text{Since, } e^{-\frac{3.887\times 10^3}{350}} = 1.577\times 10^{-5}$$

$\therefore$ $(N_e)_{Ge} = \simeq 2.097\times 10^{19}$ electrons m^{-3}

b. For Silicon

$E_g = 1.1$ eV $= 1.1\times 1.6\times 10^{-19}$ J

$E_F = 0.55$ eV $= 0.55\times 1.6\times 10^{-19}$ J

$m_e^* = 0.31\, m_e$

$(N_e)_{Si}$ at 250 K, 300 K and 350 K = ?

We know that,

$$= \frac{1}{4}\left(\frac{2m_e^* k_B T}{\hbar^2 \pi}\right)^{\frac{3}{2}} e^{-\frac{(E_c - E_F)}{k_B T}}$$

$$= \frac{1}{4}\left(\frac{2\times 9.1\times 10^{-31}\times 0.31\times 1.38\times 10^{-23}\,T}{(1.05\times 10^{-34})^2\times \pi}\right)^{\frac{3}{2}} e^{-\frac{(1.1-0.55)\times 1.6\times 10^{-19}}{1.38\times 10^{-23}T}}$$

$$= 8.445\times 10^{20}\, T^{\frac{3}{2}}\, e^{-\frac{6.382\times 10^3}{T}}$$

Now, $(N_e)_{Si}$ at 250 K

$$(N_e)_{Si}\; 8.445\times 10^{20}\,(250)^{\frac{3}{2}}\, e^{-\frac{6.382\times 10^3}{250}} = 2.73\times 10^{13} \text{ electrons m}^{-3} \qquad \left[\because e^{-\frac{6.382\times 10^3}{250}} = 8.19\times 10^{-12}\right]$$

Again, $(N_e)_{Si}$ at 300 K

$$(N_e)_{Si}\; 300\text{ K} = 8.445\times 10^{20}\,(300)^{\frac{3}{2}}\, e^{-\frac{6.382\times 10^3}{300}} = 2.53\times 10^{15} \text{ electron m}^{-3}$$

And, $(N_e)_{Si}$ at 350 K

$$(N_e)_{Si} \text{ at } 350\text{ K} = 8.445\times 10^{20}\,(350)^{\frac{3}{2}}\, e^{-\frac{6.382\times 10^3}{350}} = 6.66\times 10^{16} \text{ electrons m}^{-3}.$$

Hence, number of electrons per unit volume in the condition band for germanium and Silicon are obtained as above.

4. Suppose that the effective mass of hole in a material is four times that of electrons. At what temperature would the Fermi level shifted by 10% from the middle of the forbidden energy gap? Let, E_g = 1 eV. ***[TU Microsyllabus 2074, P; 25.2]***

Solution:

Here, given, effective mass of holes (m_h^*) = 4 × effective mass of electron (m_e^*)

Temperature (T) = ?

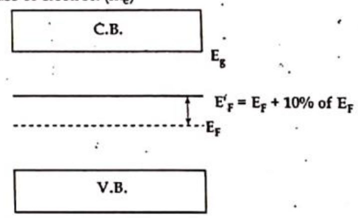
Energy gap (E_g) = 1 eV

According to question,

Fermi level shifted by 10%

$E'_F = E_F + 10\%\, E_F$

$= \frac{E_g}{2} + 10\%\, \frac{E_g}{2}$



[Fig. 47: Energy band diagram]

We know that,

$$E'_F = \frac{E_g}{2} + \frac{3}{4} k_B T\, \ell n\left(\frac{m_h^*}{m_e^*}\right)$$

or, $E_F + 10\%\, E_F = \frac{E_g}{2} + \frac{3}{4} k_B T\, \ell n\left(\frac{m_h^*}{m_e^*}\right)$

or, $0.5\times 1.6\times 10^{-19} + \frac{10}{100}\times 0.5\times 1.6\times 10^{-19} = \frac{1\times 1.6\times 10^{-19}}{2} + \frac{3k_BT}{4}\, \ell n\left(\frac{4m_e^*}{m_e^*}\right)$

or, $(0.5 + 0.05)\times 1.6\times 10^{-19} = 0.5\times 1.6\times 10^{-19} + \frac{3k_BT}{4}\, \ell n\,(4)$

or, $0.05\times 1.6\times 10^{-19} = \frac{3k_BT}{4}\, \ell n\,(4)$

or, $T = \frac{4\times 0.05\times 1.6\times 10^{-19}}{3\times 1.38\times 10^{-23}\times \ln 4}$

$= 558.058$

$\therefore$ $T \simeq 558.1$ K

Hence, at 558.1 K temperature the Fermi level shifted by 10% from the middle of the forbidden energy gap.

5. The energy gap in germanium is 0.67 eV. The electron and the hole effective masses are 0.12 m and 0.23m respectively, where m is the free electron mass. Calculate (a) the Fermi Energy, (b) the electron density, and (c) the hole density at T = 300 K. ***[TU Microsyllabus 2074, P; 25.3]***

Solution:

Here, given energy gap in germanium $(E_g)_{Ge} = 0.67$ eV $= 0.067\times 1.6\times 10^{-19}$ J

Effective mass of electron $(m_e^*) = 0.12$ m $= 0.12\times 9.1\times 10^{-31}$ kg

Effective mass of hole $(m_h^*) = 0.23$ m $= 0.23\times 9.1\times 10^{-31}$ kg

a. The Fermi energy (E_F) = ?

b. Electron Density (N_e) = ?

c. Hole density (N_h) = ?

Temperature (T) = 300 K

We know that,

a. $E_F = \frac{E_g}{2} + \frac{3}{4} k_BT \ln\left(\frac{m_h^*}{m_e^*}\right)$

$= \frac{0.67 \times 1.6 \times 10^{-19}}{2} + \frac{3}{4} \times 1.38 \times 10^{-23} \times 200 \ln\left(\frac{0.23\ m}{0.12\ m}\right)$

$= 5.562 \times 10^{-20}$ J

$= 3.472 \times 10^{-1}$ eV

$\therefore\ E_F = 0.347$ eV

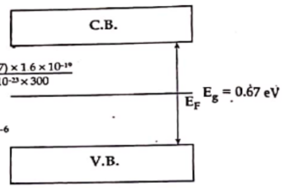
We have a relation,

b. $N_e = \frac{1}{4}\left(\frac{2m_e^* k_BT}{\hbar^2\pi}\right)^{\frac{3}{2}} e^{-\frac{(E_g - E_F)}{k_BT}}$

$= \frac{1}{4}\left(\frac{2 \times 0.12 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(1.05 \times 10^{-34})^2 \times \pi}\right)^{\frac{3}{2}} e^{-\frac{(0.67 - 0.347) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}}$

$= \frac{1}{4}\left(\frac{2 \times 0.12 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(1.05 \times 10^{-34})^2 \times \pi}\right)^{\frac{3}{2}} \times 3.478 \times 10^{-6}$

$= 3.96 \times 10^{18}\ m^{-3}$



[Fig. 48: Representation of energy gap and Fermi levels]

c. Now hole density

$N_h = \frac{1}{4}\left(\frac{2m_h^* k_BT}{\hbar^2\pi}\right)^{\frac{3}{2}} e^{-\frac{(E_g - E_F)}{k_BT}}$

$= \frac{1}{4}\left(\frac{2 \times 0.23 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(1.05 \times 10^{-34})^2 \times \pi}\right)^{\frac{3}{2}} \times 3.478 \times 10^{-6}$

$\therefore\ N_h = 1.05 \times 10^{19}$ holes m^{-3}.

Hence, required Fermi energy, electron and hole densities are 0.347 eV, 3.96×10^{18} electrons m^{-3} and 1.05×10^{19} holes m^{-3} respectively.

6. **A certain intrinsic Semiconductor has a band gap E_g is 0.2 eV. Measurement shows that it has a resistivity at room temperature 300 K of 0.3 Ωm. What would you predict its resistivity to be at 350 K?** ***[TU Microsyllabus 2074, P; 25.13]***

Solution:

Here, given band gap of Semiconductor $(E_g) = 0.2\ eV = 0.2 \times 1.6 \times 10^{-19}$ J

Initial temperature $(T_1) = 300$ K

Final temperature $(T_2) = 350$ K

Initial resistivity $(\rho_1) = 0.3\ \Omega$ m

Final resistivity $(\rho_2) = ?$

We know that, for an intrinsic Semiconductor,

$$N_i = N_e = N_h = 2\left(\frac{2\pi k_BT}{h^2}\right)^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} e^{-\frac{E_g}{2k_BT}}$$

Conductivity $(\sigma) = eN_i(\mu_e + \mu_h)$

Resistivity $(\rho) = \frac{1}{eN_i(\mu_e + \mu_h)}$

Then, $\frac{\rho_1}{\rho_2} = \frac{(N_i)_2\, e(\mu_e + \mu_h)}{(N_i)_1\, e(\mu_e + \mu_h)}$

or, $\frac{\rho_1}{\rho_2} = \frac{\left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{2\pi k_BT_2}{h^2}\right)^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} e^{-\frac{E_g}{2k_BT_2}}}{\left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{2\pi k_BT_1}{h^2}\right)^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} e^{-\frac{E_g}{2k_BT_1}}}$

or, $\frac{\rho_1}{\rho_2} = \left(\frac{T_2}{T_1}\right)^{\frac{3}{2}} e^{-\frac{E_g(T_1 - T_2)}{2k_BT_1T_2}}$

or, $\frac{0.3}{\rho_2} = \left(\frac{350}{300}\right)^{\frac{3}{2}} e^{-\frac{0.2 \times 1.6 \times 10^{-19}(300 - 350)}{2 \times 1.38 \times 10^{-23} \times 300 \times 350}}$

or, $\frac{0.3}{\rho_2} = 2.189$

or, $\rho_2 = \frac{0.3}{2.189}$

$\therefore\ \rho_2 = 0.136\ \Omega$ m

Hence, required resistivity at 350 K is 0.136 Ω m.

7. **The energy gap in Silicon is 1.1 eV, where as in diamond it is 6 eV. What conclusion can you draw about the transparency of the two materials to visible, light (4000 Å to 7000Å).** ***[TU Microsyllabus 2074, P; 25.16]***

Solution:

Here, given energy gap in Silicon $(E_g)_{Si} = 1.1\ eV = 1.1 \times 1.6 \times 10^{-19}$ J

Energy gap in diamond $(E_g)_{dia} = 6\ eV = 6 \times 1.6 \times 10^{-19}$ J

Wavelength of visible light = 4000 Å to 7000 Å

We know that,

Band gap energy, $(E_g) = \frac{hc}{\lambda_c}$

For Silicon, $(E_g)_{Si} = \frac{hc}{(\lambda_c)_{Si}}$

or, $(\lambda)_{Si} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.1 \times 1.6 \times 10^{-19}}$

$= 1.1290 \times 10^{-10} = 1129$ nm

$\therefore\ (\lambda)_{Si} = 11290$ Å

Hence all visible lights are absorbed since $(\lambda_c)_{Si} < \lambda_{visible}$ But it can transmit infrared light having wavelength $= 1.1 \times 10^{-6}$ m.

For diamond, $(\lambda_c)_{dia} = \frac{hc}{(E_g)_{dia}}$

$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{6 \times 1.6 \times 10^{-19}}$

$= 2.066 \times 10^{-7}\ m = 2066 \times 10^{-10}$ m

$\therefore\ (\lambda_c)_{dia} = 2066$ Å $= 206$ nm

$(\lambda_c)_{dia} < \lambda_{visible\ i.e.}\ E_{photon}$ (visible light) $< E_g$

Hence, in the case of diamond all visible lights are transmitted by such type of non metallic materials.

Thus, Diamond appears transparent and colourless.

8. **The current through p-n junctions is 1×10^{-8} A when a reverse bias voltage of 10 V is applied across the junction at T = 300 K. What will be the current through the diode when a forward bias voltage of (a) 0.1 V, (b) 0.3 V and (c) 0.5 V is applied?** ***[TU Microsyllabus 2074, P; 26.1]***

Solution:

Here, given current through p-n junction $(i_r) = 1 \times 10^{-8}$ A

Reverse bias voltage $(V_r) = 10$ V

Temperature (T) = 300 K

Current through p-n junction diode at different voltage

$(i_f)_{0.1V} = ?$

$(i_f)_{0.3V} = ?$

$(i_f)_{0.5V} = ?$

We know that diode equation

$$i = i_0\left[e^{\frac{|e|V_r}{k_BT}} - 1\right]$$

Where, i_0 = current associated with the flow of minority carriers (electrons) from the P side to the N side called reverse saturation current.

For reverse biased

$$i_r = i_0\left[e^{\frac{|e|V_r}{k_BT}} - 1\right]$$

or, $$1 \times 10^{-8} = i_0\left[e^{\frac{1.6\times10^{-19}\times10}{k_BT}} - 1\right].$$

$$= i_0\left[e^{\frac{1.6\times10^{-19}\times10}{1.38\times10^{-23}\times300}} - 1\right]$$

$$= i_0\,[0 - 1]$$

$\therefore\ i_0 = -1\times10^{-8}$ A.

Negative sign indicates that the net electron flow is from P side nN side.

For forward biased case

Reverse saturation current (i_0) = 1×10^{-8} A

a. Forward voltages $(V_f)_a$ = 0.1 V

Then forward current through the diode

$$(i_f)_a = i_0\left[e^{-\frac{|e|V_a}{k_BT}} - 1\right]$$

$$= 1\times10^{-8}\left[e^{-\frac{1.6\times10^{-19}\times0.1}{1.38\times10^{-19}\times300}} - 1\right]$$

$$= 1\times10^{-8}\,[47.85 - 1]$$

$$= 4.68\times10^{-7}\text{ A}$$

$$= 0.468\ \mu\text{A}$$

b. When $(V_f)_b$ = 0.3 V

Forward current $(V_f)_b$ $= i_0\left[e^{-\frac{|e|V_b}{k_BT}} - 1\right]$

$$= 1\times10^{-8}\left[e^{\frac{1.6\times10^{-19}\times0.3}{1.38\times10^{-19}\times300}} - 1\right]$$

$$= 1.078\times10^{-3}\text{ A}$$

$$= 1.08\text{ mA}$$

c. When $(V_f)_c$ = 0.5V

Forward current $(i_f)_c$ $= i_0\left[e^{-\frac{|e|V_c}{k_BT}} - 1\right]$

$$= 1\times10^{-8}\left[e^{\frac{1.6\times10^{-19}\times0.5}{1.38\times10^{-23}\times300}} - 1\right]$$

$$= 2.509\text{ A}$$

Hence, required current at forward voltage 0.1 V, 0.3 V and 0.5 V are 0.468μA, 1.08 mA and 2.509 A respectively, From above it is concluded that, if the diode is forward biased, the current increase very rapidly with increasing voltage.

9. In the ideal diode the reverse saturation current should be as small as possible. Considering the fact that E_g for Si is 1.1 eV and E_g for Ge is 0.67 eV. Which material is better suited for the fabrication of p-n junction diodes? *[TU Microsyllabus 2074, P; 26.2]*

Solution:

Here, given band gap of Si $(E_g)_{Si}$ = 1.1 eV

Band gap of Ge $(E_g)_{Ge}$ = 0.67 eV

We know that diode equation,

$$i = i_0\left(e^{\frac{|e|V_r}{k_BT}} - 1\right)$$

$$= i_0\left(e^{\frac{E_g}{k_BT}} - 1\right)$$

For Si

$$i_{Si} = i_0\left(e^{\frac{1.1\times1.6\times10^{-19}}{1.38\times10^{-23}\times T}} - 1\right)$$

Let at normal temperature T = 300 K

$$= 1\times10^{-8}\left(e^{\frac{1.1\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}} - 1\right)$$

$$i_{Si} = i_0\,[3.01\times10^{11} - 1]$$

$$= 3.01\times10^{18}\,i_0$$

Again, for Ge

$$i_{Ge} = i_0\left(e^{\frac{0.67\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}} - 1\right)$$

$$= i_0\,[1.80\times10^{11} - 1]$$

$$\simeq 1.80\times10^{11}\,i_0$$

Hence, Silicon is better suited for the fabrication of p-n junction diode, Silicon is much better for high current application as it has very high forward current whereas germanium diode have very small forward current.

10. The reverse saturation current of a Silicon diode is $i_0 = 5\times10^{-9}$ A. The voltage across the diode when forward biased is 0.45 V. (a) What is the current through the diode at T = 27°C? (b) If the voltage across the diode is held constant, and we assume that i_0 does not change with temperature, what is the current through the diode at T = 47° C? *[TU Microsyllabus 2074, P; 26.3]*

Solution:

Here, given reverse saturation current (i_0) = 5×10^{-9} A

Forward biased voltage (V) = 0.45 V

a. Current through diode at 27°C (i_{27}) = ?

Initial temperature (T_1) = 27°C = 300 k

b. Current through diode at 47°C (i_{47}) = ?

We know that diode equation,

$$i = i_0\left[e^{+\frac{|e|V}{k_BT}} - 1\right]$$

Then $$i_{27} = i_0\left[e^{\frac{1.6\times10^{-19}\times0.45}{1.38\times10^{-23}\times300}} - 1\right]$$

$$= 5\times10^{-9}\left[e^{\frac{1.6\times10^{-19}\times0.45}{1.38\times10^{-23}\times300}} - 1\right]$$

$$= 1.81\times10^{-1}$$

$$= 0.181\text{ A}$$

Again,

$$i_{47} = 5\times10^{-9}\left[e^{\frac{1.6\times10^{-19}\times0.45}{1.38\times10^{-23}\times320}} - 1\right]$$

$$= 6.11\times10^{-2}\text{ A}$$

$$= 0.0611\text{ A}$$

Hence, required current across the diode at 27°C and 47°C are 0.181 and 0.0611 A respectively.

11. In problem 10, we assumed that the reverse saturation current i_0 remains constant when the temperature changes. (a) Show that this assumption is highly incorrect by calculating i_0 at T = 47°C when $i_0 = 5 \times 10^{-9}$ A at 27°C. Assume that the Fermi level on the p-side of the junction is 1 eV below the bottom of the conduction band? (b) If the voltage across the forward biased diode is 0.45 V, as in problem 26.3, what is the current through the diode at T = 47°C? *[TU Microsyllabus 2074, P; 26.4]*

Solution:

Here is given,

Initial temperature (T_1) = 27°C = 300 K

Final temperature (T_2) = 47°C = 320 K

Saturation current at T_1, $i_o = (T = 300\ K) = 5 \times 10^{-9}$ A

Fermi level on the p-side of the junction lies below the bottom of the conduction band.

(E_1) = 1 eV = $1 \times 1.6 \times 10^{-19}$ J

Voltage across the forward biased diode = 0.45 V

Saturation current at 320 K

i_o (T = 320 K) = ?

Current through the diode at 320 K

i_o (T = 320 K) = ?

a. We know that,

Saturation current

$$(i_o)_{300K} = A\, e^{\frac{E_1}{K_B T_1}}$$

or, $5 \times 10^{-9} = Ae^{-\frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}}$

or, $A = 3.0 \times 10^{8}$ A

Now,

$$i_o\ (T = 45°C) = 3.04 \times 10^{8}\, e^{\frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}} = 5.59 \times 10^{-8}\ A$$

∴ i_o (T = 320 K) = 5.6×10^{-8} A

b. For current through the diode at 320 K

We know that,

$$i = i_o \left[e^{\frac{|e|V}{K_B T}} - 1\right]$$

$$= 5.6 \times 10^{-8} \left[e^{\frac{1.6 \times 10^{-19} \times 0.45}{1.38 \times 10^{-23} \times 320}} - 1\right]$$

∴ i = 0.674 A

Hence, from above it is concluded that saturation current varies with temperature i.e., increases with increasing temperature. The current through the diode at 47°C was found to be 0.674A.

12. The reverse saturation current of a Silicon diode doubles when the temperature changes from 27°C to 33°C. What is the position of the Fermi level on the p-side of the junction?

[TU Microsyllabus 2074, P; 26.5]

Solution:

Here is given,

Reverse saturation current at 27°C

$(i_o)_{27}$ = i (say)

Reverse saturation current at 33°C

$(i_o)_{33}$ = 2i

T_1 = 27°C = 300 K

T_2 = 273 + 33 = 306 K

Now, we know that

$i_o = A\,e^{-\frac{E_1}{K_BT}}$

Again, $(i_o)_{T_1} = Ae^{-\frac{E_1}{K_BT_1}}$

$(i_o)_{T_2} = Ae^{-\frac{E_1}{K_BT_2}}$

Now, $\dfrac{(i_o)_{T_1}}{(i_o)_{T_2}} = \dfrac{Ae^{-\frac{E_1}{K_BT_2}}}{Ae^{-\frac{E_1}{K_BT_1}}}$

or, $2 = e^{\frac{E_1}{K_B}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)}$

or, $ln\,2 = \dfrac{E_1}{K_B}\left(\dfrac{1}{T_1}-\dfrac{1}{T_2}\right)$

or, $E_1 = \dfrac{T_1T_2\,ln\,2\,K_B}{T_2 - T_1}$

$= \dfrac{306 \times 300 \times ln2 \times K_B}{306 - 300}$

$\therefore$ $E_1 = 10605.15 \times 1.38 \times 10^{-23}$

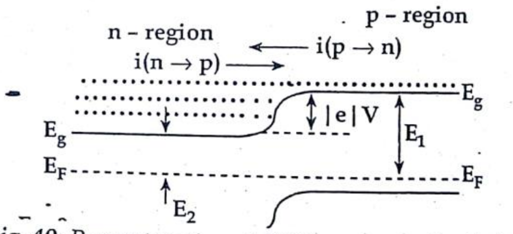
$E_1 = 1.46 \times 10^{-19}$ Joule = 0.91 eV

Here, $E_1 = E_g - E_f$ of the p-side

or, $E_f = E_g - E_1$

or, $E_f = 1.1\text{ eV} - 0.91\text{ eV} = 0.2\text{ eV}$

Hence, required position of Fermi level 0.2 eV.



[Fig. 49: Representation of different energy levels in N-type and P-type Semiconductor junction]

Since, for Silicon diode E_g = 1.1 eV.

Universal Gates and Physics of Integrated Circuits

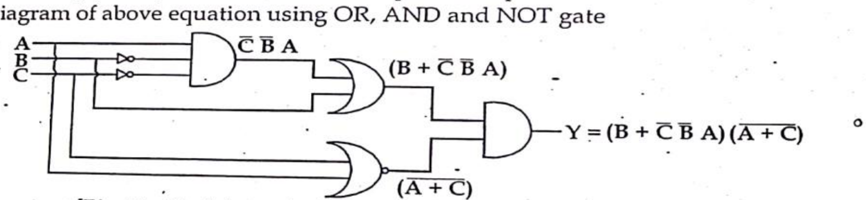
1. The output of a digital circuit Y is given by the expression $Y = (B + \bar{C}\,\bar{B}\,A)\,(\overline{A + C})$, where A, B and C represent inputs. Draw circuit of above equation using OR, AND and NOT gate. Find its truth table. *[TU Exam, 2074]*

Solution:

Here, given the output of a digital circuit Y

$Y = (B + \bar{C}\,\bar{B}\,A)\,(\overline{A + C})$, where A, B and C represent inputs.

The circuit diagram of above equation using OR, AND and NOT gate



[Fig. 51: Digital circuit diagram using OR, AND and NOT gate]

Following is the truth table for above circuit.

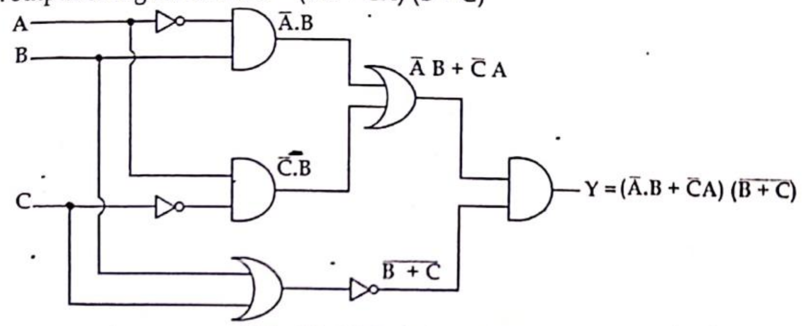
Truth table

Inputs			Intermediate					Output
A	B	C	$\bar{B}$	$\bar{C}$	$\bar{C}\,\bar{B}\,A$	$B + \bar{C}\,\bar{B}\,A$	$\overline{A + C}$	$(B + \bar{C}\,\bar{B}\,A)\,(\overline{A + C})$
0	0	0	1	1	0	0	1	0
0	0	1	1	0	0	0	0	0
0	1	0	0	1	0	1	1	1
0	1	1	0	0	0	1	0	0
1	0	0	1	1	1	1	0	0
1	0	1	1	0	0	0	0	0
1	1	0	0	1	0	1	0	0
1	1	1	0	0	0	1	0	0

2. **The output of a digital circuit Y_1 is given by this expression: $Y = (\bar{A}B + \bar{C}A)\,(\overline{B + C})$, where A, B and C represent inputs. Draw a circuit of above equation using OR, AND and NOT gate and hence find its truth table.** *[TU Model 2074]*

Solution:

Here, given output of digital circuit $Y = (\bar{A}B + \bar{C}A)\,(\overline{B + C})$



[Fig. 52: Digital circuit diagram]

Truth table

Inputs			Intermediate						Output
A	B	C	B+C	$\overline{B+C}$	$\bar{A}$	$\bar{C}$	$\bar{A}B$	$\bar{C}A$	$Y = (\bar{A}B + \bar{C}A)\,(\overline{B+C})$
0	0	0	0	1	1	1	0	0	0
0	0	1	1	0	1	0	0	0	0
0	1	0	1	0	1	1	1	0	0
0	1	1	1	0	1	0	1	0	0
1	0	0	0	1	0	1	0	1	1
1	1	0	1	0	0	1	0	1	0
1	1	1	1	0	0	0	0	0	0

3. **Make the appropriate truth tables to prove the following distributive law of Boolean algebra; A (B + C) = AB + AC** *[TU Microsyllabus 2074, P; 27.1]*

Solution:

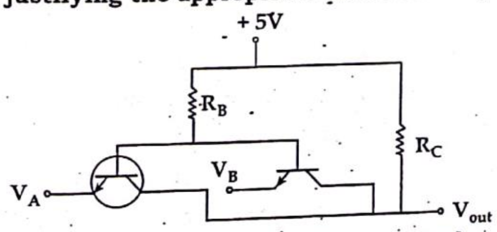
The given Boolean algebra is A (B + C) = AB + AC.

Here, we want to prove the distribution law of Boolean algebra by using truth table;

A	B	C	A (B + C)	AB	AC	AB + AC
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
1	0	0	0	0	0	0
1	0	1	1	0	1	1
1	1	0	1	1	0	1
1	1	1	1	1	1	1

Hence, given Boolean algebra was verified.

4. **Analyze the circuit shown in figure 53. Determine the logic function performed by the circuit by making and justifying the appropriate truth table.** *[TU Microsyllabus 2074, P; 27.6]*



[Fig. 53: Circuit diagram of logic function]

Solution:

The logic function performed by the given circuit:

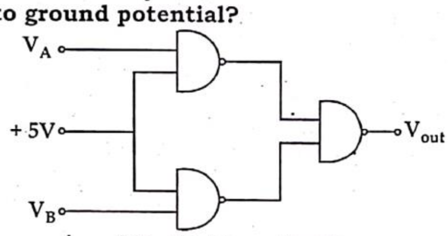
Output $Y = V_A \cdot V_B$ i.e., AND operation

Truth table;

Inputs		Output
V_A(V)	V_B(V)	$Y = V_A \cdot V_B$ (V)
0	0	0
0	5	0
5	0	0
5	5	5

5. **(a) Find the truth table for the circuit shown in figure 54. What logic function does the circuit perform?**

   (b) What logic function will the circuit perform if the constant + 5V input to the first two gates are changed to ground potential? ***[TU Microsyllabus 2074, P; 27.9]***



[Fig. 54: Given circuit]

Solution:

a. Truth table for the given circuit of logic function:

$$Y = \left[\overline{(\overline{5V. V_A}).(\overline{5V. V_B})}\right]$$
$$= V_A + V_B$$
$$= \text{OR - operation}$$

Truth table

Inputs (V)			Intermediate (V)				Output (V)
V_A	V_B	+ 5V	$5V. V_A$	$5V. V_B$	$\overline{5V. V_A}$	$\overline{5V. V_B}$	$\overline{(\overline{5V. V_A}).(\overline{5V. V_B})}$
0	0	+ 5	0	0	+ 5	0	0
0	5	+ 5	0	5	+ 5	5	5
5	0	+ 5	5	0	0	0	5
5	5	+ 5	5	5	0	5	5

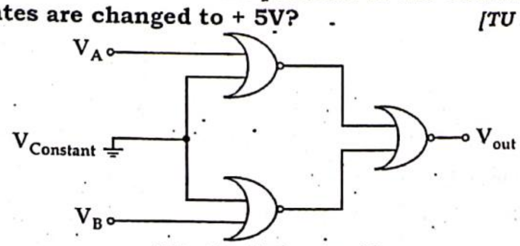
b. If the constant + 5V input to the first two gates is changed to ground potential i.e., (+ 0V) then,

$$\text{Output } Y = \overline{(\overline{5V. V_A}).(\overline{5V. V_B})}$$
$$= \overline{5V. 5V}$$
$$Y = 0V$$

Hence, none of the logic function performed by circuit.

6. **(a) Find the truth table for the circuit of given figure 55. What logic function does the circuit perform?**

   (b) What logic function will the circuit perform if the common grounded input to the first two NOR gates are changed to + 5V? ***[TU Microsyllabus 2074, P; 27.10]***



[Fig. 55: Given circuit]

Solution:

- Logic function of given circuit is

$V_{Out} = \overline{(\overline{V_A + 0}) + (\overline{V_B + 0})}$

$= \overline{(\overline{V_A} + \overline{V_B})}$

$= V_A \cdot V_B$

= AND operation

Truth Table

Inputs (V)			Intermediate (V)				Output (V)
V_A	V_B	$V_{constant}$	$V_A + 0$	$V_B + 0$	$(\overline{V_A + 0})$	$(\overline{V_B + 0})$	$V_{out} = \overline{((\overline{V_A + 0}) + (\overline{V_B + 0}))}$
0	0	0	0	0	5	5	0
0	5	0	0	5	5	0	0
5	0	0	5	0	0	5	0
5	5	0	5	5	0	0	5

b. If the common grounded input to the first two NOR gates is changed to + 5V then

$Y = \overline{((\overline{V_A + 5V}) + (\overline{V_B + 5V}))}$

$= \overline{(\overline{5V} + \overline{5V})}$

$= \overline{0V + 0V}$

= 5 V (High)

- It gives no physical meaning in the operation of the circuit.