Chapter 1 Permutation and Combination

Exercise 1.1

1. Solution:

Total no. of air flights (n_1) = 5
Total no. of buses (n_2) = 15
As from the addition rule
The no. of ways to travel from Bhairahawa to Kathmandu = 5 + 15 = 20
Hence, there are 20 ways to travel.

2. Solution:

The no. of girls and boys are 25 and 20 respectively. If a boy and a girl are to be chosen for debate competition, then
The no. of ways of selection would be 25×20 = 500 ways.

3. Solution:

If there are 5 routes from station A to station B and 4 routes from station B to C then, the no. of possible routes from A to B is 5 and from B to C is 4.

a. Here,
∴ The no. of possible routes from A to C is 5×4 = 20 routes.

b. Here,
The no. of possible routes from A to C is 20 and so as to return from C to A there is also 20 routes (i.e. 4×5).
Hence, the no. of required ways = 20×20 = 400 ways

c. Here,
The no. of routes to travel from A to C is 20. If the same route is not used more than once, the–n the no. of ways to travel and return back is 20×12 = 240 ways

4. Solution:

The no. of digits = 6
So, hundred place can be arranged in 6 ways
Tens place can be arranged in 5 ways
Units place can be arranged in 4 ways
∴ Required numbers = 6×5×4 = 120
Next, If these numbers formed must be even, the digit in the units place can be arranged in 3 ways
Ten's place can be arranged in 5 ways
Hundred place can be arranged in 4 ways
∴ Required number's place = 3×5×4 = 60 ways

5. Solution:

The no. of digits = 6

a. As we know, units place can never be filled by zero, so units place can be filled by 5 ways
Tens place can be filled by 5 ways
Hundred place can be filled by 4 ways
Thousand place can be filled by 3 ways
∴ The required no.s of 4 digit when repetition is not allowed
= 5×5×4×3 = 300

b. If the repetition is allowed, the unit place can be arranged/filled by 5 ways and then after all remaining places can be filled by 6 ways.
∴ The required no.s of 4 digit when repetition is allow = 5×6×6×6
= 1080 ways

6. Solution:

The given digits are 0, 1, 2, 3
If the digits may repeat: then
For 1 digits: For the units place, number of way = 4
For the ten's place, number of ways = 3
∴ Number of ways = 4 × 3 = 12
For 1 digit: The number of ways = 3

So, total number of ways = 12 + 3 = 15

If the digits may not repeat:

For 1 digit: Number of ways = 3

For two digits = Number of ways in tens place = 3

Number of ways in ones place = 3

$\therefore$ Number of ways = $3\times3 = 9$

So, total number of ways = 3 + 9 = 12

7. Solution:

The no. of digits = 5

The number must lies between 2000 and 3000 and so each no. should be started with 2. As the formed no. should be even each no. must be ended with 0,or 2 but here digits can be used only once.

So, units place can be filled by 1 ways

Tens place can be filled by 4 ways

Hundred place can be filled by 3 ways

Thousand place can be filled by 1 ways

$\therefore$ Required no. of digits = $1\times4\times3\times1 = 12$

8. Solution:

Here, the numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and the telephone number starts with 562

i. If repetition is not allowed: Number of ways for remaining 3 places = $7\times6\times5 = 210$

ii. If repetition is allowed: Number of ways for remaining 3 places = $10\times10\times10 = 1000$

9. Solution:

The total digit is 10 and no. of choice for unit digit is 9. So the no. digits required

= $8\times9\times9 = 648$

Now, if there is only one zero given that the repeation allowed, then, the unit place filled by 9 ways

Tens place can be filled by 10 ways

Hundred place can be filled by 9 ways

$\therefore$ No. of total digits = $9\times10\times9 = 810$

Hence, the required number = 810 – 648 = 162

Exercise 1.2

1. Solution

Given, $\dfrac{(n+1)!}{(n-1)!} = 12$

or, $\dfrac{(n+1)\,n\,(n-1)!}{(n-1)!} = 12$

or, $n(n+1) = 12$

or, $n^2 + n - 12 = 0$

or, $n^2 + 4n - 3n - 12 = 0$

or, $(n+4)(n-3) = 0$

either $n = -4$

or, $n = 3$

Since, $n \neq -4$, so $n = 3$

2. Solution:

Given, P(5, r) = 5

r = 1 [$\because$ if P(n, r) = n, then r = 1)

3. Solution:

The no. of digits (n) = 6

a. Units place can be filled only by 5 digits but the remaining 3 places can be filled 6 digits as the repetition is allowed.

$\therefore$ The required no. of 4 digits = $5\times6\times6\times6$

= 1080

b. Unit first place can be filled by 5 digit as the repletion not allowed.

2^{nd} first place can be filled by 5 digit
3^{rd} first place can be filled by 4 digit
4^{th} first place can be filled by 3 digit
$\therefore$ The required no. of 4 digit = $5\times5\times4\times3$
$= 300$

4. Solution:

There are 4 boys and 3 girls be seated in a row containing 7 seats.

$\therefore$ Required arrangement is $p(7, 7) = \frac{7!}{(7-7)!} = \frac{7!}{0!} = 5040$

Again,
If they seat alternatively, then 4 boys can set in 4! ways and 3 girls can seat in 3! ways.
$\therefore$ Required arrangement is $= 4! \times 3!$
$= 24\times6 = 144$ ways

5. Solution:

The total no. of digits = 10
The first digit can be chosen from only 1 to 9 so there is only 9 choices for first digit. The remaining 5 digits can be chosen from remaining 9 digits in p(95) ways

i.e. $\frac{9!}{(9-5)!} = \frac{15}{20}$ ways

$\therefore$ The total numbers of 6 digits is 9×15120 way = 136080

Next: For the divisible by 10. Last digit must be zero, so the last digit can be chosen from 0, so there is 1 choice for last digit. The remaining 5 digits can be chosen from 9 digits in p(95) way

i.e. $\frac{9!}{(9-5)!} = 15120$

6. Solution:

The numbers given in the question is 1, 2, 3, 4, 5
For one digit: No. of ways for even = 2
For two digits: No. of ways for ones place = 2
Number of ways for ten's place = 4
$\therefore$ Total no. of ways $2 + 2\times4 = 10$

7. Solution:

In a bracelet, beads are arrangement in circular form and the anticlockwise and clockwise arrangements are not different.
Here the total number of beds n = 9

They can be arranged in $(n-1)!$ ways $= \frac{1}{2} \times 8!$ ways = 20160

8. Solution:

The no. lying between 100 and 1000 is of 3 digit. In which at unit place can be chosen only from 5 digit and hundred place can only be chosen from 5 digit where as remaining tens place can be chosen from remaining 4 digit.

$\therefore$ The no. formed between 100 and 1000 $= \frac{5!}{(5-1)!} \times \frac{4!}{(4-1)!} \times \frac{5!}{(5-1)!}$

$= \frac{5\times4!}{4!} \times \frac{4\times3!}{3!} \times \frac{5\times4!}{4!} = 5\times4\times5 = 100$

9. Solution:

Each post cards can be posted in 4 ways
Hence, required number of ways $= n^r = 5^5 = 1024$

10. Solution:

a. Here,
PERMUTATION
Total no. of letters (n) = 11
No. of letter 'T' (p) = 2

$\therefore$ Total number of way of arrangement = $\frac{n!}{p!} = \frac{11!}{2!}$

b. INTERMEDIATE

Here, the total number of letters (n) = 12

No. of letter 'I' (p) = 2

No. of letter 'T' (q) = 2

No. of letter 'E' (r) = 3

$\therefore$ The total no. of arrangement = $\frac{n!}{p!\,q!\,r!} = \frac{12!}{2!\,2!\,3!}$

c. EXAMINATION

Here, the total number of letter (n) = 11

No. of letters 'A' (p) = 2

No. of letters 'I' (q) = 2

No. of letters 'N' (r) = 2

$\therefore$ Total no. of arrangement = $\frac{n!}{p!\,q!\,r!} = \frac{11!}{2!\,2!\,2!}$

d. CIVILIZATION

The total no. of letters (n) = 12

No. of letter 'I' (p) = 4

$\therefore$ Total no. of arrangement = $\frac{n!}{p!} = \frac{12!}{4!}$

11. Solution:

In 'ARRANGE'

Total no. of letters (n) = 7

No. of letter 'A' (p) = 2

No. off letter 'R' (q) = 2

$\therefore$ Total no. of ways of arrangement = $\frac{n!}{p!\,q!} = \frac{7!}{2!\,2!} = 1260$

If we suppose (RR) as one letter, then the no. of letters will be 6

$\therefore$ The no. of ways of arrangement when R comes together = $\frac{n!}{p!} = \frac{6!}{2!} = 360$

Thus, the required no. of ways of arrangement when two R not comes together = 1260 – 360 = 900

12. Solution:

In UNIVERSITY'

The no. of letters (n) = 10

No. of letter 'I' (p) = 2

$\therefore$ Total no. of arrangement = $\frac{n!}{p!} = \frac{10!}{2!} = 1814400$

Since the arrangement begin with U there is only. Nine letters to arrange. So, the nine letters can be arranged in

$= \frac{n!}{p!} = \frac{9!}{2!} = 181440$

$\therefore$ Required no. of arrangement = $1 \times 181440 = 181440$

Next: The total no. of ways in which the arrangement begin with U but do not end with

'Y' = $4 \times p(88) = 4 \times \frac{8!}{0!} = 161280$

13. Solution:

Total no. of countries (n) = 8

If they sit in round table then they form a circle, so its arrangement is = (n – 1)! = 7! = 5040

If Nepali and Indian always sit together, then we take it as one. Then the total no. will be 6. So, the arrangement (n–r+1)! = (7–2+1)! = 6! = 720

If they sit together, then they also can interchange there seat between themselves in 2 ways.

Hence, the required no. of arrangement = $21 \times 720 = 1440$

14. Solution:

If there is 3 candidate for the president then election can be turned in 3 ways. Similarly for 5 secretary and 2 pressure the election can be turned in 5 ways and 2 ways. Hence, the required no. ways to conduct election = $3! \times 5! \times 2! = 1440$

15. Solution:

Total no. of digit = 4

The person can try his password in p(4, 4) ways = $\frac{4!}{(4-4)!} = \frac{4!}{0!} = 24$

16. Solution:

Since 6 persons are to be arranged in row with 6 seat, so that the girls and boys are in alternate, so girl are to be arranged in odd seats and boy in even seats.

$\therefore$ The total no. of arrangement = $6! = 720$

Here, the no. of arrangement of boy restricted to occupy even seats is p(3, 3) = $\frac{3!}{0!} = 6$

The 3 boys can occupied seats in 3! by interchanging their seats.

Hence, the required no. of arrangement = $3! = 36$

17. Solution:

In 'EQUATION'

The no. of total letters (n) = 8

$\therefore$ The total no. of arrangement = $8! = 40320$

Next, The no. of vowels = 5

When we take all vowels as one then there will be total letters left = 4. Also the vowel letters be arranged themselves in 5 ways.

$\therefore$ The required no. of arrangement = $4! \times 5! = 2880$

Exercise 1.3

1. Solution:

Here, (ln, 10) = (ln, 12)

$$\Rightarrow \frac{n!}{(n-10)!\,10!} = \frac{n!}{(n-12)!\,2!}$$

$$\Rightarrow \frac{(n-12)!\,12!}{(n-10)!\,10!} = \frac{n!}{n!}$$

$$\Rightarrow \frac{(n-12)!\,12 \times 11 \times 10!}{(n-10)\,(n-11)\,(n-12)!\,10!} = 1$$

$$\Rightarrow \frac{12 \times 11}{(n-10)\,(n-11)} = 1$$

$\Rightarrow 132 = n^2 - 11n - 10n + 110$

$\Rightarrow n^2 - 21n - 22 = 0$

$\Rightarrow n^2 - 22n + n - 22 = 0$

$\Rightarrow n(n - 22)\ 11(n - 22) = 0$

$\Rightarrow (n - 22)\,(n + 1) = 0$

either n = 22

or, n = –1 (This is not possible, so rejected)

$\therefore$ n = 22

Next C(n, 6) = C(22, 6) = $\frac{22!}{(22-6)!\,6!} = \frac{22!}{16!\,16!}$

2. Solution:

Here, C(n, 8) = C(n, 6)

Then, n = 8 + 6 = 14

Now, $C(14, 2) = \frac{14!}{12!\ 2!} = 91$

3. Solution:

Given, C(n, 30) = C(n, 4)

$\Rightarrow$ $C(n, r) = C(n, r^1)$

$\Rightarrow$ $r + r^1 = n$

Then, 30 + 4 = n

$\therefore$ n = 34

Now, $C(n, 30) + C(n, 4) = \frac{34!}{(34 - 4)!\ 14!} + \frac{34!}{20!\ 14!}$

$C(n, 30) + C(n, 4) = \frac{34!}{4!30!} + \frac{34!}{30!\ 4!}$

= 46376 + 46376

= 92752

4. Solution:

Given, c(9, 2r) = c(9, 3r – 1)

$\Rightarrow$ 2r + 3r – 1 = 9 [$\because C(n, r) = C(n, r^1) = r+r^1 = n$]

$\Rightarrow$ 5r = 10

$\Rightarrow$ r = 2 or, 2r = 3r – 1

$\Rightarrow$ r = 1

$\therefore$ r = 1 or 2

5. Solution:

The no. of workers required in 3 where total applicant is 10

$\therefore$ The selection be made as $c(10, 3) = \frac{10!}{7!\ 3!}$ $\left[\because c(n, r) = \frac{n!}{(n–n!\ r!)}\right]$ = 120 Ans.

6. Solution:

To invite 7 guests out of 12 friends

Relatives (8)	Non-relative (4)	Selection
5	2	c(8, 5) × c(4, 2)

$\therefore$ The required selection is $6(8, 9) \times c(4, 2) = \frac{8!}{3!\ 5!} \times \frac{4!}{2!\ 2!}$

= 56 × 6

= 336 Ans.

7. Solution:

Here,

Group A (10)	Group B(6)	Selection
6	4	c(10, 6) × (6, 4)

$\therefore$ The required selection is $c(10, 6) \times c(6, 4) = \frac{10!}{4!\ 6!} \times \frac{6!}{2!\ 4!}$

= 210 × 15

= 3150

8. Solution:

Total number of balls = 5 and maximum number of balls to select = 3.

Thus, we can select in C(5, 0) + C(5, 1) + C(5, 2) + C(5, 3) = 1 + 5 + 10 + 10 = 26 ways.

9. Solution:

From the 4 coins, the sum can be made in the following ways:

$C(4, 1) + C(4, 2) + C(4, 3) + C(4, 4) = \frac{4!}{3!\ 1!} + \frac{4!}{2!\ 2!} + \frac{4!}{1!\ 3!} + \frac{4!}{0!\ 4!}$

= 4 + 6 + 4 + 1

= 15 Ans

10. Solution:

The no. of player in class = 15

The no. of players taken in team (r) = 11

∴ Required no. of ways of selection = C(15, 11) = $\frac{15!}{4!\ 11!}$ = 1365

a. Here, if 2 particular persons are always included then there will be 13 players in class and 9 players required to be selected.

∴ Required selection is C(13, 9) = $\frac{13!}{4!\ 9!}$ = 715

b. Here, If 2 persons are always excluded then there will be 13 players in class and 11 players to be selected.

∴ Required selection = C(13, 11) = $\frac{13!}{2!\ 11!}$ = 78

11. Solution:

Party A(5)	Party B(6)	Selection
3	5	(15, 3) × C(6, 5)
2	6	C(5, 2) × C(6, 6)

∴ Required selection =

C(5, 3) × C(6, 5) + C(5, 2) × C(6, 6)

$= \frac{5!}{2!\ 3!} \times \frac{6!}{1!\ 5!} = \frac{5!}{3!\ 2!} \times \frac{6!}{0!\ 6!}$

= 10 × 6 + 10 × 1

= 70 Ans.

12. Solution:

Group A(5)	Group B(5)	Selection
2	4	C(5, 2) × c(5, 4)
3	3	C(5, 3) × c(5, 3)
4	2	C(5, 4) × c(5, 2)

∴ The required selection = C(5, 2) × C(5,4) + C(5, 3) × C(5, 3) + C(5, 4) × C(5, 2)

$= \frac{5!}{3!\ 2!} \times \frac{5!}{1!\ 4!} + \frac{5!}{2!\ 3!} \times \frac{5!}{2!\ 3!} \times \frac{5!}{1!\ 4!} \times \frac{5!}{3!\ 2!}$

= 10 × 5 + 10 × 10 + 5 × 10

= 50 + 100 + 50

= 200 Ans.

13. Solution:

Ladies (6)	Gentle (8)	Selection
4	7	c(6, 4) × c(8, 7)

∴ Required selection is C(6, 4) × C(8, 7) = $\frac{6!}{2!\ 4!} \times \frac{8!}{1!\ 7!}$ = 15 × 8 = 120

Ladies (6)	Gentle (8)	Selection = 15×8 = 120
4	7	C(6, 4) × C(8, 7)
5	6	C(6, 5) × C(8, 6)
6	5	C(6, 6) × C(8, 5)

∴ Required selection = C(6, 4) × C(8, 7) + C(6, 5) × C(8, 6) + C(6, 6) × C(8, 5)

$= \frac{6!}{2!\ 4!} \times \frac{8!}{1!\ 7!} + \frac{6!}{1!\ 5!} \times \frac{8!}{2!\ 6!} + \frac{6!}{6!\ 1!} \times \frac{8!}{3!\ 5!}$

= 15 × 8 + 6 × 28 + 1 × 56

= 120 + 168 + 56 = 344 Ans.

14. Solution:

From the 6 question, a examinee can pass/solve one or more of questions in following ways.

A examinee can solve 1 or 2 or 3 or 4 or 5 or all

Thus, total no. of ways to solve = C(6, 1) + (6, 2) + C(6, 3) + C(6, 4) + C(6, 5) + C(6, 6)

$= \frac{6!}{5!\ 1!} + \frac{6!}{4!\ 2!} + \frac{6!}{3!\ 3!} + \frac{6!}{2!\ 4!} + \frac{6!}{1!\ 5!} + \frac{6!}{0!\ 6!}$

= 6 + 15 + 20 + 6 + 1 = 63 Ans.

15. Solution:

A candidate fails in an examination if he cannot pass either in 1 or 2 or 3 or 4 or 5 subjects

$\therefore$ Total no. of ways by which he falls = C(5, 1) + C(5, 2) + C(5, 3) + C(5, 4) + C(5, 5)

$= \frac{5!}{4!\,1!} + \frac{5!}{3!\,2!} + \frac{5!}{2!\,3!} + \frac{5!}{1!\,4!} + \frac{5!}{0!\,5!}$

= 5 + 10 + 10 + 5 + 1 = 31 Ans.