

# Electromagnetic Induction

Faraday's discovery for electromagnetic :-

Faraday did an experiment for the production of current in the coil or conductor due to relative motion b/w the coil and magnet. A/c to him, when the magnet is moved toward the coil without touching, the galvanometer ps connected to coil shows the deflection. When the magnet is moved away from the coil, the galvanometer again shows the deflection but in opposite direction to that of previous case.

The same phenomenon was observed when magnet is kept stationary but the coil is moving towards or away from the magnet. In doing so, the magnetic flux linked with the coil changes as a result emf is developed in the coil which is called induced emf. Due to this induced emf, the current ps generated in the coil called induced current.

Hence, phenomenon of development of emf in the coil due to relative motion b/w coil & magnet is called electromagnetic induction.

↓ S motion of  
↓ N magnet

↑ S

Induced  
emf

G

R

$$\text{Magnetic flux } (\phi) = BA = BA \cos \theta$$

It's unit is  $\text{Wb}$  and  $\text{m}^2$

The CGS unit of magnetic flux is maxwell (mx)

$$1 \text{ maxwell} = 1 \text{ gauss} \times 1 \text{ cm}^2$$

$$1 \text{ wb} = 1 \text{ T} \times 1 \text{ m}^2$$

$$1 \text{ wb} = 10^4 \text{ gauss} \times 10^4 \text{ cm}^2$$

$$1 \text{ wb} = 10^8 \text{ gauss cm}^2$$

$$1 \text{ wb} = 10^8 \text{ max well.}$$

Faraday's law of electromagnetic induction:-

Faraday's law of electromagnetic induction states that:-

1. whenever magnetic flux linked with the coil or conductor change and emf is produced in it which lasts as longer as the change in magnetic flux continuous.

2. Magnitude of emf is induced in a coil or conductor is directly proportional to the rate of magnetic flux linked with coil.

Let  $\phi_1$  be the magnetic flux linked with the coil at time  $t_1$  and  $\phi_2$  be the magnetic flux linked with the coil at time  $t_2$  such that  $t_2 > t_1$  and  $\phi_2 > \phi_1$ . Then acc to faraday's law of electromagnetic induction,

$$E \propto \frac{\phi_2 - \phi_1}{t_2 - t_1}$$

$\Rightarrow E = k \left( \frac{\phi_2 - \phi_1}{t_2 - t_1} \right)$  where  $k$  is proportionally constant and its value in system is found to be  $1$  and negative sign shows the opposing nature of induced emf. In general

$$E = -\frac{d\phi}{dt}$$

If coil contains  $N$  no. of turns

$$\mathcal{E}_r = N\phi$$

$$\Rightarrow \frac{d\phi}{dt} = -\frac{d\phi_r}{dt}$$

$$\Rightarrow \frac{d\phi_r}{dt} = -\frac{d(N\phi)}{dt}$$

$$\Rightarrow \frac{d\phi_r}{dt} = -N \frac{d\phi}{dt}$$

Important

2. Lenz's law :- [What is Lenz's law? How Lenz's law lead to statement:- the conservation of energy? (Explain)]

It states that the direction of induced current in the circuit is such that it always opposes the change of causes that produce it.

### Explanation

Consider a coil connected with the galvanometer. Consider a bar magnet with its N-pole facing towards the coil and moving towards the coil. Due to the motion of bar magnet, the magnetic flux linked with the coil is increasing towards the coil. Due to the motion of bar magnet, the magnetic flux linked with coil changes. As a result, an emf

is produced in the coil. Due to the induced emf a magnetic field is setup around the coil which act as induced magnet.

motion ↓ bar magnet

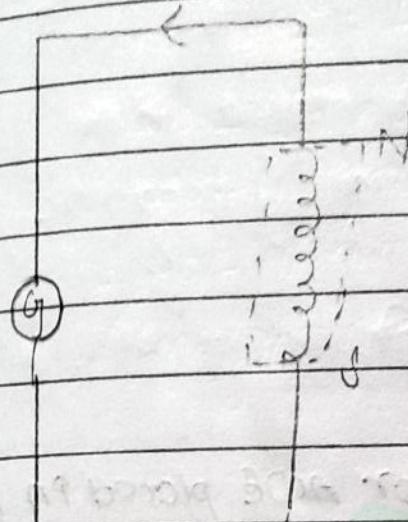


fig (a)

motion ↑ S  
N

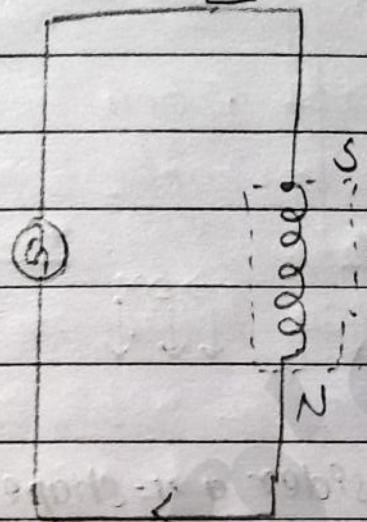
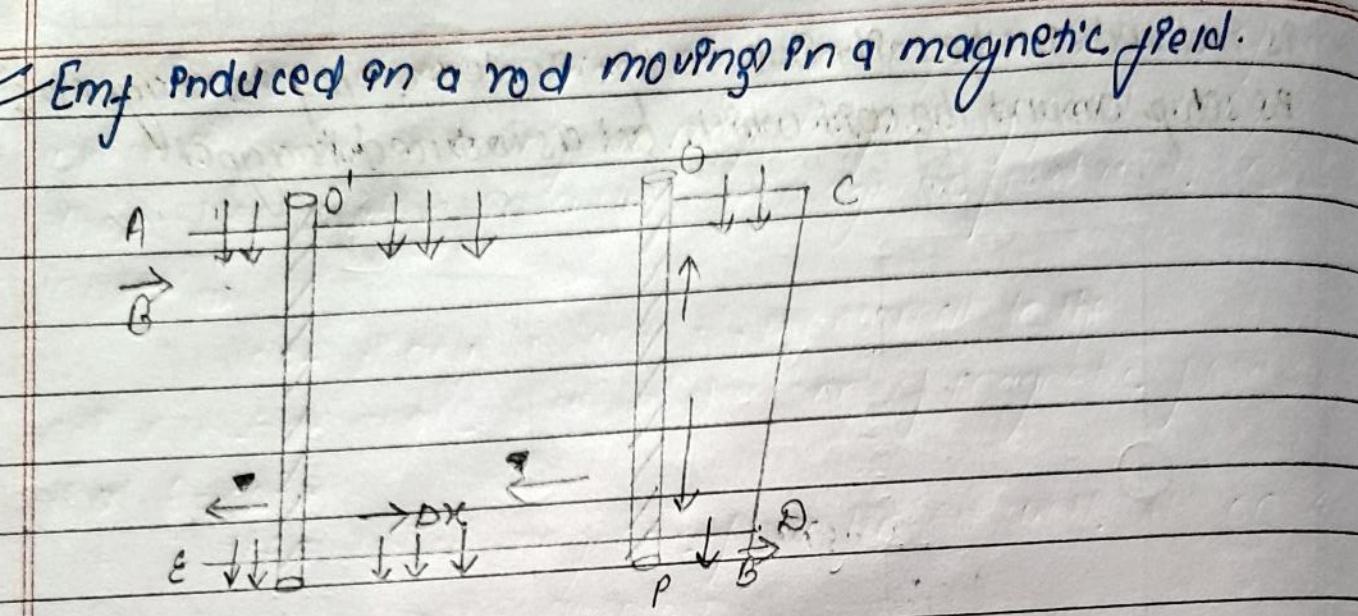


fig (b)

The north pole of induced magnet is falling forward the north pole of moving bar magnet and there exist the repulsive force between two magnets which proves that the direction of induced emf is such that which oppose that cause that produce it.

Similarly, when the bar magnet is moving away from the coil, the south pole of induced magnet is facing toward the N-pole of moving bar magnet and there exists an attractive force between two magnets which proves that the direction of induced emf is such that it oppose the motion of bar magnet.

Here, external source has to do the work to give the continuity of motion of bar magnet. The work repulsive force between bar magnet & induced magnet which appears as a electrical energy. Hence, len's law obeys law of conservation of energy.



Consider a U-shaped conductor ACD placed in uniform magnetic field of field strength B. Suppose a straight conductor of length l, area cross section 'A' placed over the U-shaped conductor. The straight conductor is moving over the U-shaped conductor in the direction perpendicular to the direction of magnetic field. Let O'P be the initial position of conductor & O''P be the final position of the conductor after time  $dt$  such that.

$$O'P = O''P = dx$$

The area swept by the conductor in the time  $dt$  is,

$$dA = O'P \times OP$$

$$= dx \times l$$

Then the rate of area swept is

$$\frac{dA}{dt} = l \frac{dx}{dt}$$

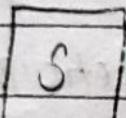
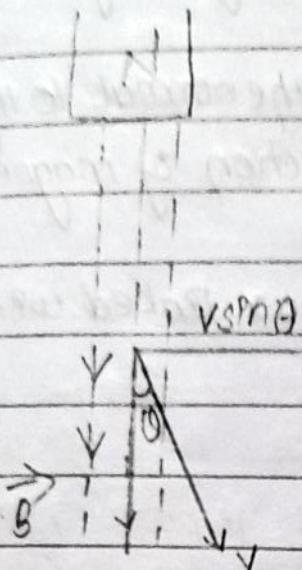
$$\frac{dA}{dt} = l \cdot v \quad \text{where } v = \frac{dx}{dt} = \text{velocity}$$

According to Faraday's law of electromagnetic induction induced emf ( $E$ ) =  $\frac{-d\phi}{dt}$ .

$$\text{But } \phi = BA$$

$$E = -\frac{d(BA)}{dt}$$

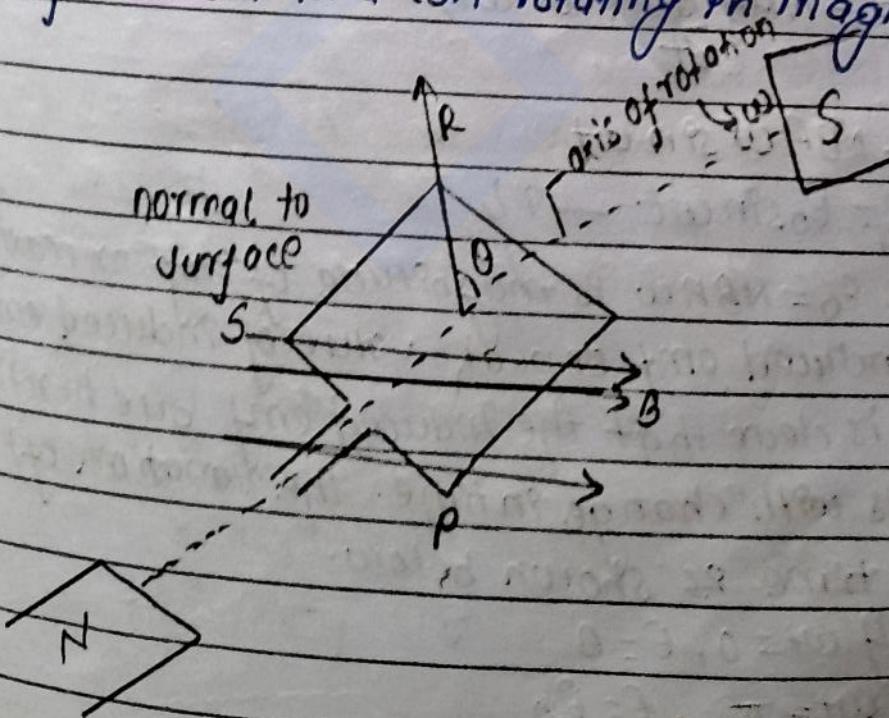
$\alpha = -BLV$  — why [(-)ve sign is not used in numerical]  
field on magnitude ( $E$ ) =  $BVI$  — why  $(N)$



If the conductor makes an angle  $\theta$  with the direction of magnetic field that effective component of velocity is  $v \sin \theta$

$\therefore$  Induced emf ( $E$ ) =  $BVI \sin \theta$   $(N)$

The emf induced in a coil rotating in magnetic field.



Consider a rectangular coil having area ' $A$ ', no. of turns 'N' which is placed in a uniform magnetic field of field strength 'B'. The coil is rotating in a uniform magnetic field with angular velocity ' $\omega$ '.

At any instant 't', the normal to the coil makes an angle ' $\theta$ ' with direction of magnetic field such that  $\theta = \cot$ .

Then, total magnetic flux linked with it on a number of turns of coil is given by,

$$\phi = NBA \cos \theta$$

$$\text{But } \theta = \omega t$$

$$\phi = NBA \cos \omega t$$

According to Faraday's law of electromagnetic induction, the induced emf is given by

$$E = -\frac{d\phi}{dt}$$

$$\Rightarrow E = -\frac{d}{dt} (NBA \cos \omega t)$$

$$\Rightarrow E = -NBA \frac{d}{dt} (\cos \omega t)$$

$$\Rightarrow E = NBA \omega \sin \omega t$$

$$\Rightarrow E = E_0 \sin \omega t - 94$$

where  $E_0 = NBA\omega$  is magnitude of emf or maximum value of induced emf or peak value of produced emf from eqn 94. It is clear that the induced emf due to rotation of coil changes with change in time. The variation of produced emf with time is shown below.

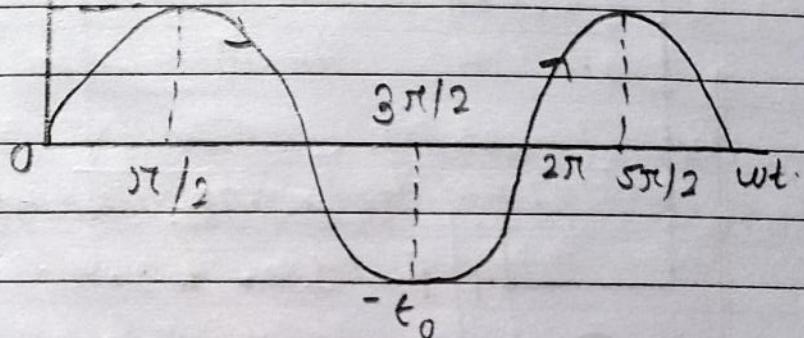
$$\text{If } \omega t = 0, E = 0$$

$$\omega t = \frac{\pi}{2}, E = E_0$$

$$\omega t = \pi, E = 0$$

$$wt = \frac{3\pi}{2}, E = -E_0$$

$$wt = 2\pi, E = 0$$



\* Disc generator (for numerical and objective)

length of conductor,  $l = \text{radius}(R)$

Average velocity of straight conductor PQ PS

$$V = \frac{R\omega + 0}{2} = \frac{R\omega}{2}$$

We know, RMF induced in the conductor PQ,

$$E = BVI$$

$$\Rightarrow E = B \frac{R\omega \cdot R}{2}$$

$$\Rightarrow E = B \frac{(2\pi f) R^2}{2}$$

$$\Rightarrow [E = BAf] \quad [\because A = \pi R^2]$$

Objective :- If  $R_1$  = interval radius of disc

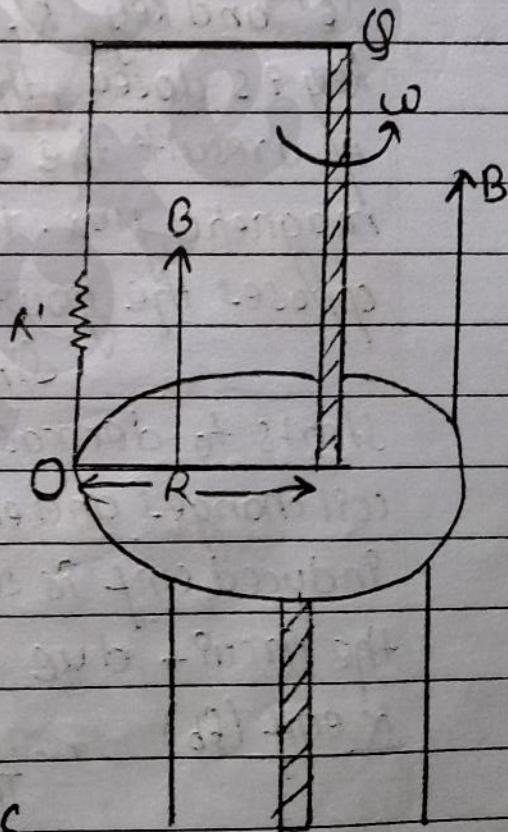
$R_2$  = external radius of disc

$$A = A_{\text{ext}} - A_{\text{int}}$$

$$= \pi R_2^2 - \pi R_1^2$$

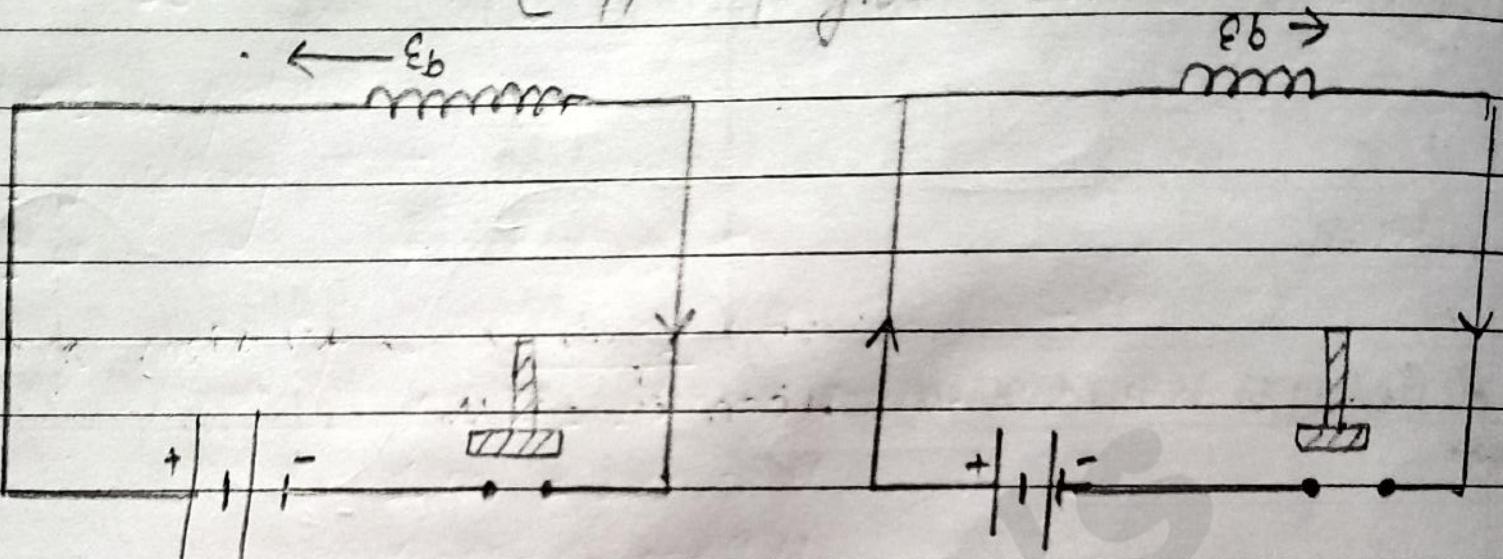
$$= \pi (R_2^2 - R_1^2)$$

$$\therefore [E = B\pi (R_2^2 - R_1^2) f] \quad (\text{For hollow disk})$$



Self Induction

loop is closed  $\rightarrow$  current grows / increases  $\rightarrow$  magnetic flux changes  $\rightarrow$  emf induced  $\rightarrow$  oppose the growth of current



fig(a)

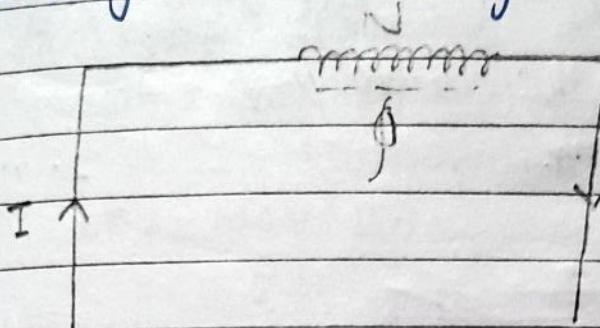
fig(b)

Consider a coil connected to the battery of emf ' $E$ ' and key 'K'. Initially let the key be opened. When the key is closed, the current through the coil starts to increase. As a result the emf is induced in the coil due to change in magnetic flux. The direction of induced emf is such that it opposes the growth of the current.

Similarly when key is opened the current starts to decrease. As a result, magnetic flux linked with the coil changes and emf is induced in the coil. The direction of induced emf is such that it opposes the decay of current in the circuit due to change in magnetic flux is called back emf ( $E_b$ )

The phenomenon of production of opposing emf in the coil due to change in current passing through it is called self-induction.

# Coefficient of Induction (self Inductance)



Let  $I$  be the current flowing through the coil and  $\phi$  be the magnetic flux linked with the coil. Then it is found that

at

$$\phi \propto I$$

$\Rightarrow \phi = LI$  where  $L$  is proportionally constant called coefficient self induction or simply self-inductance.

$$I = \frac{1}{L} \phi$$

Hence, self inductance is numerically equal to magnetic flux of current passing through it per unit current.

$$\text{Also, } E = \frac{d\phi}{dt}$$

$$\Rightarrow E = \frac{d(LI)}{dt}$$

$$\Rightarrow E = L \frac{dI}{dt} + I \cdot \frac{dL}{dt} = I A_s - L \frac{dI}{dt}$$

$$\Rightarrow E = L \frac{dI}{dt}$$

Hence, self-inductance is also defined as the emf produced in the coil if the rate of change of current in the coil is  $A_s - 1$ .

Unit of  $L$

$$E = L \frac{dI}{dt}$$

$$\Rightarrow L = \frac{E}{\frac{dI}{dt}}$$

$$\Rightarrow L = \frac{1 \text{ volt}}{\frac{1 \text{ A}}{1 \text{ s}}}$$

$$\Rightarrow L = 1 \text{ Vs A}^{-1}$$

$$\Rightarrow L = 1 \text{ Henry}$$

$$\Rightarrow 1 \text{ H}$$

Hence unit of  $L$  is Henry (H)

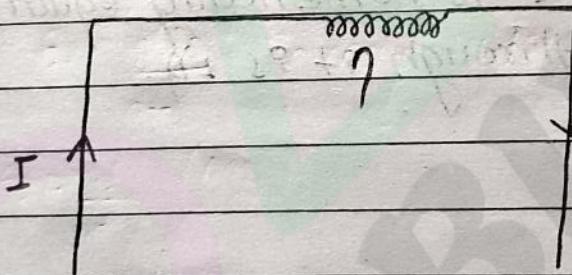
$$1 \mu\text{H} = 1 \times 10^{-6} \text{ H.}$$

$$1 \text{ mH} = 1 \times 10^{-3} \text{ H.}$$

$$1 \text{ nH} = 1 \times 10^{-9} \text{ H.}$$

$$1 \text{ pH or } 1 \text{ uH} = 1 \times 10^{-12} \text{ H.}$$

### Self production of long-solenoids.



Let us consider a long solenoid containing 'N' no of turns having cross sectional area ' $a$ ' carrying current ' $I$ '. Let ' $n$ ' be the no. of turns per unit length of solenoid. Then, total magnetic field strength inside the long solenoid :-

$$B = M_0 n I \quad \dots \text{①}$$

$$\text{But } n = \frac{N}{l}$$

$$\therefore B = \frac{M_0 N I}{l} \quad \text{--- ii.}$$

Also, total magnetic flux linked with int'l no. of turns of solenoid is ;

$$\phi = N \Phi$$

$$\Rightarrow \phi = \frac{M_0 N^2 I A}{l} \quad \text{--- iii.}$$

Also, magnet flux,  $\phi = LI - \cancel{IR}$

from eqn (ii)  $\cancel{\phi} \cancel{IR}$

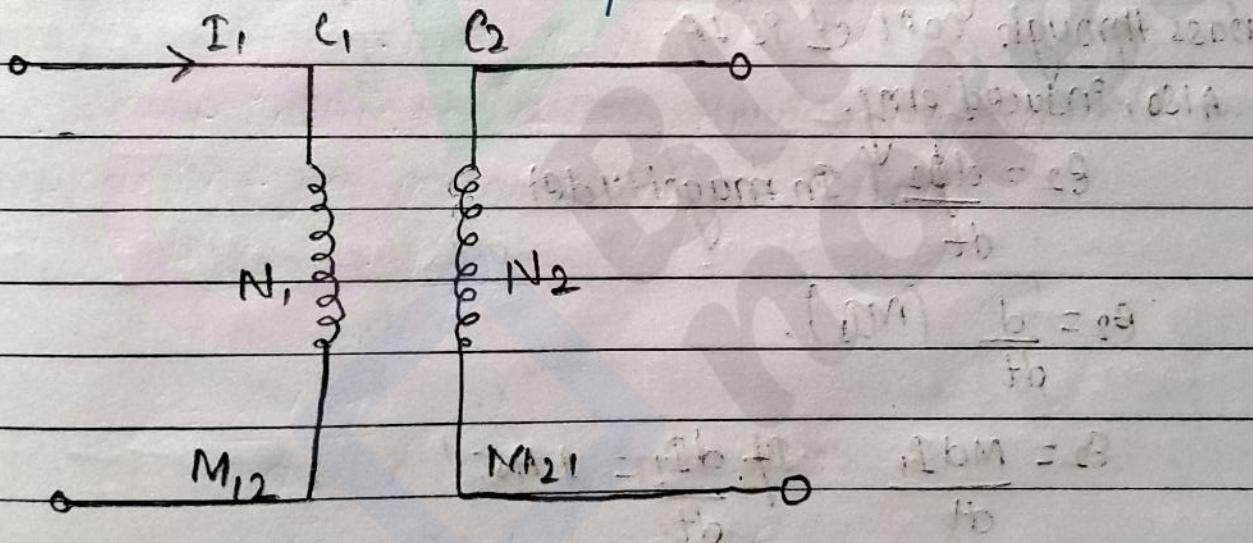
$$dI = \frac{1\mu_0 N^2 IA}{l}$$

$$\therefore L = \frac{\mu_0 N^2 IA}{l}$$

$$\therefore L = \frac{\mu_0 N^2 A}{l} - \cancel{IR}$$

This gives the required expression for self inductance of long solenoid.

### Mutual Induction



Let us consider two coils are placed very close to each other without any electrical contact & current passing through one of them is varied then emf is produced in the another coil due to change in magnetic flux. This phenomenon is called mutual induction.

Similarly, let  $I_2$  be the current flowing in the coil  $C_2$ .  
Then the magnetic field inside solenoid  $C_2$  is:-

$$B_2 = \mu_0 n_2 I_2$$

$$\text{But } n_1 = \frac{N_1}{l}$$

$$\therefore B_2 = \frac{\mu_0 N_2 I_2}{l} \quad \therefore B_2 = \frac{\mu_0 N_2 I_2}{l} - \textcircled{v}$$

In this magnetic field the coil  $C_1$  is placed so, magnetic flux linked with the coil  $C_1$  is

$$\phi_1 = N_1 B_2 A$$

$$\phi_2 = \frac{\mu_0 N_1 N_2 A I_2}{l} - \textcircled{vi}$$

Also,

$$\phi_1 \propto I_2$$

$$\phi_2 = M_{12} I_2 - \textcircled{vii}$$

when  $M_{12}$  is mutual inductance of coils  $C_1$  w.r.t to  $C_2$

from  $\textcircled{v}$  &  $\textcircled{vii}$

$$M_{12} I_2 = \frac{\mu_0 N_1 N_2 I_2 A}{l}$$

$$\therefore M_{12} = M_{21}$$

In general,  $M = \frac{\mu_0 N_1 N_2 A}{l}$  (mutual inductance of long solenoid formula).

If coil contains same no of turns i.e  $N_1 = N_2 = N$  (say)

$$M = \frac{\mu_0 N^2 A}{l}$$

Important

Energy stored in an inductor or coil.

Consider a inductor of self inductance ' $\alpha$ '. Let the resistance of the inductor is zero so that no energy is dissipated with in the inductor. Suppose the inductor is connected to the source of emf ' $\epsilon$ '. Initially, let the current through the coil is zero. When current in the coil increase from 0 to its maximum value ( $I_0$ ) emf is induced in it due to change in magnetic flux. According to lenz law, the induced emf opposes the growth of current in the coil. The external source has to do the work against the back emf to allow (growth) current in the coil. This work done by external current source to increase current in the coil is stored in the form of energy in inductor.

Let ' $I$ ' be the current flowing in an inductor &  $dI$  be the rate of flow of current in an inductor at the magnitude of back emf is given by,

$$\epsilon_b = \alpha \cdot \frac{dI}{dt} \quad \text{--- (9)}$$

The rate at which the external source does the work is equal to the power supplied by source & which is given by.

$$P = \epsilon_b I \quad (V = \epsilon_b)$$

$$P = \alpha I \frac{dI}{dt} \quad \text{--- (11)}$$

Then small amount of work done by the external source against the back emf is given by.

$$P = \frac{dw}{dt}$$

$$\Rightarrow dW = Pdt$$

$$\Rightarrow dW = \mathcal{L} \frac{dI}{dt} dt$$

$$\Rightarrow dW = \mathcal{L} I dI \quad \text{(iii)}$$

Then, total amount of work done by external source against the back emf to increase the current from 0 to maximum value ( $I_0$ ) can be determined obtained by integrating eqn (iii) from limit 0 to  $I_0$  as,

$$W = \int_0^{I_0} dW = \mathcal{L} \int_0^{I_0} IdI$$

$$\Rightarrow W = \mathcal{L} \left[ \frac{I^2}{2} \right]^{I_0}_0$$

$$\Rightarrow W = \frac{1}{2} \mathcal{L} I_0^2 \quad \text{--- (iv)}$$

This gives the required expression for energy stored in an inductor or coil.

Transformer

It is a device to transform energy from one coil to another coil. It works on the principle of mutual induction. It converts low alternating voltage at high current into high alternating voltage at low current & vice versa.

Construction:-

It consists of two insulated coils wound over a soft iron core. The coil to which AC input is given called primary coil. & the coil from which transformed output is taken is called secondary coil. [The use of soft iron core increase the magnetic flux linkage between primary & secondary coil. The core is laminated to reduce heating by adding eddy current. Lamination increases the resistance of core & reduces the eddy current] S.Q

Theory:-

Let  $N_p$  &  $N_s$  be the no. of turns in primary coil & secondary coil respectively. Both coils are wound over a same core. so, the total cross sectional area of both coils is same. Let  $A$  be the area of cross sectional area of each coil &  $B$  be the magnetic field intensity in the core at any time. Then magnetic flux linked with primary & secondary coil are

$$\Phi_p = N_p B A \quad \text{--- (i)}$$

$$\Phi_s = N_s B A \quad \text{--- (ii)}$$

According to Faraday's law of electromagnetic induction, the induced emf in primary & secondary coils are,

$$E_p = - \frac{d\Phi}{dt} = - \frac{d}{dt} (N_p B A)$$

$$E_p = - N_p A \frac{dB}{dt} \quad \text{--- (iii)}$$

$$\text{Similarly, } E_s = - N_s A \frac{dB}{dt} \quad \text{--- (iv)}$$

Rearranging eqn (iv) by (iii).

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} \rightarrow (v)$$

For ideal transformer resistance is zero. So, induced emf in primary ( $E_P$ ),  $E_P$  = Terminal pd across primary coil  
 $V_P$  [ $E_P = V_P$ ]

Similarly,  $E_S = V_S$

From eqn (v)

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = k \text{ (say)} \dots (vi) \text{ where } k = \frac{N_S}{N_P}$$

$\frac{N_S}{N_P}$  called transformation ratio.

If  $k > 1$  i.e.  $\frac{N_S}{N_P} > 1 \Rightarrow N_S > N_P$ . Such type of transformer is <sup>step up</sup>~~step down~~ transformer.

If  $k < 1$  i.e.  $\frac{N_S}{N_P} < 1 \Rightarrow N_S < N_P$ .

Then, such type of transformer is step-down transformer.

### Efficiency of transformer ( $\eta$ )

It is defined as the ratio of output power to the input power i.e.

$$\text{Efficiency } (\eta) = \frac{\text{Output power } (P_O) \times 100\%}{\text{Input power } (P_I)} \dots (vi)$$

Then, Output power ( $P_O$ ) =  $E_S I_S$ .

Input power ( $P_{in}$ ) =  $E_P I_P$ .

$$\therefore \eta = \frac{E_S I_S}{E_P I_P} \times 100\%$$

For ideal transformer,  $P_o = P_i$

$$\text{i.e. } E_S I_S = E_P I_P$$

$$\frac{E_S}{E_P} = \frac{I_P}{I_S} \quad (\text{Vipp})$$

$$\text{Finally, } \left| \begin{array}{l} \frac{E_S}{E_P} = \frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S} \\ \end{array} \right| - \varphi_x$$

S.O

### Energy losses in transformer.

1. Copper losses:-

The coils of transformer are made of copper. Due to resistance of copper, certain amt of input electrical energy  $P_i$  converted into heat energy. If heat developed  $P_i$  so high then it can break the insulation of coil.

2. Flux losses:- Total magnetic flux produced in primary coil never get linked with secondary coil. The net flux loss can be minimized by decreasing the distance b/w primary & secondary coil.

3. Hysteresis loss:-

The iron core of transformer is situated in a variable magnetic field of AC in the primary coil. This fluctuating magnetic field magnetizes & demagnetizes the core again and again over a complete cycle of AC. During magnetization & demagnetization of the core of the transformer, some energy is lost which is called Hysteresis loss.

4. Loss due to eddy current

variable magnetic  $\rightarrow$  magnetic flux  
 $\rightarrow$  faraday's law  $E = \frac{d\Phi}{dt}$   $\rightarrow$  induced current  $\rightarrow$  closed loop

As iron core of transformer is situated in a variable mag-

magnetic field of AC in the cores of transformer. Due to this variable magnetic flux, magnetic flux linked with the core changes and according to Faraday's law of electromagnetic induction, emf is produced in the core of transformer which produce induced current in the form of closed loop which is called eddy current. Hence, due to eddy current some energy is lost in the form of heat if heat produced is so high which may damage the insulation of copper winding. The losses due to eddy current can be minimized by taking lamination iron core. In laminated iron core the iron strips are quite thin and possess large resistance and it reduces eddy current produced.

## Numerical \*

① The magnetic flux passing perpendicular to the plane of a coil is given by  $\phi = 4t^2 + 5t + 2$  where  $\phi$  is in weber and  $t$  is in second. calculate the magnitude induced emf in the coil when  $t = 3$  sec.

Given →

$$\phi = 4t^2 + 5t + 2$$

$$\text{Time } (t) = 2 \text{ sec}$$

$$\text{Induced emf } (\epsilon) = ?$$

we know,

$$E = \frac{d\phi}{dt}$$

$$= \frac{d}{dt} (4t^2 + 5t + 2)$$

$$= 8t + 5$$

$$\text{where, } t = 3$$

$$\epsilon = 8 \times 3 + 5$$

$$= 29 \text{ V}$$

The induced emf in the coil is 29 V.

Q2 A conducting circular loop is placed in a uniform transverse magnetic field of 0.02 T. somehow radius of loop begin to decrease at a constant rate of 2 mm/s. Find the emf induced in the loop at the instant when radius is 2 cm.

Given

$$\text{Magnetic field } (B) = 0.02 \text{ T}$$

$$\frac{dr}{dt} = \frac{2 \text{ mm}}{\text{s}} = 2 \times 10^{-3} \text{ m/s}$$

$$\text{radius } (r) = 2 \text{ cm}$$

Now,

$$\phi = NBA \cos \theta$$

where,  $N=1, \theta=0$

$$\phi = BA$$

$$\phi = B\pi r^2 \quad (\text{where Area}(A) = \pi r^2)$$

We know,

$$E = \frac{d\phi}{dt}$$

$$= d(B\pi r^2)$$

$$\frac{dt}{dt}$$

$$= B\pi \frac{dr^2}{dt}$$

$$= B\pi \frac{dr}{dt} \times \frac{dr}{dt}$$

$$= B\pi \times 2r \times \frac{dr}{dt}$$

$$= 0.02 \times \pi \times 2 \times 2 \times 10^{-2} \times 10^{-3}$$

$$= 2.51 \times 10^{-6}$$

8) A straight wire (conductor) of length 15 cm is moving with a uniform speed of 10 m/s at an angle of  $30^\circ$ . Uniform magnetic field  $10^{-4}$  T. calculate the emf produced across the length.

⇒ Soln

$$\text{length} (l) = 15 \text{ cm} = 15 \times 10^{-2} \text{ m.}$$

$$\text{velocity} (v) = 10 \text{ m/s.}$$

$$\theta = 30^\circ$$

$$\text{Magnetic field} = 10^{-4} \text{ T.}$$

$$\text{Induced emf } (\mathcal{E}) = ?$$

we know

$$\mathcal{E} = B V l \sin \theta$$

$$= 10^{-4} \times 10 \times 15 \times 10^{-2} \times \sin 30^\circ$$

$$= 150 \times 10^{-6} \times \frac{1}{2}$$

$$= 7.5 \times 10^{-5} \text{ V.}$$

9) A coil of 100 turn each of area  $2 \times 10^{-3}$  has a resistance of  $12 \Omega$ . It lies on a horizontal plane in a vertical magnetic field of  $3 \times 10^{-3}$  wb/m<sup>2</sup>. What charge circulate through the coil if its end are short circuited and coil is rotated through  $180^\circ$  about a diameter?

⇒ Given

$$\text{No of turns } (n) = 100 \text{ turn.}$$

$$\text{Area } (A) = 2 \times 10^{-3} \text{ m}^2.$$

$$\text{Magnetic field } (B) = 3 \times 10^{-3} \text{ T.}$$

$$q = ? \text{ Cqs}$$

$$\text{Resistance } (R) = 12 \Omega.$$

Since, coil is rotated through  $180^\circ$ .

$$\text{i.e. } d\phi = \phi_2 - \phi_1$$

$$\text{or, } d\phi = NBA \cos \theta_1 - NBA \cos \theta_2$$

$$\text{or, } d\phi = NBA \cos 0^\circ - NBA \cos 180^\circ$$

$$\text{or, } d\phi = NBA + NBA.$$

$$\text{or, } d\phi = 2NBA.$$

we know,

$$\theta = \frac{d\phi}{dt}$$

$$\text{or, } IR = \frac{d\phi}{dt}$$

$$\text{or, } \frac{dq}{dt} R = \frac{d\phi}{dt}$$

$$\text{or, } dq = \frac{d\phi}{R}$$

$$\text{or, } dq = \frac{2NBA}{R}$$

$$\text{or, } dq = 2 \times 100 \times 3 \times 10^{-3} \times 2 \times 10^{-3}$$

Q 12

$$= 100 \times 10^{-6}$$

$$= 10^{-4} \text{ C}$$

- (5) A rectangular coil of 100 turns has dimensions 15cm x 10cm. It is rotated at the rate of 300 revolutions per minute in a uniform magnetic field of flux density 0.6 T. Calculate the maximum emf induced.

Given →

$$\text{Number of turns (n)} = 100.$$

$$\text{Dimension (A)} = 15 \times 10 \text{ cm}^2 = 150 \text{ cm}^2 = 150 \times 10^{-4} \text{ m}^2.$$

$$\text{frequency} = 300 \text{ rev/min}$$

$$\frac{300}{60} \text{ rev/sec} = 5 \text{ Hz}$$

$$\text{Magnetic field (B)} = 0.6 \text{ T.}$$

$$\text{Induced emf (E)} = ?$$

we know

$$E = NBA \omega \sin \omega t$$

for  $E_{\max}$  sin  $\omega t = 1$

$$E_x = NBA \omega$$

$$= NBA 2\pi f$$

$$= 100 \times 0.6 \times 150 \times 10^{-4} \times 2\pi \times 5$$

$$= 28.27 \text{ V.}$$

The maximum emf produced is 28.27 V.

- Q) When a wheel with metal spokes 1.2 m long is rotated in a uniform magnetic field of  $8 \times 10^{-5}$  tesla normal to the plane of wheel, an emf of  $10^{-2}$  V is produced between rim and axle. Find the rate of rotation of wheel.

Given,

$$\text{Radius } (r) = 1.2 \text{ m}$$

$$\text{Magnetic field } (B) = 8 \times 10^{-5} \text{ T}$$

$$\text{emf } (e) = 10^{-2} \text{ V}$$

rate of rotation of wheel ( $f$ ) = ?

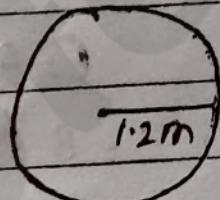
Now,

$$E = BAf$$

$$e = \frac{BA}{BA} = \frac{e}{B\pi r^2}$$

$$f = \frac{10^{-2}}{8 \times 10^{-5} \times \pi \times (1.2)^2}$$

$$= 4.4 \times 10^{-2} \text{ rev/sec.}$$



7. A wheel with 20 metallic spokes each 0.5 m long is rotated with a speed of 220 rev/min in a plane normal to the horizontal component of earth's magnetic field  $B_H$  at a place. If  $B_H = 0.4 \text{ G}$  at the place. what is the induced emf between the axle and rim of the wheel.

$\Rightarrow$  Soln →

$$\text{Radius of the wheel } (r) = \text{length of the spokes } (l) = 0.5 \text{ m}$$

$$\text{frequency of rotation } (f) = 220 \text{ rev/min} = 2 \text{ rev/sec.}$$

$$\text{Horizontal component of earth's magnetic field } (B_H) = 0.4 \text{ G}$$

$$= 0.4 \times 10^{-4} \text{ Tesla}$$

we have →

Induced emf between rim and axle of the wheel,

$$E = B A f$$

$$= B (\pi r^2) f$$

$$= 4 \times 10^{-5} \times \pi \times (0.5)^2 \times 2$$

$$= 6.28 \times 10^{-5} \text{ V.}$$

All 20 spokes are connected with their one end. For 20 spokes induced emf is same.

8. A long solenoid with 15 turns per cm has a small loop of area  $2 \text{ cm}^2$  placed inside, normal to the axis of the solenoid. If the current carried by the solenoid changes steadily from  $2 \text{ A}$  and  $4 \text{ A}$  in the  $0.1$  second, what is the induced voltage in the loop, while the current changing?

$\Rightarrow$  Soln →

No of turns per unit length  $\approx 15 \text{ turns/cm}$

$$\frac{15 \text{ turns}}{1 \text{ cm}} = 1500 \text{ turns/m.}$$

$$\text{Area}(A) = 2 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$$

current ( $I_1$ ) = 2 A.

current ( $I_2$ ) = 4 A.

$$\Delta I = I_2 - I_1 = 4 - 2 = 2 \text{ A}$$

Magnetic flux linkage ( $\Phi$ ) =  $BA$  -- (i)

But  $\Phi = M_{\text{on}} I$  -- (ii)

from (i) and (ii)

$$\Phi = M_{\text{on}} I Q$$

$$\text{Induced emf } (\epsilon) = \frac{d\Phi}{dt}$$

$$\epsilon = \frac{d M_{\text{on}} I Q}{dt}$$

$$= M_{\text{on}} Q \frac{dI}{dt}$$

$$= M_{\text{on}} Q \frac{\Delta I}{\Delta t}$$

$$= 4\pi \times 10^{-4} \times 1500 \times 2 \times 10^{-14} \times 2$$

$$= 7.54 \times 10^{-6} \text{ V.}$$

g) An air-cored solenoid with length 30 cm, area of cross-section  $25 \text{ cm}^2$  and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time  $10^{-3} \text{ sec}$ . How much is the average back emf induced across the ends of the open switch in the current.

$\rightarrow 30 \text{ cm}$

$$\text{length of core } (l) = 30 \text{ cm} = 30 \times 10^{-2} \text{ m}$$

$$\text{Area of cross-section } (A) = 25 \text{ cm}^2$$

$$= 25 \times 10^{-4} \text{ m}^2$$

$$\text{No of turns } (N) = 500$$

$$\text{current } (I_1) = 2.5 \text{ A}$$

$$\Delta t = 10^{-3} \text{ sec}$$

Current ( $I_2$ ) = 0 (The current is switched off)

$$dI = I_2 - I_1$$

$$= 0 - 2.5 = -2.5 \text{ A}$$

Self Inductance ( $\Delta$ ) =  $M0N^2A$

$$= 4\pi \times 10^{-7} \times (500)^2 \times 2.5 \times 10^{-4}$$

$$80 \times 10^{-2}$$

$$= 2.61 \times 10^{-3}$$

We know,

$$\text{Induced emf } (\mathcal{E}) = -\Delta \frac{dI}{dt}$$

$$= -2.61 \times 10^{-3} \times -0.25$$

$$10^{-3}$$

$$= \frac{6.55 \times 10^{-8}}{10^{-3}}$$

$$= 6.55 \text{ V}$$

- 10) A long solenoid of 1000 turns and cross sectional area  $2 \times 10^{-3} \text{ m}^2$  carries a current of 2 A and produces a flux density  $52 \times 10^{-3} \text{ T}$  inside it. Calculate the self inductance of the coil.

Given

$$\text{No of turns } (N) = 1000$$

$$\text{Area of cross section } (A) = 2 \times 10^{-3} \text{ m}^2$$

$$\text{current. } (I) = 2 \text{ A}$$

$$\text{magnetic flux density } (B) = 52 \times 10^{-3} \text{ T}$$

$$\text{self inductance } (L) = ?$$

We know,

$$\Phi = d \cdot I$$

$$\text{or, } NBA = dI$$

$$\text{classmate} \quad \text{or } d = \frac{NBA}{I}$$

$$1000 \times 5.2 \times 10^{-3} \times 2 \times 10^{-3}$$

2

$$= 0.052 \text{ H}$$

Over a solenoid of 50 cm length and 2 cm radius and having 500 turns, is wound another wire of 50 turns near the end. calculate the (i) mutual induction of the coil (ii) induced emf in the second coil when the current in the primary changes from 0 to 5 A in 0.02 s.

SOLN

$$\text{length of the solenoid (l)} = 50 \text{ cm} = 0.5 \text{ m}$$

$$\text{Radius (r)} = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$\text{Number of turns on first coil (N}_1\text{)} = 500 \text{ turns}$$

$$\text{Number of turns in second coil is (N}_2\text{)} = 50 \text{ turns}$$

$$\text{Mutual Inductance (M)} = ?$$

$$\text{current changes (dI)} = 0 - 5 \\ = -5 \text{ A}$$

$$dt = 0.02 \text{ sec.}$$

NOW,

$$M = \frac{M_0 N_1 N_2 A}{l}$$

$$= 4 \times \pi \times 10^{-7} \times 500 \times 50 \times \pi r^2$$

$$0.5$$

$$= 4 \times \pi \times 10^{-7} \times 500 \times 50 \times \pi r^2 (2 \times 10^{-2})^2$$

$$0.5$$

$$= 4 \times \cancel{\pi} \times 10^{-7} \quad 7.89 \times 10^{-5}.$$

Again →

$$E = M \frac{dI}{dt}$$

$$= -7.89 \times 10^{-5} \times 5$$

$$0.02$$

$$= 1.974 \times 10^{-2} \text{ V}$$

$$= 19.74 \text{ mV.}$$

12. An air filled toroidal solenoid has mean radius of 15 cm and cross sectional area of  $5\text{cm}^2$ . When the current of 12 A, the energy is 0.39 J. How many turns does the winding have?

$\Rightarrow$  Given,

$$\text{Radius of toroidal solenoid } (r) = 15\text{ cm} = 15 \times 10^{-2}\text{ m}$$

$$(\text{cross sectional area } A) = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$

$$\text{current } (I) = 12 \text{ A}$$

$$\text{Energy}(\epsilon) = 0.39$$

No of turns ( $N$ ) = ?

we know,

where,  $\lambda = \frac{M_0 N^2 A}{2\pi r}$  { because of PS toroid solenoid }  $R = 2\pi x$   $\theta$

Then in eqn ①

$$F = \frac{1}{2} \frac{M_0 n^2 A I^2}{4\pi r}$$

$$N = \sqrt{\frac{4\pi r^2}{M \Omega a^2}}$$

$$= \frac{4 \times 15 \times 10^{-2} \times 0.39}{15}$$

$$= \frac{4x\pi \times 10^{-2} + 5 \times 10^{-4} \times 12^2}{10.39}$$

$$= \sqrt{8125000}$$

~~= 8580 turns.~~

13. An inductor used in DC power supply has an inductance of 12 H and resistance of 180 Ω. It carries a current of 0.3 A.

- i) What is the energy stored in a magnetic field.  
ii) At what rate is thermal energy developed in induction?

Given →

$$\text{Inductance } (L) = 12 \text{ H}$$

$$\text{Resistance } (R) = 180 \Omega$$

$$\text{Current } (I) = 0.3 \text{ A}$$

$$\text{Energy } (E) = ?$$

$$\text{Power } (P) = ?$$

Note

$$i) P = \frac{1}{2} LI^2$$

$$= \frac{1}{2} \times 12 \times 0.3^2$$

$$= 0.54 \text{ J.}$$

ii) Power  $(P) = I^2 R$

$$= 0.3^2 \times 180$$

$$= 16.2 \text{ W.}$$

- ① The primary of an ideal step up transformer has 100 turns and the transformation ratio is 100. The input voltage and power are 220 V and 100 watt. calculate a) no of turns in secondary coil.

b) current in primary.

c) Voltage across the secondary.

d) Power in the secondary

Given,

100 of turns in primary ( $N_p = 100$  turns)

Transformation ratio ( $K = 100$ )

Input voltage ( $\epsilon_p$ ) = 220V

Input power ( $P_p$ ) = 1100 watt.

No of turns in secondary coil ( $N_s$ ) = ?

Current in primary ( $I_p$ ) = ?

(a)

We know,

$$K = \frac{N_s}{N_p}$$

$$\text{or, } N_s = K \times N_p \\ = 100 \times 100 \\ = 10,000 \text{ A.}$$

(b)

$$P_i = V_i I_p$$

$$P_i = \epsilon_p I_p$$

$$I_p = \frac{1100}{220} = 5 \text{ A.}$$

(c)

$$\frac{N_s}{N_p} = \frac{\epsilon_s}{\epsilon_p}$$

$$\text{or, } \epsilon_s = \frac{N_s \times \epsilon_p}{N_p}$$

$$= \frac{10,000 \times 220}{100}$$

$$= 2400 \text{ V.}$$

(d)

$$P_o = \epsilon_s I_s$$

for ideal transformer  $P_i = P_o$ .

$$P_i = \epsilon_s I_s$$

$$I_s = \frac{P_i}{\epsilon_s} = \frac{1100}{24000} = 0.05 \text{ A A.}$$

2. A transformer has 500 turns in the primary coil, and 100 turns in the secondary coil. What is the output voltage if the input voltage is 4000 volts. If the transformer is assumed to have an efficiency of 100%. What primary current is required to draw 2000 watt from the secondary.

Given,

No of turns in primary ( $N_p$ ) = 500 turns.

No of turns in secondary ( $N_s$ ) = 100 turn.

Input voltage ( $E_p$ ) = 4000 volts.

Efficiency ( $\eta$ ) = 100%.

Power output ( $P_o$ ) = 2000 watt.

Current in primary coil ( $I_p$ ) = ?

Now,

$$N_p = \frac{E_p}{E_s}$$

$$\text{or } \frac{500}{100} = \frac{400}{E_s}$$

$$\therefore E_s = 800 \text{ V.}$$

Again we have,

$$\eta = \frac{\text{output power}}{\text{input power}} \times 100\%$$

$$\text{or } 100\% = \frac{2000}{\text{input power}} \times 100\%$$

$$\text{or Input power} = 20 \text{ W} \quad (\text{Input power} = V_p \times I_p)$$

$$E_p \times I_p = 20 \text{ W}$$

$$\text{or } 20,000 \times I_p = 20$$

$$\therefore I_p = 0.005 \text{ A.}$$

# Bipin Khatri

## (Bipo)

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### Class 12 complete notes and paper collection.

Folders

Name ↑

 Biology	 chemistry
 English	 maths
 Nepali	 Physics



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### Feedbacks:

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