Chapter 9
Trigonometric Equations and General Values

Exercise 9

1. Solution:

a. $2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$\cos x = \cos\frac{\pi}{3}$

$\therefore \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$

b. $\sqrt{3}\sec x = 2$

$\sec x = \frac{1}{\sqrt{3}}$

$\cos x = \frac{\sqrt{3}}{2}$

$\therefore \quad \cos x = \cos\frac{\pi}{6}, \cos\left(2\pi - \frac{\pi}{6}\right)$

$\therefore \quad x = \frac{\pi}{6}, \frac{11\pi}{6}$

c. $\tan x = -\frac{1}{\sqrt{3}}$

$\tan x = \tan\left(\pi - \frac{\pi}{6}\right), \tan\left(2\pi - \frac{\pi}{6}\right)$

$\tan x = \tan\frac{5\pi}{6}, \tan\frac{11\pi}{6}$

$\therefore \quad x = \frac{5\pi}{6}, \frac{11\pi}{6}$

d. $\sin x = \frac{1}{\sqrt{2}}$

$\sin x = \sin\frac{\pi}{4}, \sin\left(\pi - \frac{\pi}{4}\right)$

$\sin x = \sin\frac{\pi}{4}, \sin\frac{3\pi}{4}$

$\therefore \quad x = \frac{\pi}{4}, \frac{3\pi}{4}$

2. Solution:

a. $\cos^2 x = \frac{1}{2}$

$\cos^2 x = \cos^2\frac{\pi}{4}$

$\therefore \quad x = n\pi \pm \frac{\pi}{4}$ (Since $\cos^2\theta = \cos^2\propto \Rightarrow \theta = n\pi \pm \propto$)

b. $\cos 3x = -\frac{1}{\sqrt{2}}$

$\cos 3x = \cos\frac{3\pi}{4}$

$\therefore$ The general solution is

$3x = 2n\pi \pm \frac{3\pi}{4} \Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{4}$

c. $\cos3x = \sin2x$

$\cos3x = \cos\left(\frac{\pi}{2} - 2x\right)$

$\therefore$ $3x = 2n\pi \pm \left(\frac{\pi}{2} - 2x\right)$ ($\because \cos\theta = \cos\propto \Rightarrow \theta = 2n\pi \pm \propto$)

$3x = 2n\pi + \frac{\pi}{2} - 2x$ or, $3x = 2n\pi - \frac{\pi}{2} + 2x$

$5x = 2x\pi + \frac{\pi}{2}$ $x = 2n\pi - \frac{\pi}{2}$

$5x = (4n + 1)\frac{\pi}{2}$ $\therefore$ $x = (4n - 1)\frac{\pi}{2}$

$5x = (4n + 1)\frac{\pi}{2}$

$\therefore$ $x = (4x + 1)\frac{\pi}{10}$

Hence, $x = (4n + 1)\frac{\pi}{10}, (4n - 1)\frac{\pi}{10}$

d. $\tan^2x = \frac{1}{3}$

$\tan^2x = \left(\frac{1}{\sqrt{3}}\right)^2$

or, $\tan^2x = \left(\tan\frac{\pi}{6}\right)^2$

$\therefore$ $x = n\pi \pm \frac{\pi}{6}$ ($\because \tan^2\theta = \tan^2\propto \Rightarrow \theta = n\pi \pm \propto$)

3.a. $\sin2x + \cos x = 0$

or, $2\sin x.\cos x + \cos x = 0$

or, $\cos x(2\sin x + 1) = 0$

Either $\cos x = 0$

or, $\sin x = -\frac{1}{2}$

$\therefore$ $x = (2x + 1)\frac{\pi}{2}$

$\sin x = \sin\left(-\frac{\pi}{6}\right)$

$\therefore$ $x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$\therefore$ $x = (2n + 1)\frac{\pi}{2}, n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

b. $\tan^3x = 3\tan x = 0$

or, $\tan x(\tan^2x - 3) = 0$

Either $\tan x = 0$ or, $\tan^2x - 3 = 0$

$\therefore$ $x = n\pi$ $\tan^2x = (\sqrt{3})^2$

$\tan^2x = \tan^2\frac{\pi}{3}$

$\therefore \quad x = n\pi \pm \frac{\pi}{3}$

$\therefore \quad x = n\pi, n\pi \pm \frac{\pi}{3}$

c. Sinax + cosbx = 0

or, –sinax = cosbx

$\cos bx = \cos\left(\frac{\pi}{2} + ax\right)$

$\therefore \quad bx = 2ns\pi \pm \left(\frac{\pi}{2} + ax\right)$ ($\because \cos\theta = \cos\propto \Rightarrow \theta = 2n\pi \pm \propto \forall n\in z$)

Taking positive sign

$bx = 2n\pi + \frac{\pi}{2} + ax$

$(b–a)\, x = 2n\pi + \frac{\pi}{2}$

$\therefore \quad x = 1(4n + 1)\, \frac{\pi}{2}$

Taking negative sign

$bx = 2n\pi - \frac{\pi}{2} - ax$

$(b + a)x = 2n\pi - \frac{\pi}{2}$

$x = \frac{1}{(b+a)}\,(4n - 1)\frac{\pi}{2}$

Hence, $x = \frac{(4n + 1)}{b - a}\,\frac{\pi}{2}, \frac{(4n - 1)}{b+a}\,\frac{\pi}{2}$

d. tanx + cotx = 2

or, $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = 2$

or, $\sin^2x + \cos^2x = 2\sin x\,.\,\cos x$

$1 = \sin 2x$

$\therefore \quad \sin 2x = \sin\frac{\pi}{2}$

$\therefore \quad 2x = n\pi \pm (-1)^n\,\frac{\pi}{2}$ ($\because \sin\theta = \sin\propto\ \theta = n\pi \pm (-1)^n \propto, \forall n$)

$x = \frac{n\pi}{2} + (-1)^n\,\frac{\pi}{4}$

4. Solution:

a. $4\cos^2x = 6\sin^2x = 5$

$4 - 4\sin^2x + 6\sin^2x = 5$

$2\sin^2x = 1$

$\sin^2x = \left(\frac{1}{\sqrt{2}}\right)^2$

$\sin^2x = \sin^2\frac{\pi}{4}$

$\therefore \quad x = n\pi \pm \frac{\pi}{4}$ ($\because \sin^2\theta = \sin^2\propto \Rightarrow \theta = n\pi \pm \propto \forall n\in z$)

b. $\cos^2x - \sin^2x + \cos x = 0$

$\cos^2x - 1 + \cos^2x + \cos x = 0$

or, $2\cos^2x + \cos x - 1 = 0$

or, $2\cos^2x + 2\cos x - \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

either $2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$\therefore$ $\cos x = \cos\frac{\pi}{3}$

$\therefore$ $x = 2n\pi \pm \frac{\pi}{3}$

or, $\cos x + 1 = 0$

$\cos x = -1$

$\cos x = \cos\pi$

$\therefore$ $x = 2n\pi \pm \pi$

c. $3\cos^2x + 5\sin^2x = 4$

or, $3-3\sin^2x + \sin^2x = 4$

$2\sin^2x = 1$

$\therefore$ $\sin^2x = \left(\frac{1}{\sqrt{2}}\right)^2$

$\sin^2x = \sin^2\frac{\pi}{4}$

$\therefore$ $x = n\pi \pm \frac{\pi}{4}$

d. $4\sin^2x - 8\cos x + 1 = 0$

$4 - 4\cos^2x - 8\cos x + 1 = 0$

or, $4\cos^2x + 8\cos x - 5 = 0$

or, $4\cos^2x + 10\cos x - 2\cos x - 5 = 0$

or, $2\cos x(2\cos x + 5) - 1(2\cos x + 5) = 0$

$(2\cos x - 1)(2\cos x + 5) = 0$

Either $2\cos x - 1 = 0$ or, $\cos x = -\frac{5}{2}$

$\cos x = \frac{1}{2}$

$\cos x = \cos\frac{\pi}{3}$

$\therefore$ $x = 2n\pi \pm \frac{\pi}{3}$

5. Solution:

a. $\cos x + \cos 2x + \cos 3x = 0$

$(\cos x + \cos 3x) + \cos 2x = 0$

or, $2\cos\left(\frac{x + 3x}{2}\right) . \cos\left(\frac{3x - x}{2}\right) + \cos 2x = 0$

$2\cos 2x . \cos x + \cos 2x = 0$

or, $\cos 2x(2\cos x + 1) = 0$

Either $\cos 2x = 0$ or, $2\cos x + 1 = 0$

$\cos 2x = \cos\frac{\pi}{2}$ $\qquad$ $\cos x = -\frac{1}{2}$

$\therefore$ $2x = (2n + 1)\frac{\pi}{2}$ $\qquad$ $\cos x = \cos\frac{2\pi}{3}$

$\therefore$ $x = (2n+1)\frac{\pi}{4}$ $\qquad$ $\therefore$ $x = 2n\pi \pm \frac{2\pi}{3}$

b. $\text{Sin}3x + \sin x = \sin^2x$

$2\sin\left(\frac{3x + x}{2}\right) . \cos\left(\frac{3x - 3}{2}\right) = \sin^2x$

or, $2\sin 2x . \cos x - \sin^2x = 0$

$\sin 2x (2\cos x - 1) = 0$

Either $\sin 2x = 0$ or, $2\cos x - 1 = 0$

$2x = n\pi$; $\cos x = \frac{1}{2}$

$\therefore x = \frac{n\pi}{3}$; $\cos x = \cos\frac{\pi}{3}$

$\therefore x = 2n\pi \pm \frac{\pi}{3}$

$\therefore x = \frac{n\pi}{2}, 2n\pi \pm \frac{\pi}{3}$

c. $\cos 3x + \cos x = \cos 2x$

or, $2\cos 2x \,.\, \cos x = \cos 2x$

$\cos 2x (2\cos x - 1) = 0$

Either $\cos 3x = 0$ or, $2\cos x - 1 = 0$

$\therefore 2x = (2n+1)\frac{\pi}{2}$; $\cos x = \frac{1}{2}$

$\therefore x = (2n+1)\frac{\pi}{4}$; $\cos x = \cos\frac{\pi}{3}$

$\therefore x = 2n\pi \pm \frac{\pi}{3}$

Hence, the general solution

$x = (2n+1)\frac{\pi}{4}, (6n \pm 1)\frac{\pi}{3}$

d. $2\tan x - \cot x = -1$

$2\tan x - \frac{1}{\tan x} = -1$

or, $2\tan^2 x - 1 = -\tan x$

or, $2\tan^2 x + \tan x - 1 = 0$

$2\tan^2 x + 2\tan x - \tan x - 1 = 0$

or, $2\tan x(\tan x + 1) - 1(\tan x + 1) = 0$

$(\tan x + 1)(2\tan x - 1) = 0$

Either $\tan x + 1 = 0$

$\tan x = -1$

$\tan x = \tan\frac{3\pi}{4}$ or, $\tan\left(-\frac{\pi}{4}\right)$

$\therefore x = n\pi + \frac{3\pi}{4}$ or, $n\pi - \frac{\pi}{4}$

or, $2\tan x - 1 = 0$

$\tan x = \frac{1}{2}$

$\therefore x = \tan^{-1}\frac{1}{2}$

$\therefore x = n\pi + \tan^{-1}\frac{1}{2}$

Hence, the general solution are

$x = n\pi - \frac{\pi}{4}, n\pi + \tan^{-1}\frac{1}{2}$

6. Solution:

a. $\sqrt{3}\sin x - \cos x = \sqrt{2}$

Dividing each term by 2

$$\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{1}{\sqrt{2}}$$

$$\sin\frac{\pi}{3}\sin x - \cos\frac{\pi}{3}\cos x = \frac{1}{\sqrt{2}}$$

$$-\cos\left(x + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

or, $\cos\left(x + \frac{\pi}{3}\right) = -\frac{1}{\sqrt{2}}$

$$\cos\left(x + \frac{\pi}{3}\right) = \cos\left(\frac{3\pi}{4}\right)$$

$\therefore\ x + \frac{\pi}{3} = 2n\pi \pm \frac{3\pi}{4}$

$$x = 2n\pi - \frac{\pi}{3} \pm \frac{3\pi}{4}$$

or, $\cos\frac{\pi}{6}\sin x - \sin\frac{\pi}{6}\cos x = \frac{1}{\sqrt{2}}$

$$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(x - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}$$

$\therefore\ x - \frac{\pi}{6} = n\pi \pm (-1)^n\frac{\pi}{4}$

$\therefore\ x = n\pi + \frac{\pi}{6} \pm (-1)^n\frac{\pi}{4}$

$$\sin x = \sin\frac{3\pi}{2}$$

$\therefore\ x = n\pi \pm (-1)^n\frac{3\pi}{2}$

or, $2\sin x = 1$

$$\sin x = \sin\frac{\pi}{6}$$

$\therefore\ x = n\pi \pm (-1)^n\frac{\pi}{6}$

b. $\tan x + \sec x = \sqrt{3}$

$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = \sqrt{3}$

or, $\sin x + 1 = \sqrt{3}\cos x$

or, $\sqrt{3}\cos x - \sin x = 1$ ... ... (i)

Dividing (i) by $\sqrt{\sqrt{3}^2 + (-1)^2} = 2$

$\therefore\ \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{1}{2}$

or, $\cos\left(x + \frac{\pi}{6}\right) = \cos\left(2x\ \pi \pm \frac{\pi}{3}\right)$

$\Rightarrow\ x = 2n\pi + \frac{\pi}{6},\ 2n\pi - \frac{\pi}{2}$

c. $\text{Sinx} + \sqrt{3}\ \text{cosx} = \sqrt{2}$

Dividing both sides by

$\sqrt{(\text{coeff. of sim})^2 + (\text{coeff. of cosx})^2} = \sqrt{1 + + (\sqrt{3})^2 = 2}$

$\frac{1}{2}\text{sinx} + \frac{\sqrt{3}}{2}\text{cosx} = \frac{1}{\sqrt{2}}$

or, $\text{sinx. sin}\frac{\pi}{6} + \text{cosx . cos}\frac{\pi}{6} = \cos\frac{\pi}{4}$

$\cos\left(x - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}$

$\therefore\ x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4}$

$\therefore\ x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$

d. $\sqrt{2}\ \text{secx} + \text{tanx} = 1$

$\sqrt{2} + \text{sinx} = \text{cosx}$

or, $\text{cosx} - \text{sinx} = \sqrt{2}$

Dividing both sides by $\sqrt{2}$

$\frac{1}{\sqrt{2}}\text{cosx} - \frac{1}{\sqrt{2}}\text{sinx} = 1$

$\cos\frac{\pi}{4}\ .\ \text{cosx} - \sin\frac{\pi}{4}\ .\ \text{sinx} = \cos 0$

$\therefore\ \cos\left(x + \frac{\pi}{4}\right) = \cos 0^o$

$\therefore\ x + \frac{\pi}{4} = 2n\pi \pm 0$

$x = 2n\pi - \frac{\pi}{4}$

e. $\text{Sinx} + \text{cosx} = -\frac{1}{\sqrt{2}}$

Dividing both sides by $\sqrt{2}$

$\frac{1}{\sqrt{2}}\text{sinx} + \frac{1}{\sqrt{2}}\text{cosx} = -\frac{1}{2}$

$\cos\left(x - \frac{\pi}{4}\right) = \cos\left(\frac{2\pi}{3}\right)$

$\therefore\ x - \frac{\pi}{4} = 2n\pi \pm \frac{2\pi}{3}$

$\therefore\ x = 2n\pi \pm 2n\pi \pm \frac{2\pi}{3} + \frac{\pi}{4}$

7. Solution:

a. $\sin 2x + \sin 4x + \sin 6x = 0$

or, $(\sin 2x + \sin 6x) + \sin 6x = 0$

or, $2\sin 4x\ .\ \cos 2x + \sin 4x = 0$

$\sin 4x\ (2\cos 2x + 1) = 0$

Either, $\sin 4x = 0$ or, $2\cos 2\ x + 1 = 0$

$\therefore\ 4x = n\pi$ $\cos 2x = -\frac{1}{2}$

$\therefore \quad x = \frac{n\pi}{4}$

$\cos 2x = \cos\left(\frac{2\pi}{3}\right)$

$2x = 2n\pi \pm \frac{2\pi}{3}$

$\therefore \quad x = n\pi \pm \frac{\pi}{3}$

$\therefore \quad x = \frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$

b. (sinx + sin5x) + sin3x = 0

or, 1sin3x . cos2x + sin3x = 0

or, sin3x (2cos2x + 1) = 0

Either sin3x = 0

$\therefore \quad 3x = n\pi$

$\therefore \quad x = \frac{n\pi}{3}$

or, 2cos2x + 1 = 0

$\cos 2x = -\frac{1}{2}$

$\cos 2x = -\frac{1}{2}$

$\cos 2x = \cos\left(\frac{2\pi}{3}\right)$

$\therefore \quad 2x = 2n\pi \pm \frac{2\pi}{3}$

$x = n\pi \pm \frac{\pi}{3}$

Hence, $x = \frac{n\pi}{3}, n\pi \pm \frac{\pi}{3}$

c. cos3x + cosx – cos2x = 0

or, 2cos2x . cosx – cos2x = 0

or, cos2x (2cosx – 1) = 0

Either cos2x = 0

$\therefore \quad 2x = (2n + 1)\frac{\pi}{2}$

$\therefore \quad x = (2n + 1)\frac{\pi}{4}$

or, 2cosx – 1 = 0

$\cos x = \frac{1}{2}$

$\cos x = \cos\frac{\pi}{3}$

$\therefore \quad x = 2n\pi \pm \frac{\pi}{3}$

Hence, $x = (2n + 1)\frac{\pi}{4}, 2n\pi \pm \frac{\pi}{3}$

d. cosx + sinx = cos2x + sin2x

or, $\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}\cos 2x + \frac{1}{\sqrt{2}}\sin 2x$

or, $\cos\frac{\pi}{4} . \cos x + \sin\frac{\pi}{4}\sin x = \cos\frac{\pi}{4}\cos 2x + \sin\frac{\pi}{4} .\sin 2x$

or, $\cos\left(x - \frac{\pi}{4}\right) = \cos\left(2x - \frac{\pi}{4}\right)$

$\therefore \quad 2x - \frac{\pi}{4} = 2n\pi \pm \left(x - \frac{\pi}{4}\right)$

$$2x - \frac{\pi}{4} = \begin{cases} 2n\pi + x - \frac{\pi}{4} \\ 2n\pi - x + \frac{\pi}{4} \end{cases}$$

Either $x = 2n\pi$

or, $3x = 2n\pi + \frac{\pi}{2}$

i.e. $x = \frac{2n\pi}{3} + \frac{\pi}{6} = (4n + 1)\frac{\pi}{6}$

Hence, $x = 2n\pi, (4nx + 1)\frac{\pi}{6}$

e. $\tan x + \tan 2x = 1 - \tan x \cdot \tan 2x$

or, $\frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x} = 1$

or, $\tan(2x + x) = 1$

$\tan 3x = \tan\frac{\pi}{4}$

$\therefore\ 3x = n\pi + \frac{\pi}{4}$ $(\because \tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha)$

$\therefore\ x = \frac{n\pi}{3} + \frac{\pi}{12}$

f. $\tan x + \tan 2x + \sqrt{3}\tan x \cdot \tan 2x = \sqrt{3}$

or, $\tan x + \tan 2x = \sqrt{3}(1 - \tan x \cdot \tan 2x)$

or, $\frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x} = \sqrt{3}$

or, $\tan(2x + x) = \sqrt{3}$

$\tan 3x = \tan\left(\frac{\pi}{3}\right)$

$\therefore\ 3x = n\pi + \frac{\pi}{3}$

$\therefore\ x = \frac{n\pi}{3} + \frac{\pi}{9}$

g. $\tan 3x + \tan 4x + \tan 7x = \tan 3x \cdot \tan 4x \cdot \tan 7x$

$\tan 3x + \tan 4x = -\tan 7x(1 - \tan 3x \cdot \tan 4)$

or, $\frac{\tan 3x + \tan 4x}{1 - \tan 3x \cdot \tan 4x} = -\tan 7x$

$\tan 7x = -\tan 7x$

$2\tan 7x = 0$ $\tan 7x = 0$

$7x = n\pi$

$\therefore\ x = \frac{n\pi}{7}$

h. $\tan^2 x + (1 - \sqrt{3})\tan x - \sqrt{3} = 0$

or, $\tan^2 x - \sqrt{3}\tan x + \tan x - \sqrt{3} = 0$

$\tan x(\tan x - \sqrt{3}) + 1(\tan x - \sqrt{3}) = 0$

$(\tan x - \sqrt{3})(\tan x + 1) = 0$

Either $\tan x - \sqrt{3} = 0$

$\tan x = \tan\frac{\pi}{3}$

$\therefore\ x = n\pi + \frac{\pi}{3}$

or, $\tan x = -1$

$\tan x = \tan\left(-\frac{\pi}{4}\right)$

$\therefore \quad x = n\pi - \frac{\pi}{4}$

Hence, $x = n\pi + \frac{\pi}{3}$, $n\pi - \frac{\pi}{4}$ $n \in z$

i. $\tan(\theta + \alpha) \cdot \tan(\theta - \alpha) = 1$

or, $\left(\frac{\tan\theta + \tan\alpha}{1 - \tan\theta \cdot \tan\alpha}\right)\left(\frac{\tan\theta - \tan\alpha}{1 + \tan\theta \cdot \tan\alpha}\right) = 1$

$\frac{\tan^2\theta - \tan^2\alpha}{1 - \tan^2\theta \cdot \tan^2\alpha} = 1$

or, $\tan^2\theta - \tan^2\alpha = 1 - \tan^2\theta . \tan^2\alpha$

or, $\tan^2\theta - \tan^2\theta + \tan^2\theta \cdot \tan^2\alpha = 1$

$\tan^2\theta + \tan^2\theta \cdot \tan^2\theta = 1 + \tan^2\alpha$

$\tan^2\theta\,(1 + \tan^2\alpha) = (1 + \tan^2\alpha)$

$\tan^2\theta = 1$

$\tan^2\theta \quad \tan^2\frac{\pi}{4}$

$\therefore \quad \theta = n\pi \pm \frac{\pi}{4}$

8. Solution:

a. $2\sin^2x + 6 - 6\sin^2x = 5$

$2\sin^2x + 6 - 6\sin^2x = 5$

$-4\sin^2x = -1$

$\sin^2x = \left(\frac{1}{2}\right)^2$

$\sin^2x = \left(\sin\frac{\pi}{6}\right)^2$

$\therefore \quad x = n\pi \pm \frac{\pi}{6}$

b. $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 4$

or, $\frac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4} \cdot \tan\theta} + \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4} \cdot \tan\theta} = 4$

or, $\frac{1 + \tan\theta}{1 - \tan\theta} + \frac{1 - \tan\theta}{1 + \tan\theta} = 4$

or, $(1 + \tan\theta)^2 + (1 - \tan\theta)^2 = 4(1 + \tan\theta)(1 - \tan\theta)$

or, $1 + 2\tan\theta + \tan^2\theta + 1 - 2\tan\theta + \tan^2\theta = 4 - 4\tan^2\theta$

or, $6\tan^2\theta = 2$

$\tan^2\theta = \left(\frac{1}{\sqrt{3}}\right)^2$

$\therefore \quad \tan^2\theta = \tan2\left(\frac{\pi}{6}\right)$

$\therefore \quad \theta = n\pi \pm \frac{\pi}{6}$, $n \in z$

c. $2\sin^2x + \sin^2 2x = 2$

or, $2\sin^2x + 4\sin^2x \cdot \cos^2x - 2 = 0$

or, $2\sin^2x + 4\sin^2x\,(1-\sin^2x) - 2 = 0$

or, $2\sin^2x + 4\sin^2x\ 4\sin^4x - 2 = 0$

or, $-4\sin^4x - 6\sin^2x - 2 = 0$

or, $2\sin^4x - 3\sin^2x + 1 = 0$

$(\sin^2x - 1)(2\sin^2x - 1) = 0$

Either

$\sin^2x - 1 = 0$ or, $2\sin^2x - 1 = 0$

$\sin^2x = 1$ $\quad$ $\sin^2x = \left(\frac{1}{\sqrt{2}}\right)^2$

$\sin^2x = \sin^2\frac{\pi}{2}$ $\quad$ $\sin^2x = \sin^2\left(\frac{\pi}{4}\right)$

$\therefore\ x = n\pi \pm \frac{\pi}{2}$ $\quad$ $\therefore\ x = n\pi \pm \frac{\pi}{4}$

Hence, $x = n\pi \pm \frac{\pi}{2}$, $n\pi \pm \frac{\pi}{4}$

d. $\tan px = \cot qx$

$\frac{\sin px}{\cos px} = \frac{\cos qx}{\sin qx}$

or, $\cos px.\cos qx - \sin px \cdot \sin qx = 0$

$\cos(px + qx) = 0$

or, $\cos(p + q)x = 0$

$\therefore\ (p + q)x = (2n + 1)\frac{\pi}{2}$

$\therefore\ x = \frac{(2n+1)}{p+q}\cdot\frac{\pi}{2}$

9. Solution:

a. $\tan^2x = \tan x\ (-\pi \le x \le \pi)$

or, $\frac{2\tan x}{1-\tan^2x} = \tan x$

$2\tan x - \tan x\,(1 - \tan^2x) = 0$

$\tan x\,(2 - 1 + \tan^2x) = 0$

$\tan x\,(1 + \tan^2x) = 0$

Either $\tan x = 0$

$\tan x = \tan 0^o, \tan\pi, \tan(-\pi)$

$\therefore\ x = 0^o, \pi, -\pi$

b. $\sin x = \frac{1}{2}$ and $\cos x = -\frac{\sqrt{3}}{2}$ $\quad (0 \le x \le 2\pi)$

Since, sine of an angle is positive and cosine of the same angle is negative, so the angle must lie in the second quadrant.

$\therefore\ x = \pi - \frac{\pi}{6}$ satisfies both equations

$\therefore\ x = \frac{5\pi}{6}$

c. $\tan x - 3\cot x = 2\tan^3x \quad (0 \le x \le 360^o)$

$\tan x - \frac{3}{\tan x} = 2\left(\frac{3\tan x - \tan^3x}{1-3\tan^2x}\right)$

or, $\frac{\tan^2x - 3}{\tan x} = \frac{6\tan x - 2\tan^3x}{1 - 3\tan^2x}$

or, $\tan^2x - 3\tan^4x - 3 + 9\tan^2x = 6\tan^2x - 2\tan^4x$

or, $\tan^3x - 4\tan^2x + 3 = 0$

$\tan^4x - 3\tan^2x - \tan^2x + 3 = 0$

$\tan^2x(\tan^2x - 3) - 1(\tan^2x - 3) = 0$

$(\tan^2x - 1)(\tan^2x - 3) = 0$

Either, $\tan^2x - 1 = 0$

$\Rightarrow$ $\tan^2x = 1$

$\tan x = \pm 1$

$\tan x = \tan\frac{\pi}{4}, \tan\frac{3\pi}{4}, \tan\frac{5\pi}{4}, \tan\frac{7\pi}{4}$

$\therefore$ $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

or, $\tan^2x = 3$

$\therefore$ $\tan x = \pm\sqrt{3}$

$\tan x = \tan\frac{\pi}{3}, \tan\frac{2\pi}{3}, \tan\frac{4\pi}{4}, \tan\frac{5\pi}{3}$

$\therefore$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Hence, $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

10. Given equations

$\cot x + \cot y = 2$ ... ... (i) and $2\sin x.\sin y = 1$ ... ... (ii)

$\Rightarrow$ $\frac{\cos x.\sin y + \sin x\cos y}{\sin x.\sin y} = 2$

or, $\sin x\cos y + \cos x.\sin y = 2\sin x.\sin y$

$\sin(x + y) = 1$ using (ii)

$\sin(x + y) = \sin 90^o$

$\therefore$ $x + y = 90^o$ ... ... (iii)

Also, $2\sin x.\sin y = 1$

or, $\cos(x - y) = \cos(x + y) = 1$

$\cos(x - y) = \cos 90^o = 1$

$\cos(x - y) = 0 = 1$

$\cos(x - y) = 1$

$\cos(x - y) = \cos 0^o$

$\therefore$ $x - y = 0^o$ ... ... (iv)

Solving (iii) and (iv) we get

$x = 45^o = \frac{\pi}{4}$ and $y = 45^o = \frac{\pi}{4}$

$\therefore$ $x = \frac{\pi}{4}, y = \frac{\pi}{4}$