Chapter – 7
Matrix Based System of Linear Equations

Exercise 7.1

1. By drawing graph or otherwise, classify each of the following system of the equations.

a. Here,

Given equations are $4x - 3y = -6$ ... (i) and $-4x + 2y = 16$ ... (ii)

Adding equation (i) and (ii), we get

$4x - 3y = -6$

$\underline{-4x + 2y = 16}$

$-y = 10$

$\therefore \quad y = -10$

Putting in equation (i),

$4x - 3x - 10 = -6$

or, $4x = -6 - 30$

$\therefore \quad x = -9$

Hence, (–9, –10) is the solution of the system. This kind of system where we get only one solution is known as consistent and independent.

b. Here,

Given equation of system are,

$2x - y = 3$ ... ... (i)

$-4x + 2y = 6$ ... ... (ii)

Multiplying by 2 in equation (i) and adding with (ii), we get

$4x - 2y = 6$

$\underline{-4x + 2y = 6}$

$0 = 12$

This is impossible result. In other word, the system has no solution. This is an inconsistent and independent.

c. Here,

Given, $-6x + 4y = 10$ ... ... (i)

$3x - 2y = -5$ ... ... (ii)

Multiplying by 2 in equation (ii)and adding with (i), we get

$6x + 4y = 10$

$\underline{6x - 4y = -10}$

$0 = 0$

So, we do not get particular value of x and y. However, the result 0 = 0 is true. In this situation, whatever be the solution of one equation satisfies the other equation as well. This kind of system, where we get infinitely many solution is known as consistent and dependent.

d. Here,

Given, $7x + 2y = 15$ ... ... (i)

$x + y = 5$ .. .. (ii)

Multiplying with 7 in equation (ii) and subtracting (i) from (ii),

$7x + 2y = 15$

$\underline{(-)\ (-) \quad (-)}$

$5y = 20$

$\therefore \quad y = 4$

Putting y = 4 in equation (ii), we get

$x + 4 = 5$

$\therefore \quad x = 1$

Hence, (1, 4) is the solution of the system. This kind of system of solution where only one solution we get is known consistent and independent.

2. Solve the following systems by using row – equivalent matrix method

a. Here,

$x + y = 5$
$2x + 3y = 12$
The augmented matrix is

$$\begin{bmatrix} 1 & 1 & : & 5 \\ 2 & 3 & : & 12 \end{bmatrix}$$

Multiplying by 2 in R_1 and subtracting from R_2.

$$\sim \begin{bmatrix} 1 & 1 & : & 5 \\ 0 & 1 & : & 2 \end{bmatrix}$$

Applying $R_2 \to R_1 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & : & 3 \\ 0 & 1 & : & 2 \end{bmatrix}$$

Hence the solution is $x = 3$ and $y = 2$

b. Here,
Augmented matrix is

$$\begin{bmatrix} 2 & 12 & : & 16 \\ 3 & 10 & : & 8 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$

$$\sim \begin{bmatrix} 3 & 10 & : & 8 \\ 2 & 12 & : & 16 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\sim \begin{bmatrix} 1 & -2 & : & -8 \\ 2 & 12 & : & 16 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$

$$\sim \begin{bmatrix} 1 & -2 & : & -8 \\ 0 & 16 & : & 32 \end{bmatrix}$$

Applying $R_2 \to \frac{1}{16} R_2$

$$\sim \begin{bmatrix} 1 & -2 & : & -8 \\ 0 & 1 & : & 2 \end{bmatrix}$$

Applying $R_1 \to R_1 + 2R_2$

$$\sim \begin{bmatrix} 1 & 0 & : & -4 \\ 0 & 1 & : & 2 \end{bmatrix}$$

Hence, the required solution is $x = -4$ and $y = 2$

c. Here,
Augmented matrix is

$$\begin{bmatrix} 1 & -3 & : & -1 \\ 4 & -1 & : & 7 \end{bmatrix}$$

Applying $R_2 \to R_2 - 4R_1$

$$\sim \begin{bmatrix} 1 & -3 & : & -1 \\ 0 & 11 & : & 11 \end{bmatrix}$$

Applying $R_2 \to \frac{1}{11} R_2$

$$\sim \begin{bmatrix} 1 & -3 & : & -1 \\ 0 & 1 & : & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 + 3R_2$

$$\begin{bmatrix} 1 & 0 & : & 2 \\ 0 & 1 & : & 1 \end{bmatrix}$$

Hence, the required solution is $x = 2$ and $y = 1$

d. Here,

The augmented matrix is

$$\begin{bmatrix} 8 & -3 & : & -31 \\ 2 & 6 & : & 26 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$

$$\sim \begin{bmatrix} 2 & 6 & : & 26 \\ 8 & -3 & : & -31 \end{bmatrix}$$

Applying $R_1 \to \frac{1}{2} R_1$

$$\sim \begin{bmatrix} 1 & 3 & : & 13 \\ 8 & -3 & : & -31 \end{bmatrix}$$

Applying $R_2 \to R_2 - 8R_1$

$$\sim \begin{bmatrix} 1 & 3 & : & 13 \\ 0 & -27 & : & -13 \end{bmatrix}$$

Applying $R_2 \to -\frac{1}{27} R_2$

$$\sim \begin{bmatrix} 1 & 3 & : & 13 \\ 0 & 1 & : & 5 \end{bmatrix}$$

Applying $R_1 \to R_1 - 3R_2$

$$\sim \begin{bmatrix} 1 & 0 & : & -2 \\ 0 & 1 & : & 5 \end{bmatrix}$$

Hence, the required solution is $x = -2$ and $y = 5$

e. Here,

The augmented matrix is

$$\begin{bmatrix} 5 & -3 & : & -2 \\ 4 & 2 & : & 5 \end{bmatrix}$$

Applying $R_1 \to \frac{1}{5} R_1$

$$\sim \begin{bmatrix} 1 & -3/5 & : & -2/5 \\ 4 & 2 & : & 5 \end{bmatrix}$$

Applying $R_2 \to R_2 - 4R_1$

$$\begin{bmatrix} 1 & -3/5 & : & -2/5 \\ 0 & 22/5 & : & 33/5 \end{bmatrix}$$

Applying $R_2 \to \frac{5}{22} R_2$

$$\sim \begin{bmatrix} 1 & -3/5 & : & -2/5 \\ 0 & 1 & : & 3/2 \end{bmatrix}$$

Applying $R_1 \to R_1 + \frac{3}{5} R_2$

$$\sim \begin{bmatrix} 1 & 0 & : & 1/2 \\ 0 & 1 & : & 3/2 \end{bmatrix}$$

Hence the required solution is $x = \frac{1}{2}$ and $y = \frac{3}{2}$

f. Here,

The augmented matrix is

$$\begin{bmatrix} 2 & 3 & : & 2 \\ 4 & -5 & : & 7 \end{bmatrix}$$

Applying $R_1 \to \frac{1}{2} R_1$

$$\sim \begin{bmatrix} 1 & 3/2 & : & 1 \\ 4 & -5 & : & 7 \end{bmatrix}$$

Applying $R_2 \to R_2 - 4R_1$

$$\sim \begin{bmatrix} 1 & 3/2 & : & 1 \\ 0 & -11 & : & 3 \end{bmatrix}$$

Applying $R_2 \to -\frac{1}{11} R_2$

$$\sim \begin{bmatrix} 1 & 3/2 & : & 1 \\ 0 & 1 & : & -3/11 \end{bmatrix}$$

Applying $R_1 \to R_1 - \frac{3}{2} R_1$

$$\sim \begin{bmatrix} 1 & 0 & : & 31/22 \\ 0 & 1 & : & -3/11 \end{bmatrix}$$

Hence, the required solution is $\frac{1}{x} = \frac{31}{22} \Rightarrow x = \frac{22}{31}$

and $\frac{1}{y} = \frac{-3}{11} \Rightarrow y = \frac{-11}{3}$

3. Use the row equivalent matrix method to solve the system of equations:

a. Here,

The augmented matrix is

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 3 & : & -1 \\ 2 & -1 & 2 & : & -4 \end{bmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - {}_2R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 2 & : & -2 \\ 0 & -3 & 0 & : & -6 \end{bmatrix}$$

Applying $R_3 \to R_3 + 3R_2$ and $R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & -1 & : & 3 \\ 0 & 1 & 2 & : & -2 \\ 0 & 0 & 6 & : & -12 \end{bmatrix}$$

Applying $R_3 \to \frac{1}{6} R_3$

$$\sim \begin{bmatrix} 1 & 0 & -1 & : & 3 \\ 0 & 1 & 2 & : & -2 \\ 0 & 0 & 1 & : & -2 \end{bmatrix}$$

Applying $R_1 \to R_1 + R_3$ and $R_2 \to R_2 - 2R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & -2 \end{bmatrix}$$

Hence, the required solution is $x = 1$, $y = 2$ and $z = -2$

b. Here,

The augmented matrix is

$$\begin{bmatrix} 1 & 4 & 1 & : & 18 \\ 3 & 3 & -2 & : & 2 \\ 0 & -4 & 1 & : & -7 \end{bmatrix}$$

Applying $R_2 \to R_2 - 3R_1$

$$\sim\begin{bmatrix} 1 & 4 & 1 & : & 18 \\ 0 & -9 & -5 & : & -52 \\ 0 & -4 & 1 & : & -7 \end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & 4 & 1 & : & 18 \\ 0 & 1 & 5/9 & : & 52/9 \\ 0 & -4 & 1 & : & -7 \end{bmatrix}$$

Applying $R_3 \to 4\,R_2 + R_3$, we get

$$\sim\begin{bmatrix} 1 & 4 & 1 & : & 18 \\ 0 & 1 & 5/9 & : & 52/9 \\ 0 & 0 & 29/9 & : & 145/9 \end{bmatrix}$$

Applying $R_3 \to \frac{9}{29} \times R_3$ we get

$$\sim\begin{bmatrix} 1 & 4 & 1 & : & 18 \\ 0 & 1 & 5/9 & : & 52/9 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

Applying $R_2 \to R_2 - \frac{5}{9} R_3$ we get,

$$\sim\begin{bmatrix} 1 & 4 & 1 & : & 18 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_3$ we get

$$\sim\begin{bmatrix} 1 & 4 & 0 & : & 13 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

Applying $R_1 \to R_1 - 4R_2$ we get

$$\sim\begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

Hence, $x = 1$, $y = 3$, $z = 5$

c. The augmented matrix is

$$\begin{bmatrix} -6 & 9 & 0 & : & 3 \\ 1 & 0 & 1 & : & 1 \\ 0 & 2 & 1 & : & 2 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$

$$\sim\begin{bmatrix} 1 & 0 & 1 & : & 1 \\ -5 & 9 & 0 & : & 3 \\ 0 & 2 & 1 & : & 2 \end{bmatrix}$$

Applying $R_2 \to R_2 + 5R_1$

$$\sim\begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 9 & 5 & : & 8 \\ 0 & 2 & 1 & : & 2 \end{bmatrix}$$

Applying $R_2 \to R_2 - 4R_3$

$$\sim\begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 1 & 1 & : & 0 \\ 0 & 2 & 1 & : & 2 \end{bmatrix}$$

Applying $R_3 \to R_3 - 2R_2$

$$\sim\begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 1 & 1 & : & 0 \\ 0 & 0 & -1 & : & 2 \end{bmatrix}$$

Applying $R_3 \to -1R_3$

$$\sim\begin{bmatrix} 1 & 0 & 1 & : & 1 \\ 0 & 1 & 1 & : & 0 \\ 0 & 0 & 1 & : & -2 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_1 - R_3$

$$\sim\begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & -2 \end{bmatrix}$$

Hence the solution is x = 3, y = 2 and z = –2

d. The augmented matrix is

$$\begin{bmatrix} 1 & -1 & 2 & : & 0 \\ 1 & -2 & 3 & : & -1 \\ 2 & -2 & 1 & : & -3 \end{bmatrix}$$

Applying $R_{12} \to R_2 - R_1$ and $R_3 \to R_3 - 2R_1$

$$\sim\begin{bmatrix} 1 & -1 & 2 & : & 0 \\ 0 & -1 & 1 & : & -1 \\ 0 & 0 & -3 & : & -3 \end{bmatrix}$$

Applying $R_2 \to -1R_2$

$$\sim\begin{bmatrix} 1 & -1 & 2 & : & 0 \\ 0 & 1 & -1 & : & 1 \\ 0 & 0 & -3 & : & -3 \end{bmatrix}$$

Applying $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & -1 & : & -1 \\ 0 & 1 & -1 & : & 1 \\ 0 & 0 & -3 & : & -3 \end{bmatrix}$$

Applying $R_3 \to -\frac{1}{3}R_3$

$$\sim\begin{bmatrix} 1 & 0 & -1 & : & -1 \\ 0 & 1 & -1 & : & 1 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 + R_3$ and $R_2 \to R_2 + R_3$

$$\sim\begin{bmatrix} 1 & 0 & 0 & : & 0 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

Hence, the required solution is x = 0, y = 2, and z = 1

e. The augmented matrix is

$$\begin{bmatrix} 2 & -1 & 4 & : & -3 \\ 1 & 0 & -4 & : & 5 \\ 6 & -1 & 2 & : & 10 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$

$$\sim\begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 2 & -1 & 4 & : & -3 \\ 6 & -1 & 2 & : & 10 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 6R_3$

$$\sim\begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 0 & -1 & 12 & : & -13 \\ 0 & -1 & 26 & : & -20 \end{bmatrix}$$

Applying $R_2 \to 1R_2$

$$\sim\begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 0 & 1 & -12 & : & 13 \\ 0 & -1 & 26 & : & -20 \end{bmatrix}$$

Applying $R_3 \to R_2 + R_3$

$$\sim\begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 0 & 1 & -12 & : & 13 \\ 0 & 0 & 14 & : & -7 \end{bmatrix}$$

Applying $R_3 \to \frac{1}{14} R_3$

$$\sim\begin{bmatrix} 1 & 0 & -4 & : & 5 \\ 0 & 1 & -12 & : & 13 \\ 0 & 0 & 1 & : & -1/2 \end{bmatrix}$$

Applying $R_2 \to R_2 + 12R_3$ and $R_1 \to R_1 + 4R_3$

$$\sim\begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 7 \\ 0 & 0 & 1 & : & -1/2 \end{bmatrix}$$

Hence, $x = 3$, $y = 7$, $z = -\frac{1}{2}$

f. The augmented matrix is

$$\begin{bmatrix} 1 & 2 & -3 & : & 9 \\ 2 & -1 & 2 & : & -8 \\ 3 & -1 & -4 & : & 3 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$$\sim\begin{bmatrix} 1 & 2 & -3 & : & 9 \\ 0 & -5 & 8 & : & -26 \\ 0 & -7 & 5 & : & -24 \end{bmatrix}$$

Applying $R_2 \to \frac{1}{5} R_2$

$$\sim\begin{bmatrix} 1 & 2 & -3 & : & 9 \\ 0 & 1 & -8/5 & : & 26/5 \\ 0 & -7 & 5 & : & -24 \end{bmatrix}$$

Applying $R_3 \to R_3 + 7R_2$ and $R_1 \to R_1 - 2R_2$

$$\sim\begin{bmatrix} 1 & 0 & 1/5 & : & -7/5 \\ 0 & 1 & -8/5 & : & 26/5 \\ 0 & 0 & -31/5 & : & 62/5 \end{bmatrix}$$

Applying $R_3 \to -\frac{5}{31} R_3$

$$\sim\begin{bmatrix} 1 & 0 & 1/5 & : & -7/5 \\ 0 & 1 & -8/5 & : & 26/5 \\ 0 & 0 & 1 & : & -2 \end{bmatrix}$$

Applying $R_1 \to R_1 - \frac{1}{5} R_3$ and $R_2 \to R_2 + \frac{1}{8} R_3$

$$\sim\begin{bmatrix} 1 & 0 & 0 & : & -1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & -2 \end{bmatrix}$$

Hence, the required solution is $x = -1$, $y = 2$ and $z = -2$

g. The augment matrix is

$$\begin{bmatrix} 3 & -2 & -3 & : & -3 \\ 2 & 1 & 1 & : & 6 \\ 1 & 3 & -2 & : & 13 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\sim \begin{bmatrix} 1 & -3 & -4 & : & -9 \\ 2 & 1 & 1 & : & 6 \\ 1 & 3 & -2 & : & 13 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & -3 & -4 & : & -9 \\ 0 & 7 & 9 & : & 24 \\ 0 & 6 & 2 & : & 22 \end{bmatrix}$$

Applying $R_2 \to R_2 - R_3$

$$\sim \begin{bmatrix} 1 & -3 & -4 & : & -9 \\ 0 & 1 & 7 & : & 2 \\ 0 & 6 & 2 & : & 22 \end{bmatrix}$$

Applying $R_1 \to R_1 + 3R_2$ and $R_3 \to R_3 - bR_2$

$$\sim \begin{bmatrix} 1 & 0 & 17 & : & -3 \\ 0 & 1 & 7: & 0 & 2 \\ 0 & 0 & -40 & : & 10 \end{bmatrix}$$

Applying $R_3 \to -\dfrac{1}{40} R_3$

$$\sim \begin{bmatrix} 1 & 0 & 17 & : & -3 \\ 0 & 1 & 7 & : & 2 \\ 0 & 0 & 1 & : & -1/4 \end{bmatrix}$$

Applying $R_1 \to R_1 - 17R_3 \to R_2 \to R_2 - 7R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 5/4 \\ 0 & 1 & 0 & : & 15/4 \\ 0 & 0 & 1 & : & -1/4 \end{bmatrix}$$

$\therefore \quad x = \dfrac{5}{4}$, $y = \dfrac{15}{4}$ and $z = -\dfrac{1}{4}$

h. The augmented matrix is

$$\sim \begin{bmatrix} 3 & 0 & -5 & : & -7 \\ 3 & 5 & 0 & : & 3 \\ 0 & -3 & 3 & : & 2 \end{bmatrix}$$

Applying $R_1 \to \dfrac{1}{3} R_1$

$$\sim \begin{bmatrix} 1 & 0 & -5/3 & : & -7/3 \\ 3 & 5 & 0 & : & 3 \\ 0 & -3 & 3 & : & 2 \end{bmatrix}$$

Applying $R_2 \to R_2 - 3R_1$

$$\sim \begin{bmatrix} 1 & 0 & -5/3 & : & -7/3 \\ 0 & 5 & 5 & : & 10 \\ 0 & -3 & 3 & : & 2 \end{bmatrix}$$

Applying $R_2 \to \dfrac{1}{5} R_2$

$$\sim \begin{bmatrix} 1 & 0 & -5/3 & : & -7/3 \\ 0 & 1 & 1 & : & 2 \\ 0 & -3 & 3 & : & 2 \end{bmatrix}$$

Applying $R_3 \to R_3 + R_3 + 3R_2$

$$\sim \begin{bmatrix} 1 & 0 & -5/3 & : & -7/3 \\ 0 & 1 & 1 & : & 2 \\ 0 & 0 & 6 & : & 8 \end{bmatrix}$$

Applying $R_3 \to \frac{1}{6} R_3$

$$\sim \begin{bmatrix} 1 & 0 & -5/3 & : & -7/3 \\ 0 & 1 & 1 & : & 2 \\ 0 & 0 & 1 & : & 4/3 \end{bmatrix}$$

Applying $R_1 \to R_1 + \frac{5}{3} R_3$ and $R_2 \to R_2 - R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & -1/9 \\ 0 & 1 & 0 & : & 2/3 \\ 0 & 0 & 1 & : & 4/3 \end{bmatrix}$$

Hence, $x = -\frac{1}{9}$, $y = \frac{2}{3}$ and $z = \frac{4}{3}$

Exercise: 7.2

1. Solution:

a. $x + y = 4$

$3x - 2y = 17$

Coe. of x	Coe. of y	Constant
1	1	4
3	–2	17

Now,

$$D = \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -2 - 3 = -5$$

$$D_1 = \begin{vmatrix} 4 & 1 \\ 17 & -2 \end{vmatrix} = -8 - 17 = -25$$

$$D_2 = \begin{vmatrix} 1 & 4 \\ 3 & 17 \end{vmatrix} = 17 - 12 = 5$$

The solution is $x = \frac{D_1}{D} = \frac{-25}{-5} = 5$

$$y = \frac{D_2}{D} = \frac{5}{-5} = -1$$

b. Let,

Coe. of x	Coe. of y	Constant
2	–1	5
1	–2	1

Now,

$$D = \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} = -4 + 1 = -3$$

$$D_1 = \begin{vmatrix} 5 & -1 \\ 1 & -2 \end{vmatrix} = -10 + 1 = -9$$

$D_2 = \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = 2 - 5 = -3$

The solution is, $x = \frac{D_1}{D} = \frac{-9}{-3} = 3$

$y = \frac{D_2}{D} = \frac{-3}{-3} = 1$

c. Let,

Coe. of x	Coe. of y	Constant
3	4	–2
15	20	24

Now,

$D = \begin{vmatrix} 3 & 4 \\ 15 & 20 \end{vmatrix} = 60 - 60 = 0$

$\therefore$ D is negative, the solution does not exist.

d. Let,

Coe. of x	Coe. of y	Constant
5	–3	20
2	5	8

Now,

$D = \begin{vmatrix} 5 & -3 \\ 2 & 5 \end{vmatrix} = 25 + 6 = 31$

$D_1 = \begin{vmatrix} 20 & -3 \\ 8 & 5 \end{vmatrix} = 100 + 24 = 124$

$D_2 = \begin{vmatrix} 5 & 20 \\ 2 & 8 \end{vmatrix} = 40 - 40 = 0$

Now, the solution is, $x = \frac{D_1}{D} = \frac{124}{31} = 4$

$y = \frac{D_2}{D} = \frac{0}{32} = 0$

e. Let,

Coe. of x	Coe. of y	Constant
$\frac{2}{3}$	1	16
1	$\frac{1}{4}$	14

Now,

$D = \begin{vmatrix} \frac{2}{3} & 1 \\ 1 & \frac{1}{4} \end{vmatrix} = \frac{1}{6} - 1 = \frac{-5}{6}$

$D_1 = \begin{vmatrix} 16 & 1 \\ 14 & \frac{1}{4} \end{vmatrix} = 4 - 14 = -10$

$D_2 = \begin{vmatrix} \frac{2}{3} & 16 \\ 1 & 14 \end{vmatrix} = \frac{28}{3} - 16 = \frac{-20}{3}$

The solution is $x = \frac{D_1}{D} = \frac{-10}{-\frac{5}{6}} = 12$

$$y = \frac{D_2}{D} = \frac{-\frac{20}{3}}{-\frac{5}{6}} = 8$$

f. Let,

Coe. of x	Coe. of y	Constant
3	4	10
–2	3	–1

$$D = \begin{vmatrix} 3 & 4 \\ -2 & 3 \end{vmatrix} = 9 + 8 = 17$$

$$D_1 = \begin{vmatrix} 10 & 4 \\ -1 & 3 \end{vmatrix} = 30 + 4 = 34$$

$$D_2 = \begin{vmatrix} 3 & 10 \\ -2 & -1 \end{vmatrix} = -3 + 20 = 17$$

The solution is $x = \frac{D_1}{D} = \frac{34}{17} = 2$

$\frac{1}{y} = \frac{D_2}{D} = \frac{1}{1} = 1$

$\therefore \quad \frac{1}{y} = 1$

$y = 1$

g. Let,

Coe. of x	Coe. of y	Constant
3	–4	–11
2	5	31

$$D = \begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix} = 15 + 8 = 23$$

$$D_1 = \begin{vmatrix} -11 & -4 \\ 31 & 5 \end{vmatrix} = -55 + 124 = 69$$

$$D_2 = \begin{vmatrix} 3 & -11 \\ 2 & 31 \end{vmatrix} = 93 + 22 = 115$$

The solution is $x = \frac{D_1}{D} = \frac{69}{23} = 3$

$$y = \frac{D_2}{D} = \frac{115}{23} = 5$$

h. Let,

Error! Bookmark not defined. Coe. of x	Coe. of y	Constant
2	1	7
1	3	11

Now,

$$D = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5$$

$$D_1 = \begin{vmatrix} 7 & 1 \\ 11 & 3 \end{vmatrix} = 21 - 11 = 10$$

$$D_2 = \begin{vmatrix} 2 & 7 \\ 1 & 11 \end{vmatrix} = 22 - 7 = 15$$

Now, the solution is, $x = \frac{D_1}{D} = \frac{10}{5} = 2$

$$y = \frac{D_2}{D} = \frac{15}{5} = 3$$

2. Solution

a. The matrix equation of given system is Ax = B

Where $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & -3 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$

Now,

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & -3 \end{bmatrix} = 1\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$$

$$= 1(-3 - 1) + 1(-3 - 2) + 1(1 - 2)$$

$$= -4 - 5 - 1 = -10$$

$\therefore$ $|A| \neq 0$, so A^{-1} exist

Let cofactor of $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$A_{11} = \begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} = -3 - 1 = -4$

$A_{12} = -\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -(-3 - 2) = 5$

$A_{13} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1$

$A_{21} = -\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = -(3 - 1) = -2$

$A_{22} = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -3 - 2 = -5$

$A_{23} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = -(1 + 2) = -3$

$A_{31} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = (-1 - 1) = -2$

$A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = -(1 - 1) = 0$

$A_{33} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 + 1 = 2$

Co. factor of $A = \begin{bmatrix} -4 & 5 & -1 \\ -2 & -5 & -3 \\ -2 & 0 & 2 \end{bmatrix}$

Adj. of $A = \begin{bmatrix} -4 & 5 & -1 \\ -2 & -5 & -3 \\ -2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -4 & -2 & -2 \\ 5 & -5 & 0 \\ -1 & -3 & 2 \end{bmatrix}$

The solution given by,

$x = A^{-1} B$

$$= \frac{1}{-10}\begin{bmatrix} -4 & -2 & -2 \\ 5 & -5 & 0 \\ -1 & -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -20 \\ 10 \\ -10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore$ $x = 2, y = -1, z = 1$

b. The matrix equation of system is AX = B

Where $A = \begin{bmatrix} 2 & -3 & -1 \\ 1 & -2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 1 \\ 9 \end{bmatrix}$

Now,

$$|A| = \begin{bmatrix} 2 & -3 & -1 \\ 1 & -2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 2\begin{vmatrix} -2 & -1 \\ -1 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$$

$$= 2(-4 - 1) + 3(2 + 1) - 1(-1 + 2)$$

$$= 2x - 5 + 3 \times 3 - 1 \times 1$$

$$= -2$$

$\therefore$ $|A| \neq 0$, A^{-1} exist

$$\text{Cofactor of } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} -2 & -1 \\ -1 & 2 \end{vmatrix} = -4 - 1 = -5$$

$$A_{12} = -\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = -(2 + 1) = -3$$

$$A_{13} = \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = -1 + 2 = 1$$

$$A_{21} = \begin{vmatrix} -3 & -1 \\ -1 & 2 \end{vmatrix} = -(-6 - 1) = 7$$

$$A_{22} = \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} = (4 \times 1) = 5$$

$$A_{23} = -\begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} = -(-2 + 3) = -1$$

$$A_{31} = \begin{vmatrix} -3 & -1 \\ -2 & -1 \end{vmatrix} = (3 - 2) = 1$$

$$A_{32} = -\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -(-2 + 1) = 1$$

$$A_{33} = \begin{vmatrix} 2 & -3 \\ 1 & -2 \end{vmatrix} = -4 + 3 = -1$$

Now,

$$\text{Co factor of } A = \begin{bmatrix} -5 & -3 & 1 \\ 7 & 5 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\text{Adj of A} = \begin{bmatrix} -5 & -3 & 1 \\ 7 & 5 & -1 \\ 1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -5 & 7 & 1 \\ -3 & 5 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

The solution given by,

$x = A^{-1} B$

$$= \frac{1}{-2}\begin{bmatrix} -5 & 7 & 1 \\ -3 & 5 & 1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 9 \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -4 \\ 2 \\ -6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 2, y = -1, z = 3$

c. The matrix equation of given system is $Ax = B$

$$\text{where } A = \begin{bmatrix} 3 & 5 & 0 \\ 2 & 0 & -3 \\ 0 & 4 & 2 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 2 \\ -7 \\ 2 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 3 & 5 & 0 \\ 2 & 0 & -3 \\ 0 & 4 & 2 \end{vmatrix} = 3\begin{vmatrix} 0 & -3 \\ 4 & 2 \end{vmatrix} - 5\begin{vmatrix} 2 & -3 \\ 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix}$$

$= 3(12) - 5(4 \neq 0) + 0 = 16$

$\therefore$ $|A| \neq 0$, A^{-1} exist

$$\text{Let cofactor of A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} 0 & -3 \\ 4 & 2 \end{vmatrix} = 12$$

$$A_{12} = -\begin{vmatrix} 2 & -3 \\ 0 & 2 \end{vmatrix} = -(4 - 0) = -4$$

$$A_{13} = \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = 8$$

$$A_{21} = \begin{vmatrix} 5 & 0 \\ 4 & 2 \end{vmatrix} = -(10) = -10$$

$$A_{22} = \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = 6$$

$$A_{23} = \begin{vmatrix} 3 & 5 \\ 0 & 4 \end{vmatrix} = -12$$

$$A_{31} = \begin{vmatrix} 5 & 0 \\ 0 & -3 \end{vmatrix} = -15$$

$$A_{32} = \begin{vmatrix} 3 & 0 \\ 2 & -3 \end{vmatrix} = -(9) = 9$$

$$A_{33} = \begin{vmatrix} 3 & 5 \\ 2 & 0 \end{vmatrix} = 0 - 10 = -10$$

$$\therefore \text{Cofactor of A} = \begin{bmatrix} 12 & -4 & 8 \\ -10 & 6 & -12 \\ -15 & 9 & 10 \end{bmatrix}$$

$$\text{Adj of A} = \begin{bmatrix} 12 & -4 & 8 \\ -10 & 6 & -12 \\ -15 & 9 & 10 \end{bmatrix}^T = \begin{bmatrix} 12 & -10 & -15 \\ -4 & 6 & 9 \\ 8 & -12 & 10 \end{bmatrix}$$

The solution given by,

$X = A^{-1}B$

$$= \frac{1}{10}\begin{bmatrix} 12 & -10 & -15 \\ -4 & 6 & 9 \\ 8 & -12 & 10 \end{bmatrix}\begin{bmatrix} 2 \\ -7 \\ 2 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 64 \\ -32 \\ 120 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 5 \end{bmatrix}$$

$\therefore$ x = 4, y = –2, z = 5

d. The matrix equation of given system is AX = B

$$\text{where } A = \begin{bmatrix} 1 & -3 & -7 \\ 2 & 3 & 1 \\ 4 & 1 & 0 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 9 \\ 7 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & -3 & -7 \\ 2 & 3 & 1 \\ 4 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} - 7\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$$

$= -1 + 3\times-4 - 7(2 - 12) = 57$

$\therefore$ $|A| \neq 0$, A^{-1} exist

$$\text{Let cofactor of A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$A_{12} = -\begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} = 4$$

$$A_{13} = \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = 2 - 12 = -10$$

$$A_{21} = \begin{vmatrix} -3 & -7 \\ 1 & 0 \end{vmatrix} = -(7) = -7$$

$$A_{22} = \begin{vmatrix} 1 & -7 \\ 4 & 0 \end{vmatrix} = 28$$

$$A_{23} = \begin{vmatrix} 1 & -3 \\ 4 & 1 \end{vmatrix} = -(1 + 12) - 13$$

$$A_{31} = \begin{vmatrix} -3 & -7 \\ 3 & 1 \end{vmatrix} = -3 + 21 = 18$$

$$A_{32} = -\begin{vmatrix} 1 & -7 \\ 2 & 1 \end{vmatrix} = -(1 + 14) = -15$$

$$A_{33} = \begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} = 3 + 6 = 9$$

$$\therefore \text{Cofactor of A} = \begin{bmatrix} -1 & 4 & -10 \\ -7 & 28 & -13 \\ 18 & -15 & 9 \end{bmatrix}$$

$$\text{Adj of A} = \begin{bmatrix} -1 & 4 & -10 \\ -7 & 28 & -13 \\ 18 & -15 & 9 \end{bmatrix}^T = \begin{bmatrix} -1 & -7 & 18 \\ 4 & 28 & -15 \\ -10 & -13 & 9 \end{bmatrix}$$

Now, the solution is given by,

$x = A^{-1} B$

$$= \frac{1}{57}\begin{bmatrix} -1 & -7 & 18 \\ 4 & 28 & -15 \\ -10 & -13 & 9 \end{bmatrix}\begin{bmatrix} 6 \\ 9 \\ 7 \end{bmatrix} = \frac{1}{57}\begin{bmatrix} 57 \\ 171 \\ -114 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

$\therefore$ x = 1, y = 3, z = –2.15

e. The matrix equation of given system is AX = B.

$$\text{where } A = \begin{bmatrix} 2 & -5 & 0 \\ 0 & 3 & 2 \\ 7 & 0 & -3 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 2 & -5 & 0 \\ 0 & 3 & 2 \\ 7 & 0 & -3 \end{vmatrix} = 2\begin{vmatrix} 3 & 2 \\ 0 & -3 \end{vmatrix} + 5\begin{vmatrix} 0 & 2 \\ 7 & -3 \end{vmatrix} + 0$$

$= 2(-9) + 5(-14)$

$= -88$

$\therefore$ $|A| \neq 0$, A^{-1} exist

$$\text{Let cofactor of A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} 3 & 2 \\ 0 & -3 \end{vmatrix} = -9$$

$$A_{12} = \begin{vmatrix} 0 & 2 \\ 7 & -3 \end{vmatrix} = 14$$

$$A_{13} = \begin{vmatrix} 0 & 3 \\ 7 & 0 \end{vmatrix} = -21$$

$$A_{21} = -\begin{vmatrix} -5 & 0 \\ 0 & -3 \end{vmatrix} = -15$$

$$A_{22} = \begin{vmatrix} 2 & 0 \\ 7 & -3 \end{vmatrix} = -6$$

$$A_{23} = -\begin{vmatrix} 2 & -5 \\ 7 & 0 \end{vmatrix} = -35$$

$$A_{31} = \begin{vmatrix} -5 & 0 \\ 3 & 2 \end{vmatrix} = -10$$

$$A_{32} = -\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = -4$$

$$A_{33} = \begin{vmatrix} 2 & -5 \\ 0 & 3 \end{vmatrix} = 6$$

$$\text{Cofactor of } A = \begin{bmatrix} -9 & 14 & -21 \\ -15 & -6 & -35 \\ -10 & -4 & 6 \end{bmatrix}$$

$$\text{Adj of } A = \begin{bmatrix} -9 & 14 & -21 \\ -15 & -6 & -35 \\ -10 & -4 & 6 \end{bmatrix}^T = \begin{bmatrix} -9 & -15 & -10 \\ 14 & -6 & -4 \\ -21 & -35 & -6 \end{bmatrix}$$

The solution is given by,

$x = A^{-1}B$

$$= \frac{1}{-88}\begin{bmatrix} -9 & -15 & -10 \\ 14 & -6 & -4 \\ -21 & -35 & 6 \end{bmatrix}\begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{88} \\ -\frac{-35}{44} \\ \frac{-15}{88} \end{bmatrix}$$

$\therefore\ x = \frac{1}{88}$, $y = \frac{-35}{44}$, $z = \frac{-15}{88}$

f. The matrix equation of system is AX = B.

$$\text{where, } A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 3 & 1 & 2 \end{bmatrix},\ x = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 7 \\ 3 \\ 8 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 1\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}$$

$= (-2 - 1) -2(4 - 3) +1(2 + 3)$

$= -3 - 2 + 5$

$= 0$

$\therefore\ |A| = 0$, A^{-1} does not exist.

3. Solution:

a.

Coe. of x	Coe. of y	coe. of z	Constant
2	–3	–1	4
1	–2	–1	1
1	–1	2	9

$$D = \begin{vmatrix} 2 & -3 & -1 \\ 1 & -2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 2\begin{vmatrix} -2 & -1 \\ -1 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$$

$= 2(-4 - 1) +3(2 +1) -1(-1 + 2)$

$= 2x - 5 + 3\times3 -1 \times 1$

$= -2$

$$D_1 = \begin{vmatrix} 4 & -3 & -1 \\ 1 & -2 & -1 \\ 9 & -1 & 2 \end{vmatrix} = 4\begin{vmatrix} -2 & -1 \\ -1 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 9 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ 9 & -1 \end{vmatrix}$$

$= 4(-4 - 1) + 3(2 + 9) -1(-1 + 18)$

$= -4$

$$D_2 = \begin{vmatrix} 2 & 4 & -1 \\ 1 & 1 & -1 \\ 1 & 9 & 2 \end{vmatrix} = 2\begin{vmatrix} 1 & -1 \\ 9 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix}$$

$= 2(2 + 9) - 4(2 + 1) - 1(9 - 1)$
$= 2$

$$D_3 = \begin{vmatrix} 2 & -3 & 4 \\ 1 & -2 & 1 \\ 1 & -1 & 9 \end{vmatrix} = 2\begin{vmatrix} -2 & 1 \\ -1 & 9 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$$

$= 2(-18+1) + 3(9 - 1) + 4(-1 + 2)$
$= -34 + 24 + 4 = -6$

The solution is $x = \frac{D_1}{D} = \frac{-4}{-2} = 2$

$y = \frac{D_1}{D} = \frac{2}{-2} = -1$

$z = \frac{D_3}{D} = \frac{-6}{-2} = 3$

b. Let,

Coe. of x	Coe. of y	coe. of z	Constant
1	1	1	–1
3	1	1	1
4	–2	2	0

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 4 & -2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 4 & -2 \end{vmatrix}$$

$= (2 + 2) - 1 (6 - 4) + (-6 - 4)$
$= -8$

$$D_1 = \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix} = -1\begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix}$$

$= -1(2 + 2) - 1(2 - 0) + 1(-2 - 0)$
$= -4 - 2 - 2$
$= -8$

$$D_2 = \begin{vmatrix} 1 & -1 & 1 \\ 3 & 1 & 1 \\ 4 & 0 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 4 & 0 \end{vmatrix}$$

$= 2 + (6 - 4) + (-4)$
$= 2 + 2 - 4$
$= 0$

$$D_3 = \begin{vmatrix} 1 & 1 & -1 \\ 3 & 1 & 1 \\ 4 & -2 & 0 \end{vmatrix} = 1\begin{vmatrix} 1 & 1 \\ -2 & 0 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 4 & 0 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 4 & -2 \end{vmatrix}$$

$= (0 + 2) - 1(-4) - 1(-6 - 4)$
$= 2 + 4 + 10$
$= 16$

The solution is $x = x = \frac{D_1}{D} = \frac{-8}{-8} = 1$

$y = \frac{D_2}{D} = \frac{0}{-8} = 0$

$z = \frac{D_3}{D} = \frac{16}{-8} = -2$

c. Let,

Coe. of x	Coe. of y	coe. of z	Constant
0	6	6	–1

8	0	6	–1
4	9	0	8

$$D = \begin{vmatrix} 0 & 6 & 6 \\ 8 & 0 & 6 \\ 4 & 9 & 0 \end{vmatrix} = -6\begin{vmatrix} 8 & 6 \\ 4 & 0 \end{vmatrix} + 6\begin{vmatrix} 8 & 0 \\ 4 & 9 \end{vmatrix}$$

$= -6(-24) + 6\times 72$

$= 144 + 432$

$= 576$

$$D_1 = \begin{vmatrix} -1 & 6 & 6 \\ -1 & 0 & 6 \\ 8 & 9 & 0 \end{vmatrix} = -1\begin{vmatrix} 0 & 6 \\ 9 & 0 \end{vmatrix} - 6\begin{vmatrix} -1 & 6 \\ 8 & 0 \end{vmatrix} + 6\begin{vmatrix} -1 & 0 \\ 8 & 9 \end{vmatrix}$$

$= -1(-54) - 6(-48) + 6(-9)$

$= 288$

$$D_2 = \begin{vmatrix} 0 & -1 & 6 \\ 8 & -1 & 6 \\ 4 & 8 & 0 \end{vmatrix} = +1\begin{vmatrix} 8 & 6 \\ 4 & 0 \end{vmatrix} + 6\begin{vmatrix} 8 & -1 \\ 4 & 8 \end{vmatrix}$$

$= -24 + 6(64 + 4)$

$= 384$

$$D_3 = \begin{vmatrix} 0 & 6 & -1 \\ 8 & 0 & -1 \\ 4 & 9 & 8 \end{vmatrix} = -6\begin{vmatrix} 8 & -1 \\ 4 & 8 \end{vmatrix} - 1\begin{vmatrix} 8 & 0 \\ 4 & 9 \end{vmatrix}$$

$= -6(64 + 4) - 1(72 - 0)$

$= -480$

The solution is $x = \frac{D_1}{D} = \frac{288}{576} = \frac{1}{2}$

$y = \frac{D_2}{D} = \frac{288}{576} = \frac{2}{3}$

$z = \frac{D_3}{D} = \frac{-480}{570} = \frac{-5}{6}$

d. Let,

Coe. of x	Coe. of y	coe. of z	Constant
1	–3	–7	6
2	3	1	9
4	1	0	7

$$D = \begin{vmatrix} 1 & -3 & -7 \\ 2 & 3 & 1 \\ 4 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} - 7\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$$

$= -1 + 3(-4) - 7(2 - 12)$

$= 57$

$$D_1 = \begin{vmatrix} 6 & -3 & -7 \\ 9 & 3 & 1 \\ 7 & 1 & 0 \end{vmatrix} = 6\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} + 3\begin{vmatrix} 9 & 1 \\ 7 & 0 \end{vmatrix} + -7\begin{vmatrix} 9 & 3 \\ 7 & 1 \end{vmatrix}$$

$= 6(-1) + 3(0 - 7) - 7(9 - 21)$

$= 57$

$$D_2 = \begin{vmatrix} 1 & 6 & -7 \\ 2 & 9 & 1 \\ 4 & 7 & 0 \end{vmatrix} = 1\begin{vmatrix} 9 & 1 \\ 7 & 0 \end{vmatrix} - 6\begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} - 7\begin{vmatrix} 2 & 9 \\ 4 & 7 \end{vmatrix}$$

$= (-7) - 6(-4) - 7(14 - 36)$

$= 171$

$$D_3 = \begin{vmatrix} 1 & -3 & 6 \\ 2 & 3 & 9 \\ 4 & 1 & 7 \end{vmatrix} = 1\begin{vmatrix} 3 & 9 \\ 1 & 7 \end{vmatrix} + 3\begin{vmatrix} 2 & 9 \\ 4 & 7 \end{vmatrix} + 6\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$$

$= (21 - 9) + 3(14 - 36) + 6(2 - 12)$

$= -114$

The solution is, $x = \frac{D_1}{D} = \frac{57}{57} = 1$

$y = \frac{D_2}{D} = \frac{171}{57} = 3$

$z = \frac{D_3}{D} = \frac{-114}{57} = -2$

4. Solution

a. Let,

Coe. of x	Coe. of y	coe. of z	Constant
1	1	1	6
2	3	5	23
7	5	–2	11

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 7 & 5 & -2 \end{vmatrix} = 1\begin{vmatrix} 3 & 5 \\ 5 & -2 \end{vmatrix} - 1\begin{vmatrix} 2 & 5 \\ 7 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 7 & 5 \end{vmatrix}$$

$= (-6 - 25) - 1\,(-4 - 35) + 2(10 - 21)$

$= -3$

$$D_2 = \begin{vmatrix} 6 & 1 & 1 \\ 23 & 3 & 5 \\ 11 & 5 & -2 \end{vmatrix} = 6\begin{vmatrix} 3 & 5 \\ 5 & -2 \end{vmatrix} - 1\begin{vmatrix} 23 & 5 \\ 11 & -2 \end{vmatrix} + 1\begin{vmatrix} 23 & 3 \\ 11 & 5 \end{vmatrix}$$

$= 6(-6 - 25) - 1(-46 - 55) + 1(115 - 33)$

$= -3$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 23 & 5 \\ 7 & 11 & -2 \end{vmatrix} = 1\begin{vmatrix} 23 & 5 \\ 11 & -2 \end{vmatrix} - 6\begin{vmatrix} 2 & 5 \\ 7 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & 23 \\ 7 & 11 \end{vmatrix}$$

$= (-46 - 55) - 6(-4 - 35) + 2(22 - 16)$

$= -6$

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & 23 \\ 7 & 5 & 11 \end{vmatrix} = 1\begin{vmatrix} 3 & 23 \\ 5 & 11 \end{vmatrix} - 1\begin{vmatrix} 2 & 23 \\ 7 & 11 \end{vmatrix} + 6\begin{vmatrix} 2 & 3 \\ 7 & 5 \end{vmatrix}$$

$= (33 - 115) - 1(22 - 161) + 6(10 - 21)$

$= -9$

The solution is $x = \frac{D_1}{D} = \frac{-3}{-3} = 1$

$y = \frac{D_2}{D} = \frac{-6}{-3} = 2$

$z = \frac{D_3}{D} = \frac{-9}{3} = 3$

b. Let,

Coe. of x	Coe. of y	coe. of z	Constant
1	4	1	18
3	3	–2	2
0	–4	1	–7

Now,

$$D = \begin{vmatrix} 1 & 4 & 1 \\ 3 & 3 & -2 \\ 0 & -4 & 1 \end{vmatrix} = 1\begin{vmatrix} 3 & -2 \\ -4 & 1 \end{vmatrix} - 4\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 3 \\ 0 & -4 \end{vmatrix}$$

$= (3 - 8) - 4(3) + (-12)$

$= -29$

$$D_1 = \begin{vmatrix} 18 & 4 & 1 \\ 2 & 3 & -2 \\ -7 & -4 & 1 \end{vmatrix} = 18\begin{vmatrix} 3 & -2 \\ -4 & 1 \end{vmatrix} - 4\begin{vmatrix} 32 & -2 \\ -7 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ -7 & -4 \end{vmatrix}$$

$= 18(3 - 8) - 4(2 - 14) + 1(-8 + 21)$

$= -29$

$$D_2 = \begin{vmatrix} 1 & 18 & 1 \\ 3 & 2 & -2 \\ 0 & -7 & 1 \end{vmatrix} = 1\begin{vmatrix} 2 & -2 \\ -7 & 1 \end{vmatrix} - 18\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 0 & -7 \end{vmatrix}$$

$= (2 - 14) - 18(3) + 1(-21)$

$= -87$

$$D_3 = \begin{vmatrix} 1 & 4 & -18 \\ 3 & 3 & 2 \\ 0 & -4 & -7 \end{vmatrix} = 1\begin{vmatrix} 3 & 2 \\ -4 & -7 \end{vmatrix} - 4\begin{vmatrix} 3 & 2 \\ 0 & -7 \end{vmatrix} + 28\begin{vmatrix} 3 & 3 \\ 0 & -4 \end{vmatrix}$$

$= (-21 + 8) - 4(-21) + 18(-12)$

$= -145$

The solution is $x = \dfrac{D_1}{D} = \dfrac{-29}{-29} = 1$

$y = \dfrac{D_2}{D} = \dfrac{-87}{-29} = 3$

$z = \dfrac{D_3}{D} = \dfrac{-145}{-29} = 5$