Set B

Group - B

12.

Soln

→. The difference between permutation and Combination are:-

Permutation	Combination.
(i) The process of arranging things in certain order is called permutation.	(i) It is the process of selecting or choosing the things where order doesn't matter called Combination.
(ii) $^nP_r = P(n,r) = \frac{n!}{(n-r)!}$	(ii) $^nC_r = \frac{n!}{(n-r)!\,r!}$
(iii) eg:-	(iii) eg:-

To prove:-

(i) $P(n,r) = r! \times C(n,r)$

From LHS.

$= r! \times \frac{n!}{(n-r)!\,r!}$

$= \frac{n!}{(n-r)!}$

$= P(n,r).$

(ii) $C(n,r) + C(n, r-1) = C(n+1, r)$.

$\frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$

Page No. ______
Date ______

13 (a).

Solution

$$1+\frac{1+2}{2!}+\frac{1+2+2^2}{3!}+\cdots$$

Let t_n be the n^{th} term.

$$t_n=\frac{1+2+2^2+\cdots+2^{n-1}}{n!}$$

General term निकालेपछि Sumको Formula लिने

$$=\frac{\frac{1(2^n-1)}{2-1}}{n!}\qquad \left[\because \frac{a(r^n-1)}{r-1}\right]$$

$$t_n=\frac{2^n}{n!}-\frac{1}{n!}\quad[\text{general term}]$$

Now,

$$S_n=\sum t_n=\sum\frac{2^n}{n!}-\frac{1}{n!}=\sum\frac{2^n}{n!}-\sum\frac{1}{n!}$$

$$=\left(\frac{2}{1!}+\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}+\cdots\right)-\left(\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots\right)$$

$$=\left[\left(1+\frac{2}{1!}+\frac{2^2}{2!}+\frac{2^3}{3!}+\cdots\right)-1\right]-\left[\left(1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots\right)-1\right]$$

$$\boxed{e^x = x + \frac{x}{1!} + \frac{x^2}{2!} \quad / \quad e^2 = 2 + \frac{2}{1!} + \frac{2^2}{2!} + \cdots}$$

$$= \left(1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \cdots\right) - \left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right)$$

$$= e^2 - e.$$

(b).

Ans:→ Any rule which assigns to each ordered pair of elements in the set J of integers a unique element of J, is called binary (bi = two) operations on J.
Symbolically:- function f is defined by

$$f : J \times J \to J.$$

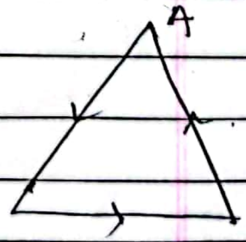
Addition binary operation

→ The operation of addition is denoted by '+' says to each pair of integers m and n, there is an integer x such that $m+n=x$.
Here the number x is the sum of m and n.
eg:- $1+2=3$, $5+4=9$, $2+(-2)=0$, etc.

14 (a).

Solution

Let O be origin then



$\vec{OA} = \vec{i} + 2\vec{j} + 3\vec{k} = (1, 2, 3)$

$\vec{OB} = -\vec{i} + \vec{j} - 8\vec{k} = (-1, 1, -8)$

$\vec{OC} = -4\vec{i} + 4\vec{j} + 6\vec{k} = (-4, 4, 6)$.

$\vec{AB} = \vec{OB} - \vec{OA} = (-2, -1, -11)$. $\therefore |\vec{AB}| = \sqrt{4+1+121} = \sqrt{126}$.

$\vec{BC} = \vec{OC} - \vec{OB} = (-3, 3, 14)$. $\therefore |\vec{BC}| = \sqrt{9+9+196} = \sqrt{214}$

$\vec{CA} = \vec{OA} - \vec{OC} = (5, -2, -3)$. $\therefore |\vec{CA}| = \sqrt{25+4+9} = \sqrt{38}$.

~~LA~~

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$\vec{AB} \times \vec{BC} = \begin{matrix} -2 & -1 & -11 & -2 & -1 \\ -3 & 3 & 14 & -3 & 3 \end{matrix}$

$= (19, 61, -9)$.

$|\vec{AB} \times \vec{BC}| = \sqrt{361+3721+81} = \sqrt{4163}$.

$\sin\theta = \frac{|\vec{AB} \times \vec{BC}|}{|\vec{AB}| \cdot |\vec{BC}|} = \frac{\sqrt{4163}}{\sqrt{126} \times \sqrt{214}} = \sqrt{\frac{4163}{26964}}$.

$\vec{AB} \times \vec{AC} = \begin{matrix} -2 & -1 & -11 & -2 & -1 \\ -5 & 2 & 3 & -5 & 2 \end{matrix}$

$= (19, 61, -9)$

$|\vec{AB} \times \vec{AC}| = \sqrt{361+3721+81} = \sqrt{4163}$.

$\sin\theta = \frac{|\vec{AB} \times \vec{AC}|}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{\sqrt{4163}}{\sqrt{126} \times \sqrt{38}} = \sqrt{\frac{4163}{4788}}$

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15.

Solution

Given, lines of regression as:-

$x+2y=5$ ---- (i) $\quad \sigma_x^2=12$

$2x+3y=8$ ---- (ii). $\quad \therefore \sigma_x=\sqrt{12}$.

(i) ~~Give~~ since given equation Interset ~~at~~ mean value so,

$\bar{x}+2\bar{y}=5$ ---- (iii)

$2\bar{x}+3\bar{y}=8$ ---- (iv)

Solving eqn (iii) and (iv).

$$2\bar{x}+4\bar{y}=10$$
$$-\underline{2\bar{x} \pm 3y = 8}.$$
$$\therefore \bar{y}=2.$$

from (iii). $\bar{x} = 5-2\times2 = 1.$

mean of ~~eqn~~ $\bar{x}$ and y series are $(\bar{x},\bar{y}) = (1,2)$.

(ii) From eqn (i)

$x+2y=5$. $\quad y = \frac{-x}{2} + \frac{5}{2}$.

$\therefore b_{yx} = -1/2$.

Regression Coeff of y on x is $b_{yx} = \frac{-1}{2}$.

(iii) From eqn (ii).

$2x+3y=8$.

$x = \frac{-3}{2}y + \frac{8}{2} \quad \therefore b_{xy} = -3/2$.

(iv) Correlation Coefficient $= \sqrt{b_{yx} \times b_{xy}}$

$= \sqrt{-\frac{1}{2} \times -\frac{3}{2}}$.

$= -\sqrt{3}/2$.

(v) Variance of y-series. (σ_y^2)

$b_{yx} = r \frac{\sigma_y}{\sigma_x}$.

$\frac{-1}{\cancel{2}} = \frac{-\sqrt{3}}{\cancel{2}} \times \frac{\sigma_y}{\sqrt{12}}$.

$\cancel{1 = \sqrt{3}\sigma_y}$ $\quad 1 = \frac{\sqrt{3}.\sigma_y}{2\sqrt{3}}$

$\therefore \sigma_y = 2$.

$\therefore \sigma_y^2 = 2^2 = 4$

16

Solution

Introducing the non-negative slack variables r & s.

$2x_1 + x_2 + r = 40$.

$x_1 + 2x_2 + s = 50$.

Now the given lp in standard form.

$2x_1 + x_2 + r + 0.s + 0.F = 40$

$x_1 + 2x_2 + 0.r + s + 0.F = 50$.

$-5x_1 - 3x_2 + 0.r + 0.s + F = 0$.

Initial simplex Tableau.

Basic Variable.	x_1	x_2	r	s	F	RHS.
r	2	1	1	0	0	40
s.	1	2	0	1	0	50
	-5	-3	0	0	1	0
	↑					

Since -5 is the most negative entry so $\frac{40}{2} < \frac{50}{1}$. so, 2 is pivot element.

Apply:- $R_1 = R_1/2$.

	x_1	x_2	r	s	f	RHS
x_1	1	1/2	1/2	0	0	20
s	1	2	0	1	0	50
	-5	-3	0	0	1	0

Apply:- $R_2 = R_2 - R_1$ and $R_3 = 5R_1 + R_3$.

	x_1	x_2	r	s	f	RHS
x_1	1	1/2	1/2	0	0	20
s	0	3/2	-1/2	1	0	30
	0	-1/2	5/2	0	1	100

Again x_2 is the pivot column and $\frac{30}{3/2} < \frac{20}{1/2}$.
So, 3/2 is pivot element.

Apply:- $R_2 = \frac{2}{3} R_2$

	x_1	x_2	r	s	f	RHS
x_1	1	1/2	1/2	0	0	20
x_2	0	1	-1/3	2/3	0	20
	0	-1/2	5/2	0	1	100

Apply:- $R_1 = \frac{1}{2}R_2 - R_1$ and $R_3 = \frac{1}{2}R_2 + R_3$.

	x_1	x_2	r	s	f	RHS
x_1	1	0	-2/3	1/3	0	-10
x_2	0	1	-1/3	2/3	0	20
	0	0	9/4	1/3	1	110

Since all the value in the last Row is (+)ve so The optimal Solution is 110 at $x_1 = -10$ and $x_2 = 20$.

17 (a).

Solution

The Curve is $(1+x) = x$.

$\therefore$ let $y = \frac{1+x}{x}$.

$$\frac{dy}{dx} = \frac{d\left(\frac{1+x}{x}\right)}{dx} = \frac{x\frac{d(1+x)}{dx} - (1+x)}{x^2}.$$

$$= \frac{x-1-x}{x^2} = \frac{-1}{x^2}.$$

$\tan$

17 (b).

Solution

Let $f(x) = \log(\cos x)$ $\quad f(x+h) = \log[\cos(x+h)]$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\log[\cos(x+h)] - \log(\cos x)}{h}.$$

$$= \lim_{h\to 0} \frac{\log\left[\frac{\cos(x+h)}{\cos x}\right]}{h}.$$

put $\cos x = u$ and $\cos(x+h) = u+k$.

$\therefore k = \cos(x+h) - \cos x$.

when $h \to 0$, $k \to 0$ from eqn (i).

$$\lim_{k\to 0} \frac{\log(u+k) - \log u}{k}$$

$$= \lim_{h \to 0} \frac{\log\left(\frac{4+k}{4}\right)}{h} = \lim_{h \to 0} \frac{\log(1+k/4)}{h}$$

$$= \lim_{h \to 0} \frac{\log(1+k/4)}{k/4} \times \frac{k}{4} \times \frac{1}{h}$$

$$= \frac{1}{4} \lim_{h \to 0} \frac{k}{h}$$

$$= \frac{1}{4} \lim_{h \to 0} \frac{\cos(x+h) - (\cos x)}{h}$$

$$= \frac{1}{4} \lim_{h \to 0} \frac{2 \sin\frac{x+h+x}{2} \sin\frac{x-x-h}{2}}{h}$$

$$= \frac{1}{4} \lim_{h \to 0} \frac{2 \sin\frac{2x+h}{2} \sin\frac{-h}{2}}{h/2} \times \frac{1}{2}$$

$$= \frac{1}{4} \lim_{h \to 0} -2 \sin\frac{2x+h}{2} \frac{\sin h/2}{h/2} \times \frac{1}{2}$$

$$= \frac{1}{4} \sin\left(\frac{2x+0}{2}\right)$$

$$= \frac{1}{4} \sin x$$

$$= \cos x \sin x$$ //.

18.

Solution.

Ans: → A first order and first degree differential equation is said to be linear differnitial equation if it can be written in the form

$$\frac{dy}{dx} + Py = Q \text{ ------ (i)}$$

where P and Q are the function of X or constant (but not of y).

Solution

$(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

Dividing both side by $(1+x^2)$.

$\frac{dy}{dx} + \frac{y}{(1+x^2)} = \frac{e^{\tan^{-1}x}}{(1+x^2)}$

Comparing with $\frac{dy}{dx} + Py = Q$.

$P = \frac{1}{1+x^2}$ and $Q = \frac{e^{\tan^{-1}x}}{(1+x^2)}$.

$\int P.dx = \int \frac{1}{1+x^2} dx = \tan^{-1}x$ ~~tc~~.

$I.F = e^{\int P dx} = e^{\tan^{-1}x}$.

Now,

$y \cdot IF = \int Q \cdot IF \cdot dx$

$y \cdot e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x}}{(1+x^2)} \times e^{\tan^{-1}x}$.

$y \cdot e^{\tan^{-1}x} = \int \frac{(e^{\tan^{-1}x})^2}{(1+x^2)}$.

Put $\tan^{-1}x = y$

$\therefore dy = dx \cdot \frac{1}{(1+x^2)}$.

$\therefore \int \frac{(e^{\tan^{-1}x})^2}{1+x^2} dx = \int e^{2y} \cdot dy$

$= \frac{e^{2y}}{2} + c.$

$e^{\tan^{-1}x} \cdot y = \frac{e^{2\tan^{-1}x}}{2} + c$

$y = \frac{e^{(\tan^{-1}x)^2}}{2(e^{\tan^{-1}x})} + c \cdot e^{-\tan^{-1}x}$.

$\therefore y = \frac{e^{\tan^{-1}x}}{2} + c e^{-\tan^{-1}x}$.

1

Group C

(n) 19

20 (a).

Solution.

Let z be the cube roots of unity i.e. 1 then.

$z^3 = 1$.

$\Rightarrow z^3 = 1 + i.0$.

$z^3 = \cos 0° + i \sin 0°$. (polar form).

$z = (\cos 0° + i \sin 0°)^{1/3}$.

$= \{\cos(k360° + 0°) + i \sin(k.360° + 0°)\}^{1/3}$.

$= \cos \frac{k360°}{3} + i \sin \frac{k360°}{3}$.

$= \cos(k.120°) + i \sin(k.120°)$

when $k = 0$. $z = \cos 120 \times 0 + i \sin 120 \times 0° = 1 + 0i = 1$.

when $k = 1$, $z = \cos 120° + i \sin 120° = -\frac{1}{2} + i\frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}i}{2}$.

when $k = 2$, $z = \cos 240° + i \sin 240° = \frac{-1}{2} - \frac{i\sqrt{3}}{2} = \frac{-1-\sqrt{3}i}{2}$.

So, the three cubic root of unity are 1, $\frac{-1-\sqrt{3}i}{2}$ and $\frac{1+\sqrt{3}i}{2}$.

(b)

Solution

Let $\alpha = \omega$ and $\beta = \omega^2$ be the cube root of unity.

$\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1}$

or, $\omega^4 + (\omega^2)^4 + \omega^{-1}\omega^{-2}$.

or $\omega^3 . \omega + (\omega^3)^2 . \omega^2 + \omega^{-3}$.

or $\omega + \omega^2 + \frac{1}{\omega^3}$.

or $\omega + \omega^2 + 1$ $[\because \omega^3 = 1]$

$\therefore 0$ proved

Page No.
Date

2) (a)

Solution

For hyperbola

focus = $(-7,0)$ = $(\pm ae, 0)$. $\therefore ae = 7$.

$\therefore e = 7/a$

eccentricity = $\frac{7}{4} = \frac{7}{a}$

$\therefore a = 4$.

Also,

$$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{b^2}{16}} = \frac{\sqrt{16+b^2}}{4} = \frac{7}{4}$$

$\sqrt{16+b^2} = 7$.

Squaring on both side

$16+b^2 = 49$

or, $b^2 = 33$

$\therefore b = \sqrt{33}$.

Now,

Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

$$\frac{x^2}{16} - \frac{y^2}{33} = 1.$$

(b)

Solution

→ The Condition of parallelism when drs are a_1, b_1, c_1 and a_2, b_2, c_2 is. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Again

The line Joining point $(1,2,3)$ and $(-1,-2,-3)$ is

Parallel with line joining (2,3,4) and (5,9,13) if it satisfy: Condition of parallelism.

$$\frac{-1-1}{5-2} = \frac{-2-2}{9-3} = \frac{-3-3}{13-4}$$

$$\frac{-2}{3} = \frac{-4}{6} = \frac{-6}{9} \text{ (True)}.$$

$$\frac{-2}{3} = \frac{-2}{3} = \frac{-2}{3}.$$

Hence. They are parallel.

(c).

Solution

Given relations are:-

$al+bm+cn=0$ ------- (i) i.e. $n = \frac{-al-bm}{c}$.

$fmn+gnl+hlm=0$ ---- (ii).

Eliminating n between (i) and (ii) we have.

$$fm\left(\frac{-al-bm}{c}\right)+g\left(\frac{-al-bm}{c}\right)l+hlm=0.$$

$$\frac{-alfm-fbm^2-agl^2-bmgl+hlmc}{c}=0.$$

$$-agl^2-lm(af+bg-ch)-bfm^2=0.$$

Dividing both side by m^2.

$$-ag\left(\frac{l}{m}\right)^2-\left(\frac{l}{m}\right)(af+bg-ch)-bf=0 \text{ ------- (iii)}$$

which is quadratic in $\frac{l}{m}$. Let the two roots be $\frac{l_1}{m_1}$ & $\frac{l_2}{m_2}$.

Page No.
Date

$$\frac{l_1}{m_1} \times \frac{l_2}{m_2} = \frac{bf}{ag} \quad \left[\text{Product of root} = \frac{c}{a}\right]$$

$$\frac{l_1 l_2}{bf} = \frac{m_1 m_2}{ag}$$

$$\frac{l_1 l_2}{f/a} = \frac{m_1 m_2}{g/b} \quad \text{------ (iv)}$$

Similarly if we eliminate l between (i) and (ii)

$$\frac{m_1 m_2}{g/b} = \frac{n_1 n_2}{h/c} \quad \text{---- (v)}$$

from (iv) and v.

$$\frac{l_1 l_2}{f/a} = \frac{m_1 m_2}{g/b} = \frac{n_1 n_2}{h/c} = k \text{ (suppose)}.$$

Then,

$$l_1 l_2 = k\frac{f}{a}, \quad m_1 m_2 = k\frac{g}{b}, \quad n_1 n_2 = k\frac{h}{c}$$

The two lines will be perpendicular if.

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0.$$

$$k\frac{f}{a} + k\frac{g}{b} + k\frac{h}{c} = 0.$$

$$\therefore \frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0 \cdot \text{proved}.$$

22 (a)

Solution

$$f(x) = \sqrt{25 - x^2}.$$

For every value of x such that $-5 \leq x \leq 5$, $f(x)$ has a definite value so $f(x)$ is Continuous for $[-5, 5]$.

Again, $$f'(x) = \frac{d(25-x^2)^{1/2}}{d(25-x^2)} \times \frac{d(25-x^2)}{dx} = \frac{-2x}{2\sqrt{25-x^2}} = \frac{-x}{\sqrt{25-x^2}}$$

which exists for all x such that $(-5, 5)$.

$\therefore f(x)$ is differentiable in $(-5, 5)$.

Now, $f(-5) = \sqrt{25-(-5)^2} = 0$. (ii)

$f(5) = \sqrt{25-5^2} = 0$.

$\therefore f(-5) = f(5)$.

$\therefore$ all the Condition of Rolle's theorem are satisfied.
Hence there exist at least a point such that
point $c \in (-5,5)$ such as:

$f'(c) = 0$.

$$\frac{-c}{\sqrt{25-c^2}} = 0.$$

$\therefore c = 0$ which belongs to $(-5,5)$

Hence, Rolles theorem is verified.

(b) $$\int \frac{dx}{2+\cos x}$$

$$= \int \frac{dx}{2(\cos^2 x/2 + \sin^2 x/2) + (\cos^2 x/2 - \sin^2 x/2)}$$

$$= \int \frac{dx}{3\cos^2 x/2 + \sin^2 x/2}$$

Dividing numerator and denominator by $\cos^2 x/2$.

$$\int \frac{\sec^2 x/2 \, dx}{3 + \tan^2 x/2}$$

put $y = \tan x/2$.

$$\frac{dy}{dx} = \frac{\sec^2 x/2}{1/2}.$$

$2dy = dx \sec^2 x/2$.

Now,

$$I = 2\int \frac{dy}{(\sqrt{3})^2 + y^2}$$

$$= 2\left[\frac{1}{\sqrt{3}} \tan^{-1}\frac{y}{\sqrt{3}}\right] + c.$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\tan x/2}{\sqrt{3}}\right) + c.$$