Chapter 18: Linear Programming

Exercise 18

1. Solution:

a. Here, Max. $z = 3x + 5y$

Subject to the constraints

$x + 2y \leq 5$

$2x - 3y \leq 7$

$x, y \geq 0$

Let, r, s be any two non-negative slack variables. Then,

$x + 2y + r = 5$

$2x - 3y + 5 = 7$ and

$z = 3x + 5y$

The reformulation of LP into standard form as

$x + 2y + r + 0.s + 0.z = 5$

$2x - 3y + 0.r + s + 0.z = 7$

$-3x - 5y + 0.r + 0.s + z = 0$

$x, y, r, z \geq 0$

b. Here, Max $z = 10x + 15y$

Subject to

$x + 2y = \leq 20$

$x + y \leq 16$

$x, y \geq 0$

Let, r, s be any two non-negative slack variables.

$x + 2y + r = 20$

$x + y + s = 16$

The reformulation of LP into standard form as;

$x + 2y + r + 0.s + 0.z = 20$

$x + y + 0.r + s + 0.z = 16$

$-10x - 15y + 0.r + 0.5 + z = 0$

$x, y, r, s \geq 0$

c. Here, Min. $z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$

Let, r, s be any two non–negative slack variables. Then,

$2x_1 + x_2 + r = 4$

$x_1 + 7x_2 + s = 7$

The reformulation of LP into standard form as;

$2x_1 + x_2 + r + 0.s + 0.z = 4$

$x_1 + 7x_2 + 0.r + s + 0.z = 7$

$-x_1 - x_2 + 0.r + 0.s + z = 0$

$x_1, x_2, r, s \geq 0$

d. Here, Min. $z = 7x + 5y$

Subject to $4x + 3y \leq 12 \Rightarrow -4x - 3y \geq -12$

$x + 2y \leq 6 \Rightarrow -x - 2y \geq -6$

Let, r, s be any two non–negative slack variables.

$-4x - 3y + r = -12$

$-x - 2y + s = -6$

The reformulation of LP problem into standard forms

$-4x - 3y + r + 0.s + 0.z = -12$

$-x - 2y + 0.r + s + 0.z = -6$

$-7x - 5y + 0.r + 0.s + z = 0$

$x, y, r, s \geq 0$

2. Solution:

a. Here, given equations are $x + y + z = 6$

$4x + 3y + z = 12$

There are 3 variables and 2 equations so, there are two basic solution and one non-basic.

Solution:

Case – I: if $z = 0$, then,

$x + y = 6$ ... ... (i)

$4x + 3y = 12$ ... ... (ii)

Solving equation (i) and (ii)

$\therefore$ $y = \frac{12}{5}$ (basic)

$\therefore$ $x = \frac{6}{5}$ (basic)

$\therefore$ $z = 0$ (non–basic)

Case – II: if $y = 0$

$x + z = 6$ ... ... (iii)

$4x + z = 12$ ... ... (iv)

Solving (iii) and (iv0

$\therefore$ $x = 2$ (basic)

$\therefore$ $z = 4$ (basic)

$\therefore$ $y = 0$ (non–basic)

Case–III: if $x = 0$

$y + z = 6$ ... ... (v)

$3y + z = 12$ ... ... (vi)

Solving (v) and (vi)

$\therefore$ $y = 6$ (basic)

$\therefore$ $z = -6$ (basic)

$\therefore$ $x = 0$ (non basic)

b. Here,

Given equations are

$x + 2y + z = 4$

$2x + y + 5z = 5$

There are 3 variables in 2 equations among them 2 are basic and 1 is non-basic.

Case–I: if $z = 0$

$x + 2y = 4$ ... ... (i)

$2x + y = 5$ ... ... (ii)

Solving equation (i) and equation (ii)

$\therefore$ $y = 1$ (basic)

$\therefore$ $x = 2$ (basic)

$\therefore$ $z = 0$ (non–basic)

Case–II: if $y = 0$

$x + z = 4$ ... ... (iii)

$2x + 5z = 5$ ... ... (v)

Solving (iii) and (iv)

$\therefore$ $z = -1$ (basic)

$\therefore$ $y = 0$ (non–basic)

$\therefore$ $x = 3$ (basic)

Case–III: if $x = 0$

$2y + z = 4$ ... ... (v)

$y + 5z = 5$ ... ... (vi)

Solving (v) and (vi)

$\therefore$ $z = \frac{2}{3}$ (basic)

$\therefore$ $y = \frac{5}{3}$ (basic)

$\therefore$ $x = 0$ (non–basic)

3. Solution:

a. Given equations are

$x + 2y - z = 3$

$x - y + z = 5$

There are3 variables and 2 equations. So, among them 2 are basic and 1 is non-basic.

Case–I: if $z = 0$

$x + 2y = 3$ ... ... (i)

$x - y = 5$ ... ... (ii)

Solving (i) and (ii)

$\therefore$ $x = \frac{13}{3}$ (basic)

$\therefore$ $y = \frac{-2}{3}$ (basic)

$\therefore$ $z = 0$ (non–basic)

Case–II: if $y = 0$

$x - z = 3$ ... ... (iii)

$x + z = 5$ ... ... (iv)

Solving (ii) and (i)

$\therefore$ $x = 4$ (basic)

$\therefore$ $z = 1$ (basic)

$\therefore$ $y = 0$ (non–basic)

Case–III: if $x = 0$

$2y - z = 3$ ... ... (v)

$\underline{-y + z = 5 \text{ ... ... (vi)}}$

Solving (v) and (vi)

$\therefore$ $y = 8$ (basic)

$\therefore$ $z = 13$ (basic)

$\therefore$ $x = 0$ (non-basic)

Since, the case II and III are non-negative, so they give basic feasible solution.

$\therefore$ The basic feasible solution are (4, 0, 1) and (0, 8, 13)

b. Here, the given equations are

$2x + 3y + z = 12$

$x + 2y - 3z = 5$

There are 3 variables and 2 equations. Among them 2 are basic and 1 is non-basic.

Case–I: if $z = 0$

$2x + 3y = 12$ ... ... (i)

$x + 2y = 5$ ... ... (ii)

Solving (i) and (ii)

$\therefore$ $y = -2$ (basic)

$\therefore$ $x = 9$ (basic)

$\therefore$ $z = 0$ (non–basic)

Case–II: if $y = 0$

$2x + z = 12$ ... ... (iii)

$x - 3z = 5$ ... ... (iv)

Solving (iii) and (iv)

$\therefore$ $z = \frac{2}{7}$ (basic)

$\therefore \quad x = \frac{41}{7}$ (basic)

$\therefore \quad y = 0$ (non–basic)

Case III: If x = 0,

3y + z = 12 .... (v)

2y – 3z = 5 .... (vi)

solving (v) and (vi), we get

$y = \frac{41}{11}$ and $z = \frac{9}{11}$

Since, the cases II and III are non-negative, so the basic feasible solution are

$\left(\frac{41}{7}, 0, \frac{3}{7}\right)$ and $\left(0, \frac{41}{11}, \frac{9}{11}\right)$

4. Solution:

a. Here, max. z = 2x + 3y

Subject to $x + 2y \le 10$

$2x + y = \le 14$

$x, y \ge 0$

Let, r, s be any two non–negative slack variables.

x + 2y + r = 10

2x + y + s = 14 and

z = 2x + 3y

$\Rightarrow \quad x + 2y + r + 0.s + 0.z = 10$

$2x + y + 0.r + s + 0.z = 14$

$-2x - 3y + 0.r + 0.s + z = 0$

The simplex tableau;

Basic variables	x	y	r	s	z	RHS
r	1	2	1	0	0	10
s	2	1	0	1	0	14
	–2	–3	0	0	1	0

The most negativity entry is –3 so, y column is pivot column. Then, $\frac{10}{2} = 5, \frac{14}{1} = 14$

Here, 5 < 14 so 2 is pivot element.

$R_1 \to \frac{1}{2} R_1$

Basic variables	x	y	r	s	z	RHS
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	5
x	2	1	0	1	0	14
	–2	–3	0	0	1	0

$R_2 \to R_1$, $R_3 \to 3R_1$

Basic variables	x	y	r	s	z	RHS
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	5
s	$\frac{3}{2}$	0	$\frac{-1}{2}$	1	0	9
	$-\frac{1}{2}$	0	$\frac{3}{2}$	0	1	15

All the values in last row is not positive. So, it is not optimal solution.

Here, the most negativity entry is $\frac{-1}{2}$ so x column is pivot column. Then,

$\frac{5}{\frac{1}{2}} = \frac{5\times2}{10}, \frac{9}{\frac{3}{2}} = \frac{9\times2}{3} = 6$

Here, 6 < 10 so, $\frac{3}{2}$ is pivot element.

$R_2 \rightarrow \frac{2}{3} R_2$

Basic variables	x	y	r	s	z	RHS
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	5
x	1	0	$\frac{-1}{3}$	$\frac{2}{3}$	0	6
	$\frac{-1}{2}$	0	$\frac{3}{2}$	0	1	15

$R_1 \rightarrow R_1 \frac{-1}{2} R_2,\ R_3 \rightarrow \frac{1}{2} R_2 + R_3$

Basic variables	x	y	r	s	z	RHS
y	0	1	$\frac{5}{6}$	$\frac{-1}{3}$	0	2
x	1	0	$\frac{-1}{3}$	$\frac{2}{3}$	0	6
	0	0	$\frac{4}{3}$	$\frac{1}{3}$	1	18

Here, all the elements in R_3 are positive so, it is optimal solution. The maximum value is 18 at x = 6 and y = 2.

b. Here, max. z = 9x + y

Subject to $2x + y \leq 8$

$4x + 3y \leq 18$

$x, y \geq 0$

Let, r, s be any two non-negative slack variables. Then,

$2x + y = r = 8$

$4x + 3y + 5 = 18$

$z = 9x + y$

$\Rightarrow 2x + y + r + 0.s + 0.z = 8$

$4x + 3y + 0.r + s + 0.z = 18$

$-9x - y + 0.r + 0.s + z = 0$

The simplex tableau is

Basic variables	x	y	r	s	z	RHS
r	②	1	1	0	0	8
s	4	3	0	1	0	18
	–9	–1	0	0	1	0

The most negativity entry is –9 so, x column is pivot column. Then, $\frac{8}{2} = 4$, $\frac{18}{4} = 4.5$

Here, 2 is pivot element.

$R_1 \rightarrow \frac{1}{2} R_1$

Basic variables	x	y	r	s	z	RHS
x	1	2	$\frac{1}{2}$	0	0	4
s	4	3	0	1	0	18
	–9	–1	0	0	1	0

$R_2 \rightarrow R_2 - 4R_1$

Basic variables	x	y	r	s	z	RHS
x	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	4
s	0	1	–2	1	0	2

	–9	–1	0	0	1	0

$R_3 \to R_3 + 9R_1$

Basic variables	x	y	r	s	z	RHS
x	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	4
s	0	1	–2	1	0	2
	0	$\frac{7}{2}$	$\frac{9}{2}$	0	1	36

Here, all the element in R_3 are positive so, it is optimal solution.
The maximum value is 36 at x = 4 is y = 0.

c. Here, max. $f = 6x_1 - 9x_2$
Subject to $2x_1 - 3x_2 \le 6$
$x_1 + x_2 \le 20$
$x_1, x_2 \ge 0$
Let, r, s be any two non–negative slack variables then,
$2x_1 - 3x_2 + r = 6$
$x_1 + x_2 + s = 20$ and
$f = 6x_1 - 9x_2$
$\Rightarrow$ $2x_1 - 3x_2 + r + 0.s + 0.f = 6$
$x_1 + x_2 + 0.r + s + 0.f = 20$
$-6x_1 + 9x_2 + 0.r + 0.s + f = 0$
The initial simplex tableau is;

Basic variables	x	y	r	s	z	RHS
r	②	–3	1	0	0	6
s	1	1	0	1	0	20
	–6	9	0	0	1	0

The most negativity entry is –6 so, x column is pivot column. Then, $\frac{6}{2} = 3$, $\frac{20}{1} = 20$.

Here, 3 < 20 so, 2 is pivot column.

$R_1 \to \frac{1}{2} R_1$

Basic variables	x	y	r	s	z	RHS
x	1	$\frac{-3}{2}$	$\frac{1}{2}$	0	0	3
s	1	1	0	1	0	20
	–6	9	0	0	1	0

Now, $R_2 \to R_2 - R_1$, $R_3 \to R_3 + 6R_1$

Basic variables	x	y	r	s	z	RHS
x	1	$\frac{-3}{2}$	$\frac{1}{2}$	0	0	3
s	0	$\frac{5}{2}$	$\frac{-1}{2}$	1	0	17
	0	0	3	0	0	18

Here, all the elements in last row are positive so, the maximum value is 18 at x = 3 and y = 0.

d. Here, max. $z = 7x_1 + 5x_2$
Subject to $x_1 + 2x_2 = 6$
$4x_1 + 3x_2 \le 12$
$x_1, x_2 \ge 0$
Let, r, s be any two non–negative slack variables. Then,
$x_1 + 2x_2 + r = 6$
$4x_1 + 3x_2 + 5 = 12$ and

$z = 7x_1 + 5x_2$

$\Rightarrow$ $x_1 + 2x_2 + r + 0.s + 0.z = 6$

$4x_1 + 3x_2 + 0.r + s + 0.z = 12$

$-7x_1 - 5x_2 + 0.r + 0.s + z = 0$

The simplex tableau is;

Basic variables	x_1	x_2	r	s	z	RHS
r s	1 ④	2 3	1 0	0 1	0 0	6 12
	–7	–5	0	0	1	0

The most negativity entry is –7 so, x_1 column is pivot column. Then,

$\frac{6}{1} = 6, \frac{12}{4} = 3$

Here, 3 < 6 so, 4 is pivot element.

$R_2 \to \frac{1}{4} R_2$

Basic variables	x_1	x_2	r	s	z	RHS
r x_1	1 1	2 $\frac{3}{4}$	1 0	0 $\frac{1}{4}$	0 0	6 3
	–7	–5	0	0	1	0

$R_1 \to R_1 - R_2$, $R_3 \to R_3 + 7R_2$

Basic variables	x_1	x_2	r	s	z	RHS
r x_1	0 1	$\frac{5}{4}$ $\frac{3}{4}$	1 0	$\frac{-1}{4}$ $\frac{1}{4}$	0 0	3 3
	0	$\frac{1}{4}$	0	$\frac{7}{4}$	1	21

Here, all the elements in last row are positive so, it is optimal solution.

The maximum value is 21 at $x_1 = 3$ and $x_2 = 0$.

5. Solution:

a. Here, mix. W = 3x + 2y

Subject to $2x + y \geq 6$

$x + y \geq 4$

$x, y \geq 0$

The augmented matrix is A =

$$\left(\begin{array}{cc|c} 2 & 1 & 6 \\ 1 & 1 & 4 \\ \hline 3 & 2 & 0 \end{array}\right)$$

Then, the augmented dual problem is

A^T =

$$\left(\begin{array}{cc|c} 2 & 1 & 3 \\ 1 & 1 & 2 \\ \hline 6 & 4 & 0 \end{array}\right)$$

Hence, max. $z = 6x_1 + 4y_1$ s.t.

$2x_1 + y_1 \leq 3$

$x_1 + y_1 = \leq 2$

$x_1, y_1 \geq 0$

Let, x and y be two non-negative slack variables,

$2x + y + x = 3$

$x_1 + y_1 + y = 2$ and

$z = 6x_1 + 4y_1$

$\Rightarrow \quad 2x_1 + y_1 + x + 0.y + 0.z = 3$

$x_1 + y_1 + 0.x + y + 0.z = 2$

$-6x_1 - 4y_1 + 0.x + 0.y + z = 0$

The simplex tableau is

Basic variables	x_1	x_2	r	s	z	RHS
x	②	1	1	0	0	3
y	1	1	0	1	0	2
	–6	–4	0	0	1	0

The most negativity entry is –6 so, x_1 column is pivot column. Then,

$\frac{3}{2} = 1.5 < \frac{2}{1} = 2$

So, 2 is pivot element.

$R_1 \to \frac{1}{2} R_1$

Basic variables	x	y	r	s	z	RHS
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{3}{2}$
y	1	1	0	1	0	2
	–6	–4	0	0	1	0

$R_2 \to R_2 - R_1$, $R_3 \to R_3 + 6R_2$

Basic variables	x_1	y_1	r	s	z	RHS
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{3}{2}$
y	0	$\frac{1}{2}$	$\frac{-1}{2}$	1	0	$\frac{1}{2}$
	0	–1	3	0	1	9

Here, all the elements in last raw are not positive so it is not optimal solution.

Here, the most negativity entry is –1 so, y_1 column is pivot column. Then,

$$\frac{\frac{3}{2}}{\frac{1}{2}} = 3 > \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

so, $\frac{1}{2}$ is pivot element.

$R_2 \to 2R_2$

Basic variables	x_1	y_1	r	s	z	RHS
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{3}{2}$
y_1	0	1	–1	2	0	1
	0	–1	3	0	1	9

$R_1 \to R_1 - \frac{1}{2} R_2$, $R_3 \to R_3 + R_2$

Basic variables	x	y	r	s	z	RHS
x_1	1	0	1	–1	0	1
y_1	0	1	–1	2	0	1
	0	0	2	2	1	10

Here, all the elements in last row are positive so, it is optimal solution.

The maximum value is 10 at $x_1 = 1$ and $y_1 = 1$.

Hence, the corresponding min. W = 10 at x = 2 and y = 2

b. Here, min. $W = 18x + 12y$

Subject to $2x + y \geq 8$

$6x + 6y \geq 36$

$\Rightarrow \quad x + y \geq 6$

$x, y \geq 0$

The augmented matrix A =

$$\left(\begin{array}{cc|c} 2 & 1 & 8 \\ 1 & 1 & 6 \\ \hline 18 & 12 & 0 \end{array}\right)$$

The augmented dual problem is

$A^T =$

$$\left(\begin{array}{cc|c} 2 & 1 & 18 \\ 1 & 1 & 12 \\ \hline 8 & 6 & 0 \end{array}\right)$$

Hence, max $z = 8x_1 + 6y_1$, $2x_1 + y_1 \leq 18$ s.t.

$x_1 + y_1 \leq 12$

$x_1, y_1 \geq 0$

Let, x, y be any two non–negative slack variables then,

$2x_1 + y_1 + x = 18$

$x_1 + y_1 + y = 12$ and

$z = 8x_1 + 6y_1$

$\Rightarrow \quad 2x_1 + y_1 + x + 0.y + 0.z = 18$

$x_1 + y_1 + 0.x + y + 0.z = 12$

$-8x_1 - 6y_1 + 0.x + 0.y + z = 0$

Basic variables	x_1	y_1	x	y	z	RHS
x	2	1	1	0	0	18
y	1	1	0	1	0	12
	–8	–6	0	0	1	0

The most negatively entry is –8 so x_1, column is pivot column. Then,

$\frac{18}{2} = 9 < \frac{12}{1} = 12$

Here, 2 is pivot element.

$R_1 \rightarrow \frac{1}{2} R_1$

Basic variables	x_1	y_1	x	y	z	RHS
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	9
y	1	1	0	1	0	12
	–8	–6	0	0	1	0

$R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 + 8R_1$

Basic variables	x_1	y_1	x	y	z	RHS
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	9
y	0	$\frac{1}{2}$	$\frac{-1}{2}$	1	0	3
	0	–2	4	0	1	72

All the elements in last row are not positive. So, it is not optimal solution.

The most negative entry is –2 so y_1 column is pivot column. Then,

$\frac{9}{\frac{1}{2}} = 18 > \frac{3}{\frac{1}{2}} = 6$

Here, $\frac{1}{2}$ is pivot element.

$R_2 \rightarrow 2R_2$

Basic variables	x_1	y_1	x	y	z	RHS
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	9
y_1	0	1	–1	2	0	–6
	0	–2	4	0	1	72

$R_1 \rightarrow R_1 - \frac{1}{2} R_2,\ R_3 \rightarrow R_3 + R_2$

Basic variables	x_1	y_1	x	y	z	RHS
x_1	1	0	1	–1	0	6
y_1	0	1	–1	2	0	6
	0	0	2	4	1	84

Hence, the min. W = 84 at (2, 4)

c. Here, min. $f = x + 4y$

Subject to $x + 2y \geq 8$

$3x + 2y \geq 12$

The augmented matrix is A =

$$\left|\begin{array}{cc|c} 1 & 2 & 8 \\ 3 & 2 & 12 \\ \hline 1 & 4 & 0 \end{array}\right|$$

The augmented dual problem is

$A^T =$

$$\left|\begin{array}{cc|c} 1 & 3 & 1 \\ 2 & 2 & 4 \\ \hline 8 & 12 & 0 \end{array}\right|$$

Hence, max. $G = 8x_1 + 12y_1$, $x_1 + 3y_1 \leq 1$

$x_1 + y_1 \leq 2$

$x_1\ y_1 \leq 0$

Let x, y are two non–negative slack variables then,

$\Rightarrow\quad x_1 + 3y_1 + x = 1$

$\quad x_1 + y_1 + y = 2$

$\quad -8x_1 - 12y_1 + G = 0$

Then,

$x_1 + 3y_1 + x + 0.y + 0.G = 1$

$x_1 + y_1 + 0.x + 0.y + G = 0$

The simplex tableau is

Basic variables	x_1	y_1	x	y	G	RHS
x	1	3	1	0	0	1
y	1	1	0	1	0	2
	–8	–12	0	0	1	0

The most negativity entry is –12 so, y_1 column is pivot column. Then,

$\frac{1}{3} = 0.33 < \frac{2}{1} = 2$

So, 3 is pivot element.

$R_1 \rightarrow \frac{1}{3} R_1$

Basic variables	x_1	y_1	x	y	G	RHS
y_1	$\frac{1}{3}$	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$
y	1	1	0	1	0	2
	–8	–12	0	0	1	0

$R_2 \to R_2 - R_1$, $R_3 \to R_3 + 12R_1$

Basic variables	x_1	y_1	x	y	G	RHS
y_1	$\frac{1}{3}$	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$
y	$\frac{2}{3}$	0	$\frac{-1}{3}$	0	0	$\frac{5}{3}$
	–4	0	4	0	1	4

All the elements in last row are not positive. So it is not optimal solution. The most negativity entry is –4 so, x_1 column is pivot column. Then,

$$\frac{\frac{1}{3}}{\frac{1}{3}} = 1 < \frac{\frac{5}{3}}{\frac{2}{3}} = 2.5$$

So $\frac{1}{3}$ is pivot element.

$R_1 \to 3R_1$

Basic variables	x_1	y_1	x	y	G	RHS
x_1	1	3	1	0	0	1
y	$\frac{2}{3}$	0	$\frac{-1}{3}$	0	0	$\frac{5}{3}$
	–4	0	4	0	1	4

$R_2 \to R_2 - \frac{2}{3} R_1$, $R_3 \to R_3 + 4R_1$

Basic variables	x_1	y_1	x	y	G	RHS
x_1	1	3	1	0	0	1
y	0	–2	$\frac{-11}{9}$	0	0	$\frac{1}{9}$
	0	12	8	0	1	8

The maximum value is 8 at $x_1 = 1$ and $y_1 = 0$.

Hence, the corresponding min. F = 8 at x = 8, and y = 0

d. Here, min. W = 14x + 20y

Subject to $7x + 6y \geq 20$

$x + 2y \geq 4$

$x, y \geq 0$

The augmented matrix is A =

$$\left|\begin{array}{cc|c} 7 & 6 & 20 \\ 1 & 2 & 4 \\ \hline 14 & 20 & 0 \end{array}\right|$$

The augmented dual problem is A^T =

$$\left|\begin{array}{cc|c} 7 & 1 & 14 \\ 6 & 2 & 20 \\ \hline 20 & 4 & 0 \end{array}\right|$$

Max $z = 20x_1 + 4y_1$, $7x_1 + y_1 \leq 14$

$6x_1 + 2y_1 \leq 20$

$x, y \geq 0$

Let, x, y be any two non–negative slack variables.

$7x_1 + y_1 + x = 14$

$6x_1 + 2y_1 + 0.x + y + 0.w = 20$

$-20x_1 - 4y_1 + 0.x + 0.y + w = 0$

The simplex tableau is

Basic variables	x_1	y_1	r	s	z	RHS
x	7	1	1	0	0	14
y	6	2	0	1	0	20

	–20	–4	0	0	1	0

The most negativity entry is –20 so, x_1 column is pivot column. Then,

$\frac{14}{7} = 2 < \frac{20}{6} = 3.3$

$R_1 \to \frac{1}{7} R_1$

Basic variables	x_1	y_1	r	s	z	RHS
x_1 y	1 6	$\frac{1}{7}$ 2	$\frac{1}{7}$ 0	0 1	0 0	2 20
	–20	–4	0	0	1	0

$R_2 \to R_2 – 6R_1$

Basic variables	x_1	y_1	r	s	z	RHS
x_1 y	1 0	$\frac{1}{7}$ $\frac{8}{7}$	$\frac{1}{7}$ $\frac{-6}{7}$	0 1	0 0	2 8
	–20	–4	0	0	1	0

$R_3 \to R_3 + 20R_1$

Basic variables	x_1	y_1	x	y	z	RHS
x_1 y	1 0	$\frac{1}{7}$ $\frac{8}{7}$	$\frac{1}{7}$ $\frac{-6}{7}$	0 1	0 0	2 8
	0	$\frac{-8}{7}$	$\frac{20}{7}$	0	1	40

All the elements in last row are not positive so, it is not optimal solution. The most negativity element is $\frac{-8}{7}$. So, y_1 column is pivot column. Then,

$\frac{2}{\frac{1}{7}} = 14, \frac{8}{\frac{8}{7}} = 7$

Here, $\frac{8}{7}$ is pivot element.

$R_2 \to \frac{7}{8} R_2$

Basic variables	x_1	y_1	r	s	z	RHS
x_1 y_1	1 0	$\frac{1}{7}$ 1	$\frac{1}{7}$ $\frac{-3}{4}$	0 $\frac{7}{8}$	0 0	2 7
	0	$\frac{-8}{7}$	$\frac{20}{7}$	0	1	40

$R_1 \to R_1 – \frac{1}{7} R_2$

Basic variables	x_1	y_1	r	s	z	RHS
x_1 y_1	1 0	0 1	$\frac{1}{4}$ $\frac{-3}{4}$	$\frac{-1}{8}$ $\frac{7}{8}$	0 0	1 7
	0	$\frac{-8}{7}$	$\frac{20}{7}$	0	1	40

$R_3 \rightarrow R_3 + \frac{8}{7} R_2$

Basic variables	x_1	y_1	r	s	z	RHS
x_1	1	0	$\frac{1}{4}$	$\frac{-1}{8}$	0	1
y_1	0	1	$\frac{-3}{4}$	$\frac{7}{8}$	0	7
	0	0	2	1	1	48

All the elements in last row are positive so it is optimal solution.
The maximum value is 48 at $x_1 = 1$ and $y_1 = 7$.
Hence, the corresponding min. w = 48 at x = 2 and y = 1