

Thermodynamics

Thermodynamics:- Thermodynamics is concerned with energy relationship and it deals with transformation of heat into mechanical work and vice-versa.

Thermodynamic system:-

A definite quantity of matter bounded by a closed surface is known as system. (e.g. air in a tube or in cylinder fitted with movable piston). The thermodynamic system is one which interacts with its surrounding by transfer of heat Q and out of the system and by doing work by and on the system.

A system which exchanges energy and matter with its surroundings is known as open system. A system which exchanges only energy with surroundings, is called closed system and that which exchanges nothing is called isolated system.

Work done during volume change.

Let us consider a certain mass of a gas enclosed in a cylinder fitted with a movable, frictionless piston of cross-sectional area 'A' as in fig (1).

External work done by the system (gas) when piston moves from B to B' through small displacement dx will be

$$dW = \text{Force on piston} \times \text{displacement}$$

$$= F dx = (PA) dx \quad [\because P = F/A] \quad \text{fig - (1)}$$

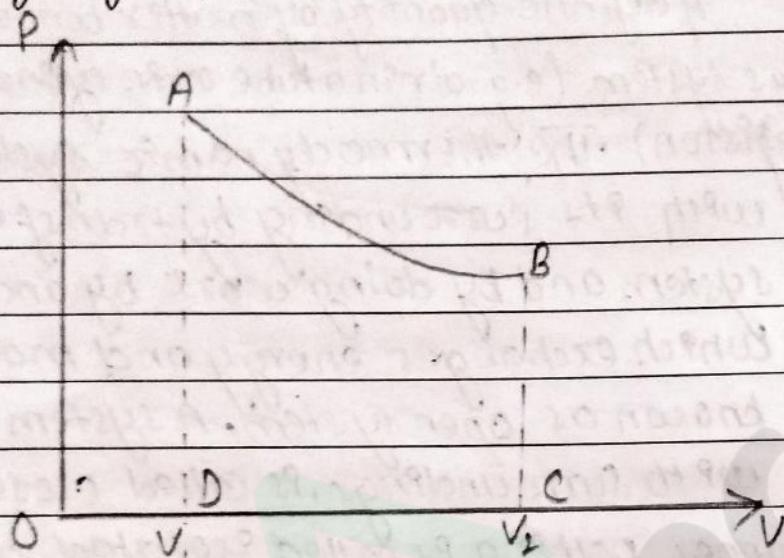
$$= P dV \quad [\text{where } dV = Adx \text{ is the volume change}]$$

∴ Total work done during the expansion of gas from volume V_1 to V_2 will be

$$W = \int dW = \int_{V_1}^{V_2} P dV$$

Notes:-

1. Work done by the system during expansion P_s taken positive.
2. Work done on the gas during compression P_s taken negative.
3. the graph between pressure P and volume V of a system undergoing a process P_s called indicator diagram (shown in fig 2)



$$W = \int_{V_1}^{V_2} P dV$$

= area ABCD under
P.V diagram.

fig(2) Indicator diagram.

* Internal energy of gas (U):-

The sum of internal kinetic energy (due to the thermal motion of gas molecules) and the internal potential energy (due to intermolecular attraction) of gas P_s called internal energy of gas. Change in internal energy of ideal gas depends on temp. change only and P_s given by

$$dU = nC_V dT = mC_V dT$$

where; n = number of moles.

C_V = molar heat capacity at constant volume.

m = mass of gas

C_u = specific heat capacity at constant volume.

Insulating wall

- * First law of thermodynamics: → If heat dQ is given to the system then from the law of conservation of energy.
- $$dQ = dU + dw \quad (1)$$

Eqn (1) is the mathematical statement of first law of thermodynamics

which may be stated as "If the quantity of heat supplied to the system is capable of doing work, then the quantity of heat absorbed by the system is equal to the sum of the internal energy of the system and the external work done by it."

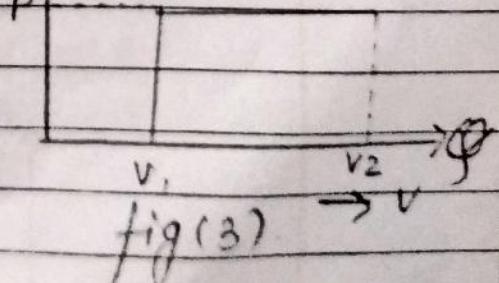
* Thermodynamic Processes:

A process by which one or more parameters (P, V, T) undergo a change is called thermodynamic process.

- (1) Isobaric process: → A thermodynamic process that takes place at constant pressure P is called isobaric process shown in fig (3)

∴ work done

$$W = \int P dV = P \int_{V_1}^{V_2} dV = P(V_2 - V_1)$$



(ii)

Isochoric process:- A thermodynamic process that takes place at constant volume is called Isochoric process (fig 4).

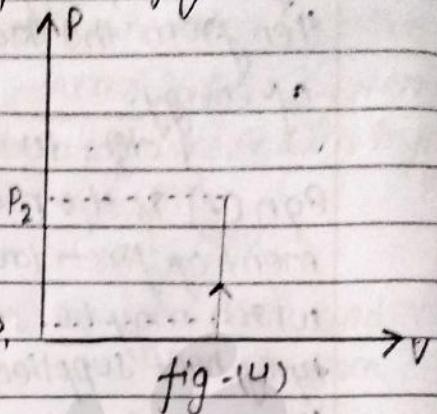
∴ Work done

$$W = \int P dV = 0$$

(∴ V = constant)

$$\therefore dQ = dU + dW$$

$$= dU = nC_V dT = mC_V dT$$

(iii) **Isothermal process:-**

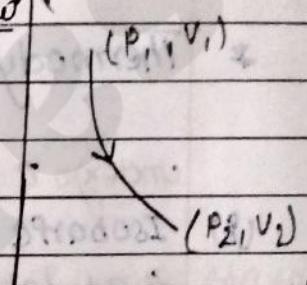
A thermodynamic process in which temp. remains constant is called Isothermal process (fig 5).

Thus Isothermal process obeys Boyle's law.

Also for Isothermal process

$$dQ = dU + dW = nC_V dT + PdV.$$

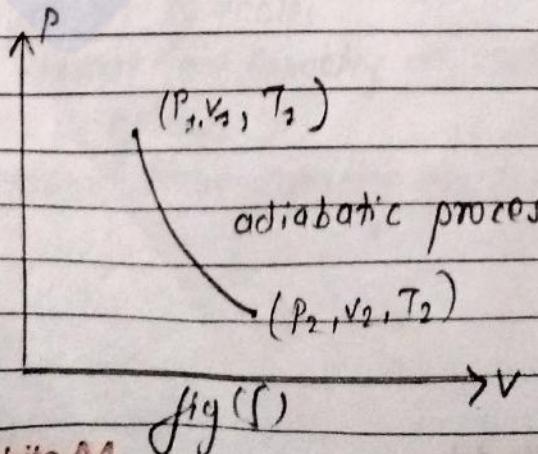
$$\therefore dQ = PdV (\because dT = 0).$$

(iv) **Adiabatic process:-**

The thermodynamic process in which neither heat comes in nor goes out, but is called adiabatic process. Such process is very quick and heat change is zero.

$$\text{Thus } dQ = dU + dW = 0$$

$$dU = -dW.$$



* Work done in an isothermal process:-

Let us consider that 1 mole of an ideal gas is expanded isothermally in a cylinder fitted with movable piston such that the state changes from (P_1, V_1) to (P_2, V_2) at constant temp. T as shown in fig. 6)

For one mole of ideal gas

$$PV = RT$$

$$P = \frac{RT}{V} \quad \text{--- (1)}$$

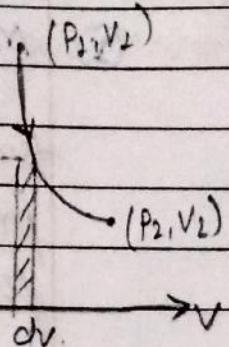


fig :- (6.1)

$$\therefore \text{Work done} = W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{RT}{V} dV$$

$$W = RT \int_{V_1}^{V_2} \frac{1}{V} dV = RT \ln \frac{V_2}{V_1} \quad \text{--- (2)}$$

Also for isothermal process $P_1 V_1 = P_2 V_2$

$$\therefore \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

Hence eqn (2) can be also written as

$$W = RT \ln \left(\frac{P_1}{P_2} \right) \quad \text{--- (3)}$$

$$\text{Thus } W = RT \ln \left(\frac{V_2}{V_1} \right) = RT \ln \left(\frac{P_1}{P_2} \right)$$

* Heat capacity at constant pressure and at constant volume:-

(1) Molar heat capacity at constant pressure (C_p):-

The amount of heat needed to increase the temperature of 1 mole of gas by 1 degree ~~at~~ at constant pressure is known as molar heat capacity at constant pressure for that gas. Its unit is Joule ~~rate~~ $\text{mole}^{-1}\text{K}^{-1}$.

(2) Molar heat capacity at constant volume (C_v):-

The amount of heat needed to increase the temperature of 1 mole of gas by 1 degree at constant volume is known as molar heat capacity at constant volume for that gas. Its unit is Joule $\text{mol}^{-1}\text{K}^{-1}$.

(3) Specific heat capacity at constant pressure (C_p):-

The amount of heat needed to increase the temp of unit mass of a gas by 1 degree at constant pressure is known as specific heat capacity at constant pressure for that gas. Its unit is $\text{J kg}^{-1}\text{K}^{-1}$.

(4) Specific heat capacity at constant volume (C_v):-

The amount of heat needed to raise the temp of unit mass of a gas by 1 degree at constant volume is called specific heat capacity of that gas at constant volume. Its unit is $\text{J kg}^{-1}\text{K}^{-1}$.

* **Gas constant per unit mass (or specific gas constant (r))**

The ratio of the universal gas constant ' R ' and the molar mass of a gas is known as specific gas constant for that gas. Thus

$$r = \frac{R}{M}$$

$$\text{Its unit is } \frac{\text{Joule} \cdot \text{mol}^{-1} \text{K}^{-1}}{\text{kg mol}^{-1}} = \text{Joule kg}^{-1} \text{K}^{-1}$$

Note:- $PV = nRT = \frac{m}{M} RT = MRT \therefore P = \frac{M}{M} RT$

* **Relation between two heat capacities of a gas.**

When 1 mole of an ideal gas is heated through 1 degree at constant volume (as in fig. 1)

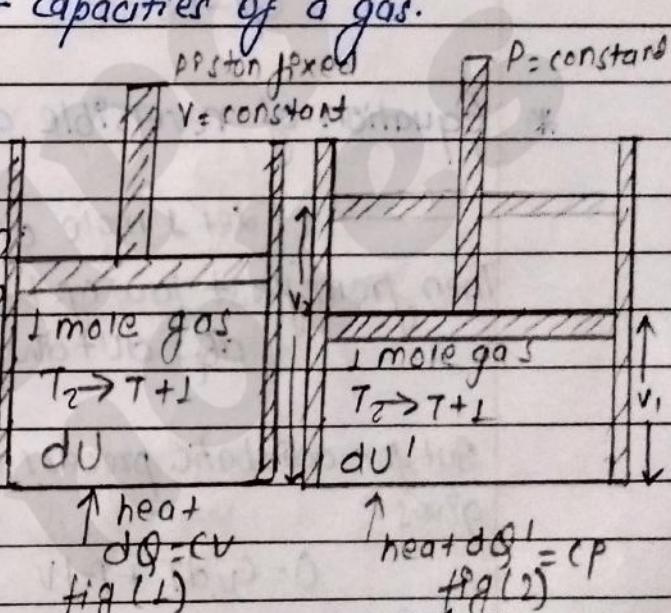
then from first law of thermodynamics gives

$$dQ = dU + dW$$

$$\therefore C_V = dU + PDV$$

$$= dU + 0$$

$$= dU \quad \dots \textcircled{1}$$



And when one mole of the same gas is heated through 1 degree at constant pressure (as in fig. 2) then heat given to gas will be

$$dQ' = dU' + PDV$$

$$C_P = dU' + P(V_2 - V_1) = dU' + PV_2 - PV_1$$

$$C_P = dU' + RT_2 - RT \quad [\because PV = RT \text{ for 1 mole gas}]$$

$$= dU' + R(T_2 - T_1)$$

$$= dU' + R[T + 1 - T]$$

$$C_P = dU' + R \quad \dots \textcircled{2}$$

Because internal energy changes do not depend on volume change and depends on temp. change only, therefore

$$dU = dU' + CV \quad (\text{from eqn 1})$$

\therefore eqn ② gives

$$CP = CV + R \quad \therefore [CP - CV = R]$$

where,

CP = molar heat capacity of gas at constant pressure

CV = molar heat capacity of gas at constant volume.

Note:- Taking 1 kg gas in place of 1 mole we can find the difference capacities as

$$CP - CV = R$$

* Equation of reversible adiabatic process:-

Let 1 mole of an ideal gas is heated adiabatically

Then from first law of thermodynamics

$$dQ = dU + dW = nCV dT + PdV = CV dT + PdV$$

$(\because n = 1 \text{ mole})$

But for adiabatic process $dQ = 0$, therefore above equation gives.

$$0 = CV dT + PdV$$

$$PdV = -CVdT \quad \dots \textcircled{1} \quad \text{or } dT = -\frac{PdV}{CV}$$

Also for 1 mole gas

$$PV = RT$$

$$\therefore PdV + VdP = RdT \quad [\text{differentiated both sides}]$$

$$\text{Or, } dT = \frac{PdV + VdP}{R} = \frac{PdV + VdP}{(CP - CV)} \quad (\because R = (CP - CV))$$

$$\text{Or, } -\frac{PdV}{CV} = \frac{PdV + VdP}{(CP - CV)}$$

$$\text{or, } -CV(PdV) + CV(dP) = -CP(PdV) + CV(PdV)$$

$$\text{or, } \frac{VdP}{CV} + CPdV = 0.$$

$$\text{or, } VdP + \gamma PdV = 0.$$

$$\text{or, } \frac{VdP}{PV} + \frac{\gamma PdV}{PV} = 0. \quad [\text{divided both sides by } PV]$$

$$\text{or, } \frac{1}{P} dP + \gamma \frac{1}{V} dV = 0 \dots \textcircled{1}$$

Integrating eqn (1) we get

$$\int \frac{1}{P} dP + \gamma \int \frac{1}{V} dV = \text{constant.}$$

$$\log P + \gamma \log V = \text{constant.}$$

$$\log P + \log V^\gamma = \text{constant.}$$

$$\log PV^\gamma = \text{constant.}$$

$$\Rightarrow PV^\gamma = \text{constant} \quad \text{--- } \textcircled{2}$$

$$\text{or, } P_1 V_1^\gamma = P_2 V_2^\gamma \quad \text{--- } \textcircled{2}$$

Also putting $P=RT$ in eqn ② we get

$$RT \cdot V^\gamma = \text{constant}$$

$$RTV^{\gamma-1} = \text{constant}$$

$$\therefore TV^{\gamma-1} = \text{constant} \quad \text{--- } \textcircled{3}$$

$$\text{or, } T_1 V_1^{\gamma-1} = P_2 V_2^{\gamma-1} \quad \text{--- } \textcircled{4}$$

And putting $V=RT$ in eqn ② we get

$$\text{or, } P \cdot \left[\frac{RT}{P} \right]^\gamma = \text{constant}$$

$$\frac{P R^Y \cdot T^Y}{P^Y} = \text{constant}$$

$$\Rightarrow T^{\gamma} p^{\gamma-1} = \text{constant} \dots \dots \quad (4)$$

* Work done during adiabatic process:-

Let 1 mole of an ideal gas
is expanded adiabatically
from initial volume V_1 to volume
 V_2 as shown in Fig.

Total work done by the gas
will be

$$W = \int_{V_1}^{V_2} P dV$$

$$= \int_{V_1}^{V_2} \left(\frac{P}{V^Y} \right) dV \quad [\because PV^Y = \text{constant} = k]$$

$$= k \int_{V_1}^{V_2} V^{-\gamma} dV$$

$$= k \left[v^{-\gamma+2} \right]_{v_1}^{v_2} = k \frac{(1-\gamma)}{(\gamma-1)} \left[v_2^{-\gamma+2} - v_1^{-\gamma+2} \right]$$

$$= \frac{1}{(1-\gamma)} \left[\frac{k}{V_1^{\gamma-1}} - \frac{k}{V_1^{1-\gamma}} \right]$$

$$\frac{1}{(1-\gamma)} \left[\frac{P_2 V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1 V_1^\gamma}{V_1^{\gamma-1}} \right]$$

$$= \frac{1}{(1-\gamma)} \left(P_2 V_2 - P_1 V_1 \right)$$

$$W = R \frac{(T_2 - T_1)}{1-\gamma}$$

Chapter:- First law of thermodynamics

Formulae

- i. $dQ = dU + dW = nC_V dT + PdV = mC_V dT + PdV$
2. $dW = PdV = \text{external work done.}$
3. $dU = \text{change in internal energy of gas}$
 $\stackrel{\text{defn}}{=} nC_V dT; \text{ for } n \text{ mole of gas.}$
 $\stackrel{\text{defn}}{=} mC_V dT; \text{ for mass of gas.}$
4. Work done in isothermal process for n moles of gas
$$W = nRT \ln \left(\frac{V_2}{V_1} \right)$$
5. $C_p - C_V = R = \text{difference of two molar heat capacities.}$
6. $C_p - C_V = r = \text{difference of two specific heat capacities.}$
7. Adiabatic equation
$$P_1 V_1^\gamma = P_2 V_2^\gamma = \text{constant} \quad \text{and} \quad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad (\text{constant})$$

where, $\gamma = \frac{C_p}{C_V} = \frac{C_p}{C_V} = \text{adiabatic constant.}$

8. Work done in adiabatic process.

$$W = \frac{R}{\gamma-1} (T_1 - T_2)$$

9. $PV = mRT; \text{ where } r = \frac{R}{M} = \text{constant per unit mass (or specific gas constant)}$

$$\text{or, } P = \left(\frac{m}{V} \right) rT$$

$$\text{or, } P = \rho rT$$

$$\therefore r = \frac{P}{\rho T}$$

NOTES:-
1. For air and diatomic gas. $\gamma = 1.4$
2. For monoatomic gas $\gamma = 1.66$

Numericals: Based on first law of thermodynamics.

- 2) Five moles of an ideal gas are kept at constant temp 53°C and pressure is increased from 1 atm to 3 atm. Calculate work done [2022 set C]

Given

$$\text{number of moles } (n) = 5$$

$$T = (273 + 53) \text{ K} = 326 \text{ K}$$

$$P_1 = 1 \text{ atm} = 1 \times 1.01 \times 10^5 \text{ Nm}^{-2}$$

$$P_2 = 3 \text{ atm} = 3 \times 1.01 \times 10^5 \text{ Nm}^{-2}$$

$$= 3.03 \times 10^5 \text{ Nm}^{-2}$$

$$\text{work done } (w) = ?$$

Soln

As temp is constant hence the process is isothermal

$$\therefore w = nRT \ln \left(\frac{V_2}{V_1} \right) - q$$

Also at const. temp

$$P_1 V_1 = P_2 V_2$$

$$\therefore \frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{1 \text{ atm}}{3 \text{ atm}} = \frac{1}{3}$$

Hence eqn 1 gives

$$w = 5 \times 8.31 \times 326 \ln \left(\frac{1}{3} \right) \quad \begin{cases} \text{where } \\ R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1} \end{cases}$$

$$= -14881 \text{ joules}$$

- (2) A pt is compressed adiabatically to half its volume, calculate the change in its temperature [2022 set C]

Soln

For adiabatic process

$$\eta, V_1^{r-1} = P_2 V_2^{r-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{r-1}$$

$$\text{or } \frac{P_2}{T_1} = \left[\frac{V_1}{V_1 - 2} \right]^{1.4-1} \cdot \left[\therefore V_2 = \frac{V_1}{2}, \text{ given} \right]$$

$$\text{or } \frac{T_2}{T_1} = (2)^{0.4}$$

$$\text{or } \frac{P_2}{T_1} = 1.3195$$

$$\text{or } P_2 = 1.3195 T_1$$

$$\begin{aligned} \therefore \text{change in temp } (T_2 - T_1) &= 1.3195 T_1 - T_1 \\ &= (1.3195 - 1) T_1 \\ &= 0.3195 T_1 \end{aligned}$$

$$\begin{aligned} \therefore \text{Percentage change in temp} &= \left(\frac{T_2 - T_1}{T_1} \right) \times 100\% \\ &= \frac{0.3195 T_1}{T_1} \times 100\% \\ &= 31.95\% \approx 32\% \end{aligned}$$

(9) An monoatomic gas ($P_1 = 1.5 \times 10^5 \text{ Hz}$, $V_1 = 0.08 \text{ m}^3$) is compressed adiabatically to volume $V_2 = 0.04 \text{ m}^3$

(i) $P_2 = ?$ (ii) $\omega = ?$ (iii) $T_2 = ?$ (20 to supp)

Soln

For monoatomic gas $\gamma = 1.6$

$$(i) T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\text{or } \frac{T_2}{T_1} = \left(\frac{0.08}{0.04} \right)^{1.66-1}$$

$$\text{or } \frac{T_2}{T_1} = 2^{0.66}$$

$$\therefore T_2 = 1.58$$

$$(i) P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \quad (\text{For adiabatic process})$$

$$\text{or } P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{\gamma}$$

$$= 1.5 \times 10^5 \left[\frac{0.08}{0.04} \right]^{1.66}$$

$$= 1.5 \times 10^5 \times 2^{1.66}$$

$$= 4.74 \times 10^5 \text{ Nm}^{-2}$$

$$= 4.75 \times 10^5 \text{ Pa.}$$

(ii) For adiabatic process, work done

$$W = \frac{R}{\gamma - 1} (T_1 - T_2)$$

$$= \frac{R}{\gamma - 1} [T_1 - 1.58 T_1]$$

$$= \frac{RT_1 \times 0.58}{\gamma - 1}$$

$$= \frac{-0.58 \times 8.31 T_1}{1.66 - 1}$$

$$= -7.3 T_1$$

(4) If the ratio of specific heat capacities of a gas is 1.4 and its density at STP is 0.9 kgm⁻³. calculate the value of specific heat capacities at constant pressure and at constant volume. (2009 supp)

Given

$$P = 1.01 \times 10^5 \text{ Nm}^{-1} \text{ at STP}$$

$$T = 273 K$$

$$\rho = 0.9 \text{ kgm}^{-3}$$

$$\gamma = 1.4$$

for mass 'm' of ideal gas

$$\text{or, } PV = mRT$$

$$\text{Or, } P = \frac{m}{V} RT$$

$$\text{Or, } P = gRT \quad (\because g = \frac{m}{V})$$

$$\text{Or, } \gamma = \frac{P}{gT}$$

$$\text{Or, } \gamma = \frac{1.01 \times 10^3}{0.9 \times 273}$$

$$\text{Q4} \quad \therefore \gamma = 411.07 \text{ Joule kg}^{-1} \text{ K}^{-1}$$

$$\text{But } CP - CV = \gamma \rightarrow$$

$$\text{Also, } \frac{CP}{CV} = \gamma.$$

$$\text{Or, } CP = \gamma CV$$

$$\therefore CP = 1.4 CV \rightarrow \text{Eqn 1}$$

From Eqn 1 & Eqn 2 gives

$$CP - CV = \gamma$$

$$\text{Or, } 1.4 CV - CV = \gamma$$

$$\text{Or, } 0.4 CV = \gamma.$$

$$\text{Or, } CV = \frac{\gamma}{0.4}$$

$$\text{Or, } CV = \frac{411.07}{0.4}$$

$$\therefore CV = 1027.68 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$\therefore CP = 1.4 \times 1027.68 = 1438.75 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$\text{Thus, } CP = 1438.75 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$CV = 1027.68 \text{ J kg}^{-1} \text{ K}^{-1}$$

5) A 1 litre of air, initially at 20°C and at 760 mm of pressure at constant pressure until its volume is doubled
 Find i) final temp. ii) external work done in expansion
 iii) quantity of heat supplied (cp heat cap at constant volume)
 $714 \text{ J kg}^{-1} \text{ K}^{-1}$ and density of air at STP is 1.293 kg m^{-3} [2067]

\Rightarrow Given

$$T_1 = (273 + 20) \text{ K} = 293 \text{ K}$$

$$V_1 = 1 \text{ litre} = 1 \times 10^{-3} \text{ m}^3$$

$$P_1 = 760 \text{ mm of Hg} = 1.01 \times 10^5 \text{ N m}^{-2}$$

$$P_2 = P_1 = P \text{ (constant)}$$

$$V_2 = 2V_1 = 2 \times 10^{-3} \text{ m}^3$$

$$(i) T_2 = ?$$

$$(ii) dN = ?$$

$$(iii) dQ = ?$$

Soln

$$i) \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad [\because P_1 = P_2]$$

$$\therefore T_2 = \frac{T_1 V_2}{V_1} = \frac{293 \times (2V_1)}{V_1} = 586 \text{ K}$$

$$ii) dN = pdv$$

$$= 1.01 \times 10^5 [V_2 - V_1]$$

$$= 1.01 \times 10^5 [2 \times 10^{-3} - 1 \times 10^{-3}]$$

$$= 101 \text{ Joules.}$$

iii) quantity of heat supplied will be

$$dQ = dw + dN$$

$$= mc_v dT + 101. \quad [\because dw = 101 \text{ Joule from } ii)]$$

$$= mc_v (T_2 - T_1) + 101 \rightarrow$$

$$\frac{P_1 V_1}{T_1} = \frac{P_{STP} V_{STP}}{T_{STP}} \quad [\because P_1 = P_{STP} = 760 \text{ mm of Hg}]$$

$$\therefore V_{STP} = \frac{V T_{SPP}}{T_1}$$

$$= \frac{1 \times 10^{-3} \times 293}{293} m^3$$

As mass remain constant

$$\therefore \rho_{STP} = \text{density of air at STP} = \frac{m}{V_{STP}}$$

$$\therefore m = \rho_{STP} \times V_{STP} = 1.293 \times \left(\frac{273}{293} \times 10^{-3} \right)$$

$$= 1.2047 \times 10^{-3} \text{ kg.}$$

Now, from eqn 8), we get

$$dq = m C_v (T_2 - T_1) + 101.$$

$$= 1.2047 \times 10^{-3} \times 714 \times (586 - 293) + 101.$$

$$= 353 \text{ Joules.}$$

6. Gas in a cylinder initially at 17°C and pressure $1.01 \times 10^5 \text{ Nm}^{-2}$ is to be compressed adiabatically to one eighth of its volume. find the final temp and pressure of gas. [2061 sopp]

Given → For adiabatic process,

$$T_1 = (293 + 17) \text{ K} = 290 \text{ K}$$

$$P_1 = 1.01 \times 10^5 \text{ Nm}^{-2}$$

$$V_2 = \frac{V_1}{8}$$

i) $T_2 = ?$

ii) $P_2 = ?$

$\alpha = r = 1.4$

For adiabatic process

$$i) T_1 V_1^{r-1} = T_2 V_2^{r-1}$$

$$\therefore T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{r-1}$$

$$= 290 \left(\frac{\frac{V_1}{8}}{V_1} \right)^{1.4-1}$$

$$= 290 \times (8)^{0.4}$$

$$= 666.24 \text{ Pa}$$

ii) $P_1 V_1 T = P_2 V_2 T$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^T$$

$$= 1.01 \times 10^5 \left(\frac{V_1}{V_2} \right)^{1.4}$$

$$= 1.01 \times 10^5 \times 8^{1.4}$$

$$= 18.56 \times 10^5 \text{ Nm}^{-2}$$

$$= 1.856 \times 10^6 \text{ Nm}^{-2}$$

7) An ideal gas initially at 4 atm and 300 K is expanded adiabatically to twice its initial volume. Find the final pressure and temp.

8. For monoatomic gas

For diatomic gas with $(\nu = \frac{5}{2}) R$

~~join~~

i) For monoatomic gas, $\gamma = 1.66$

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{\gamma} \quad \rightarrow \quad \text{i}$$

$$\therefore P_2 = 4 \text{ atm} \times \left(\frac{1}{2} \right)^{1.66}$$

$$= 4 \times (0.5)^{1.66} \text{ atm}$$

$$= 1.266 \text{ atm}$$

$$\text{and } P_1 V_1^{\gamma-1} = P_2 V_2^{\gamma-1}$$

$$\Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} \quad \rightarrow \quad \text{ii}$$

$$\Rightarrow T_2 = 300 (0.5)^{1.66-1}$$

$$\Rightarrow T_2 = 300 \times (0.5)^{0.66}$$

$$= T_2 = 189.86 \text{ K}$$

ii) According to question for diatomic gas

$$C_V = \frac{5}{2} R$$

$$C_V = 2.5 R$$

$$\text{But } C_P - C_V = R$$

$$\text{or, } C_P = 2.5 R + R$$

$$\text{or, } C_P = R + 2.5 R$$

$$\therefore C_P = 3.5 R$$

Hence for diatomic gas

$$\gamma = \frac{C_P}{C_V} = \frac{3.5 R}{2.5 R} = 1.4$$

Therefore, from eqn 1) P_2 in this case

$$P_2 = 4 \times (0.5)^{1.4} = 1.52 \text{ atm}$$

and from eqn ii) we get

$$T_2 = 300 (0.5)^{1.4-1}$$

$$\text{or, } T_2 = 300 \times (0.5)^{0.4}$$

$$\therefore T_2 = 227.36 \text{ K}$$

8) The density of an ideal gas is 1.6 kgm^{-3} at 27°C and 10^5 Nm^{-2} pressure. If its specific heat capacity at constant vol is $312 \text{ J kg}^{-1} \text{ K}^{-1}$ find the ratio of ponder (2065)

\Rightarrow here

$$\rho = 1.6 \text{ kgm}^{-3} = \text{density of ideal gas.}$$

$$T = (273+27) \text{ K} = 300 \text{ K}$$

$$P = 10^5 \text{ Nm}^{-2} = \text{pressure.}$$

$$C_V = 312 \text{ J kg}^{-1} = \text{sp. heat capacity at const. vol}$$

$$\gamma = \frac{C_P}{C_V} = ?$$

For m^3 mass of ideal gas

$$PV = mRT$$

$$\therefore P = mRT$$

$$\delta r_1 \gamma^0 = \delta r T \quad [\because \gamma = m/v]$$

$$\gamma = \frac{P}{\delta T}$$

$$\text{on } \gamma = 1.6 \times 10^5$$

$$1.6 \times 800$$

$$\text{on } \gamma = 208.33 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$\text{therefore, } C_p - C_v = \gamma$$

$$C_p = C_v + \gamma$$

$$= 312 + 208.33$$

$$= 520.33 \text{ J kg}^{-1} \text{ K}^{-1}$$

Now,

$$\gamma = \frac{C_p}{C_v}$$

$$= 520.33$$

$$312$$

$$= 1.667$$

9. One mole of an ideal gas is slowly compressed at constant temp of 50°C to one half its original volume. In this process 80 cal of heat was given. calculate work done and the internal energy change (2062)

Given,

$$n = 1 \text{ mole}$$

$$T = (273 + 50) \text{ K} = 323 \text{ K} = \text{const. temp}$$

$$V_2 = \frac{V_1}{2} = 0.5 V_1$$

$$dq = 80 \text{ cal} = 80 \times 4.2 \text{ J} = 336 \text{ Joules.}$$

i) $dW = ?$ work done.

ii) $dU = ?$ change of internal energy.

Soln 9) As temp is constant (isothermal)

∴ work done will be

$$\Delta W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$= 1 \times 8.31 \left[\ln\left(\frac{0.5V_1}{V_1}\right) \right] \times 323$$

$$= 8.31 \ln(0.5) \times 323$$

$$= -1860.5 \text{ Joules.}$$

ii) $dQ = dU + dW$

$$\therefore dU = dQ - dW$$

$$= 323 - (-1860.5)$$

$$= 323 + 1860.5$$

$$= 2196.5 \text{ Joules.}$$

- 10) 16 gm of oxygen having volume 0.02 m^3 at a temp of 27°C and pressure of $2 \times 10^5 \text{ Nm}^{-2}$ is heated at constant pressure until its volume increase to 0.03 m^3 . calculate external work done and internal energy increase of mol. heat cap at constant volume is $0.3 \text{ mol}^{-1} \text{ K}^{-1}$ and mol. mass of oxygen is 32. (2060)

Given

$$\text{Pressure } (P) = 2 \times 10^5 \text{ Nm}^{-2}$$

$$(V_1) = 0.02 \text{ m}^3$$

$$(V_2) = 0.03 \text{ m}^3$$

$$T_1 = (273 + 27) \text{ K} = 300 \text{ K}$$

$$(v = 0.87 \text{ mol}^{-1} \text{ K}^{-1})$$

$$\eta = \frac{16 \text{ gm}}{32 \text{ gm}} = 0.5 \text{ mol}$$

∴ mass of 1 mole of $O_2 = 32 \text{ gm.}$

i) External work done at constant pressure is given by
 $dW = PdV$
 $= P(V_2 - V_1)$
 $= 2 \times 10^5 \times (0.03 - 0.02)$
 $= 2 \times 10^3 \text{ joules.}$

ii) Internal energy change

$dU = nC_V dT$

$= nC_V (T_2 - T_1) \rightarrow$

Now,

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad [\because P_1 = P_2 = P \text{ (const. pressure)}]$

$\therefore T_2 = \frac{T_1V_2}{V_1} = \frac{300 \times 0.03}{0.02} = 450 \text{ K}$

\therefore Eqn 9 gives

$$\begin{aligned} dU &= 0.5 \times 0.8 \times (450 - 300) \\ &= 0.5 \times 0.8 \times 150 \\ &= 60 \text{ joules.} \end{aligned}$$

ii) A gasoline engine takes in air at 25°C and 1 atm pressure and compresses adiabatically to one tenth of its original volume. Find the final temp and pressure. ($\gamma = 1.4$) (2019)

Given

For adiabatic process

$T_1 = (273 + 25) \text{ K} = 298 \text{ K}$

$P_1 = 1 \text{ atm} = 1.01 \times 10^5 \text{ Nm}^{-2}$

$V_2 = \frac{V_1}{10}$

$\gamma = 1.4$

i) $T_2 = ?$

ii) $P_2 = ?$

$$\textcircled{1} \quad P_1 V_1^{r-1} = P_2 V_2^{r-1}$$

$$\therefore T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{r-1}$$

$$= 298 \times \left(\frac{V_1}{\frac{V_1}{10}} \right)^{1.4-1}$$

$$= 298 \times 10^{0.4}$$

$$= 748.5 \text{ K}$$

$$\textcircled{2} \quad P_1 V_1^r = P_2 V_2^r$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^r$$

$$= 1 \text{ atm} \left(\frac{\frac{V_1}{V_1}}{\frac{V_1}{10}} \right)^{1.4}$$

$$= 1 \text{ atm} \times 10^{1.4} = 25.118 \text{ atm.}$$

12. A mass of air occupying initially a volume $2 \times 10^{-3} \text{ m}^3$ at a pressure of 760 mm of Hg and at temp of 20°C is expanded adiabatically and reversibly to twice its volume and then compressed isothermally and reversibly to a volume of $3 \times 10^{-3} \text{ m}^3$. find the final pressure ($r=1.4$) [2018]

\Rightarrow Given,

$$\text{adiabatic change (i)} \quad (P_1, V_1, T_1) \rightarrow (P_2, V_2, T_2) \quad \text{isothermal change (ii)} \quad \rightarrow (P_3, V_3, T_3)$$

$$P_3 = ?$$

where,

$$P_1 = 760 \text{ mm of mercury}$$

$$V_1 = 2 \times 10^{-3} \text{ m}^3$$

$$T_1 = 293 \text{ K}$$

$$V_2 = 2 V_1 = 2 \times (2 \times 10^{-3}) \text{ m}^3$$

$$V_3 = 3 \times 10^{-3} \text{ m}^3 = \text{final volume}$$

$$T_3 = T_2 \text{ (due to isothermal change)}$$

Soln For adiabatic process (T)

$$P_1 V_1^r = P_2 V_2^r$$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^r$$

$$= 760 \left(\frac{V_1}{2V_1} \right)^{1.4}$$

$$= 760 \times 0.5^{1.4}$$

$$= 287.99 \text{ mm of Hg.}$$

Now, for isothermal process (change is)

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \quad [\because T_2 = T_3]$$

$$\frac{P_3}{V_3} = \frac{P_2 V_2}{8 \times 10^{-3}} = \frac{287.99 \times 4 \times 10^{-3}}{8 \times 10^{-3}}$$

$$= 383.99 \text{ mm of mercury.}$$

$$= 384 \text{ mm of mercury}$$

- 13) A certain volume of dry air at NTP is allowed to expand 4 times its original volume adiabatically. Find final pressure & temp. [2076 set B - 10 (c)]

→ Given For air, ($\therefore r = 1.4$)

for adiabatic process

$$P_1 = 1.01 \times 10^5 \text{ Nm}^{-2} \quad [\text{NTP given}]$$

$$T_1 = 273 \text{ K}$$

$$V_2 = 5V_1$$

$$i) P_2 = ?$$

$$ii) T_2 = ?$$

$$iii) P_1 V_1^r = P_2 V_2^r$$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^r$$

$$= 1.01 \times 10^5 \left(\frac{V_1}{5V_1} \right)^{1.4}$$

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(Bipo)

Class 12 complete notes and paper collection.

Folders

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