Chapter 21: Dynamics

Exercise 21.1

1. a. An average sized onion has a mass 50g. Find the weight of the apple in Newton? ($g = 9.8m/s^2$)
   b. A bicycle of mass 20 kg is accelerated at $2m/sec^2$. Find the force acting on it.
   c. Find the acceleration produced when a force of 5hg wt. acts on a mass of 1 kg.

Solution:

a. Here, mass (m) = 50g = $\frac{50}{100}$ kg = 0.05 kg
   $g = 9.8\ m/s^2$
   Weight (w) = ?
   $\therefore$ $w = mg = 0.05 \times 9.8 = 0.49N$ Ans.

b. Here, mass (m) = 20kg
   Acceleration (a) = $2\ m/s^2$
   Force (f) = ?
   $\therefore$ $f = ma = 20 \times 2 = 40N$

c. Here, acceleration (a) = ?
   Force (f) = 5kg = 50N
   Mass (m) = 1kg
   $\therefore$ F = ma
   or, $a = \frac{f}{m} = \frac{50}{1}$
   $\therefore$ $a = 50\ m/s^2$

2. a. A bicycle has mass 50kg.If its velocity increases from 2m/sec to 5m/sec in 6 seconds, find the force exerted on it.
   b. A body of mass 10kg falling from a certain height is brought to rest after striking the ground with a speed of 5m/sec. If the resistance force of the ground is 200N, find the duration of contact.
   c. A car is pushed on a frictional smooth plane with an average force of 50N for 10 sec. If the car with mass 500 kg is at rest in the beginning, find the velocity acquired by the car.

Solution:

a. Mass of bicycle (m) = 50 kg
   Initial velocity (u) = 2 m/s
   Final velocity (v) = 5 m/s
   Time taken (t) = 6 sec
   Force exerted (f) = ?
   $\therefore$ $F = \frac{m(v-u)}{t} = \frac{50(5-2)}{6} = \frac{50 \times 3}{6} = 25N$
   $\therefore$ F = 25N

b. Here, mass of the body (m) = 10kg
   Initial velocity (u) = 5m/s
   Final velocity (v) = 0
   Force on the ground (f) = 200N
   Duration of contact (t) = ?
   Now, applying the formula, $f = \frac{mv - mu}{t}$
   or, $t = \frac{m(v-u)}{f} = \frac{10(0-5)}{200} = -0.25$
   $\therefore$ t = 0.25 sec

c. Here, Average force (f) = 50N
Mass of car (m) = 500kg
Time taken (t) = 10 sec
Initial velocity (u) = 0
Final velocity (v) = ?

Now, we have, $f = \frac{m(v-u)}{t}$

or, $v = \frac{ft}{m} + u$

or, $v = \frac{50 \times 10}{500} + 0$

$\therefore$ v = 1 m/s

3. a. A horse directs a horizontal Jet of water, moving with a velocity of 30 m/sec on a vertical wall. If the mass of water per second striking the wall is 3kg/ sec, find the force on the wall.
   b. Sand allowed to fall vertically at a steady rate hits a horizontal floor with a speed $0.04ms^{-1}$. If the force exerted on the floor is 0.004N, find the mass of sand falling per second.
   c. Rain drops falling vertically on ground at the rate of 0.3 kgs^{-1} come to rest after hitting the ground. If the resistance force of the ground is 3N, find the velocity of rain drops just before hitting the ground.

Solution:

a. Mass of water per second $\left(\frac{m}{t}\right)$ = 3 kg/sec.
Initial velocity (u) = 30 m/s
Final velocity (v) = 0
Force on the wall (f) = ?

Now, apply, $f = \frac{mv - mu}{t}$

or, $f = \frac{m(v-u)}{t} = \frac{m}{t}(v-u) = 3(0-30) = -90$

$\therefore$ f = 90N

b. Here, initial velocity (u) = 0
Fin al velocity (v) = 0.04 m/s
Force exerted on the floor (f) = 0.004N
Mass of sand falling per second (m/t) = ?

Now, applying $f = \frac{mv - mu}{t} = \frac{m}{t}(v-u)$

or, $\frac{m}{t} = \frac{f}{v-u} = \frac{0.004}{(0.04-0)} = 0.1$

$\therefore$ $\frac{m}{t} = 0.1$ kg/s

$\therefore$ Mass of sand falling per second = 0.1kg/sec

c. Quantity of rain falling per second $\left(\frac{m}{t}\right)$ = 0.3 kg/s
Force of the ground (f) = 3N
Velocity before hitting the ground (u) = ?
Velocity after hitting the ground (v) = 0

We know, $f = \frac{mv - mu}{t} = \frac{m}{t}(v-u)$

or, 3 = 0.3 (0 – u)
or, u = –10 m/s
$\therefore$ u = 10 m/s

4. a. A force 1 kg wt. acts on a body continuously for seconds and causes it to describe one metre in that time, find the mass of the body.
b. A body of, mass 25kg is acted upon by a force of 200N. How long will it take to move the body from rest through 64m?
c. A force of 520N acting on a body for 30 secs increases its velocity from 290 m/sec to 350 m/sec. Find the mass of the body.
d. A bullet of mass 20g fired into a wall with a velocity of 30m/sec loses its velocity in penetrating into a wall through 3cms. Find the average force exerted by the wall.

e. How large a force required to bring a motorbike of mass 500 kg moving with a velocity of $50ms^{-1}$ to rest at
i. a distance of 50m   ii. in 10 secs
f. A constant force of 20N acting on an object reduces if velocity from $30ms^{-1}$ to $10ms^{-1}$ in 3 secs. Find the mass of the object.
g. A car of mass 1000 kg travelling at 36 km/hr is brought to rest over a distance of 20m. Find the average braking force.
h. Find the velocity of a 5kg shot that will just penetrate through a wall 20 cms thick the resistance being 40 tons wt.

Solution:

a. Suppose m be the mass of an object and a be the acceleration. So that

$f = ma$ $[\because f = 1\times 9.8 = 9.8N]$

or, $9.8 = ma$

or, $a = \frac{9.8}{m}$

Now, $s = \frac{1}{2}at^2$

or, $l = \frac{1}{2} \times \frac{9.8}{m} \times 10^2$

or, $l = \frac{9.8 \times 50}{m}$

or, $m = 490$ kg

b. Mass of body (m) = 25kg
Force (f) = 200N
Time taken (t) = ?
Initial velocity (u) = 0
Distance covered (s) = 64m
we have, f = ma

or, $a = \frac{f}{m} = \frac{200}{25} = 8$

$\therefore$ $a = 8\ m/s^2$

Now, $s = ut + \frac{1}{2}at^2$

or, $64 = 0 + \frac{1}{2} \times 8 \times t^2$

or, $4t^2 = 64$

or, $t^2 = 16$

$\therefore$ $t = 4$ sec

c. Here, force (f) = 520N
Time (t) = 30 sec
Initial velocity (u) = 290 m/s
Final velocity (v) = 350 m/s
Mass of body (m) = ?

We have, $f = \frac{m(v-u)}{t}$

or, $m = \frac{ft}{v-u} = \frac{520 \times 30}{350 - 290} = \frac{520 \times 30}{60} = 260$

$\therefore$ m = 260 kg

d. Mass of bullet (m) = 20gms = 0.02 kg
Initial velocity (u) = 30 m/s
Distance (s) = 3 cms = 0.03m
Average force (f) = ?
Final velocity (v) = 0
$\therefore$ Applying, $v^2 = u^2 + 2as$
or, $0 = 30^2 + 2a \times 0.03$
or, $-900 = 0.06a$
or, $a = -\frac{900}{0.06} = -15000$
$\therefore$ $a = —15,000$ m/s^2
Now, force, f = ma = 0.02×(–15,000) = –300
$\therefore$ f = 300N

e. Mass of motor bike (m) = 500kg
Initial velocity (u) = 50m/s
Final velocity (v) = 0
Force (f) = ?
(i) A distance of 50m
Since, s = 50m
Now, using, $v^2 = u^2 + 2as$
or, $0^2 = 50^2 + 2a \times 50$
or, $2500 = 100a$
or, a = –25 where a is retardation
$\therefore$ a = 25 m/s^2
$\therefore$ f = mass × retardation
= 500 × 25
= 12500N
(ii) In 10 seconds
Now, v = u + at
or, $0 = 50 + a \times 10$
or, $10a = -50$
a = –5 m/s^2, where a is retardation
$\therefore$ f = mass × retardation = 500 × 5 = 2500 N

f. Force (f) = 20N
Initial velocity (u) = 30 m/s
Final velocity (v) = 10 m/s
Time taken (t) = 3 sec
Mass (m) = ?
$\therefore$ $F = \frac{m(v-u)}{t}$
or, $m = \frac{Ft}{v-u} = \frac{20 \times 3}{(10 - 30)} = \frac{20 \times 3}{-20}$
$\therefore$ m = 3kg

g. Final velocity (v) = 0
Distance (s) = 20m
Average force (f) = ?
We have,
$\therefore$ $v^2 = u^2 + 2as$

or, $0^2 = 10^2 + 2.a \times 20$

or, $-100 = 40a$

or, $a = -\frac{100}{40} = -2.5$ m/s^2 where a is retardation.

Now, f = mass × retardation

= 1000 × 2.5

= 2,500N

h. Mass of shot (m) = 5kg

Penetrating space (S) = 20cms = 0.2m

Resistance (f) = 40 tones

= 40×1000×9.8N

If a is the retardation produced by the wall then, f = –ma

or, $a = -\frac{f}{m} = -\frac{40 \times 1000 \times 9.8}{5} = -78400$ m/s^2

Let u is initial velocity and v be final velocity then u = 2, v = 0

Using the formula,

$v^2 = u^2 + 2as$

or, $0^2 = u^2 + 2(-78400) \times 0.2$

or, $u^2 = 31,360$

or, $u = \sqrt{31360}$

∴ u = 177.08 m/s

5. Find the velocity of 4 kg shot that will just penetrate through a wall 16 cms thick, the resistance being 4 metric tons weight.

Solution:

Here, mass of the shot (m) = 4kg

Penetrated space (S) = 16cms = 0.16m

Resistance (F) = 4 metric tons

= 4×1,000×9.8 = 392000N

If a is retardation produced by the wall then, F = –ma

or, $a = -\frac{F}{m} = -\frac{39200}{4} = -9800$ m/s^2

Let u is the initial velocity and v is final velocity then, u = 2, v = 0

Using the formula,

$v^2 = u^2 + 2as$

or, $0^2 = u^2 + 2(-9800) \times 0.16$

or, $u^2 = 3136$

or, $u = \sqrt{3136} \Rightarrow u = 56$m/s

6. A resultant force of 25N acts on a mass of 0.5 kg starting from rest. Find.

a. the acceleration   b. the final velocity after 20 secs

c. the distance moved (g = 10 m/ sec^2)

Solution:

Here, force acting (f) = 25N

Mass of body (m) = 0.50 kg

Initial velocity (u) = 0

a. The acceleration in ms^{-2}

Now, f = ma

or, $a = \frac{F}{m} = \frac{25}{0.50} = 50$ m/s^2

b. The final velocity after 20 sec.

⇒ Let v be the velocity after 20 sec.

Then, using v = u + at

or, $v = 0 + 50 \times 20$

$\therefore$ v = 1,000 m/s

Distance of penetration of the target

If a is the retardation of the system, then F = ma

$\Rightarrow$ $a = \frac{F}{m} = \frac{72}{0.006} = 12{,}000 \text{ m/s}^2$

If S is the required distance of penetration of target then, $s = \frac{1}{2}at^2 = \frac{1}{2} \times 12{,}000 \times (0.01)^2$

$\therefore$ S = 0.6m

c. The distance moved in 20 sec.

If S is required distance moved in 20 sec.

Then, $S = ut + \frac{1}{2}at^2$

or, $S = 0 + \frac{1}{2} \times 50 \times (20)^2$

or, S = 10,000m

S = 10km

7. A body of mass 1 kg is falling under gravity at the rate of 28m/secs. What is the uniform force that will stop it in (a) 0.1 sec (b) 20 cms

Solution:

Here, mass of the body (m) = 1kg

Initial velocity (u) = 28 m/s

Final velocity (v) = 0

Let P be the uniform force applied in upward direction to stop the body. Then force acting on the body are

a. Weight of the body, w = mg = lg in downward direction

b. The force p in upward direction

$\therefore$ Resultant force, F = P – lg ... ... (i)

Case – I:

When time taken t = 0.1 second.

Now, if this resultant force (i) produces the retardation a.

Then, v = u + at

or, $0 = 28 - a \times 0.1 \Rightarrow a = \frac{28}{0.1} = 280 \text{ m/s}^2$

Then by second law of motion,

We get,

P – mg = ma

or, $P - 1\times10 = 1\times280 \Rightarrow P = 290\text{N} = \frac{290}{10} \text{ kg} = 29\text{kg}$

Case – II

When distance is S = 20cm = 0.2m

Now, if the resultant force (i) produce the retardation a,

Then, $v^2 = u^2 + 2aS$

or, $0^2 = (28)^2 + 2\times a\times0.2 \Rightarrow a = \frac{784}{0.4} = 1960\text{m/s}^2$

Then, by second law of motion

P – mg = ma

or, P = 1(10 + 1960)

$P = 1970\text{N} = \frac{1970}{10} \text{ kg} = 197\text{kg}$

8. a. A body of mass 20kg falls 10m form rest and is then brought to rest penetrating 0.5 m into sand. Find the resistance of the sand on it in kg wt.

b. A mass of 4kg falls 200cms from rest and is then brought to rest by penetrating 20cms into some sand. Find the average thrust of the sand on it.

Solution:

a. Mass of body (m) = 20kg

Distance covered (s) = 10m

Initial velocity (u) = 0

$\therefore$ $v^2 = u^2 + 2gh \Rightarrow v^2 = 20g$ ... ... (i)

The velocity given by (i) is reduced to zero when the body goes to 0.5m into sand. If a is the retardation of the system then,

$(0)^2 = v^2 - 2\times a\times 0.5 \Rightarrow a = v^2$

$\Rightarrow a = 20g \text{ m/s}^2$

Let T be the average thrust of the sand on the body. Now, when the body is penetrating into the sand, then the force acting on the body are

a. A force TN of the sand acting upward

b. The weight 20gN of the body acting downward.

Resultant upward force = (T – 20g)N

Then applying Newton's second law of motion, we have,

$T - mg = ma$

or, $T - 20g = 20\times log$

or, $T = 20g + 200g$

$T = 220$ kgwt

b. Suppose V is the velocity of the body when it falls 200cms from rest under gravity.

Then u = 0, v = v, h = 2m

$\therefore$ $v^2 = u^2 + 2gh$

$v^2 = 0 + 2g \times 2 \Rightarrow v^2 = 4g$ ... ... (i)

The velocity given by (i) is reduced to zero when the body goes to 20cms = 0.2m into sand. If a is the retardation of the system, then

$O^2 = v^2 - 2\times a\times 0.2$

or, $a = \dfrac{v^2}{0.4} = \dfrac{4g}{0.4} = 10g \text{ m/s}^2$

Let T be the average thrust of the sand on the body.

Now when the body is penetrating into the sand, then the force acting on the body are

a. A force TN of the sand acting upward

b. The weight 4gN of the body acting downward

$\therefore$ Resultant upward thrust = (T – 4g)N

Then apply Newton's second law of motion,

$T - mg = ma$

or, $T = 4g = 4\times 10g$

or, $T = 40g \Rightarrow T = 40$kg wt

9. A man of mass 70 kg stands on a lift which moves with a uniform acceleration of 2m /sec^2. Find the reaction of the floor when the lift moves (a) up (b) down

Solution:

Here, two forces act on a mass one, the weight mg of the man acting vertically downwards and other, the reaction R of the floor on a man acting vertically upward.

a. Since, the lift is moving upward, so the resultant upward force = R – mg, by Newton's second law.

$R - mg = ma$ (where a is acceleration of lift)

or, $R = m(g + a) = 70(9.8 + 2) = 826$N

b. Since the lift is moving downwards, so the resultant downward force = mg – R

By Newton's second law,

$mg - R = ma$

or, $R = mg - ma$

$= m(g - a)$

$= 70(9.8 - 2)$

$= 546$N

10. A bullet of mass 0.006 kg travelling at 120 m/sec penetrates deeply into a fixed target & is brought to rest in 0.01 secs. Calculate.
    The average retarding force exerted on the bullet. ($g = 10ms^{-2}$)

Solution:

Mass of the bullet (m) = 0.006kg
Final velocity of the bullet (v) = 120m/s
Time taken (t) = 0.01 sec.
Initial velocity (u) = 0
If F be the average retarding force on bullet then,

$$F = \frac{\text{Change in momentum}}{\text{Time taken}} = \frac{m(v-u)}{t} = \frac{0.006 \times (120 - 0)}{0.01} = 72N$$

11. A resultant force of 12N acts for 5 sec on a mass of 2kg. What is the change in momentum of the mass? What would be the change in momentum of a mass of 10kg under the same condition?

Solution:

Given, Force act (F) = 12N
Time taken (t) = 5 sec
Change in momentum = ?

Case–I

When mass m is 2kg
Let a is acceleration of the system then, F = ma
or, $12 = 2\times a \Rightarrow a = 6\ m/s^2$
Let v be the velocity then v = u + at
or, $v = 0 + 6 \times 6$
$\Rightarrow$ v = 30 m/s
Change in momentum = mv – mu
$= 2\times30 - 2\times0 = 60\ kgms^{-1}$

Case–II

When mass is m = 10kg
Under the same condition, v = 30 m/s, u = 0 m/s
Change in momentum = 10 (30 – 0) = 300 kgm/s

12. Solution:

a. Mass of bullet (M) = 0.02kg
Mass of rifle (m) = 10kg
Muzzle velocity of bullet (v) = 1,000 m/s
Recoil velocity of rifle (v) = ?
We know,
Mass of the bullet × Muzzle velocity = Mass of rifle × Recoil velocity
or, $0.02 \times 1{,}000 = 10\times v$
or, $v = \frac{0.2 \times 1{,}000}{10} = 20$ m/s

b. Here, momentum of the bullet = mv = 330×0.1
Momentum of the gun = Mv = $10\times v$
$\therefore$ Momentum of bullet = Momentum of gun
or, $330 \times 0.1 = 10\times v$
or, $v = \frac{330\times0.1}{10} = 3.3$ m/s
Since initial velocity of gun and bullet = 0 m/s
Total momentum before firing = mu + Mu
$= 0.1\times0 + 10\times0 = 0$

c. Here, m = Mass of shot = 700kg
v = Velocity of shot = 600 m/s

M = Mass of the gun = 40 metric tons = (40×1000)kg
v = Velocity of the gun = ?
Momentum of the shut = mv = 700 × 600
Momentum of the gun = Mv = (40 × 1000) v
By the principal of conservation of linear momentum.
Momentum of shot = momentum of gun (in magnitude)
or, $700 \times 600 = (40 \times 1000)v$
or, $v = \dfrac{700 \times 600}{40 \times 1000} = 10.5$ m/s
∴ Velocity of gun = 109.5 m/s

d. Let v be the recoil velocity of the gun
Then moment of the shut = 10×245
Momentum of the gun = 5000 × v
But we know momentum of the shot = Momentum of the gun
or, $10 \times 245 = 5000 \times v$
or, v = 0.49 m/s
The gun recoils with velocity 0.49 m/s. Apply a constant force to the gun so that it will stop after recoiling at time $1\frac{1}{4} = \frac{5}{4}$ seconds.
Let a be the retardation then $0 = v - at \Rightarrow a = \dfrac{v}{t}$
If f is required constant force to be applied then, $f = ma = m \times \dfrac{v}{t} = \dfrac{5000 \times 0.49}{5/4}$
$$= \frac{5000 \times 0.49 \times 4}{5} = 1960\text{N}$$

e. Let v be the velocity of the gun then momentum of the shot = 400 × 400
Momentum of the gun = 80,000 × v
∴ Momentum of gun = Momentum of shot
or, $80,000 \times v = 400 \times 400$
or, v = 2 m/s
The gun recoils with velocity 2 m/s. Applying a constant force to the gun so that it will stop after recoiling at distance 2 meters.
⇒ Let a be the retardation, then $0^2 = v^2 - 2as \Rightarrow a = \dfrac{v^2}{25}$
If f is the required constant force to be applied then, $f = ma = m \times \dfrac{v^2}{25}$
$$= 80,000 \times \frac{2^2}{2\times2} = 80,000\text{N}$$

f. Let v be the recoil velocity at the gun then, momentum of the shot = 40 × 140
Momentum of the gun = 7,000 × v
But, momentum of gun = Momentum of shot 7,000 × v = 40 × 140
or, v = 0.8 m/s
The gun recoil with velocity 0.8 m/s. Applying a constant force to the gun so that it wll stop after recoiling at distance 6.4m.
Let a be the retardation, then, $0^2 = v^2 - 2as \Rightarrow a = \dfrac{v^2}{25}$
If f be the required constant force to be applied then, f = ma
$$= 7,000 \times \frac{(0.8)^2}{2 \times 6.4}$$
$$f = 350\text{N}$$

g. Let v be the recoil velocity at gun then momentum of bullet = 2 × 250
Momentum of gun = 100 × v
But, momentum of gun = Momentum of bullet

$100 \times v = 2 \times 250$

$v = 5$ m/s

13. A cricket ball of mass 150g is moving with a velocity of 12m/sec & it hit by a bat so that the ball is turned back with a velocity of 20m/sec. The force of blow acts for 0.01 secs on the ball. Find the average force exerted by the bat on the ball.

Solution:

Here, mass of ball (m) = 150g = 0.15kg

Initial velocity (u) = 12 m/s

Final velocity (v) = 20 m/s

Time duration (t) = 0.01 sec

Let R be the average force exerted on ball by bat then, impulse of force = Change in momentum

$R \times t = m(12 - x - 20)$

$R = \dfrac{0.15 \times 32}{0.01} = \dfrac{4.8}{0.01} = 480N$

14. a. A body of mass 2kg moving with a uniform velocity of 40m/sec collides with another at rest. If the two together begin to move with a uniform velocity of 25m/sec. Find the mass of the other.

b. Two bodies of masses 10 kg and 5 kg move along the opposite directions with velocity $15ms^{-1}$ and $-6ms^{-1}$ respectively collide and stick together. Find their common velocity.

c. A 20g bullet enters a block of wood of mass 980g with a velocity of $300ms^{-1}$. Find the common velocity the bullet and the wood.

Solution:

a. Mass of first body (m_1) = 2kg

Initial velocity (u_1) = 40 m/s

Mass of second body (m_2) = ?

Initial velocity second body (u_2) = 0 m/s

Le their common velocity of collision be v, then v = 25m/s

Then total momentum before collision = $m_1u_1 + m_2u_2 = 2\times40 + m_2\times0 = 80$

On collision,

$m_1u_1 + m_2u_2 = (m_1 + m_2)\, v$

or, $80 = (2 + m_2)\, 25$

or, $2 + m_2 = \dfrac{80}{25} = m_2 = 1.2$ kg

b. Let v be the velocity just after collision of two bodies whose combined mass = (10 + 5) kg = 15kg

$\therefore$ Its momentum = (10 + 5)v = 15v

Also, total momentum before collision = $m_1u_1 + m_2u_2$

$= (10\times15 + 5\times- 6) = 150 - 30 = 120$

Now, by the principal of conservation of linear momentum, the total momentum of the system remains constant. Hence momentum after collision = momentum before collision

or, $15v = 120$

or, $v = 8$ m/s

c. Mass of bullet (m_1) = 20g = 0.02 kg

Mass of block (m_2) = 980g = 0.98 kg

initial velocity of bullet (u_1) = 300 m/s

Initial velocity of block (u_2) = 0 m/s

Let v be the common velocity of bullet and block, v = ?

Now, by the principal of conservation of linear momentum

We have, $(m_1u_1 + m_2u_2) = (m_1 + m_2)\, v$

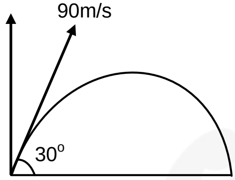
or, $(0.02 \times 300 + 0.98 \times 0) = (0.02 + 0.98)\, v$

or, $(6 + 0) = 1 \times v$

$\Rightarrow$ v = 6 m/s

Exercise 21.2

1. A ball is thrown with a velocity of 98 m/sec at an elevation of 30°, find
   a. the horizontal range,
   b. time of light
   c. magnitude and direction of the velocity after 2 seconds.
   d. position after 2 seconds.



Solution:

Initial velocity (u) = 98m/s

Angle of elevation (θ) = 30^o

a. Horizontal range (R) = $\dfrac{u^2 \sin^2 \theta}{g}$

$= \dfrac{(98)^2 . \sin^2 .30}{10} = \dfrac{9604 \times 50.866}{10} = 831.7\text{m}$

b. Time of flight (T) = $\dfrac{2u \sin\theta}{g} = \dfrac{2 \times 98 \times \sin 30^o}{10} = 9.8$ sec

c. Let v be the striking velocity of the ball making an angle θ with horizontal.

$\therefore$ v_x = horizontal component = $v\cos\theta = 98.\cos 30^o = \dfrac{98\sqrt{3}}{2} = 49\sqrt{3}$ m

v_y = vertical component = $u \sin\alpha - gt = 98 \sin 30^o - 10\times 2$

$= \dfrac{98}{2} - 20 = 49 - 20 = 29\text{m}$

$\therefore$ Now, $v^2 = v_x^2 + v_y^2 = (49\sqrt{3})^2 - (29)^2$

$= 7202.58 - 841 = 6361.58$

$\therefore$ v = 79.76 m/s

Direction, $\text{Tan}\theta = \dfrac{v_y}{v_x} = \dfrac{29}{49\sqrt{3}} = 0.342$

$\theta = \text{Tan}^{-1}(0.342)$

$\theta = 20^o$

d. If (x, y) be position of projectile after time t = 2 sec

$\therefore$ $x = u\cos\alpha\, t = 98 \times \cos 30^o \times 2$

$= 98 \times \dfrac{\sqrt{3}}{2} \times 2 = 98\sqrt{3}$ m

$y = u\sin\alpha\, t - \dfrac{1}{2} gt^2$

$= 98 \times \sin 30^o \times 2 - \dfrac{1}{2} \times 10 \times 2^2$

$= 98 \times \dfrac{1}{2} \times 2 - \dfrac{40}{2} = 98 - 20 = 78\text{m}$

$\therefore$ Position (x, y) = $(98\sqrt{3}, 78)$

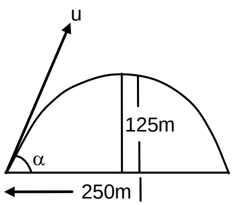
2. a. Find the velocity and the direction of projection of a shot which passes in a horizontal direction just over the top of a wall which is 250 m off and 125m high (g = 9.8ms^{-2}]
   b. A shot is seen to pass horizontally just over a vertical wall 64m high and 96m off. Find the magnitude and direction of the velocity of the shot with which it was fired.

Solution:

a. Let u be the velocity of the projection of a shot making an angle α with the horizon. Since the shot just passes the top of the building, it moves horizontally.

$\therefore$ Max. height (H) = 125m

Horizontal range (R) = 2 × 250m = 500m



$\therefore \quad H = \dfrac{u^2 \sin^2\alpha}{2g} \Rightarrow 125 = \dfrac{u^2 \sin^2\alpha}{2g}$ ... ... (i)

and, $R = \dfrac{u^2 \sin^2\alpha}{g} \Rightarrow 500 = \dfrac{u^2 \sin^2\alpha}{g}$ ... ... (ii)

Dividing (i) by (ii)

$\dfrac{1}{4} = \dfrac{\sin^2\alpha}{2\sin\alpha \cdot \cos\alpha} \Rightarrow \text{Tan}\alpha = 1 = \text{Tan}45^o \Rightarrow \alpha = 45^o$

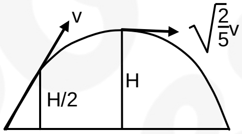
Velocity of projection:

Substituting the value of α in (i) $\Rightarrow 125 = \dfrac{u^2}{2g} \times \left(\dfrac{1}{\sqrt{2}}\right)^2$

$\Rightarrow u^2 = 500 \times 9.8 = u = 70\text{m/s}$

b. The velocity of a particle when at its greatest height is $\sqrt{\dfrac{2}{5}}$ of its velocity when at half its greatest height. Show that the angle of projection is 60^o.

Let u be the velocity and α, angle of projection off a particle. Let H be the greatest height. If v be the velocity at $\dfrac{H}{2}$, then the velocity at H is $\sqrt{\dfrac{2}{5}}$ v.



$\therefore \quad \sqrt{\dfrac{2}{5}}\, v = u \cos\alpha$

or, $v^2 = \dfrac{5}{2} u^2 \cos^2\alpha$ ... ... (i)

Also, $H = \dfrac{u^2 \sin^2 \alpha}{2g}$ ... ... (ii)

And, $v^2 = u^2 - 2g\dfrac{H}{2}$

or, $\dfrac{5}{2} u^2 \cos^2\alpha = u^2 - g\dfrac{u^2 \sin^2 \alpha}{2g}$ (From (i) and (ii)

or, $\dfrac{5}{2}\cos^2\alpha = 1 - \dfrac{\sin^2\alpha}{2}$

or, $5\cos^2\alpha = 2 - \sin^2\alpha$

or, $5 - 5\sin^2\alpha = 2 - \sin^2\alpha$

or, $-4\sin^2\alpha = -3$

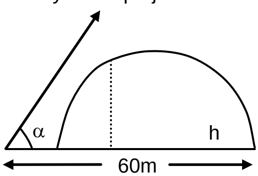
$\sin^2\alpha = \dfrac{3}{4}$

$\sin\alpha = \dfrac{\sqrt{3}}{2}$

$\alpha = 60^o$

3. A projectile thrown from a point in a horizontal plane come back to the plane in 4 secs at a distance of 58.8m from the point of projection, find the velocity of the projectile.

Solution:



Here, time of projection (t) = 4 sec

Horizontal range (R) = 58.8m

Velocity of projection (u) = ?

Let α be the angle of projection, then

We know,

$t = \dfrac{2\, u\sin\alpha}{g}$ and $R = \dfrac{u^2 \sin^2\alpha}{g}$

or, $4 = \dfrac{2u \sin\alpha}{10}$ $60R = \dfrac{v^2\, 2\sin\alpha \cos\alpha}{g}$

or, $u \sin\alpha = 20$ ... ... (i) $u^2\sin\alpha \cdot \cos\alpha = 300$ ... ... (ii)

from (i) and (ii)

$\left(\frac{20}{\sin\alpha}\right)^2 \sin\alpha.\cos\alpha = 300 \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{3}{4}$

or, $\tan\alpha = \frac{4}{3}$, $\therefore \sin\alpha = \frac{4}{5}$

Substituting the value of $\sin\alpha$ in (i)

$u\sin\alpha = 20$

$u . \frac{4}{5} = 20 \Rightarrow u = 25$ m/s

4. Find the angle of projection when the range on a horizontal plane is 4 times the greatest height attained.

Solution:

Angle of projection (α) = ?

Given, Horizontal range = 4 maximum height

or, $\frac{u^2 \sin^2 \alpha}{g} = 4 . \frac{u^2 \sin^2\alpha}{2g}$

or, $2\sin\alpha.\cos\alpha = \frac{4\sin^2\alpha}{2}$

or, $1 = \frac{\sin\alpha}{\cos\alpha} \Rightarrow \text{Tan}\alpha = \text{Tan}45^o$

$\therefore \alpha = 45^o$

5. The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Find the angle of projection.

Solution:

Angle of projection (α) = ?

Given, Horizontal range = $4\sqrt{3}$ max. height

$\frac{u^2 \sin^2\alpha}{g} = 4\sqrt{3}\, \frac{u^2\, \alpha\sin^2\alpha}{2g}$

or, $2\sin\alpha.\cos\alpha = 2\sqrt{3}\sin^2\alpha$

or, $\tan\alpha = \frac{1}{\sqrt{3}} = \tan30^o$

$\therefore \alpha = 30^o$

6. From the top of a tower 144m high, a particle is projected horizontally with a velocity of 60m/sec. Find its velocity when it reaches the ground.

Solution:

Here, for horizontal projectile, just before hitting the ground,

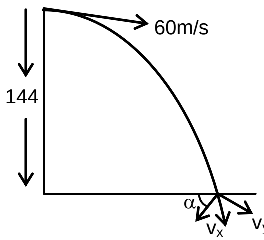
$h_{max} = 144$m

$u = 60$m/s, $v = ?$, $g = 10$m/s^2

Let T = time of flight

v = Velocity with which it hits the ground

α = angle made by $\vec{v}$ with positive x–axis



Now,

$u_x = u = 60$m/s, $u_y = 0$

$v_x = ux = 60$m/s, $v_y = uy + gT = 0 + gT$

We have, $T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 144}{10}}$

T = 5.37 sec

Again, we have, $v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{u^2 + (gT)^2}$

$= \sqrt{(60)^2 + (10 \times 5.37)^2} = \sqrt{3600 + 53.7} = 60.45$ m/s

7. A stone is projected from the top of a tower 72.5m high at an angle of 45° which strikes the ground at a distance of 50m from the foot of the tower. Find the velocity of projection.

Solution:

For horizontal projectile, just before hitting the ground hmax = 72.5m, R = 50m, v = ?, g = $10 m/s^2$, $\alpha = 45^o$.

Let u be the velocity with which body be projected t be the we taken by the body to reach the ground. Now, taking upward direction as positive.

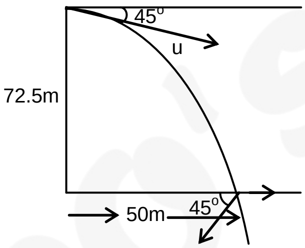
We have,

$-h = u\sin\alpha \,.\, t - \frac{1}{2} \times 10 \times t^2$

or, $-72.5 = \frac{ut}{2} - 5t^2$ ... ... (i)

The particle hits at a distance of 50m from the base of the tower, so that

$s = u \cos\alpha . t \Rightarrow \frac{ut}{\sqrt{2}} = 50$ ... ... (ii)



from (i) and (ii)

$-72.5 = \frac{50\sqrt{2}}{2} = 5t^2$

$-72.5 = 35.46 - 5t^2$

$5t^2 = 107.96$

$t^2 = 21.59$

$t = 4.65$

Again, from (ii)

$\frac{u \times 4.65}{\sqrt{2}} = 50$ $\qquad$ $u = \frac{50\sqrt{2}}{4.65} = 15.21 m/s$

Hence, required projected velocity = 15.21 m/s

8. A ball is projected from a point with a velocity 64m/sec from the top of a tower 128m high in direction making an angle 30° with the horizon. Find when and at what distance from the foot of the tower it will strike the ground.

Here, initial velocity (u) = 64 m/s

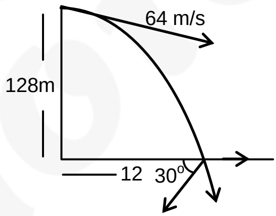
Angle of projection (θ) = 30^o

Height fallen (H) = 128m

Time of flight (T) = ?

Horizontal range (R) = ?

We have,



$H = \frac{1}{2} gT^2 \Rightarrow T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 128}{10}} = 5.06$ sec

Horizontal range (R) = uT

$= 64 \times 5.06 = 323.8m$

9. A canon ball has the same range R on a horizontal plane for two different angles of projection. If H and H' are the greatest heights and t_1 and t_2 are the time of flights in two paths for which this is possible, prove that

 a. $R^2 = 16 HH'$ $\qquad$ b. $R = \frac{1}{2}$ gtt'

Solution:

a. Let α and α_1 be two different angle of projections having the same range R.

$R = \frac{u^2 \sin^2\alpha}{g} = \frac{u^2 \sin^2\alpha_1}{g}$

or, $\frac{\sin^2\alpha}{2\alpha} = \sin 2\alpha_1$

or, $2\alpha = 180 - 2\alpha_1 \Rightarrow \alpha = 90 - \alpha_1$

So, that, $H = \frac{u^2 \sin\alpha^2}{2g}$ and, $H^1 = \frac{u^2 \sin^2 (90 - \alpha)}{2g} = \frac{u^2 \cos^2\alpha}{2g}$

$t_1 = \frac{2u\sin\alpha}{g}$ and $t_2 = \frac{2u\sin(90-\alpha)}{g} = \frac{2u\cos\alpha}{g}$

Now, $R^2 = \left(\frac{u^2\sin^2\alpha}{g}\right) = \frac{u^2 4\sin^2\alpha.\cos^2\alpha}{g^2}$

$= 4\frac{u^2\sin^2\alpha}{g}.\frac{u^2\cos^2\alpha}{g} = 4.4\frac{u^2\sin^2\alpha}{2g}.\frac{u^2\cos^2\alpha}{2g}$

= 16HH, Hence proved.

b. Again, $R = \frac{u^2\sin^2\alpha}{g} = \frac{u^2 . 2\sin\alpha.\cos\alpha}{g}$

$= \frac{1}{2}g\frac{2u^2\sin\alpha}{g}\times\frac{2u\cos\alpha}{g} = \frac{1}{2}gtt^1$ Hence proved.

10. If 't' be the time in which a projectile reaches a point P of its path and t be the time from P till it strikes the horizontal plane through the point of projection, show that the height of P above the plane is $\frac{1}{2}$ gt t'.

Solution: ***Please see the answer of 9(b)***

11. A particle is projected with a velocity u. If the greatest height attained by the particle be H, prove that the range R on the horizontal plane through the point of projection is

$R = 4\sqrt{H\left(\frac{u^2}{2g} - H\right)}$

Solution:

If α is the angle of projection, then

H = greatest height = $\frac{u^2\sin\alpha}{2g}$

and, R = horizontal range = $\frac{u^2\sin^2\alpha}{g}$

Then, $\frac{u^2}{2g} - H = \frac{u^2}{2g} - \frac{u^2\sin^2\alpha}{2g} = \frac{u^2(1-\sin^2\alpha)}{2g} = \frac{u^2\cos^2\alpha}{2g}$

Now, $4\sqrt{H\left(\frac{u^2}{2g} - H\right)} = 4\sqrt{\frac{u^2\sin^2\alpha}{2g}.\frac{u^2\cos^2\alpha}{2g}}$

$= \frac{4 . u^2\sin\alpha.\cos\alpha}{2g} = \frac{u^2\sin^2\alpha}{g} = R$

$\therefore\quad R = 4\sqrt{H\left(\frac{u^2}{2g} - H\right)}$

12. If R be the horizontal range and T, the time of flight of a projection, show that $\tan\alpha = \frac{gT^2}{2R}$, where α is the angle of projection.

Solution:

Let u be the velocity of the projection, then

We know,

R = horizontal range = $\frac{u^2\sin^2\alpha}{g}$ ... ... (i)

T = time of flight = $\frac{2u\sin\alpha}{g}$ ... ... (ii)

from (i) and (ii)

$\frac{gT^2}{2R} = g\left(\frac{2u\sin\alpha}{g}\right)^2 = \frac{g.4u^2\sin^2\alpha}{g} = \frac{4u^2\sin^2\alpha}{4u^2\sin\alpha.\cos\alpha} = \tan\alpha$

$\therefore\quad \text{Tan}\alpha = \frac{gT^2}{2R}$ Hence proved.

13. A ball is projected with a velocity of 49m/s, find the two directions along which the ball must be projected so as to have arrange of 122.5m.

Solution: Here, u = 49 m/s, R = 122.5m

Angle of projection (α) = ?

We know that, $R = \frac{u^2 \sin^2\alpha}{g}$

or, $\sin^2\alpha = \frac{Rg}{u^2} = \frac{122.5 \times 9.8}{49 \times 49}$

or, $\sin^2\alpha = \frac{1}{2}$

or, $\sin^2\alpha = \sin30^o$ or $\sin150^o$

$\therefore$ $\alpha = 15^o$ or, 75^o

14. A body is thrown from the top of a tower 96m high with a velocity 80m/sec at an angle of 30° above the horizon. Find the horizontal distance from the foot of the tower to the point where it hits the ground.

Solution:

Here,

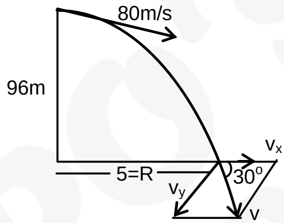
Initial velocity (u) = 80 m/s

Height fallen (H) = 96m

Time of flight (T) = ?

Horizontal range (R) = ?

Angle of projection (Q) = 30^o



We have, $H = \frac{1}{2} gT^2$

$T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 96}{10}}$

T = 4.38 sec

Horizontal range (R) = u.T

$= 80 \times 4.38 = 350.54$m

15. A ball thrown by a player from a height of 2m at an angle of 30° with the horizon with a velocity of 18m/sec is caught by another player at a height of 0.4m from the ground. How far apart were the two players?

Solution:

Distance between the players (x) = ?

Initial velocity (u) = 18m/s

Angle (θ) = 30^o

Vertical distance to the traveler (y) = 2 – 0.4 = 1.6m

We have, $y = \frac{1gT^2}{2}$

$t^2 = \frac{2y}{g} = \frac{2\times1.6}{10}$, t = 0.57 sec

$\therefore$ x = u.t

$= 18 \times 0.57 = 10.18$m