

## Chapter 14

### Probability

#### Exercise 14.1

4.  $n$  = Total no. of cards = 52
  - a. No. of club = 13  
No. of diamond = 13  
 $m$  = No. of possible cases =  $13 + 13 = 26$   
 $P(\text{Either a club or diamond}) = \frac{m}{n} = \frac{26}{52} = \frac{1}{2}$
  - b. There are four kings  
 $\therefore$  No. of possible cases =  $52 - 4 = 48$   
 $\therefore P(\text{Not of king}) = \frac{48}{52} = \frac{12}{13}$
  - c. There are 12 face cards and 13 club cards.  
 $\therefore m$  = no. of cases =  $12 + 13 - 3 = 22$   
 $\therefore P(\text{Either a face or a club}) = \frac{m}{n} = \frac{22}{52} = \frac{11}{26}$
5. From 20 tickets marked from 1 to 20, one is drawn at random. Find the probability that
  - a. It is an odd number
  - b. A multiple of 4 or 5

#### Solution:

- a.  $P(\text{Odd number}) = ?$   
Among 20 tickets, there are 10 tickets marked with odd number.  
 $\therefore P(\text{Odd number}) = \frac{m}{n} = \frac{10}{20} = \frac{1}{2}$
- b.  $P(\text{A multiple of 4 or 5}) = ?$   
There are 5 tickets marked with multiple of 4 and 4 tickets marked with multiple of 5.  
 $= P(\text{Multiple of 4}) + P(\text{multiple of 5}) - P(\text{Multiple of 4}) \times P(\text{Multiple of 5})$   
 $= \frac{5}{20} + \frac{4}{20} - \frac{5}{20} \times \frac{4}{20} = \frac{2}{5}$
6. Given that,
 
$$P(A) = \text{Probability that A solves the problem} = \frac{1}{2}$$

$$P(B) = \text{Probability that B solves the problem} = \frac{1}{3}$$

$$P(C) = \text{Probability that C solves the problem} = \frac{1}{4}$$

$$P(D) = \text{Probability that D solves the problem} = \frac{1}{5}$$

$$P(\overline{A}) = \text{Probability that A not solve the problem} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(\overline{B}) = \text{Probability that B not solve the problem} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(\overline{C}) = \text{Probability that C not solve the problem} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(\overline{D}) = \text{Probability that D not solve the problem} = 1 - \frac{1}{5} = \frac{4}{5}$$

- b. Probability that A, B, C, D not solve the problem  $= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{1}{5}$
- a. Probability that A, B, C, D solve the problem  $= 1 - \frac{1}{5} = \frac{4}{5}$
7. Suppose 4 cards are drawn from a well-shuffled deck of 52 cards.
- What is the probability that all 4 are spade?
  - What is the probability that all 4 are black?

**Solution:**

- a. There are 13 spades

Now,  $n$  = Total no. of possible cases

= No. of selection of 4 cards out of 52

$$= {}^{52}C_4$$

= No. of favourable cases

= No. of selection of 4 spades out of 13 =  ${}^{13}C_4$

$$P(4 \text{ are spades}) = ?$$

$$\text{Now, } P(4 \text{ are spades}) = \frac{m}{n} = \frac{{}^{13}C_4}{{}^{52}C_4} = \frac{13!}{9!4!} \times \frac{48! \times 41}{52!} = \frac{11}{4165}$$

- b. There are 26 black. So, we have to choose 4 black among 26 blacks.

Now,  $n$  = Total no. of possible cases

= No. of selection of 4 cards out of 52 =  ${}^{52}C_4$

$m$  = No. of favourable cases

= No. of selection of 4 black out of 26 black

$$= {}^{26}C_4$$

$$P(4 \text{ are black}) = ?$$

$$\text{Now, } P(4 \text{ are black}) = \frac{m}{n} = \frac{{}^{26}C_4}{{}^{52}C_4} = \frac{46}{833}$$

**8. Solution:**

- a. Two cards can be drawn from a pack of 52 playing cards in  ${}^{52}C_2$  ways

$$\text{i.e. } \frac{52 \times 51}{2} = 1326 \text{ ways}$$

The event that two kings appear in a single draw can appear in  ${}^4C_2$  ways

$\therefore$  The probability that the two cards drawn from a pack of 52 cards are kings

$$= \frac{6}{1326} = \frac{1}{221}$$

- b. One king and one queen can be selected as  $\frac{4}{52} \times \frac{4}{51}$  ways.

One queen and one king can be selected as  $= \frac{4}{52} \times \frac{4}{51}$  ways

$$\text{Total no. of ways} = \frac{4}{52} \times \frac{4}{51} + \frac{4}{52} \times \frac{4}{51} = \frac{8}{663}$$

9. Here,

Total no. of candidates = 9

Total no. of men = 6

Total no. of women = 3

Total no. of vacancy = 2

∴ Out of 2, one man and one woman can be selected in the following ways.

$$m = {}^{bc}_1 \times {}^{9c}_1$$

∴ Total no. of vacancy can be chosen from total no. of candidates as

$$n = {}^{9c}_2$$

$$\therefore P(1 \text{ man and } 1 \text{ woman}) = \frac{{}^{6c}_1 \times {}^{9c}_1}{{}^{9c}_2} = \frac{18}{36} = 0.5$$

10. Since the bag consists of 7 white and 9 black balls.

$$\therefore \text{Total balls} = 7 + 9 = 16$$

Total number of possible cases means the number of selection of 2 balls out of 16.

Since, the selection of 1 white and 1 black. So, the number of favourable cases is the selection of balls with 1 white and 1 black

$$\therefore m = \text{No. of favourable cases}$$

= No. of selection of 1 white out of 7 and 1 black out of 9

$$= {}^{7c}_1 \times {}^{9c}_1$$

$$n = \text{Total no. of possible cases}$$

= No. of selection of 2 balls out of 16.

$$= {}^{16c}_2$$

$$\therefore P(1 \text{ white and } 1 \text{ black}) = \frac{m}{n} = \frac{{}^{7c}_1 \times {}^{9c}_1}{{}^{16c}_2} = \frac{63}{120}$$

11. There are  $6 + 8 = 14$  balls (Total)

a.  $P(\text{both white}) = ?$

$$P(\text{First white}) = \frac{6}{14} \text{ and } P(\text{second white}) = \frac{5}{13}$$

$$P(\text{Both white}) = \frac{6}{14} \times \frac{5}{13} = \frac{15}{91}$$

b.  $P(\text{Both red}) = ?$

$$P(\text{First red}) = \frac{8}{14}, P(\text{Second red}) = \frac{7}{13}$$

$$\therefore P(\text{Both red}) = \frac{8}{14} \times \frac{7}{13} = \frac{4}{13}$$

c. Since balls are drawn one after another without replacement.

$P(\text{One red and one white}) = ?$

$$\therefore P(\text{First red}) = \frac{8}{14}, P(\text{Second white}) = \frac{6}{13}$$

$$P(\text{First white}) = \frac{6}{14}, P(\text{Second red}) = \frac{8}{13}$$

$$\therefore P(\text{One red and one white}) = \frac{6}{14} \times \frac{8}{13} + \frac{8}{13} \times \frac{6}{13} = \frac{48}{91}$$

12. Given,  $P(A) = 0.40$ ,  $P(B) = 0.80$ ,  $P(B/A) = 0.60$ ,  $P(A/B) = ?$   $P(A \cup B) = ?$

$$\therefore P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B/A) = 0.40 \times 0.60 = 0.24$$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.80} = 0.3$$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.40 + 0.80 - 0.3 \end{aligned}$$

$$= 0.9$$

13. A box contains six red and four black balls. Two balls are drawn one at a time without replacing the first ball compute  $P(R_2/131)$ . Also find the probability that both are red balls.

**Solution:**

$P(R_2/B_1)$  = Probability of getting a red ball given that the first ball is black.

First black ball

$n$  = Total no. of possible cases

Total no. of balls =  $6 + 4 = 10$

$m$  = No. of favourable cases

= No. of black balls

$$= 4$$

$$P(B_1) = \frac{m}{n} = \frac{4}{10}$$

### Second Red Ball

One black ball which is drawn is not replaced.

$n$  = Total no. of possible cases

= No. of remaining balls

$$= 6 + 3 = 9$$

$m$  = No. of favourable cases

= No. of red balls = 6

$$P(R_2/B_1) = \frac{m}{n} = \frac{6}{9} = \frac{2}{3}$$

$P(R_1)$  = Probability of getting a red ball

$$= \frac{m}{n} = \frac{6}{10}$$

$P(R_2/R_1)$  Probability that second ball is red when first also red =  $\frac{m}{n} = \frac{5}{9}$

$$\therefore P(R_1 \cap R_2) = P(R_1) \cdot P(R_2/R_1)$$

$$= \frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$$

14. A lot contains 12 items of which 5 are defective. If 5 items are chosen from the lot at random. One after another without replacement. Find the probability that all the five are defective.

**Solution:** We have,

No. of total items = 12

No. of defective items = 5

$$\therefore \text{Probability of getting first item defective, } P(A) = \frac{5}{12}$$

Since, second item is drawn without replacement of first items.

$$\text{So, probability of getting second item defective } P(B) = \frac{4}{11}$$

Similarly,

$$\text{Probability of getting 3}^{\text{rd}} \text{ item defective, } P(C) = \frac{3}{10}$$

$$\text{Probability of getting 4}^{\text{th}} \text{ item defective, } P(D) = \frac{2}{9}$$

$$\text{Probability of getting 5}^{\text{th}} \text{ item defective, } P(E) = \frac{1}{8}$$

Probability of getting all items defective

$$\begin{aligned}
 &= P(A) \times P(B) \times P(C) \times P(D) \times P(E) \\
 &= \frac{5}{12} \times \frac{4}{11} \times \frac{3}{11} \times \frac{1}{9} \times \frac{1}{8} \\
 &= \frac{1}{792}
 \end{aligned}$$

15. A bag contains 3 white, 2 black and 4 red balls. Two balls are drawn, the first replaced before the second is drawn, what is the probability that

- They will be of same colour?
- They will be of different colour?

**Solution:** We have,

No. of white balls = 3

No. of black balls = 2

No. of red balls = 4

Total no. of balls = 9

Let  $P(W)$  = Probability of getting a white ball =  $\frac{3}{9} = \frac{1}{3}$

$P(B)$  = Probability of getting black ball =  $\frac{2}{9}$

$P(R)$  = Probability of getting red ball =  $\frac{4}{9}$

a.  $P(\text{They will be of same colour}) = P(WW \text{ or } BB \text{ or } RR)$

$$\begin{aligned}
 &= P(WW) + P(BB) + P(RR) \\
 &= P(W) \times P(W) + P(B) \times P(B) + P(R) \times P(R) \\
 &= \frac{1}{3} \times \frac{1}{3} + \frac{2}{9} \times \frac{2}{9} + \frac{4}{9} \times \frac{4}{9} \\
 &= \frac{29}{81}
 \end{aligned}$$

b. Probability of getting different colours, there should be either WB or BW or BR or RB or WR or RW

$\therefore P(\text{That they are of different colour})$

$= P(WB \text{ or } BW \text{ or } BR \text{ or } RB \text{ or } WR \text{ or } RW)$

$= P(W) \times P(B) + P(B) \times P(W) + P(B) \times P(R) + P(W) \times P(R) + P(R) \times P(W)$

$$\begin{aligned}
 &= \frac{1}{3} \times \frac{2}{9} + \frac{2}{9} \times \frac{1}{3} + \frac{2}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{2}{9} + \frac{1}{3} \times \frac{4}{9} + \frac{4}{9} \times \frac{1}{3} \\
 &= \frac{52}{81}
 \end{aligned}$$

### Exercise 14.2

1. We have, mean =  $np = 25 \dots \dots \dots$  (i)

Variance =  $npq = 5 \dots \dots \dots$  (ii)

from (i) and (ii)

$25q = 5$

$$\text{or, } q = \frac{5}{25} = \frac{1}{5}$$

$$\therefore p = 1 - q = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\therefore \text{from (ii), } n \cdot \frac{4}{5} \cdot \frac{1}{5} = 5$$

$$\therefore p = \frac{4}{5}, q = \frac{1}{5}$$

2. We have,

$$p = \frac{3}{5}, n = 50$$

$$\therefore q = 1 - p = 1 - \frac{3}{5} = \frac{2}{5}$$

$$\therefore \text{Mean} = np = 50 \times \frac{3}{5} = 30$$

$$\text{S.D.} = \sqrt{npq} = \sqrt{50 \times \frac{3}{5} \times \frac{2}{5}} = 2\sqrt{3}$$

3. We have, mean = np = 4 ... .. (i)

$$\text{S.D.} = \sqrt{npq} = \sqrt{3}$$

$$\text{or, } npq = 3 \dots \dots \dots \text{(ii)}$$

$$\therefore \text{from (i) and (ii)}$$

$$4q = 3$$

$$\Rightarrow q = \frac{3}{4}$$

$$\therefore p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\therefore \text{from (i), } n \times \frac{1}{4} = 4 \Rightarrow n = 16$$

$$\therefore \text{Binomial distribution} = (q + p)^n = \left(\frac{3}{4} + \frac{1}{4}\right)^{16}$$

4. We have,

$$\text{Mean} = np = 7 \dots \dots \dots \text{(i)}$$

$$\text{Variance} = npq = 11 \dots \dots \dots \text{(ii)}$$

$$\therefore \text{from (i) and (ii)}$$

$$7 \times q = 11$$

$$\text{or, } q = \frac{11}{7} = 1.57 > 1$$

Since, q is probability of failure, which cannot be greater than 1. So, the given statement is not correct.

5. Here,

$$p = \text{probability of getting ahead} = \frac{1}{2}$$

$$q = \text{probability of getting a tail} = \frac{1}{2}$$

$$n = \text{no. of trials} = 4$$

$$p(r) = \text{probability of } r \text{ success in } n \text{ trials} \\ = {}^n C_r p^r q^{n-r}$$

- a.  $p(2) = \text{probability of 2 heads in 4 trials}$

$$= {}^4 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} = \frac{4 \times 3}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{3}{8}$$

- b.  $p(\text{at least two heads}) = p(2) + p(3) + p(4)$

$$= {}^4 C_2 p^2 q^2 + {}^4 C_3 p^3 q + {}^4 C_4 p^4 \\ = \frac{4 \times 3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + 4 \left(\frac{1}{2}\right)^3 \frac{1}{2} + 1 \left(\frac{1}{2}\right)^4 \\ = \frac{11}{16}$$

- c.  $\text{Plat least one head} = p(1) + p(2) + p(3) + p(4)$

$$= {}^4 C_1 p^1 q^3 + {}^4 C_2 p^2 q^2 + {}^4 C_3 p^3 q + {}^4 C_4 p^4 \\ = {}^4 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + {}^4 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + {}^4 C_3 \left(\frac{1}{2}\right)^3 \frac{1}{2} + 1 \cdot \left(\frac{1}{2}\right)^4 \\ = \frac{1}{4} + \frac{11}{16} \\ = \frac{15}{16}$$

6. Let p be the probability of getting 3 or 6.

$$\therefore p = \frac{2}{6} = \frac{1}{3}, q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

$n$  = no. of trials = 4

Now, probability of  $r$  success out of  $n$  trials is given by

$$p(r) = n_{cr} p^r q^{n-r} = 4_{cr} \left(\frac{1}{3}\right)^r \cdot \left(\frac{2}{3}\right)^{4-r} = r \frac{1}{81} 4_{cr}$$

$$\begin{aligned} \text{a. Probability of getting at least one success} &= p(\geq 1) = p(1) + p(2) + p(3) + p(4) \\ &= 4_{c1} p^1 q^{4-1} + 4_{c2} p^2 q^{4-2} + 4_{c3} p^3 q^{4-3} + 4_{c4} p^4 q^{4-4} \\ &= \frac{4!}{3! 1!} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3 + \frac{4!}{2! 2!} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 + \frac{4!}{3! 1!} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1 + \frac{4!}{0! 4!} \left(\frac{1}{3}\right)^4 \\ &= \frac{4 \times 8}{81} + \frac{6 \times 4}{81} + \frac{4 \times 2}{81} + \frac{1}{81} \\ &= \frac{65}{81} \end{aligned}$$

$$\begin{aligned} \text{b. Probability of exactly two success} &= p(2) \\ &= 4_{c2} p^2 q^{4-2} \\ &= \frac{4!}{2! 2!} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 \\ &= \frac{4 \times 3}{2} \times \frac{4}{81} = \frac{8}{27} \end{aligned}$$

7. Let  $X$  represents the number of diamond cards among the five cards drawn. Since the drawing off cards is with replacement, the trials are Bernoulli trial.  
In a well-shuffled deck of 52 cards, there are 13 diamond cards.

$$\Rightarrow p = \frac{13}{52} = \frac{1}{4}, q = 1 - \frac{1}{4} = \frac{3}{4}$$

$X$  has a Binomial distribution with  $n = 5$  and  $p = \frac{1}{4}$

$$\begin{aligned} \therefore p(x = x) &= n_{cx} p^x q^{n-x}, \text{ where } x = 0, 1, 2, \dots, n \\ &= 5_{cx} \left(\frac{3}{4}\right)^{5-x} \left(\frac{1}{4}\right)^x \end{aligned}$$

- a.  $p(\text{all 5 cards are diamond}) = p(x = 5)$   
 $= 5_{c5} \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$   
 b.  $p(\text{only 3 cards are diamond}) = p(x = 3)$   
 $= 5_{c3} \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^3 = \frac{45}{512}$   
 c.  $p(\text{none is spade}) = p(x = 0)$   
 $= 5_{c0} \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^0 = \frac{243}{1024}$   
 8. Let  $p$  = the event of getting a head 10 coins being tossed simultaneously is the same as one coin being tossed 10 times.

$$p(x = r) = 10_{cr} p^r q^{10-r} = 10_{cr} \left(\frac{1}{2}\right)^{10}$$

$$\begin{aligned} \text{a. } p(\text{exactly 6 heads}) &= 10_{c6} \left(\frac{1}{2}\right)^{10} \\ &= \frac{10!}{6! 4!} \times \frac{1}{1024} = \frac{105}{512} \end{aligned}$$

$$\begin{aligned} \text{b. } p(\text{at least 7 heads}) &= p(7 \text{ heads or } 8 \text{ heads or } 9 \text{ heads or } 10 \text{ heads}) \\ &= 1 - [p(x = 0) + p(x = 1) + p(x = 2) + p(x = 3) + p(x = 4) + p(x = 5) + p(x = 6)] \left(\frac{1}{2}\right)^{10} \\ &= 1 - (1 + 10 + 45 + 120 + 210 + 252 + 210) \frac{1}{1024} \\ &= \frac{176}{1024} \end{aligned}$$

$$\begin{aligned}
 \text{c. } p(\text{not more than 3 heads}) &= p(x \leq 34) \\
 &= p(x = 0) + p(x = 1) + p(x = 2) + p(x = 3) \\
 &= (10_{c_0} + 10_{c_1} + 10_{c_2} + 10_{c_3}) \cdot \left(\frac{1}{2}\right)^{10} \\
 &= (1 + 10 + 45 + 120) \cdot \frac{1}{1024} = \frac{11}{64}
 \end{aligned}$$

9. Given,

Probability of getting a six in one throw ( $p$ ) =  $\frac{1}{6}$

$$\therefore q = 1 - p = \frac{5}{6}$$

No. of trials ( $n$ ) = 4

Now, probability of  $r$  success in 4 trials is given by

$$p(r) = n_{c_r} p^r \cdot q^{n-r} = 4_{c_r} \left(\frac{1}{6}\right)^r \cdot \left(\frac{5}{6}\right)^{4-r} \dots \dots \dots (i)$$

$$\text{a. } p(\text{no six}) = p(0) = 4_{c_0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{4-0} = \frac{625}{1296}$$

$$\text{b. } p(\text{exactly 1 six}) = p(1) = 4_{c_1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{4-1} = \frac{125}{324}$$

$$\text{c. } p(\text{exactly two sixes}) = p(2) = 4_{c_2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{4-2} = \frac{25}{216}$$

$$10. \text{ Probability of fail} = \frac{40}{100} = \frac{2}{5} = q$$

$$\text{Probability of pass} = 1 - \frac{2}{5} = \frac{3}{5} = p$$

$$n = 6, q = \frac{2}{5}$$

$x \rightarrow$  R.V.

We have Binomial condition,

$$P(x = r) = n_{c_r} p^r \cdot q^{n-r}$$

$$p(x \geq 4) = ?$$

$$\begin{aligned}
 \therefore p(x \geq 4) &= p(x = 4) + p(x = 5) + p(x = 6) \\
 &= 6_{c_4} \left(\frac{3}{4}\right)^4 \left(\frac{2}{5}\right)^4 + 6_{c_5} \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right) + 6_{c_6} \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^0 \\
 &= \frac{6!}{4! 2!} \times \frac{3^4 \times 2^2}{5^6} + \frac{6!}{5! 1!} \times \frac{3^5 \times 2}{5^6} + \frac{6!}{6!} \times \frac{3^6}{5^6} \\
 &= \frac{1701}{3125}
 \end{aligned}$$

11. Given,

$$p = 60\% = \frac{60}{100} = \frac{3}{5}$$

$$\therefore q = 1 - p = 1 - \frac{3}{5} = \frac{2}{5}$$

$n$  = number of trials = 10

Now, probability of  $r$  successes in 10 trials is given by

$$p(r) = 10_{c_r} p^r \cdot q^{10-r} = 10_{c_r} \left(\frac{3}{5}\right)^r \left(\frac{2}{5}\right)^{10-r}$$

a.  $P(\text{None of them male}) = p(0)$

$$\begin{aligned}
 &= 10_{c_0} \left(\frac{3}{5}\right)^0 \left(\frac{2}{5}\right)^{10-0} \\
 &= \frac{10!}{0! 10!} \times 1 \times \frac{2^{10}}{5^{10}} = 0.0001049
 \end{aligned}$$

b.  $P(\text{Exactly three male}) = p(3) = 10_{c_3} \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^{10-3}$



$$= \frac{10!}{7! 3!} \times \frac{3^3 \times 2^7}{5^3 \times 5^7} = 0.04246$$

c.  $P(\text{More than 4 are male}) = P(r > 4)$   
 $= p(5) + p(6) + p(7) + p(8) + p(9) + p(10)$   
 $= 10c_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^{10-5} + 10c_6 \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^{10-6} + 10c_7 \left(\frac{3}{5}\right)^7 \left(\frac{2}{5}\right)^{10-7}$   
 $+ 10c_8 \left(\frac{3}{5}\right)^8 \left(\frac{2}{5}\right)^{10-8} + 10c_9 \left(\frac{3}{5}\right)^9 \left(\frac{2}{5}\right)^{10-9} + 10c_{10} \left(\frac{3}{5}\right)^{10} \left(\frac{2}{5}\right)^0$   
 $= \frac{10!}{5! 5!} \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^5 + \frac{10!}{6! 4!} \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^4 + \frac{10!}{7! 3!} \left(\frac{3}{5}\right)^7 \left(\frac{2}{5}\right)^3 + \frac{10!}{8! 2!} \left(\frac{3}{5}\right)^8 \left(\frac{2}{5}\right)^2$   
 $+ \frac{10!}{9! 1!} \left(\frac{3}{5}\right)^9 \left(\frac{2}{5}\right)^1 + \frac{10!}{10!} \left(\frac{3}{5}\right)^{10} = 0.9447$

12. Here,

$$p = \text{Probability of hitting a target} = \frac{1}{5}$$

$$q = 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$$

$n$  = No. of hitting = 6

$p(r)$  = Probability of  $r$  successful hitting =  $n_c r p^r q^{n-r}$

a.  $p(\text{Exactly once}) = p(1) = ?$

$$= 6c_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^{6-1} = \frac{6!}{5! 1!} \times \frac{1}{5} \times \left(\frac{4}{5}\right)^5 = 0.3932$$

b.  $p(\text{Exactly twice}) = p(2)$

$$= 6c_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^{6-2} = \frac{6!}{4! 2!} \times \left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)^4 = 0.24576$$

13. Given,

$$p = \text{Probability that a bomb dropped} = \frac{1}{4}$$

$$q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

$n$  = no. of dropped = 5

a.  $P(\text{None will strike target}) = p(0) = n_c r p^r q^{n-r}$

$$= 5c_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{5-0}$$

$$= \frac{5!}{2! 3!} \times \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 = 0.879$$

c.  $p(\text{At least three will strike target}) = p(x \leq 3)$

$$= p(3) + p(4) + p(5)$$

$$= 5c_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{5-3} + 5c_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{5-4} + 5c_5 \left(\frac{1}{4}\right)^5$$

$$= 5c_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 + \frac{5!}{4! 1!} \times \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^1 + \frac{5!}{5!} \left(\frac{1}{4}\right)^5$$

$$= 0.1035$$

14. Given,

$$p = \text{detective products} = 20\% = \frac{20}{100} = \frac{1}{5}$$

$$q = 1 - p = 1 - \frac{1}{5} = \frac{4}{5}, n = 4$$

Now, the probability of  $r$  defective in 4 trials is given by

$$p(r) = 4c_r p^r q^{4-r} = 4c_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{4-r}$$

a.  $p(\text{No chip is defective}) = p(0)$

$$= 4c_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{4-0}$$

$$= \frac{4!}{4! 0!} \times 1 \times \left(\frac{4}{5}\right)^4 = 0.4096$$

- b.  $p(\text{One chip is defective}) = p(1)$

$$= {}^4C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^{4-1}$$

$$= \frac{4!}{1! 3!} \times \frac{1}{5} \times \left(\frac{4}{5}\right)^3 = \frac{4 \times 4 \times 4 \times 4}{5 \times 5 \times 5 \times 5} = 0.4096$$

- c.  $p(\text{more than one chip are defective})$

$$= p(2) + p(3) + p(4)$$

$$= {}^4C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^{4-2} + {}^4C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^{4-3} + {}^4C_4 \left(\frac{1}{5}\right)^4$$

$$= \frac{4!}{2! 2!} \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^2 + \frac{4!}{3! 1!} \times \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right) + \frac{4!}{4!} \left(\frac{1}{5}\right)^4 = 0.1808$$