# Chapter 10: Conic Section

## Exercise 10.1

1.

a. $\frac{x^2}{16} + \frac{y^2}{4} = 1 \ldots \ldots \ldots (1)$

Comparing (1) with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$a^2 = 16, b^2 = 4$

$\therefore \quad a = 4, b = 2$

Now, eccentricity (e) $= \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{16}} = \frac{\sqrt{3}}{2}$

Co-ordinate of vertices = $(\pm a, 0) = (\pm 4, 0)$

Co-ordinate of foci = $(\pm ae, 0) = \left(14 . \sqrt{3}2 , 0\right)$

$= (\pm 2\sqrt{3}, 0)$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{4} = 2$

Major axis = $2a = 2\times4 = 8$

Minor axis = $2b = 2\times2 = 4$

b. $\frac{x^2}{9} + \frac{y^2}{25} = 1 \ldots \ldots \ldots (1)$

Compare (1) with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$a^2 = 9, b^2 = 25$

$\therefore \quad a = 3, b = 5$

Now, eccentricity (e) $= \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

Co–ordinate of vertices = $(0, \pm b) = (0, \pm 5)$

Co-ordinate of foci = $(0, \pm be)$

$= \left(0, \pm \times \frac{4}{5}\right) = (0, \pm 4)$

Length of latus rectum $= \frac{2a^2}{b} = \frac{2\times9}{5} = \frac{18}{5}$

Major axis = $2b = 2\times5 = 10$

Minor axis = $2a = 2\times3 = 6$

c. $3x^2 + 4y^2 = 36$

or, $\frac{3x^2}{36} + \frac{4y^2}{36} = 1$

or, $\frac{x^2}{12} + \frac{y^2}{9} = 1 \ldots \ldots \ldots (1)$

Comparing (1) with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$a^2 = 12, b^2 = 9$

$\therefore \quad a = 2\sqrt{3}, b = 3$

Now, eccentricity (e) $= \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{12}} = \frac{\sqrt{3}}{\sqrt{12}} = \frac{1}{2}$

Co–ordinate of vertices = $(\pm a, 0) = (\pm 2\sqrt{3}, 0)$

Co–ordinate of foci = $(\pm ae, 0) = (\pm 2\sqrt{3}\,\frac{1}{2}, 0)$

$= (\pm\sqrt{3}, 0)$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{2\sqrt{3}} = 3\sqrt{3}$

Major axis = $2a = 2\times2\sqrt{3} = 4\sqrt{3}$

Minor axis = $2b = 2\times3 = 6$

***d, e are similar to a, b, c***

**2.**

a. Focus = (–2, 0), vertex = (5, 0)

**Solution:**

Here, a = 5, ae = 2

$\therefore \quad 5e = 2 \Rightarrow e = \frac{2}{5}$

Now, using $b^2 = a^2 (1 - e^2)$

$= 25 \left(1 - \frac{4}{25}\right) = 21$

So, the equation of the ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

or, $\frac{x^2}{25} + \frac{y^2}{21} = 1$

b. Vertex = (0, 10), eccentricity = $\frac{4}{5}$

**Solution:** Here, major axis is along the y–axis.

So, $b = 10 \Rightarrow b^2 = 100$ and $e = \frac{4}{5}$

Now, using $a^2 = b^2 (1 - e^2) = 100 \left(1 - \frac{16}{25}\right) = 36$

So, the equation of ellipse is $\frac{x^2}{36} + \frac{y^2}{100} = 1$

c. Foci = (± 2, 0), eccentricity = $\frac{1}{2}$

**Solution:** Here, foci = (± 2, 0) = (± ae, 0)

$\Rightarrow \quad ae = 2$ and $e = \frac{1}{2}$

$\Rightarrow \quad a = \frac{2}{1/2} = 4$

and $e^2 = 1 - \frac{b^2}{16} \Rightarrow \frac{1}{4} = \frac{16 - b^2}{16}$

$\Rightarrow 4 = 16 - b^2$

$\Rightarrow b^2 = 12$

Using equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

or, $3x^2 + 4y^2 = 48$

d. Vertex = (0, 8) and passing through $\left(3, \frac{32}{5}\right)$

**Solution:** Here, major axis is along the y–axis.

So, b = 8

The equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{64} = 1$

which passes through $\left(3, \frac{32}{5}\right)$, so

$\frac{9}{a^2} + \frac{(32/5)^5}{64} = 1$

or, $\frac{9}{a^2} + \frac{1025}{25 \times 64} = 1$

or, $\frac{9}{a^2} + \frac{16}{25} = 1$

or, $\frac{9}{a^2} = \frac{9}{25} \Rightarrow a^2 = 25$

$\therefore$ The equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{64} = 1$

e. Passing through the points (1, 4) and (–3, 2)

**Solution:** Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ … … … (1)

Since, (1) passes through (1, 4) and (–3, 2), so

$\frac{1}{a^2} + \frac{16}{b^2} = 1$ and $\frac{9}{a^2} + \frac{4}{b^2} = 1$

Solving these two equations, we get

$a^2 = \frac{140}{12} = \frac{35}{3}$

and $b^2 = \frac{140}{8} = \frac{35}{2}$

From equation (1), equation of ellipse is $\frac{x^2}{35/3} + \frac{y^2}{35/2} = 1$

or, $\frac{3x^2}{35} + \frac{2y^2}{35} = 1$

or, $3x^2 + 2y^2 = 35$

**3.**

a. $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$ … … … (1)

**Solution:** Comparing (i) with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

We get, $h = -2$, $k = 5$, $a^2 = 16$, $b^2 = 9$

$\therefore$ $a = 4$, $b = 3$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

The co–ordinate of vertices = $(h \pm a, k)$

= $(-2 \pm 4, 5)$ = (–6, 5) and (2, 5)

So, the co–ordinate of centre = $\left(-\frac{6+2}{2}, \frac{5+5}{2}\right)$ = (–2, 5)

And co–ordinate of foci = $(h \pm ae, k)$

$$= \left(-2 \pm 4 \cdot \frac{\sqrt{7}}{4}, 5\right) = (-2 \pm \sqrt{7}, 5)$$

b. $\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$

We have,

$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$ which is in the form of $\frac{(x-h)^2}{9} + \frac{(y-k)^2}{25} = 1$

where $h = 3$, $k = 5$, $a^2 = 9$ and $b^2 = 25$

$\therefore$ $a = 3$ and $b = 5$

Since (b) (a) 0. So, the ellipse in along y–axis.

eccentricity (e) = $\sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

Co–ordinate of the center = (h, k) = (3, 5)

Foci of the ellipse = $(h, k \pm be) = \left(3, 5 \pm 5 \times \frac{4}{5}\right)$ = (3, 1) and (3, 9)

c. $x^2 + 4y^2 - 4x + 24y + 24 = 0$

or, $(x-2)^2 + 4(y+3)^2 = 4 + 36 - 24 = 16$

or, $\frac{(x-2)^2}{16} + \frac{(y+3)^2}{4} = 1$ … … … (1)

Comparing (i) with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, we get

$a^2 = 16$, $b^2 = 4$, $h = 2$, $k = -3$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{16}} = \frac{\sqrt{3}}{2}$

Foci = (h ± ae, k) = $\left(2 \pm 4.\frac{\sqrt{3}}{2}, -3\right) = (2 \pm \sqrt{3}, -3)$

and centre (h, k) = (2, –3)

d. We have,

$9x^2 + 5y^2 - 30y = 0$

$\Rightarrow\ 9x^2 + 5(y^2 - 6y) = 0$

$\Rightarrow\ 9x^2 + 5(y^2 - 2.y.3 + 3^2 - 3^2) = 0$

$\Rightarrow\ 9x^2 + 5[(y-3)^2 - 9] = 0$

$\Rightarrow\ 9x^2 + 5(y-3)^2 = 45$

Dividing by 45 on both sides, we get

$\frac{x^2}{5} + \frac{(y-3)^2}{9} = 1$ which is in the form of $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$; where h = 0, k = 3, $a^2 = 5$ and $b^2 = 9$

Since b > a > 0. So, the ellipse is along y–axis.

Hence,

Eccentricity (e) = $\sqrt{1 - a^2/b^2} = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$

Co–ordinate of the center = (h, k) = (0, 3)

Foci of the ellipse = (h, k ± be) = $\left(0, 3 \pm 3 \times \frac{2}{3}\right)$ = (0,5) and (0, 1)

e. We have,

$9x^2 + 4y^2 + 40y + 18x + 73 = 0$

$\Rightarrow\ (9x^2 + 18x) + (4y^2 + 40y) + 73 = 0$

$\Rightarrow\ 9[x^2 + 2.x.1 + 1^2 - 1^2] + 4[y^2 + 2.5.y + 5^2 - 5^2] + 73 = 0$

$\Rightarrow\ 9[(x+1)^2 - 1] + 4[(y+5)^2 - 25] + 73 = 0$

$\Rightarrow\ 9(x+1)^2 - 9 + 4(y+5)^2 - 100 + 73 = 0$

$\Rightarrow\ 9(x+1)^2 + 4(y+5)^2 = 36$

$\Rightarrow\ \frac{(x+1)^2}{4} + \frac{(y+5)^2}{9} = 1$; which is in the form of $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$; where h = –1, k = –5, $a^2 = 4$ and $b^2 = 9$

$\therefore$ a = 2 and b = 3

Since b > a > 0. So, the ellipse is along y–axis

Hence,

eccentricity (e) = $\sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

Co–ordinate of the center (h, k) = (–1, –5)

Foci of the ellipse = (h, k ± be) = $\left(-1, -5 \pm 3 \times \frac{\sqrt{5}}{3}\right) = (-1, -5 \pm \sqrt{5})$

**4.**

a. Major axis is twice its minor axis and which passes through the point (0, 1)

**Solution:**

The equation of the ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ … … … (1)

It is given that a = 2b and ellipse passes through (0, 1)

So, $\frac{0}{a^2} + \frac{y^2}{b^2} = 1$

or, $b^2 = 1$ $\therefore$ b = 1

and a = 2b = 2

from (1), $\frac{x^2}{4} + \frac{y^2}{1} = 1$

or, $x^2 + 4y^2 = 4$ is the required equation of an ellipse.

b. Latus rectum 3 and eccentricity is $\frac{1}{\sqrt{2}}$

**Solution:** Here, equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It is given, length of latus rectum = 3

or, $\frac{2b^2}{a} = 3 \quad \therefore b^2 = \frac{3a}{2}$

Using $e^2 = 1 - \frac{b^2}{a^2}$ or, $\frac{1}{2} = 1 - \frac{3a}{2a^2}$

or, $a = 2a - 3$ or, $a = 3$

and $b^2 = \frac{3.3}{2} = \frac{9}{2}$

So, the equation of ellipse is $\frac{x^2}{a} + \frac{y^2}{9/2} = 1$

or, $x^2 + 2y^2 = 9$

c. Distance between the two foci is 8 and the semi-latus rectum is 6.

**Solution:**

Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let a > b distance between foci = 8

i.e, 2ae = 8

$\therefore$ ae = 4

and semi latus rectum = $\frac{b^2}{a} = 6$

or, $b^2 = 6a$

Using $b^2 = a^2 (1–e^2)$

or, $6a = a^2\left(1 - \frac{16}{a^2}\right)$ $(\therefore e = 4/a)$

or, $6 = a\left(\frac{a^2 - 16}{a^2}\right)$

or, $a^2 - 6a - 16 = 0$

or, $a = 8, -2$ (but $a \neq -2$)

So, $b^2 = 6\times8 = 48$

$\therefore$ The equation of ellipse is $\frac{x^2}{64} + \frac{y^3}{48} = 1$

or, $3x^2 + 4y^2 = 192$

d. Latus rectum is equal to the half its major axis and which passes through the point (4, 3).

**Solution:**

Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ … … … (1)

which passes through (4, 3)

So, $\frac{16}{a^2} + \frac{9}{b^2} = 1$ … … … (ii)

Also, $\frac{2b^2}{a} = \frac{1}{2}2a$

or, $ab^2 = a^2$

Put $a^2 = 2b^2$ in (ii), then

$\frac{16}{2b^2} + \frac{9}{b^2} = 1$

or, $8 + 9 = b^2 \quad \therefore b^2 = 17$

and, $a^2 = 2\times b^2 = 34$

So, from (1), equation of ellipse is $\frac{x^2}{34} + \frac{y^2}{17} = 1$

or, $x^2 + 2y^2 = 34$

e. Foci are at (± 2, 0) and length of latus rectum is 6.

**Solution:** Here, foci = (± ae, 0) = (± 2, 0)

$\Rightarrow$ ae = 2 $\quad \therefore e = \frac{2}{a}$

and length of latus rectum $\frac{2b^2}{a} = 6$

or, $b^2 = 3a$

Also, $e = \sqrt{1 - \frac{b^2}{a^2}}$

or, $\frac{2}{a} = \sqrt{1 - \frac{3a}{a^2}}$

or, $\frac{4}{a^2} = 1 - \frac{3}{a}$

or, $\frac{4}{a} = a - 3$

or, $4 = a^2 - 3a$

or, $a^2 - 3a - 4 = 0$

or, $a^2 - 4a + a - 4 = 0$

or, $a(a - 4) + 1(a - 4) = 0$

$\therefore$ $a = -1, 4$ (but $a \neq -1$)

and $b^2 = 3\times4 = 12$

Hence, the equation of ellipse is

$\frac{x^2}{16} + \frac{y^2}{12} = 1$

## Exercise 10.2

1.

a. $\frac{x^2}{25} - \frac{y^2}{16} = 1$ … … … (1)

Compare (1) with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$a^2 = 25, b^2 = 16$ $\therefore$ $a = 5, b = 4$

Now, eccentricity (e) = $\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{25}} = \frac{\sqrt{41}}{5}$

Co–ordinate of vertices $(\pm a, 0) = (\pm 5, 0)$

Co–ordinate of foci $(\pm ae, 0) = (\pm 5. \frac{\sqrt{41}}{5}, 0)$

$= (\pm\sqrt{41}, 0)$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2\times16}{5} = \frac{32}{5}$

Length of transverse axis = $2a = 2\times5 = 10$

Length of conjugate axis = $2b = 2\times4 = 8$

b. $\frac{x^2}{9} - \frac{y^2}{25} = 1$

We have,

$\frac{x^2}{9} - \frac{y^2}{25} = 1$ which is in the form of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; where $a^2 = 9$ and $b^2 = 25$

$\therefore$ $a = 3$ and $b = 5$

Since the hyperbola is along y–axis

Hence,

eccentricity (e) = $\sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{9}{25}} = \frac{\sqrt{34}}{5}$

Co–ordinate of the vertices = $(0, \pm b) = (0, \pm 5)$

Foci of the hyperbola = $(0, \pm be) = \left(0, \pm \times \frac{\sqrt{34}}{5}\right) = (0, \pm\sqrt{34})$

Lenth of the latus rectum = $\frac{2a^2}{b} = \frac{2 \times 9}{5} = \frac{18}{5}$

Length of transverse axis = $2b = 2\times b = 2\times5 = 10$

Length of conjugate axis = $2a = 2\times3 = 6$

c. $3x^2 - 4y^2 = 36$

or, $\frac{x^2}{12} - \frac{y^2}{9} = 1$ which is in the form of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; where $a^2 = 12$, $b^2 = 9$

$\therefore$ a = $2\sqrt{3}$ and b = 3

Since the hyperbola is along x–axis

Hence, eccentricity (e) = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{9}{12}} = \frac{\sqrt{21}}{2\sqrt{3}} = \frac{\sqrt{7}}{2}$

Co–ordinate of the vertices = $(\pm a, 0) = (\pm 2\sqrt{3}, 0)$

Foci of the hyperbola = $(\pm ae, 0) = \left(\pm 2\sqrt{3}\,.\,\frac{\sqrt{7}}{2}, 0\right) = (\pm\sqrt{21}, 0)$

Length of the latus rectum = $\frac{2b^2}{a} = \frac{2\times 9}{2\sqrt{3}} = 3\sqrt{3}$

Length of the transverse axis = 2a = $2\times2\sqrt{3} = 4\sqrt{3}$

Length of the conjugate axis = 2b = $2\times3 = 6$

**2.**

a. $\frac{(x+1)^2}{144} - \frac{(y-1)^2}{25} = 1$ … … … (1)

Compare (1) with $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, we get

$h = -1, k = 1, a^2 = 144, b^2 = 25$

Now, eccentricity (e) = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{25}{144}} = \frac{13}{12}$

Cor–ordinate of vertices = $(h \pm a, k)$

= $(-1 \pm 12, 1) = (-13, 1)$ and $(11, 1)$

Co–ordinate of foci = $(h \pm ae, k) = \left(-1 \pm 12 \times \frac{13}{12}, 1\right)$

= $(-14, 1)$ and $(12, 1)$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2\times 25}{12} = \frac{25}{6}$

Length of conjugate axis = 2b = 2 × 5 = 10

Length of transverse axis = 2a = 2 × 12 = 24

b. $5x^2 - 20y^2 - 20x = 0$

or, $x^2 - 4y^2 - 4x = 0$

or, $(x-2)^2 - 4y^2 = 4$

or, $\frac{(x-2)^2}{4} - \frac{y^2}{1} = 1$ … … … (1)

Compare (1) with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, then (h, k) = (2, 0), a = 2, b = 1

Now, eccentricity (e) = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{2}$

Co–ordinate of vertices = $(h \pm a, k) = (2 \pm 2, 0)$

= (4, 0)| and (0, 0)

Co–ordinate of foci = $(h \pm ae, k)$

= $\left(2 \pm 2.\frac{\sqrt{5}}{2}, 0\right) = (2 \pm \sqrt{5}, 0)$

Length of latus rectum = $\frac{2b^2}{a} = 2\,.\,\frac{1}{2} = 1$

Length of transverse axis = 2a = 2.2 = 4

Length of conjugate axis = 2b = 2.1 = 2

c. $\mathbf{16x^2 - 9y^2 + 96x - 72y + 144 = 0}$

or, $16(x^2 + 6x) - 9(y^2 + 8y) + 144 = 0$

or, $16(x+3)^2 - 9(y+4)^2 + 144 - 144 + 144 = 0$

or, $16(x+3)^2 - 9(y+4)^2 = -144$

or, $\frac{(x+3)^2}{9} - \frac{(y+4)^2}{16} = -1$ … … … (1)

Compare (1) with $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = -1$, we get

(h, k) = (–3, –4), $a^2 = 9$, $b^2 = 16$. (b > a)

Now, eccentricity (e) = $\sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Co–ordinate of vertices = (h, k ± b) = (–3, –4 ± 4)
= (–3, 0) and (–3, –8)

Co–ordinate of foci = (h, k ± be)

$= (-3, -4 \pm 4.\frac{5}{4}) = (-3, 1)$ and $(-3, -9)$

Length of latus rectum = $\frac{2a^2}{b} = 2 . \frac{9}{4} = \frac{9}{2}$

Length of transverse axis = 2b = 2×4 = 8

Length of conjugate axis = 2a = 2×3 = 6

**3.**

a. Transverse and conjugate axis are respectively 4 and 5.

**Solution:**

Let the equation of hyperbola be

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ … … … (1)

Where, 2a = 4 and 2b = 5 ⇒ a = 2, b = $\frac{5}{2}$

∴ from (1), equation of hyperbola is

$\frac{x^2}{4} - \frac{y^2}{25/4} = 1$

or, $\frac{x^2}{4} - \frac{4y^2}{25} = 1$

b. Foci = (± 3, 0), eccentricity (e) = $\frac{3}{2}$

Here, e = 3 and ae = 3

⇒ a = $\frac{3x^2}{3} = 2$

Using, $b^2 = a^2(e^2 - 1)$

or, $b^2 = 4\left(\frac{9}{4} - 1\right) = 5$

∴ The equation of hyperbola is

$\frac{x^2}{4} - \frac{y^2}{5} = 1$

**c. Latus rectum is 4 and eccentricity is 3**

**Solution:** Here, e = 3 and $\frac{2b^2}{a} = 4$

Now, $b^2 = \frac{4a}{2}$

Using $b^2 = a^2(e^2 - 1)$

$\frac{4a}{2} = a^2(9 - 1)$

or, 2 = 8a ⇒ a = $\frac{1}{4}$ ∴ $a^2 = \frac{1}{16}$

and $b^2 = \frac{4a}{2} = \frac{4\frac{1}{4}}{2} = \frac{1}{2}$

So, the equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

or, $\frac{x^2}{1/16} - \frac{y^2}{1/2} = 1$

or, $16x^2 - 2y^2 = 1$

**d. Vertex at (0, 8) and passing through $(4, 8\sqrt{2})$**

**Solution:** Here, vertex = $(0, \pm b) = (0, 8)$

$\Rightarrow$ b = 8

Let the equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ … … … (1)

Which passes through $(4, 8\sqrt{2})$, then

$$\frac{4^2}{a^2} - \frac{(8\sqrt{2})^2}{64} = -1$$

or, $\frac{16}{a^2} - \frac{128}{64} = -1$

or, $\frac{16}{a^2} = -1 + 2 \Rightarrow a^2 = 16$

Hence, from (1), $\frac{x^2}{16} - \frac{y^2}{64} = -1$

**e. Vertices at $(0, \pm 7)$, $e = \frac{4}{3}$**

**Solution:** Here, b = 7, $e = \frac{4}{3}$

Using $a^2 = b^2(e^2 - 1)$

$$= 49\left(\frac{16}{9} - 1\right) = 49 \times \frac{7}{9} = \frac{343}{9}$$

Hence, the equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

or, $\frac{x^2}{343/9} - \frac{y^2}{49} = -1$

or, $9x^2 - 7y^2 = -343$

or, $9x^2 - 7y^2 + 343 = 0$

**f. Focus at (6, 0) and a vertex at (4, 0)**

**Solution:** Here, ae = 6 and a = 4

Then, $e = \frac{6}{4} = \frac{3}{2}$

Using $b^2 = a^2(e^2 - 1)$

$b^2 = 16\left(\frac{9}{4} - 1\right) = 16 \times \frac{5}{4} = 20$

Now, the equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

or, $\frac{x^2}{16} - \frac{y^2}{20} = 1$