Model Questions - 2078 (2022)

Subject: **Mathematics (0081)** Full Marks: 75

Time: 3 hrs.

SET A

Candidates are required to give answers in their own words as far as practicable. The figures in the margin indicate full marks.

GROUP – A

Attempt all questions. **[11×1=11]**

Rewrite the correct option in your answer sheet.

1. In how many ways 4 players out of 11 players can be selected when two particular players are always included?

   A. C(11, 4) B. C(11, 2)
   C. C(9, 4) D. C(9, 2)

2. The value of $1-\frac{\log 2}{1!}+\frac{(\log 2)^2}{2!}-\frac{(\log 2)^3}{3!}+... =$

   A. 2 B. $\frac{1}{2}$
   C. 1 D. 0

3. The general solution of $\sin\frac{3\theta}{2}=0$ is

   A. $\theta=n\pi, n\in Z$ B. $\theta=\frac{n\pi}{3}, n\in Z$
   C. $\theta=\frac{2n\pi}{3}, n\in Z$ D. $\theta=\frac{3n\pi}{2}, n\in Z$

4. What is the value of λ, so that the vectors $\left(2\vec{i}+6\vec{j}+27\vec{k}\right)\times(\vec{i}+3\vec{j}+\lambda\vec{k})=0$?

   A. 3 B. 27
   C. $\frac{27}{2}$ D. $\frac{20}{27}$

5. The angle between the planes 3x – 4y + 5z = 0 and 2x – y – 2z = 5 is

   A. $\frac{\pi}{6}$ B. $\frac{\pi}{3}$
   C. $\frac{\pi}{4}$ D. $\frac{\pi}{2}$

6. The latus rectum of hyperbola $16x^2 - 9y^2 = 144$ is

A. $\frac{32}{9}$ B. $\frac{32}{3}$

C. $\frac{8}{3}$ D. $\frac{4}{3}$

7. Four unbiased coins are tossed successively. The mean and variance of the distribution is differed by

A. 1 B. 2

C. 3 D. 4

8. If x changes from 4 to 4.1, then actual change in the function $x^2 + x$ is

A. 0.91 B. 0.9

C. 0.1 D. 0.81

9. $\int \frac{dx}{(x+3)\sqrt{x+2}}$ equals

A. $2\tan^{-1}\sqrt{x+2} + C$ B. $2\tan^{-1}\sqrt{x+3} + C$

C. $2\tan^{-1}\sqrt{x^2+2} + C$ D. $2\tan^{-1}x + C$

10. The forward elimination step of Gaussian elimination method, the coefficient matrix of a system of equation is obtained as $\begin{pmatrix} 0 & 2 & 5 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{pmatrix}$, then the system of equation has

A. unique solution B. no solution

C. infinitely many solutions D. finite solutions

11. A body of 5 kg falling from a certain height is brought to rest hitting the ground with the speed of 10 ms^{-1}. Find the duration of contact when the resistance force of the ground is 500 N?

A. 1 s B. 0.1 s

C. 0.01 s D. 0. 5 s

OR

What is the elasticity of demand for the demand function $Q = 150 - P^2$ at $P = 5$ when the price is increased by 10%?

A. 0. 1 B. – 1

C. 0. 4 D. – 0.4

GROUP – B

Attempt all questions. **[8×5=40]**

12. State De'moivres theorem. Write the Euler's formula representing the complex number $\cos\theta + i\sin\theta$. Using De'moivres theorem find the square root of $-1+\sqrt{3}i$. [1+1+3]

13. a) If α and β are the roots of $px^2 + qx + q = 0$, prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$ [3]

    b) Using inverse matrix method solve the system of equations:
    $-2x + 4y = 3,\ 3x - 7y = 1.$ [2]

14. a) Using vector method, show that $\sin(A + B) = \sin A \cos B + \cos A \sin B$ [3]

    b) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, then prove $x^2 + y^2 + z^2 + 2xyz = 1$. [2]

15. The marks obtained by 8 students in Mathematics and Physics are given below

S. N.	1	2	3	4	5	6	7	8
Marks in Maths	40	60	35	68	70	96	70	84
Marks in Physics	48	62	28	52	85	90	52	73

Find the rank correlation coefficient between the marks in Mathematics and Physics and interpret the rank correlation coefficient hence obtained. [4+1]

16. Using simplex method, Maximize $F = 4x - 6y$ subject to the constraints,
    $2x - 3y \le 8$
    $x + y \le 24$
    $x, y \ge 0$ [5]

17. a) Find the derivative of $\left(\cosh \frac{x}{a}\right)^{\log x}$ [3]

    b) Using L.Hospital rule evaluate: $\lim_{x \to 0} \frac{(e^x - 1)\tan x}{x^2}$ [2]

18. Evaluate: (a) $\int \sqrt{x^2 - 9}\, dx$ (b) $\int \frac{1}{(a^2 + x^2)(b^2 + x^2)} dx$ [1+4]

19. a) A stone is thrown horizontally with the velocity $\sqrt{2gh}$ from the top of the of height h. Find where it will strike the level of ground through the foot of tower? [2]

b) P, Q are like parallel forces. If P is moved parallel to itself through a distance x, show that the resultant of P and Q moves a distance $\frac{Px}{P+Q}$. [3]

OR

Define consumer and producer surplus. The demand and supply function under perfect competition are $P_d = 16 - x^2$ and $P_s = 2x^2 + 4$ respectively. Then find

i) market price

ii) consumer surplus

iii) producer surplus [5]

Group 'C'

Attempt all questions. **[3×8=24]**

20. a) Find the general term and sum upto n terms of the series:
$1.n + 2.(n-1) + 3.(n-2) + \ldots\ldots\ldots$ [4]

b) State binomial theorem. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, then show
$C_0C_n + C_1C_{n-1} + C_2C_{n-2} + \ldots + C_nC_0 = \frac{2n!}{n!n!}$. [1+3]

21. a) Define direction cosine of a line. If OX, OY and OZ are three mutually perpendicular axis then

i. What are the direction cosines of these coordinate axes? [1+1]

ii. What are the projection of line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ on the coordinate axes? [2]

iii. Prove that the sum of squares of the projection in coordinate axes is square length of AB. [1]

b) Consider an equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b > 0 and e be its eccentricity then

i. What are the coordinates of vertices?

ii. What are the coordinates of foci?

iii. What are the equation of directrices? [3]

22. a) Verify Lagrange's mean value theorem for the curve f(x) = x(x-2) in [1, 4]. Also find the point on the curve prescribed by the theorem [5]

b) A differential equation is given by $\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$

i. State the type of differential equation and its order [1]

ii. Obtain the general solution of the differential equation [2]

SET B

GROUP – A

Attempt all questions. **[11×1=11]**

Rewrite the correct option in your answer sheet.

1. Let a sequence of group of natural numbers is (2,3), (4,5,6), (7,8,9,10) .... Which one of the following is the first term of the n^{th} group of sequence?

   A. $\frac{1}{2}(n^2+n+2)$ B. $\frac{1}{2}(n^2-n+2)$

   C. $n(n+1)$ D. none

2. The quadratic equation $ax^2 + bx + c = 0$ have reciprocal root if

   A. $b = 0$ B. $c = a$

   C. $c = 0$ D. $b = c = 0$

3. Which one of the following is the general value solution for $\sin x = -1$?

   A. $2n\pi, n \in Z$ B. $(2n-1)\frac{\pi}{2}, n \in Z$

   C. $(4n+1)\frac{\pi}{2}, n \in Z$ C. $(4n-1)\frac{\pi}{2}, n \in Z$

4. Two non-zero vectors are such that their cross product is zero, then the vectors are

   A. coplanar B. collinear

   B. perpendicular C. co-initial

5. What are the direction ratios of normal to the plane $2x - 7y + 2z + 1 = 0$?

   A. 2, 1 2 B. 1, $\frac{-1}{2}$, 1

   C. 1, –2, 1 D. None of the above.

6. How far apart are the directrix of the curve $25x^2 + 9y^2 - 300x - 144y + 1251 = 0$?

   A. 12.5 B. 14.2

   C. 13.2 D. 15.2

7. Let A and B be any two dependent events with $P(A)\neq0$ and $P(B)\neq0$. Then $P(A/B)$ is

   A. $P(A\cap B)$ B. $P(A).P(B)$

   C. $\frac{P(A\cap B)}{P(A)}$ D. $\frac{P(A\cap B)}{P(B)}$

8. The normal to a given curve is parallel to x-axis if

A. $\frac{dy}{dx}=0$ B. $\frac{dy}{dx}=1$

C. $\frac{dx}{dy}=0$ D. $\frac{dx}{dy}=1$

9. The value of $\int \frac{(\tan^{-1}x)^2\,dx}{(1+x^2)}$ is

A. $2\tan^{-1}x + c$ B. $\frac{(\tan^{-1}x)^3}{1+x^2}+c$

C. $\frac{1}{3}(\tan^{-1}x)^3+c$ D. $\frac{1}{(1+x)^2}+c$

10. A system of equations: $x + y + 2z = 7$, $3x + 4y - 5z = -5$, $2x - y + 3z = 12$; are solved using the partial pivoting of Gaussian elimination, then which one of the following are the pivots for eliminating x and y respectively?

A. 3, $-\frac{11}{3}$ B. 3, $-\frac{11}{6}$

C. 2, $-\frac{1}{3}$ D. 1, $-\frac{8}{3}$

11. The maximum horizontal range of a particle thrown with a certain velocity is 10 m. Find the velocity of projection. ($g = 10ms^{-2}$)

A. 5 ms^{-1} B. 20 ms^{-1}

C. 10 ms^{-1} D. 2.5 ms^{-1}

OR

The demand function $P = 45 - 2Q$, then the consumer surplus at $Q = 10$ is

A. 45 B. 25

C. 100 D. 10

GROUP – B

Attempt all questions. **[8×5=40]**

12. Distinguish between permutation and combination with suitable examples. For $0 < r \le n$ prove that

(i) $P(n,r) = r! \times C(n,r)$

(ii) $C(n,r) + C(n, r-1) = C(n+1,r)$ [2+1+2]

13. a) Show that: $1+\frac{1+2}{2!}+\frac{1+2+2^2}{3!}+\ldots\ldots\ldots\ldots=e^2-e$ [3]

b) Define binary operation. Show that the usual operation addition is binary on the set of integers Z. [2]

14. a) If position vectors of three vertices of a triangle are $\vec{i}+2\vec{j}+3\vec{k}$, $-\vec{i}+\vec{j}-8\vec{k}$ and $-4\vec{i}+4\vec{j}+6\vec{k}$ then find the angles of the triangle. [2]

b) Show that $\sin^{-1}x=\tan^{-1}\frac{1}{\sqrt{1-x^2}}$, for $-1<x<1$. Further using the result show that $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$. [3]

15. Two lines of regression are x + 2y = 5 and 2x + 3y = 8 and $\sigma_x{}^2$ =12.

Calculate

a) Means of x and y – series [1]

b) Regression coeff. of x on y [1]

c) Regression coeff. of x on y [1]

d) Correlation coefficient between x and y [1]

e) Variance of y-series. [1]

16. Solve following LP by simplex method,

Maximize (F) = $5x_1+3x_2$

Subject to $2x_1+x_2\le 40$, $x_1+2x_2\le 50$; $x_1,x_2\ge 0$ [5]

17. a) What is derivative of a curve? At what angle does the curve (1+x)=x cut the x-axis? [2]

b) Find the derivative of log (cosx), using definition. [3]

18. Define a linear differential equation. Solve the linear differential equation

$$(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$$ [1+4]

19. a) Derive the resultant of two like parallel forces acting on a rigid body [3]

b) Find the velocity of 4 kg shot that will just penetrate through a wall 25 cms thick, the resistance being 36 tonnes wt. [2]

OR

A difference equation $y_{t+1}=1.8y_t$ is given

a) Solve the equation for the periods 2, 3, 4 given that the income in the year 1 is Rs. 12500.

b) Obtain the sequence of the solutions of the difference equation for t = 2, 3, 4 in terms of y_t.

c) Find the general solution for y_t in terms of t.

d) Evaluate y_6 when y_1 = Rs. 12500.

GROUP – C

Attempt all questions. **[3×8=24]**

20. a) Using De'moivres theorem find cube roots of unity. If α and β are complex cube roots of unity then show $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = 0$ [3+2]

b) A system of equations is given by x – 3y – 7z = 6, 2x + 3y + z = 9, 4x + y = 7. Solve the system using Crammer's method. [3]

21. a) Find the equation of hyperbola in the standard form with focus at (–7, 0) and eccentricity 7/4. [2]

b) Write the condition that the lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are parallel? Using the condition show that the line joining (1, 2, 3) and (–1,–2,–3) is parallel to the line joining (2, 3, 4) and (5, 9, 13). [2]

c) Prove that lines whose direction cosines are given by the relation al + bm + cn = 0 and fmn + gnl + hlm= 0 are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ [4]

22. a) Verify Rolle's theorem for the curve f(x) = $\sqrt{25 - x^2}$ in [- 5, 5]. [3]

b) Evaluate: i) $\int \frac{dx}{2 + \cos x}$ ii) $\int \frac{1 - x}{x^2 + x^3}\, dx$ [2+3]

SET C

GROUP – A

Attempt all questions. **[11×1=11]**

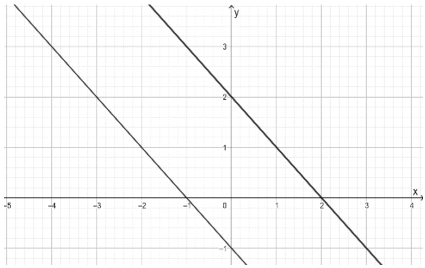
Rewrite the correct option in your answer sheet.

1. What is the value of n, if ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$?

   A. 3 B. 2
   C. 5 D. 6

2. The graph of a simultaneous equation is given aside, then the system of the equation is

   A. consistent and dependent
   B. inconsistent and independent
   C. consistent and independent
   D. inconsistent and dependent



3. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{3}$ then $\cos^{-1}x + \cos^{-1}y =$

   A. $-\frac{\pi}{3}$ B. $\frac{\pi}{3}$
   C. $\frac{2\pi}{3}$ D. 0

4. Which one of the following is the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}\times\vec{b}| = \vec{a}.\vec{b}$?

   A. 0 B. $\frac{\pi}{2}$
   C. π D. $\frac{\pi}{4}$

5. The distance of point (2, 3, 5) from yz-plane is

   A. 2 unit B. 3 unit
   C. 5 unit D. 1 unit

6. The foci of the ellipse $9x^2 + 5y^2 - 30y = 0$ are equal to

   A. $(0,\ 3\pm2)$ B. $(2\pm3,\ 0)$
   C. $\left(\sqrt{5},\ 2\pm\frac{3}{2}\right)$ D. $(\sqrt{5},\ 3\pm2)$

7. If $3x+2y=26$ and $6x+y=21$ be two regression lines, then correlation coefficient r_{xy} is

A. – 0.2 B. 0.2

C. 0.5 D. – 0.5

8. A function f(x) is said to be continuous for $x \in R$ if

A. limit of f(x) exists for all $x \in R$

B. It is discontinuous at x = 0

C. It is differentiable for $x \in R$

D. It is differentiable for $x \neq 0$

9. The value of $\int \frac{1}{\sqrt{a^2 - x^2}} dx$ is equal to

A. $\frac{1}{a}\tan^{-1}\frac{x}{a}+c$ B. $\tan^{-1}\frac{x}{a}+c$

C. $\frac{1}{a}\sin^{-1}\frac{x}{a}+c$ D. $\sin^{-1}\frac{x}{a}+c$

10. Maximum of Z = $5x_1 + 3x_2$ is subjected to the constraints $x_1+2x_2 \le 10$, $x_1 - x_2 \le 8$; $x_1, x_2 \ge 0$. In the starting simplex tableau x_1, x_2 are the non-basic variables and Z is zero. The value of Z in next simplex tableau is

A. 24 B. 40

C. 15 D. 80

11. The least velocity with which a cricket ball can be thrown 10 m horizontally is

A. 10 ms^{-1} B. 20 ms^{-1}

C. 100 ms^{-1} D. 200 ms^{-1}

OR

According to the principle of dynamics of market price, the rate of change of price is

A. directly proportional to the excess demand

B. inversely proportional to the excess demand

C. directly proportional to the excess supply

D. none of the above

GROUP – B

Attempt all questions. **[8×5=40]**

12. Consider a complex number $\dfrac{1+3i}{1-2i}$.
    a) Express the complex number in the form of a + ib [1]
    b) Represent the complex number in its polar form and hence write their modulus and principal argument. [1]
    c) Using De'moivres theorem find square roots of the complex number. [3]
13. a) State principle of mathematical induction. Applying the principle of mathematical induction prove that $3^{2n} - 1$ is divisible by 8. [3]
    b) If one root of quadratic equation $ax^2 + bx + c = 0$ is double of the other then show $2b^2 = 9ac$. [2]
14. a) Find the general value solution for cos 3x + cos x = cos 2x. [2]
    b) Find the equation of plane thorough (3, 2, 1) and is perpendicular to the line joining points (–5, 3, 7) and (2, – 4, 5). [3]
15. In a class there are 25 out of 40 students are girls. If two students are chosen at random what is the probability of getting
    a) both are girls? [1.5]
    b) First is boy and second is girl? [1.5]
    Further, what will be the probability of one student is boy and other will be girl when students are selected with replacement method? [2]
16. Write down the steps solving a system of equations using the Gaussian elimination method? Solve the following simultaneous equations using Gaussian elimination method with partial pivoting: $2x_1 + 2x_2 + x_3 = 6$, $4x_1 + 2x_2 + 3x_3 = 4$, $x_1 - x_2 + x_3 = 0$. [1+4]
17. a) Examine the applicability of Rolle's Theorem for $f(x) = 1-(x-1)^{2/3}$ in $[0, 2]$. [2]
    b) Find the derivative of $x^{\sinh^2 x/a}$ [3]
18. Evaluate the integrals: a) $\displaystyle\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$ b) $\displaystyle\int \frac{dx}{2\sin x + 3\cos x}$. [2+3]
19. a) A man carries a bundle at the end of a stick 75 cm long which is placed over his shoulder. What should be the distance between his hand and shoulder so that the pressure on the shoulder may be three times the

weight of the bundle? [2]

b) Find the velocity and the direction of projection of a shot which passes in a horizontal direction just over of a building which is 50 meter off and 25 meter high. ($g = 9.8\ ms^{-2}$) [3]

OR

Given that demand function and supply function are $Q_{s,t} = 100 + 4p_{t-1}$ and $Q_{d.t} = 170 - 5p_t$ respectively. Using equilibrium condition, find expressions for P_t and Q_t when $P_0 = 36$. Also find the equilibrium price and quantity. Is the price level stable? [5]

GROUP – C

Attempt all questions. **[3×8=24]**

20. a) Define binomial coefficients. Write one property of binomial coefficients in the expansion of $(1+x)^n$. Find the coefficient of x^6 in the expansion of $\left(3x^2 - \frac{1}{3x}\right)^9$ [1+1+3]

b) If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots\ldots\ldots\ldots + C_nx^n$

i. Find the coefficient of x^{n-2} in the product of $(1+x)^n.(x+1)^n$ [1]

ii. Find the coefficient of x^{n-2} in the expansion of $(1+x)^{2n}$ [1]

iii. Should the coefficient of x^{n-2} in (i) always be equal to the coefficient of x^{n-2} in (ii)? Give your reason. [1]

21. a) Using vector method prove: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ [4]

b) Find the angle between the lines whose direction cosines l, m, n satisfies the relation $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$. [4]

22. a) Find from first principle, the derivative of $e^{\tan x}$. [4]

b) The temperature T of a cooling objects drops at the rate of which is proportional to the difference (T – S), where S is the constant temperature of the surrounding medium. Thus, $\frac{dT}{dt} = -k(T - S)$, where K is positive constant and t is the time. Find the solution of the differential equation at $T(0) = 100$. [4]

SET D

GROUP – A

Attempt all questions. **[11×1=11]**

Rewrite the correct option in your answer sheet

1. The Euler's form of the complex number $i-\sqrt{3}$ is

A. $2e^{i\frac{\pi}{2}}$ B. $2e^{-i\frac{\pi}{6}}$

C. $2e^{-i\frac{\pi}{3}}$ D. $2e^{i\frac{5\pi}{6}}$

2. The n^{th} term of the series 5 + 7 + 13 + 31 + 85 + ……….. is

A. $4 + 3^n$ B. $5 + 3^n$

C. $5 + 3^{n-1}$ D. $5 + n^3$

3. Which one of the following is the general value for θ satisfying the equations $\sin\theta = \frac{1}{2}$ and $\cos\theta = -\frac{\sqrt{3}}{2}$?

A. $n\pi + \frac{\pi}{6}$ B. $2n\pi + \frac{\pi}{6}$

C. $2n\pi + \frac{5\pi}{6}$ D. $2n\pi \pm \frac{2\pi}{3}$

4. If $\vec{a}$ and $\vec{b}$ are any two adjacent sides of a parallelogram, then what will be the area of parallelogram having $2\vec{a}-\vec{b}$ and $\vec{a}+2\vec{b}$ as its diagonals?

A. $\vec{a}\times\vec{b}$ B. $5\left(\vec{a}\times\vec{b}\right)$

C. $\frac{5}{2}\left(\vec{a}\times\vec{b}\right)$ D. $2\left(\vec{a}\times\vec{b}\right)$

5. The projection of line OP on the co-ordinate axes are 6, 2, 3 respectively, then distance of OP is

A. 7 B. 11

C. 15 D. $\sqrt{11}$

6. A pair of dice is tossed once and a total of 8 has appeared. What is the chance that odd number appears on each dice?

A. $\frac{2}{9}$ B. $\frac{2}{5}$

C. $\frac{1}{4}$ D. $\frac{3}{5}$

7. Let y = f(x) be any function then approximate change in y is defined by

A. $dy = f'(x)dx$

B. $dy = f'(x)\Delta x$

C. $dy = f'(y)\Delta x$

D. $dy = f(x+\Delta x) - f(x)$

8. $\int \frac{x\,dx}{(x-1)(x-2)}$ equals

A. $\log \frac{(x-1)^2}{|x-2|} + C$

B. $\log \frac{(x-2)^2}{|x-1|} + C$

C. $\log \left(\frac{x-1}{x-2}\right)^2 + C$

D. $\log|(x-1)(x-2)| + C$

9. The slope of tangent to the curve at a point is twice to the slope of the line joining that point to the origin, then the curve is

A. circle

B. parabola

C. hyperbola

D. ellipse.

10. In Gauss-elimination method, the coefficient of variable of equation a_{ij} where i=j are known as

A. basic elements

B. non basic elements

C. pivot elements

D. common elements

11. If two like parallel forces of $\frac{P}{Q}$ Newtons and $\frac{Q}{P}$ Newtons have resultant of 2 Newtons, then

A. P = Q

B. P = 2Q

C. 2P = Q

D. $P^2 = Q$

OR

The cost function and the revenue function are given by the equations $TC = 7 + 2x + x^2$, $TR = 10x$, where x is level of output, then the break-even point (s)

A. 1, 7

B. 1

C. 7

D. -1, -7

GROUP – B

Attempt all questions. **[8×5=40]**

12. If α and β are two roots of equation $ax^2 + bx + c = 0$, find the equation whose roots are $(\alpha - \beta)^2$ and $(\alpha + \beta)^2$. Also, If sum of roots of the equation be equal to the sum of their squares, show $2ac = ab + b^2$. [3+2]

13. a) Using , matrix method solve the system of equations:
$x + 2y - z = -5$, $2x - y + z = 6$, $x - y - 3z = -3$. [3]

b) Define group. If (G, o) is a group, then show $(aob)^{-1} = b^{-1} \text{ o } a^{-1}$ [2]

14. a) Prove that $4(\cot^{-1}3 + \text{cosec}^{-1}\sqrt{5}) = \pi$. [3]

b) Find the equation of the ellipse whose latus rectum is half the major axis and focus is at (3, 0). [2]

15. a) Calculate the correlation coefficient from the following data: [3]

X:	10	12	14	20	22
Y:	8	9	7	14	13

b) 20% of the bulbs produced by a machine are non-defective, In the sample of 4 bulbs determine the probability of getting at least one bulb are defective. [2]

16. Use simplex method to solve the following LPP
Maximize (W) = $5x + 3y$
Subject to the constraints: $2x + y \le 40$, $x + 2y \le 50$; $x, y \ge 0$ [5]

17. State first mean value theorem. Interpret the statement geometrically. Using the theorem, find the point on the curve $f(x) = x^2 - 6x + 1$ at which tangent drawn is parallel to the chord joining the points (1,- 4) and (3,- 8). [1+2+2]

18. Prove $\int \text{cosec}\, x dx = \log\left(\tan\frac{x}{2}\right) + c$ and using the result evaluate:

$$\int \frac{dx}{2\sin x + 3\cos x}.$$ [2+3]

19. a) A straight weightless rod 60 cm in length, rests in a horizontal position between two pegs places at a distance of 6 m apart, one peg being at one end of the rod, and weight of 2 N is suspended from the other end; find the pressure on the pegs. [3]

b) A particle is projected with a velocity u. If the greatest height attained by the particle be H, prove that the range R on the horizontal plane through the point of projection is $R = 4\sqrt{H\left(\frac{u^2}{2g} - H\right)}$ [2]

OR

The demand function for a product is $Q = f(P) = 22500 - 75P$, where Q is measured in units and P in rupees.

(i) Determine the quadratic total revenue function R = f(P). [1]

(ii) What is the concavity of the graph of the revenue function? [1]

(iii) What is the total revenue at a price of Rs. 40? [1]

(iv) How many units will be demanded at this price? [1]

(v) At what price will the total revenue be maximum and what will be the maximum revenue? [1]

GROUP – C

Attempt all questions. **[3×8=24]**

20. a) How many words, with or without meaning, can be formed from the letters of the word 'MONDAY' if,

i) 4 letters are used at a time [1]

ii) all letters are used at a time [1]

iii) all letters are used but first letter is a vowel [1]

iv) all letters are used but first letter is consonant. [1]

b) In the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x^2}\right)^{12}$

i) Find the general term and using it compute the term which is independent of x.

ii) Find the middle terms in the expansion.

21. a) Derive the formula for angle between two lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2. Further state the condition for the lines to be (i) Parallel (ii) Perpendicular [4+1+1]

b) If the position vectors of points A and B are given by (1, 4, 6) and (-2, 5, 1) then find the unit vectors perpendicular to the plane containing both $\overrightarrow{OA}$ and $\overrightarrow{OB}$. [2]

22. a) State L. Hospital rule. Using the rule evaluate: $\lim_{x \to 0} \frac{x - \sin x \cos x}{x^3}$ [1+2]

b) Find the derivative of $x^{\sinh^2 x/a}$ [2]

c) Solve differential equation: $(x^2+1)\frac{dy}{dx} + 2xy = 3x^2$ [3]

SET E

GROUP – A

Attempt all questions. **[11×1=11]**

Rewrite the correct option in your answer sheet.

1. In how many ways the numbers of 3 digits divisible by 5 can be formed using the digits 0, 1, 2, 3, 4, 5?

   A. 320 B. 40

   C. 36 D. 20

2. What is the value of k, so that the roots of $2x^2 + (4 - k)x - 17 = 0$ has the roots equal in magnitude but opposite in sign?

   A. 2 B. 4

   C. – 4 D. 17

3. Which one of the following is not true?

   A. $\vec{a}\times\vec{b}=\vec{b}\times\vec{a}$ B. $\vec{a}\times\vec{a}=0$

   C. $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ D. $(\vec{a}\times\vec{b})^2+(\vec{a}.\vec{b})^2=a^2b^2$

4. Which one of the following is the principal value of $\sin^{-1}\left[\sin\frac{2\pi}{3}\right]$?

   A. $\frac{2\pi}{3}$ B. $-\frac{\pi}{3}$

   C. $\frac{\pi}{6}$ D. $\frac{\pi}{3}$

5. A die is rolled two times then what is the probability that the numbers in each face shows the odd numbers given that the sum of numbers appeared in the faces of die is observed to be 8?

   A. $\frac{8}{9}$ B. $\frac{1}{3}$

   C. $\frac{2}{5}$ D. $\frac{1}{4}$

6. If α,β,γ are the angles made by the line with coordinate axes then value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$

   A. 1 B. 2

   C. 0 D. $\frac{1}{2}$

7. Which one of the following is the equation of plane through the intersection of the planes x + y + z = 6 and 2x + 3y + 4z + 5 = 0 and is perpendicular to the plane 4x + 5y – 3z = 8?

   A. x + 7y + 13z – 96 = 0 B. x + 7y - 13z – 96 = 0

   C. x + 7y + 13z + 96 = 0 D. x - 7y + 13z – 96 = 0

8. Let f(x) = $\log\left(x+\sqrt{x^2+1}\right)$ then f′(x) is equal to

A. x^2+1 B. $\frac{x}{\sqrt{x^2+1}}$

C. $1+\frac{1}{\sqrt{x^2+1}}$ D. $\frac{1}{\sqrt{x^2+1}}$

9. If $\int\frac{dx}{5+4\cos x}=k\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$ + C, then value of k is

A. $\frac{3}{2}$ B. $\frac{1}{3}$

C. $\frac{1}{2}$ D. $\frac{2}{3}$

10. The forward elimination step of Gaussian elimination reduces the coefficient matrix of system of equation into

A. Diagonal matrix B. Identity matrix

C. Upper triangular Matrix D. Lower triangular matrix

11. If a stone is projected at an angle α to the horizontal with the initial velocity u then which one of the following is the time taken by the projectile to attain its maximum height?

A. $u\sin\alpha$ B. $\frac{u\sin\alpha}{g}$

C. $\frac{2u\sin\alpha}{g}$ D. $\frac{u^2\sin2\alpha}{g}$

OR

What will be the consumer surplus if the demand function is p = 16 − q^2 and the market price is 12?

A. $\frac{14}{3}$ B. $\frac{16}{3}$

C. 5 D. $\frac{17}{3}$

GROUP – B

Attempt all questions. **[8×5=40]**

12. State principle of mathematical induction. Using the statement prove that: $2^n < (n+1)!$. [1+4]

13. Write any three properties of cube roots of unity. Prove any one of them. Using the properties prove that: $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)=9$. [2+1+2]

14. a) If $\vec{a}$ and $\vec{b}$ are any two vectors, then derive the expression for $\sin\theta$ given by $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$. [3]

b) Find the general value solution for $\sin 2x \cdot \tan x + 1 = \sin 2x + \tan x$ [2]

15. From the following data:

	Price (Rs.)	Demand of commodity
Arithmetic mean	36	85
Standard deviation	11	8

coefficient of correlation = 0.66

(i) Find the regression coefficients. [2]

(ii) Find the equation of regression lines. [2]

(iii) Estimate the likely price of commodity when quantity of demanded commodity is 75. [1]

16. Using simplex method solve The following LPP:

Max $z = 3x + 5y$

Subject to the constraints: $x + 2y \le 20$, $x + y \le 16$; $x, y \ge 0$ [5]

17. a) Using definition, find the derivative of $\tan^{-1} x$ [3]

b) Find the equation of Tangent and normal to the curve $x^2 - y^2 = 7$ at (4, 3). [2]

18. Let $y = f(x)$ be any function. Present graphically, the differential dx of the independent variable x and dy of the dependent variable y. If x be length of sides of a square such that its area be given by $y = x^2$, compute dx and dy when x changes from 2 to 2.01. [3+2]

19. a) A cricket ball of mass 150 g is moving with the velocity of 12 ms^{-1} and is hit by the bat so that the ball is turned back with a velocity of 20ms^{-1}. The force blow acts for 0.01 s. Find the impulse and the average force exerted on the ball by the bat. [3]

b) Find the velocity and direction of projection of shot which passes in a horizontal direction just over the top a wall which is 250m off and 125m high. ($g = 9.8\ ms^{-2}$) [2]

OR

The following table shows the inter-relationship between the product of to industries A and B in a year

Industry	A	B	Consumer's Demand	Total Output
A	45	50	55	250
B	30	40	30	100

i) Find the coefficient of output matrix. [3]

ii) Find the gross output of the two industries A and B to satisfy the demands of 72 and 48 units. [2]

GROUP – C

Attempt all questions. **[3×8=24]**

20. a) If three consecutive coefficients in the expansion of $(1 + x)^n$ be 45, 120, 210 then find n. [4]

b) Write the expansion of e^x. Using it prove that:

$$\frac{1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots.}=\frac{e^2+1}{e^2-1}$$ [1+3]

21. a) Define ellipse. Find the co-ordinate of centre, vertices, eccentricity foci and the equation of the directix of the ellipse $x^2 + 5y^2 + 3x - 10y - \frac{71}{4} = 0$ [5]

b) Find the direction cosines of line which is perpendicular to the lines whose direction cosines are proportional to 3, - 1, 1 and – 3, 2, 4 [3]

22. a) Evaluate the following integrals:

i) $\int \frac{dx}{\sqrt{1+e^{-2x}}}$ ii) $\int \frac{dx}{x-\sqrt{x^2-4}}$ [2+2]

b) Solve the differential equations:

i) $\frac{dy}{dx}+\frac{1+\cos 2y}{1-\cos 2x}=0$ ii) $y(1 + xy)\, dx - x\, dy = 0$ [2+2]