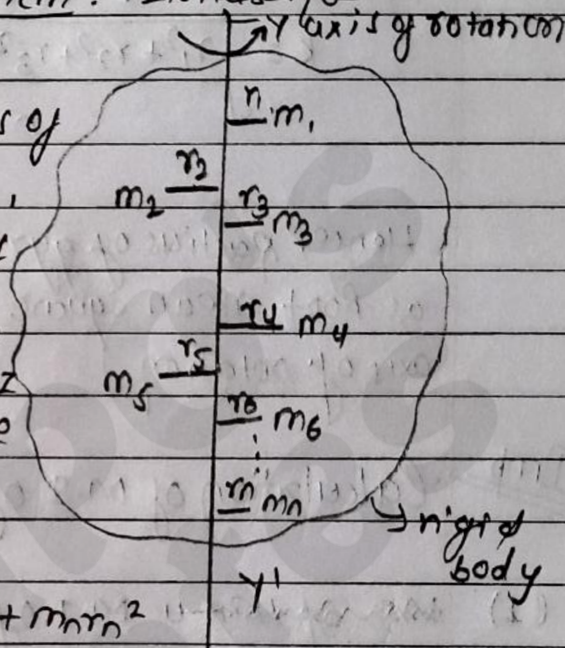
* Rotationaly dynamics:-

Moment of intertia of a rigid body (I):-

Momentum inertia of a given body about a given axis of rotation is defined as product of mass and square of perpendicular distance from axis of rotation.

If 'm' be the mass of the body and 'r' be the perpendicular distance from axis of rotation then M.I of rigid body is $I = mr^2$. It's unit is kgm^2 or $gmcm^2$. ~~It has no~~



Let consider a rigid body consists of n' no. of particles of masses $m_1, m_2, m_3, \ldots m_n$ rotating about an axis of rotation yy' at the distance $r_1, r_2, r_3 \ldots r_n$ from axis of rotation Then M.I of whole rigid body is equal to the sum of M.I of individual particles.

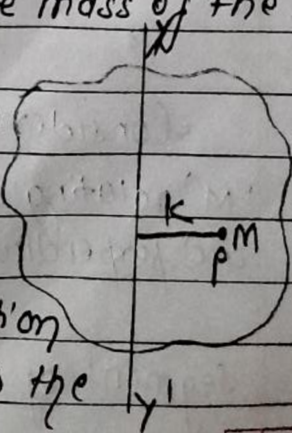
i.e $I = I_1 + I_2 + I_3 + \ldots + I_n$

or $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2$

or $I = \left[\sum_{i=1}^{n} m_i r_i^2\right]$ where $i = 1, 2, 3, \ldots n$.

* Radius of gyration of a rigid body (k)

The radius of gyration of a rigid body about a given axis of rotation is define as the distance from axis of rotation to a point on the rigid body at which whole mass of the body is supposed to be concentrated.



Consider a rigid body consist of 'n' no of particles of same mass (m) rotating about an axis of rotation YY' at the distance of $r_1, r_2, r_3 \ldots r_n$ from axis of rotation Then M.I of whole rigid body is equal to the

Sum of M.I of individual ~~protes~~ particles.

i.e $I = I_1 + I_2 + I_3 + \dots + I_n$

or, $I = mr_1^2 + mr_2^2 + mr_3^2 + \dots + mr_n^2$

or, $I = m(r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2)$

or, $I = mn\left(\dfrac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}\right)$

$$I = MK^2$$

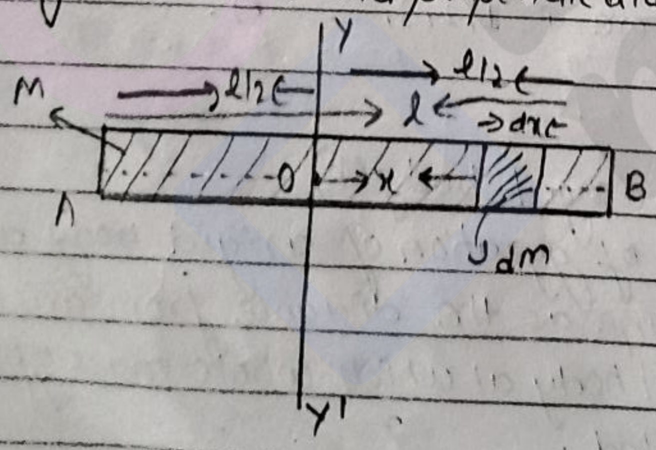
where $M = mn$ = total mass of rigid body and

$K = \sqrt{\dfrac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}}$ is called the radius of gyration of rigid body also defined as (Root mean square)

Hence, Radius of gyration of a rigid body (K) is also defined as Root mean square of distances of particles of body from axis of rotation.

Imp Calculation of M.I of different rigid body :-

(1) ~~....~~ M.I of thin uniform rod about an axis passing through its centre and perpendicular to its length.



Consider a thin uniform rod 'AB' having length 'l' and mass 'M' rotating about on axis of rotation YY' passing through centre and perpendicular to its length.

To calculate M.I of whole rod consider a small segment of the rod at the distance 'x' from axis of rotation having

classmate

thickness dx and mass (dm).

Then, mass of small segment of rod is

$$dm = \frac{m}{l} dx \quad \text{--- (1)}$$

$$\left[\begin{array}{l} l \rightarrow m \\ 1 \rightarrow M/l \\ dx \rightarrow \frac{M}{l} dx \\ dm = \frac{M}{l} dx \end{array}\right]$$

Then M.I of small segment

$$dI = dm x^2$$

$$\text{or, } dI = \frac{M}{l} x^2 dx \quad \text{--- (2)}$$

Then, M.I of whole rod can be obtained by integrating equation (2) from limit $-l/2$ to $(+) l/2$ as,

$$I = \int_{-l/2}^{+l/2} dI$$

$$\text{or, } I = \frac{M}{l} \int_{-l/2}^{l/2} x^2 dx$$

$$\text{or, } I = \frac{M}{l} \left[\frac{x^3}{3}\right]_{-l/2}^{+l/2}$$

$$\text{or, } I = \frac{M}{3l} \left[\left(\frac{l}{2}\right)^3 - \left(-l/2\right)^3\right]$$

$$\text{or, } I = \frac{M}{3l} \left[\frac{l^3}{8} + \frac{l^3}{8}\right]$$

$$\text{or, } I = \frac{M}{3l} \times \frac{2l^3}{8}$$

$$\therefore \boxed{I = \frac{Ml^2}{12}} \text{ [going through center]}$$

Note:- when a axis of rotation passes through its end and perpendicularly then,

$$I = \int_0^l dI$$

$$I = \int_0^l \frac{M}{l} x^2 dx$$

$$I = \frac{M}{l} \left[\frac{x^3}{3}\right]_0^l$$

$$I = \frac{M}{l} \left[\frac{l^3}{3}\right]$$

$$I = \frac{M}{3l} l^3$$

$$I = \frac{1}{3} Ml^2$$

Also, $I = MK^2$

we have, $I = \frac{Ml^2}{12}$

$$\therefore MK^2 = \frac{Ml^2}{12}$$

$$\text{or, } K^2 = \frac{l^2}{12}$$

$$K = \frac{l}{\sqrt{12}}$$

when axis passing through its end.

$$I = \frac{Ml^2}{3}$$

$$\cancel{M}K^2 = \frac{\cancel{M}l^2}{3}$$

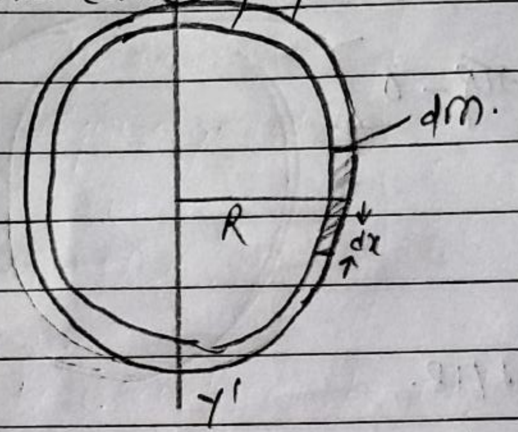
$$K^2 = \frac{l^2}{3}$$

$$K = \sqrt{\frac{l^2}{3}}$$

$$\boxed{\therefore K = \frac{l}{\sqrt{3}}}$$

classmate

2) Moment of inertia of thin uniform ring about an axis passing through its centre & perpendicular to its plane.



Consider a circular ring of radius 'R', mass rotating about an axis of rotation yy' passing through its centre and perpendicular to its plane.

To calculate M.I of ring, consider a small segment of the ring of mass 'dm', thickness / length 'dx' and the distance 'R' from centre of ring (from axis of rotation)

Then M.I of small segment having length dx at the distance R from axis of rotation is $dI = dmR^2$ — 1

but $dm = \frac{M}{2\pi R} \times dx$

$2\pi R \to M$

$I = \frac{M}{2\pi R}$

$dx \to \frac{M}{2\pi R} \times dx$

$\Rightarrow dm = \frac{M}{2\pi R} \times dx$

From eqn 1,

$$dI = \frac{M}{2\pi R} R^2 dx \quad \text{—} (2)$$

then, the total M.I of whole ring can be obtained by integrating eqn (2) from limit 0 to $2\pi R$ as

$$I = \int_0^{2\pi R} dI$$

$$I = \frac{MR^2}{2\pi R} \int_0^{2\pi R} dx$$

$$I = \frac{MR^2}{2\pi R}\left[x\right]_0^{2\pi R}$$

$$I = \frac{MR^2}{2\pi R}\left[2\pi R - 0\right]$$

$$\boxed{\therefore I = MR^2}$$

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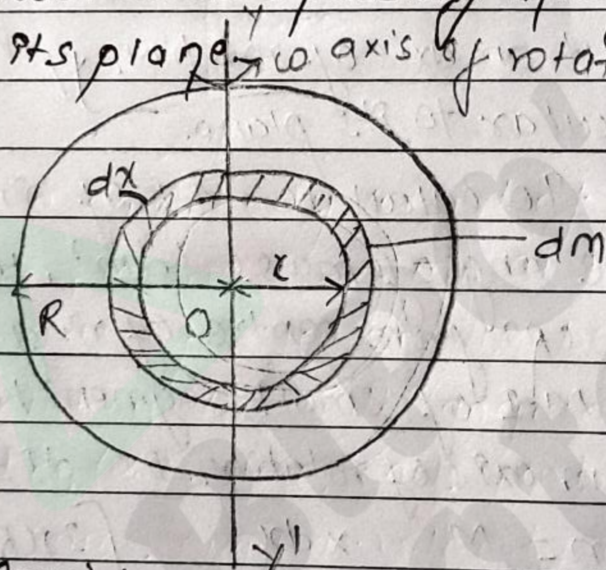
(3) M.I of a disc about an axis passing through its centre and perpendicular to its plane. axis of rotation.



Consider a circular disc of radius R and mass (M) rotating about an axis of rotation YY' passing through its centre. Since disc is supposed to be made of by the combination of infinite number of circular ring. Consider one such a ring of thickness 'dx' and radius (x) having mass (dm)

Now →

Area of disc = πR^2

Area of small circular ~~ring~~ disc = $2\pi x\, dx$

$\therefore$ Mass of small circular ~~ring~~ disc is $(dm) = \frac{M}{\pi R^2} \times 2\pi x\, dx$

$$dm = \frac{2M}{R^2} \cdot x\, dx$$

Now,

M.I of small circular ~~ring~~ disc is

$$dI = dm x^2$$

$$dI = \frac{2M}{R^2} x^3 dx \quad \text{—} (1)$$

Now, total M.I of whole disc is

$$I = \int_0^R dI$$

$$= \frac{2M}{R^2} \int_0^R x^3 dx$$

$$I = \frac{2m}{R^2} \left[\frac{x^4}{4}\right]_0^R$$

$$I = \frac{2M}{4R^2} \left[R^4 - 0^4\right]$$

$$\boxed{I = \frac{1}{2} MR^2}$$

* Torque ($\vec{\tau}$) :- The torque ($\vec{\tau}$) of a rigid body about a given axis of rotation is defined as the product of force ($\vec{F}$) and perpendicular distance ($\vec{r}$) from axis of rotation.

In vector form

$$\text{Torque } (\vec{\tau}) = \vec{r} \times \vec{F}$$

$$\text{or, } \vec{\tau} = rF \sin\theta \, \hat{n}$$

In magnitude, $\tau = rF \sin\theta$

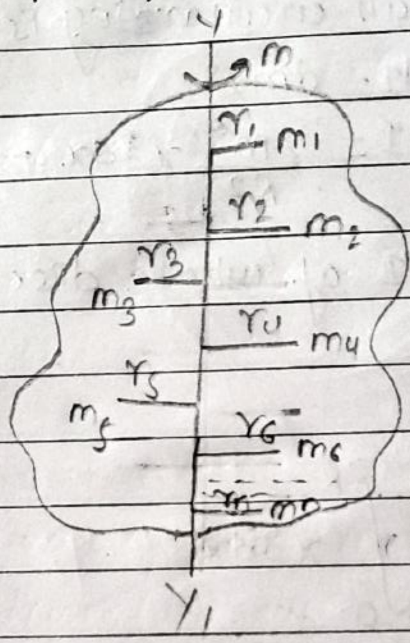
a) If $\theta = 90^\circ$

$$\tau_{max} = rf$$

b) If $\theta = 0^\circ$ or 180°

$$\tau_{min} = 0.$$

* Relation between torque (τ) and moment of inertia (I)



Consider a rigid body consist of 'n' number of particle having masses $m_1, m_2, m_3, \ldots m_n$ at the distance $r_1, r_2, r_3, \ldots r_n$ from axis of rotation. The body is rotating about an axis of rotation YY' with angular velocity (ω) and angular acceleration (α). Here, each particle in the body is rotating with same angular velocity and same angular acceleration α. Then,

magnitude of torque acting on the first particle of mass (m_1) is:

$\tau_1 = r_1 F_1$ but $F_1 = m_1 a_1$

$\tau_1 = r_1 m_1 a_1$ but $a_1 = r_1 \alpha$

$\tau_1 = m_1 r_1^2 \alpha$

$$\left[\begin{array}{l} v = r\omega \\ \dfrac{dv}{dt} = r\dfrac{d\omega}{dt} \\ a = r\alpha \end{array}\right]$$

Similarly, torque acting on the other remaining particles are

$\tau_2 = m_2 r_2^2 \alpha$

$\tau_3 = m_3 r_3^2 \alpha$

$\vdots$

$\tau_n = m_n r_n^2 \alpha$.

Now, total torque acting on the body is

$\tau = \tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n$

or, $\tau = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2)\alpha$

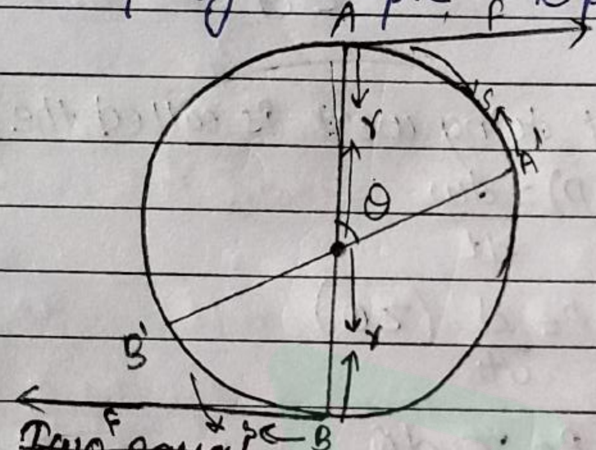
or $\tau = \left[\sum_{i=1}^{n} m_i r_i^2\right] \alpha$ where $i = 1, 2, 3 \cdots n$

or $\tau = I\alpha$ — (1)

where $I = \sum_{i=1}^{n} m_i r_i^2$ is M.I of whole rigid body. Eqn (1) gives relation between τ and I which is similar to $F = ma$

$$\tau = I\alpha.$$

* Work done by a couple and power:-



~~Two equal~~

Two equal and opposite forces acting at two different points on the rigid body constitute a couple.

Consider a wheel of radius 'r' centre O. Then two equal and opposite forces F acting on the rigid body at point A and B and due to these force the wheel is rotating about its centre 'c'.

Then work done by the force (F) to move from A to A' by the force f is

$$\omega_1 = F \times AA'$$

but $AA' = s = r\theta$

$$\therefore \omega_1 = Fr\theta \quad \text{— (1)} \quad \left[\theta = \frac{AA'}{r}\right] = [AA' = r\theta]$$

Similarly work done by the same force F to move two wheel from B to B'

$\omega_2 = F \times BB'$ but $BB' = s = r\theta$

$\omega_2 = Fr\theta$ — (2)

Now, total work done in rotating the wheel is

$$W = W_1 + W_2$$

$$W = Fr\theta + Fr\theta = 2Fr\theta$$

$$W = (F \times 2r)\theta$$

$$W = \tau\theta \quad \text{---} (3)$$

where $\tau = F \times 2r$ is the torque. Then work done by couple, $W = \tau\theta$ which is similar to $W = F S$
$\tau \quad \theta$.

* Power (P) :- The rate of doing work is called the power. i.e

$$\text{Power } (P) = \frac{dW}{dt}$$

$$\text{or, } P = \frac{d}{dt}(\tau\theta)$$

$$\text{or, } P = \tau\frac{d\theta}{dt}$$

$$P = \tau\omega \quad \text{where } \omega = \frac{d\theta}{dt} = \text{angular velocity}$$

This gives power which is similar to $P = FV$

* Angular momentum (L) of a rigid body :-

The angular momentum of the rigid body about an given axis of rotation is defined as the product of linear momentum and perpendicular distance from axis of rotation.

In vector form angular momentum of the rigid body is given by

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\text{or } \vec{L} = rp\sin\theta\,\hat{n}$$

In magnitude,

$$L = rp\sin\theta$$

(i) If $\theta = 90^0$

$L_{max} = rp$

but $p = mv$

but, $v = r\omega$

$L_{max} = mr^2\omega$

$L_{max} = I\omega$.

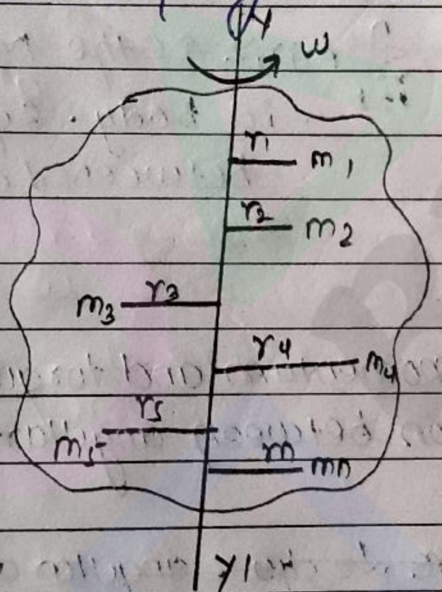
(ii) If $\theta = 0^0$ or $\theta = 180$

$L_{min} = 0$.

The unit of L is kgm^2s^{-1} and its dimensional formula is $[ML^2T^{-1}]$

$\left[\begin{matrix} r.mv \\ m\,kg\,ms^{-1} \\ kg\,m^2s^{-1} \end{matrix}\right.$

* Relation between angular momentum and moment of inertia.



Consider a rigid body consist of 'n' number of particles of masses $m_1, m_2, m_3 \dots m_n$, rotating with an angular velocity 'ω' ~~at the~~ about an axis of rotation YY' at the distance $r_1, r_2, r_3 \dots r_n$ from axis of rotation.

Then magnitude of maximum value of angular momentum of 1^{st} particle of mass m_1 is

$L_1 = r_1 P_1$ but $P_1 = m_1 v_1$.

$L_1 = m_1 r_1 v_1$ Also $v_1 = r_1\omega$.

$L_1 = m_1 r_1^2 \omega$.

Similarly angular momentum of other remaining particles are

$$L_2 = m_2 r_2^2 \omega ,$$

$$L_3 = m_3 r_3^2 \omega .$$

$$\vdots$$

$$L_n = m_n r_n^2 \omega .$$

Hence total angular momentum (L) of the whole rigid body is given by

$$L = L_1 + L_2 + L_3 + \dots + L_n$$

$$L = (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2)\omega .$$

$$L = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega \text{ where } i = 1, 2, 3, \dots n$$

or, $L = I\omega$ where $I = \sum_{i=1}^{n} m_i r_i^2$ is the M.I of whole rigid body. Eqn (1) gives relation between L and I.

* Relation between angular momentum and torque:-

we know, relation between angular momentum (L) and moment of inertia (I) is

$$L = I\omega \quad \text{— (1)}$$ ~~where $\alpha = \frac{d\omega}{dt}$ = angular acceleration~~

Diff eqn (1) w.r. to 't',

$$\frac{dL}{dt} = I\frac{d\omega}{dt}$$

$$\text{or, } \frac{dL}{dt} = I\alpha \quad \text{— (2)}$$

where $\alpha = \frac{d\omega}{dt}$ = angular acceleration.

Also, we know relation between torque and M.I is

$$\tau = I\alpha \quad \text{--- (3)}$$

From eqn (2) and (3)

$$\tau = \frac{dL}{dt}$$

Hence the torque is also defined as rate of change of angular momentum of the body.

Principle of Conservation of angular momentum:-

<u>Statement:-</u> It states that if there is no external torque is acting on the system then total angular momentum of the system remain conserved.

i.e L = constant.

or, $I\omega$ = constant.

Proof:- we know, torque (τ) is defined as rate of change of angular momentum of the system i.e

$$\tau = \frac{dL}{dt} \quad \text{--- (1)}$$

If there is no external torque is acting on the system, $\tau = 0$

from eqn (1) $\frac{dL}{dt} = 0$

$$dL = 0$$

Integrating both sides,

$$\int dL = \int 0$$

L = constant

or, $I\omega$ = constant proved

For Numerical

$I\omega = \text{constant}$

or, $I_1\omega_1 = I_2\omega_2$

or $I_1 2\pi f_1 = I_2 \cdot 2\pi f_2$

or, $I_1 f_1 = I_2 f_2$

or $\frac{I_1}{T_1} = \frac{I_2}{T_2}$

for short question / objective

$I\omega = \text{constant}$

$\omega \propto \frac{1}{I}$

$I \propto \frac{1}{\omega}$

i.e $\omega_{less} \propto \frac{1}{I_{large}}$

$\omega_{large} \propto \frac{1}{I_{less}}$

$\omega_{large} \propto \frac{1}{mr^2_{less}} \Rightarrow \omega_{large} \propto \frac{1}{r^2_{less}}$

also,

$\omega_{less} \propto \frac{1}{r^2_{large}}$

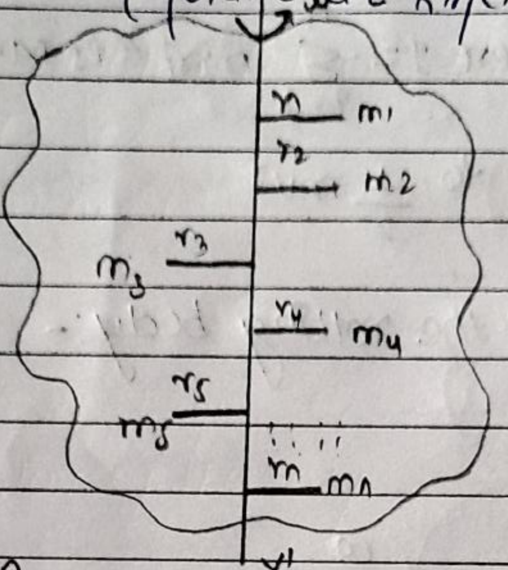
Also, $I \propto \frac{1}{\omega}$

or, $I \propto \frac{1}{\frac{2\pi}{T}} \Rightarrow I \propto T$

i.e $I_{large} \propto T_{large}$

$I_{small} \propto T_{small}$

Kinetic energy of the rotating body:
(Rotational kinetic energy)



Consider a rigid body consist of 'n' number of particles of masses $m_1, m_2, m_3 \ldots m_n$, rotating with an angular velocity 'ω' about an axis of rotation YY'

Let $v_1, v_2, v_3 \ldots v_n$ be the linear velocity of respective particles. Then, kinetic energy of the 1st particle of mass m_1 having linear ~~kinetic~~ velocity v_1 is,

$$E_1 = \frac{1}{2} m_1 v_1^2 \quad \text{but } v_1 = r_1 \omega$$

$$\text{or, } E_1 = \frac{1}{2} m_1 r_1^2 \omega^2$$

Similarly rotational k.E of the other remaining particles are,

$$E_2 = \frac{1}{2} m_2 r_2^2 \omega^2$$

$$E_3 = \frac{1}{2} m_3 r_3^2 \omega^2$$

$$\vdots$$

$$E_n = \frac{1}{2} m_n r_n^2 \omega^2$$

Then total kinetic energy of whole rigid body is

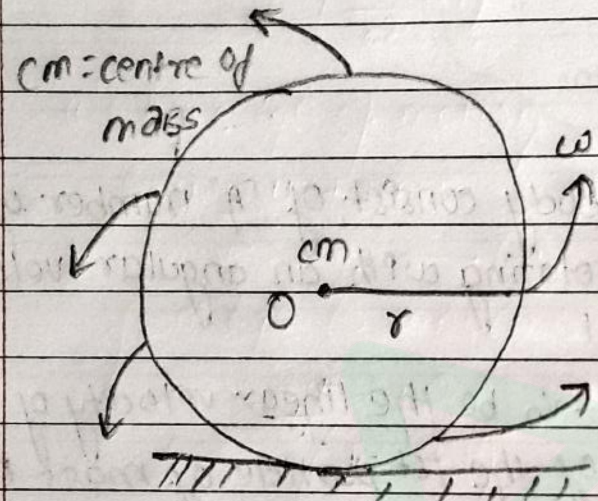
$$E_{rot} = E_1 + E_2 + E_3 + \ldots + E_n$$

$$\text{or, } E_{rot} = \frac{1}{2} (m_1 r^2 + m_2 r_2^2 + \ldots + m_n r_n^2) \omega^2 :$$

or, $E_{rot} = \frac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2$

or $E_{rot} = \frac{1}{2} I\omega^2$ where $I = \sum_{i=1}^{n} m_i r_i^2$ is M.I of whole rigid body, which is similar to $\frac{1}{2}mv^2$

* Kinetic energy of the rolling body:-



A rolling body has two types of energy. One is translational kinetic energy and rotational kinetic energy. When the body rolls its rotates about an horizontal axis. At the same time its centre of mass moves linearly.

Consider a wheel of mass 'm' radius 'r' rolling over a horizontal surface with angular velocity 'ω' such that it centre of mass moves linearly with velocity 'v'. Hence,

Total kinetic energy of a rolling body,

$E_{roll} = E_{trans} + E_{rot}$.

or $E_{roll} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

But $v = r\omega$ & $I = mk^2$ where k is the radius of gyration.

$\therefore E_{roll} = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}mk^2\omega^2$

or, $E_{roll} = \frac{1}{2} mr^2\omega^2 \left[\frac{1+k^2}{r^2}\right]$

$$\boxed{\text{or, } E_{roll} = \frac{1}{2} mv^2 \left(1+\frac{k^2}{r^2}\right)} \text{ — (1).}$$

This gives required expression k.E of rolling body.

For eg if disc is rolling

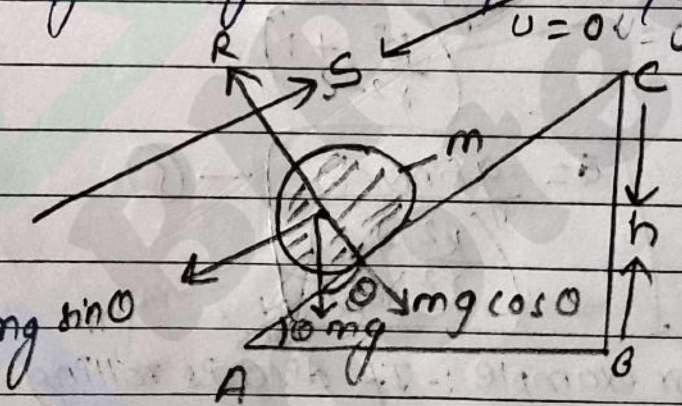
$E_{disc} = \frac{1}{2} mv^2 \left(1+\frac{1}{2}\right)$.

$$\left[\begin{array}{l} I = mk^2 \\ I = \frac{1}{2} mr^2 \\ \cancel{m}k^2 = \frac{1}{2}\cancel{m}r^2 \\ \frac{k^2}{r^2} = \frac{1}{2} \end{array}\right]$$

or, $E_{disc} = \frac{1}{2} mv^2 \times \frac{3}{2}$

$\therefore \boxed{E_{disc} = \frac{3}{4} mv^2}$

Acceleration of a body rolling down an inclined plane.



Consider a body of mass m and radius 'r' rolling down an inclined plane of height 'h', length of inclination 's' and angle of inclination 'θ'. When the body is rolling it acquires a linear velocity 'v' and here kinetic energy of the body is goes on increasing and potential energy of body is goes on decreasing

Hence, Gain in k.E by the rolling body is

$k.E = \frac{1}{2} mv^2 \left(\frac{k^2}{r^2} + 1\right)$ — (1)

Loss in P.E = mgh

In ΔABC $\sin\theta = \frac{h}{s}$

$h = s\sin\theta$

$\therefore$ Loss in P.E $= mgs\sin\theta$ — (2)

This gain in K.E comes from loss in P.E

Gain in K.E = loss in P.E

$$\frac{1}{2}mv^2\left(\frac{k^2}{r^2}+1\right) = mgs\sin\theta$$

$$\text{or, } v^2 = \frac{2gs\sin\theta}{\left(\frac{k^2}{r^2}+1\right)} \quad \text{— (3)}$$

We have eqn of motion

$$v^2 = u^2 + 2as$$

$$\therefore v^2 = 2as \quad \text{— (4)} \quad \because u = 0$$

from eqn (3) and (4)

$$2as = \frac{2gs\sin\theta}{\left(\frac{k^2}{r^2}+1\right)}$$

$$\boxed{a = \frac{g\sin\theta}{\left(\frac{k^2}{r^2}+1\right)}} \quad \text{— (5)}$$

for example :- If disc is rolling, $I = Mk^2$ — (1)

$I = \frac{1}{2}mr^2$ — (2)

from (1) and (2)

$$\cancel{m}k^2 = \frac{1}{2}\cancel{m}r^2$$

$$\frac{k^2}{r^2} = \frac{1}{2}$$

In eqn (5)

$$a = \frac{g\sin\theta}{\left(\frac{1}{2}+1\right)}$$

$$= \frac{g\sin\theta}{\frac{3}{2}} = \frac{2g\sin\theta}{3}$$

$m = I$
$\tau = F$
$a = \alpha$

DATE

(1) A disc is rolling along a horizontal plane has a moment of inertia $4 kgm^2$ about its centre and mass of 5 kg. The velocity along the plane is $2ms^{-1}$. If the radius of disc is 2m. find.

(i) Angular velocity (2) The total energy of disc.

Ans 1 rad/s (b) 12 J.

Given

$I = 4 kgm^2$

$m = 5kg$

$V = 2ms^{-1}$

$\omega = ?$

$E_{total} = ?$

we know,

$V = r\omega$

$\omega = \frac{v}{r} = \frac{2}{2} = 1$ radius

$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$E = \frac{1}{2} \times 5 \times 4 + \frac{1}{2} \cdot 4 \times 1$

or $E = 12$

(2) A constant torque of 500 Nm turns a wheel which has a moment of inertia $20 kgm^2$ about its centre: Find angular velocity gained in 2 sec and kinetic energy gained.

Ans: 50 rad/s (ii) 25000J.

Given

$\tau = 500 Nm$

$I = 20 kgm^2$

$t = 2 sec$

$\omega = ?$

$K.E = ?$

Now,

$\tau = I\alpha$

$\alpha = \frac{\tau}{I} = \frac{500}{20} = 25 \text{ rad/s}.$

$\omega = \omega_0 + \alpha t$

but $\omega_0 = 0$

$\omega = \alpha t$

$= 25 \times 2$

$= 50 \text{ rad/sec}$

Now,

$KE = \frac{1}{2} I\omega^2$

$= \frac{1}{2} \times 20 \times 2500$

$= 25000 J$

(3) A constant torque of 200 Nm turns a wheel which has a moment of inertia 100 kgm2 about its centre. find angular velocity gained in 4 sec and kinetics energy gained after 20 revs. Ans:- 8 rad/s. = 25132.7 J.

Given

$\tau = 200 Nm$

$I = 100 kgm^2$

$t = 4 sec$

$\omega = ?$

Now,

$\tau = I\alpha$

$\alpha = \frac{\tau}{I} = \frac{200}{100} = 2.$

$\omega = \omega_0 + \alpha I$

but $\omega_0 = 0$

$\omega = \alpha t$

$= 2 \times 4$

$= 8$ rad/sec.

Now

No. of rev(n) = 20 rev.

$\omega^2 = \cancel{\omega_0^2}^{0} + 2\alpha\theta$

$\omega^2 = 2\alpha \times 2\pi n$

$= 4\pi\alpha n$

$= \cancel{4 \times \frac{22}{7} \times 2 \times 20} \; \frac{22}{7} \times 4\pi \times 2 \times 20$

$= 160\pi.$

Again

$K.E = \frac{1}{2} I \omega^2$

$= \cancel{\frac{1}{2} \times 100 \times 4 \times \frac{22}{7} \times 2 \times 20.} = \frac{1}{2} \times 100 \times 160\pi$

$= 8000\pi.$

$= 25132.7$ J.

(4). A wheel stands from rest and accelerates with a constant angular acceleration to an angular velocity of 15 revolutions per sec in 10 sec. calculate the angular acceleration and angle which the wheel has rotated at the end of 2 sec.

Given,

$f = 15$ rev/s

$\omega = 2\pi f$

$= 2\pi \times 15$

$= 30\pi$ rad/s

$t = 10$ sec

$\omega = \cancel{\omega_0}^{0} + \alpha t$

$$\alpha = \frac{\omega}{t} = \frac{30\pi}{10} = 3\pi \text{ rad/sec.}$$

Now, $t' = 2\text{sec}$.

$$\theta = \omega_0 t + \frac{1}{2}\alpha t'^2$$

$$= 0 \times \cancel{20} + \frac{1}{2} \times 3\pi \times \cancel{10^2}\, 2^2$$

$$= \cancel{150\pi}\; 6\pi$$

(5) A fly wheel of moment of inertia 0.32 kgm^2 is rotated at 120 rad/sec by 50 watt electric motor find the kinetic energy and angular momentum of flywheel and frictional couple (τ) opposing the rotation?

Given

$I = 0.32\ kgm^2$

$\omega = 120\ \text{rad/s}$.