6th poush, Tuesday

Modes of vibration in organ pipes:

① Modes of vibration in closed pipe:-

In closed organ pipe stationary waves are formed with anti-node at the mouth (open end of pipe) and node at the closed end. The lowest frequency that can be produced in a closed pipe is known as fundamental frequency. The following figures (i, ii, iii) show first mode of vibration, second mode of vibration and third mode of vibration in a closed pipe of length 'l'.

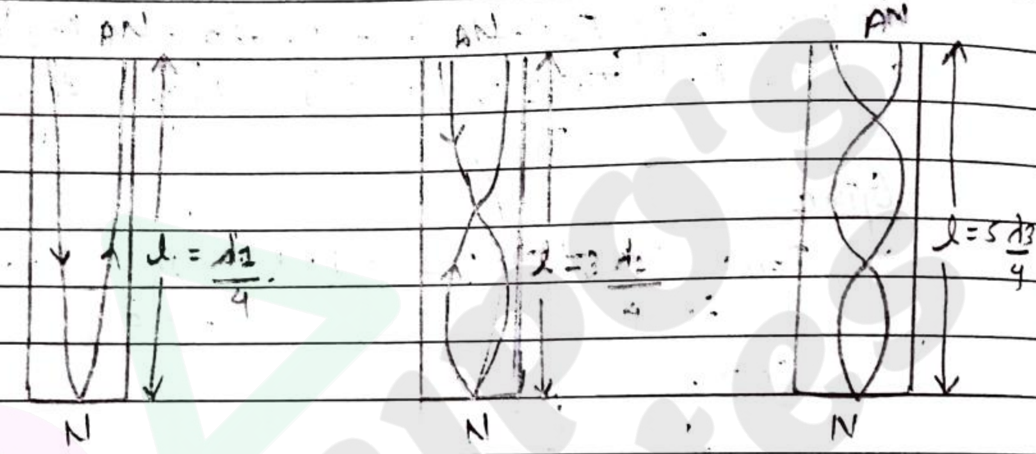

fig (i) First mode fig (ii) Second fig (iii) Third mode

If 'v' is the velocity of sound in air and f_1 and λ_1 be fundamental frequency and the corresponding wavelength then from fig (i)

$$l = \frac{\lambda_1}{4}$$

$$\lambda_1 = 4l$$

$$\therefore v = f_1 \lambda_1$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4l}$$

$$\therefore f_1 = \frac{v}{4l} = \text{fundamental frequency} \quad \text{--- (i)}$$

Similarly, if f_2 is the frequency and λ_2 is the corresponding wavelength in the same pipe/tube then from fig (2)

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$l = \frac{3\lambda_2}{4}$

or, $\lambda_2 = \frac{4l}{3}$

we have,

$$\therefore F_2 = \frac{v}{\lambda_2} = \frac{4}{\left(\frac{4l}{3}\right)} = 3\left(\frac{v}{4l}\right) = 3f_1 \text{ [from eqn (1).}$$

Thus $\boxed{F_2 = 3F_1}$

Similarly in the third mode of vibration in the same tube from fig (3)

$l = \frac{5\lambda_3}{4}$

or, $\lambda_3 = \frac{4l}{5}$

we have.

$$\therefore F_3 = \frac{v}{\lambda_3} = \frac{v}{\left(\frac{4l}{5}\right)} = 5\left(\frac{v}{4l}\right)$$

$$\therefore f_3 = 5f_1 \text{ [from eqn (9)]}$$

Similarly $\boxed{F_n = (2n-1) f_1}$

Thus in the closed organ pipe only odd multiples of fundamental frequency or produce / present.

* Note:

f_2 = frequency of second mode (or first overtone) or third harmonic.

f_3 = frequency of third mode (or second overtone) or fifth harmonic.

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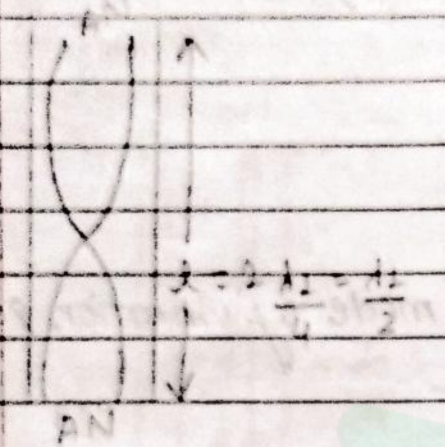
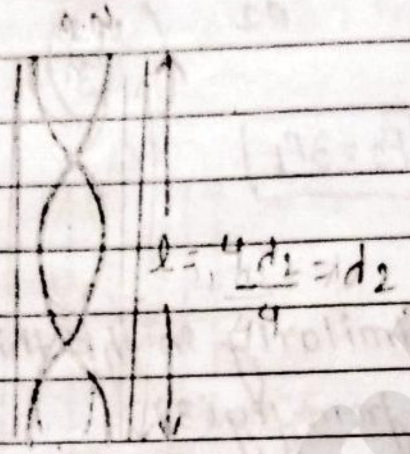
Modes of vibration in Opened pipe:-

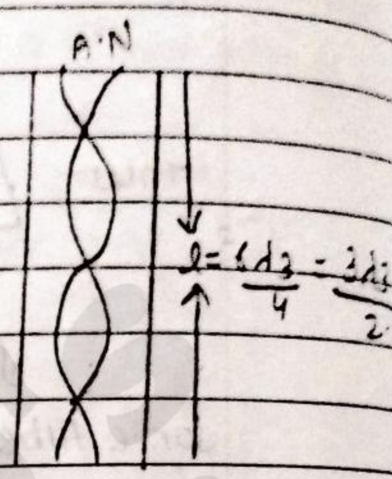
In open pipe during stationary wave formation anti-nodes are formed at two open's ends. Thus, in the ~~fundamental mode~~ first mode of vibration in an open organ pipe of length 'l' there must be 1 node and 2 anti-nodes as shown in fig ① below.



fig① first mode



fig② second mode



fig③ Third mode

If 'l' is the length of the open pipe and f_1 is the fundamental frequency with λ_1 is corresponding wave length then from fig (i)

$$l = \frac{\lambda_1}{2}$$

or, $\lambda_1 = 2l$

we have

$$\because v = f_1 \lambda_1 \Rightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{2l}.$$

or, $f_1 = \frac{v}{2l}$ — ① where f is fundamental frequency

Similarly in the second mode in the same open pipe from fig (ii)

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{l} = \frac{2 \times v}{2l} = 2f_1$$

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Thus, $f_2 = 2f_1$ -- (2).

Similarly in the third mode in the same pipe from fig (3)

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\left(\frac{2l}{3}\right)} = 3\left(\frac{v}{2l}\right) = 3f_1.$$

thus,

$f_3 = 3f_1$. —— (3) [from eqn 1].

$f_n = nf_1$, where $n = 1, 2, 3$.

Thus in the opened organ pipe only ~~even~~ multiples of fundamental frequency or product present.

Note:
f_2 = frequency of second mode (or first overtone) or second harmonic.

End correction in pipes (correction)

It is observed that the antinode is formed slightly above the open end of the tube. The distance of this antinode from the mouth of the tube is known as end correction.

According to Rayleigh if 'd' is internal diameter of the tube then the end correction at the open end will be x or $c = 0.3D$. Closed organ pipe has only 1 end correction. Hence, end correction is $x = 0.3D$ but in case of open organ pipe there are 2 end corrections x at each end correction

$\therefore 2x = 0.3D$.

End correction in resonance tube experiment:

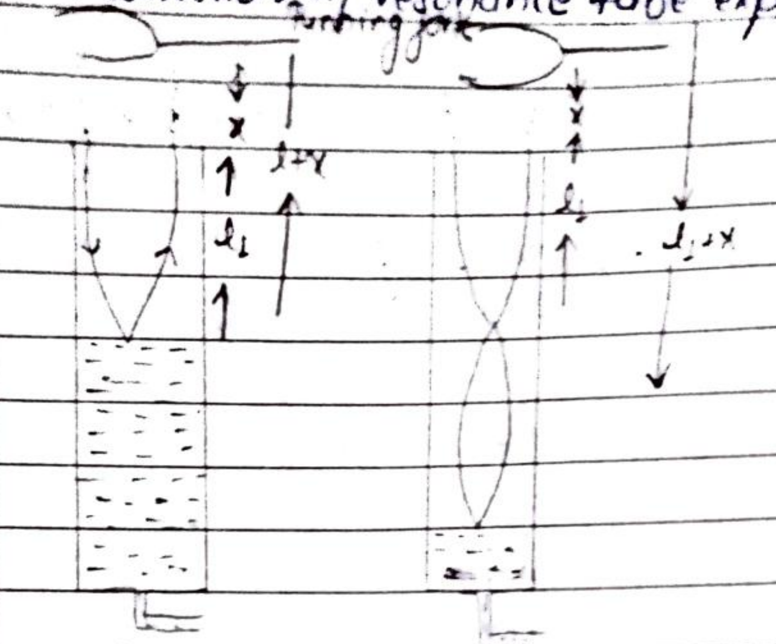

fig ① resonance   fig ⑪ resonance.

If l_1 and l_2 be the lengths of air column for first and second resonance respectively then

$$(l_1+x) = \frac{\lambda}{4} \quad \text{—— ① from fig ①}$$

and $$(l_2+x) = \frac{3\lambda}{4} \quad \text{—— ②}$$

eqn ② subtracting ① gives,

$$(l_2 - l_1) = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{2\lambda}{4} = \frac{\lambda}{2}$$

$$\therefore \lambda = 2(l_2 - l_1) \quad \text{—— ③}$$

$$\therefore v = f \cdot \lambda$$

$$\therefore \boxed{v = 2f(l_2 - l_1)}$$

Multiplying both sides of eqn ① with 3 eqn ③ we get,

$$3(l_1+x) = \frac{3\lambda}{4} \quad \text{— ⑤}$$

Equating eqn ② and eqn ⑤ we get

$$3l_1 + 3x = l_2 + x$$

$$2x = l_2 - 3l_1$$

$$\boxed{x = \frac{l_2 - 3l_1}{2}} \quad \text{— ⑥}$$

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or, $3l_1 + 3x = 3\lambda/4$

$- \quad l_2 + -x = 3\lambda/4$

$(3l_1 - l_2) + 2x = 0$

or, $2x = -3l_1 + l_2$

$\therefore x = \frac{l_2 - 3l_1}{2}$ --- (6)

Numerical

(39) In a resonance air column appartus, the first and secondary resonance position were observed at 18 cm and 56 cm respectively. The frequency of tunning fork used was 480 Hz. calculate the velocity of sound in air and correction of the tube.

Given,

In resonance tube experiment

$l_1 = 18$ cm = length of air column for first resonance.

$l_2 = 56$ cm = length of air column for second resonace.

$f = 480$ Hz = frequency of tuning fork.

(i) $v = ?$ velocity of sound in air at lap temperatur.

(ii) $x = ?$

soln,

(i) $V = 2f(l_2 - l_1)$

$= 2 \times 480 \times [56 - 18]$

$= 2 \times 480 \times 38$

$= 36480$ cm s^{-1}

$= \frac{36480}{100}$ ms^{-1}

$= 364.8$ ms^{-1}

(ii) End correction $(x) = ?$

$x = \frac{l_2 - 3l_1}{2}$

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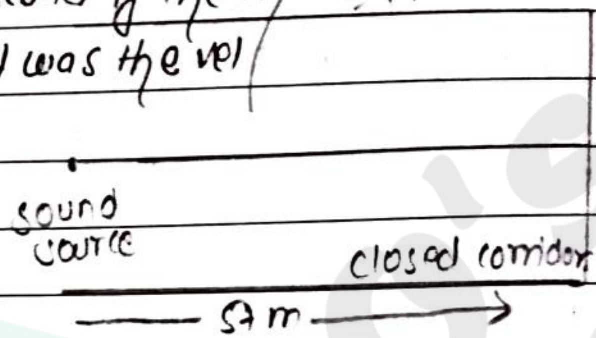
$= \frac{56 - 3\times18}{2}$

$= 1$ cm

57

(46) A man standing at one end of a closed corridor 57 m long blow a short blast on a whistle. He found that the time from the blast to the sixth echo was 2 second if the temperature was 17°C, what was the velocity of sound 0°C?.

temp $= 17°C = 290K = T_1$



Given,

$t = 2$ sec = time taken to hear six echoes.

$V_0 = ?$ = velocity of sound in air at 0°C.

Soln:

V_1 = velocity at temp T_1 = $\frac{\text{Total distance in 6 echoes}}{\text{time taken}}$

$= \frac{6\times(2\times57\text{ m})}{2}$

$= 342\ ms^{-1}$

Now,

$$\frac{V_1}{V_0} = \sqrt{\frac{T_1}{T_0}}$$

$$V_0 = V_1\sqrt{\frac{T_0}{T_1}}$$

$$\text{or} = 342\sqrt{\frac{273}{290}}$$

$= 331\ ms^{-1}$.

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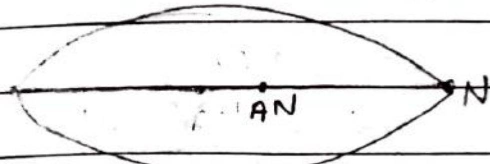
Modes of vibration in a streched string

For a strecthed string, stationary wave is formed with nodes at two fixed ends. fig (1), (2), 3 below show the first, second and third mode of vibration in a strethed string of length l.



$\longleftarrow l = \frac{\lambda}{2} \longrightarrow$

fig (i) first mode

$\longleftarrow l = \lambda_2 \longrightarrow$

fig (ii) second mode

fig (iii) Third mode $l = 3\lambda_3/2 \longrightarrow$

If ~~the~~ 'v' is the velocity of wave in the strecthed string then from fundamental frequency (f_1) will be

$$f_1 = \frac{v}{\lambda_1}$$

$$\therefore f_1 = \frac{v}{2l} \quad \text{---- eqn (1)}$$

Similarly, from fig (2) in second mode of vibration in the same strecthed string f_2 will be.

$$f_2 = \frac{v}{\lambda_2}$$

$$\text{or, } f_2 = \frac{v}{l}$$

$$\text{or, } f_2 = \frac{2v}{2l}$$

$$\therefore f_2 = 2f_1 \text{ [from (i)]}$$

Similarly, from fig (3) in third mode of vibration the same streatched string f_3 will be:-

$$f_3 = \frac{v}{\frac{3\lambda}{3}} = 3\left(\frac{v}{2l}\right) = 3f_1$$

$$\therefore f_3 = 3f_1$$

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Thus, $f_n = n f_1$

In the stretched string both the even and odd harmonic are present.

* Velocity of wave in a stretched string:

The frequency of a vibration of a stretched string depends on the mass per unit length (m) and the tension 'T' (in the force on string).

Let

$V = k\, m^x \cdot T^y$ — ① where k = dimensionless constant.

Writing the dimensions on both sides of equation(1) we get,

$[M^0 L T^{-1}] = [M L^{-1}]^x [M L T^{-2}]^y$

$[M^0 L T^{-1}] = [M^{x+y} \cdot L^{-x+y} T^{-2y}]$

On comparing, we get

$x + y = 0$ – ② and $1 = -2y$ --- ④

or, $-x + y = 1$ – ③ $\quad \therefore y = 1/2$

putting value of y in eqn ①

$x = 1/2$.

Now,

Therefore, eqn ① can be written as

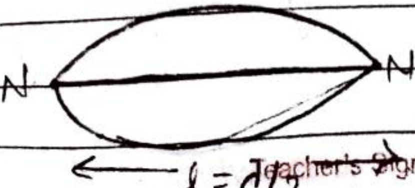
$V = k\, m^x \cdot T^y$

$V = k m^{-1/2} \cdot T^{1/2} = k \dfrac{\sqrt{T}}{\sqrt{M}}$

$$\boxed{\therefore V = k\sqrt{\frac{T}{M}}} \quad --- ⑤$$

In fundamental mode of vibration of stretched string of length 'l'

$\lambda = 2l$ ---- ⑥



$\therefore V = f \cdot \lambda$

$l = \lambda/2$

$$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Thus $\boxed{f = \frac{1}{2l}\sqrt{\frac{T}{m}}}$ --- (7)

Note: If L is the length of the wire or string and 'M' is its mass then its density will be ;

$$\rho = \frac{Mass}{Volume}$$

$$= \frac{M}{V} = \frac{M}{\pi r^2 L}$$

$$= \frac{M}{\pi r^2}$$ where $m = \frac{M}{L}$ = mass per unit length.

$\therefore m = \pi r^2 \rho$. — (8)

from eqn (8) eqn (7) gives.

$\therefore \boxed{f = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}}$

(i) $f \propto \sqrt{T}$

(ii) $f \propto \frac{1}{\sqrt{m}}$

— 14th poush, Wednesday. Date: / /

Numerical. (2076 set c).

(96) A source of sound of frequency 550 Hz emits wavelength of 60 cm in air at 20°C. What would be the wavelength of sound from the source in air at 0°C?

Given,

$f = 550\ Hz$ (frequency)

$\lambda_1 = 60\ cm = 0.60\ m$ (waves of wavelength)

$T_1 = 293\ K$

$\lambda_2 = ?$

at $T_2 = 273\ K$.

Now
soln,

$$\frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2}$$

or, $\lambda_2 = \lambda_1 \left(\frac{v_2}{v_1}\right)$

$$= \lambda_1 \sqrt{\frac{T_2}{T_1}} \quad [\because v \propto \sqrt{T}]$$

$$= 0.60 \sqrt{\frac{273}{293}}$$

$$= 0.60 \sqrt{9.31 \times 10^{-1}}$$

$$= 1.83 \times 10^{-1}$$

(37) What is the differences between the speed of longitudinal waves in air at 27°C and at -13°C? What is the speed at 0°C?

Given,

$T_1 = 300\,K$

$T_2 = 260\,K$.

(i) $(v_1 - v_2) = ?$

(ii) $v_0 = ?$

soln;

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

or, $$\frac{v_1}{v_2} = \sqrt{\frac{300}{260}}$$

or, $$\frac{v_1}{v_2} = 1.07$$

or, $v_1 = 1.07\,v_2$

or, $(v_1 - v_2) = 1.07\,v_2 - v_2 = 0.07\,v_2$ —— (1).

ii) $$v_0 = \sqrt{\frac{\gamma P}{\rho}} \quad \text{at STP}$$

$$= \sqrt{\frac{1.4 \times 1.01 \times 10^5}{1.293}}$$

$= 332\,ms^{-1}$

Now,

$$\frac{v_0}{v_2} = \sqrt{\frac{273}{T_2}}$$

$$v_2 = v_0\sqrt{\frac{T_2}{273}} = 332\sqrt{\frac{260}{273}}$$

$= 324\,ms^{-1}$

$\because$ from eqn ①

$$v_1 - v_2 = 0.07 v_2$$

$$v_1 - 324 = 0.07$$

$$v_1 = 0.07 v_2 + 324$$

$$v_1 = 324.07 v_2.$$

(38) (28) At what temperature the velocity of sound in air is increased by 50% to that at 27°C?

⇒ Soln;

Here

$T_2 = ?$ (At which

$$v_2 = v_1 + \frac{50}{100} v_1$$

$$= 1.5 v_1$$

where v_1 = velocity of sound in air.

$T_1 = 300\ K$

Now,

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

or, $$\frac{v_1}{1.15 v_1} = \sqrt{\frac{300}{T_2}}$$

squaring on both sides,

$$\left(\frac{v_1}{1.5 v_1}\right)^2 = \left(\sqrt{\frac{300}{T_2}}\right)^2$$

$$\left(\frac{1}{1.5}\right)^2 = \frac{300}{T_2}$$

or, $T_2 = 300 \times (1.5)2$

$T_2 = 300 \times 2.25$

$= 675\ K.$

So, at 675 K temperature the velocity of sound in air is increased, Page:

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(39) In a resonance air vaccum appartus, the first and second resonance position were observed at 18 cm and 56 cm respectively. The frequency of tuning fork used was 480 Hz. calculate the velocity of sound in air and end correction of the tube.

→ given

In a resonance tube experiment

$l_1 = 18$ cm (first resonance position)

$l_2 = 56$ cm (second resonance position)

$f = 480$ Hz (frequency)

(i) $v = ?$

(ii) end correction $(x) = ?$

Now,

(i) $V = 2f(l_2 - l_1)$

$= 2 \times 480 (56 - 18)$

$= 2 \times 480 \times 38$

$= \frac{36480}{100}$ ms^{-1}

$= 364.8$ ms^{-1}

(ii) end correction $(x) = \frac{l_2 - 3l_1}{2}$

$= \frac{56 - 3 \times 18}{2}$

$= \frac{2}{2}$

$= 1/100$ cm

$= 0.01$ m.

Therefore the velocity of ~~tube~~ sound in air is $364.8\ ms^{-1}$ and end correction of the tube is 0.01 m.

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(40) Calculate the bulk modulus of a liquid in which longitudinal waves with frequency of 250 Hz have the wavelength of 8m if the density of liquid is 900 kgm-3.

→ Given,

$k = ?$ (bulk modulus of a liquid)

$f = 250\,Hz$ (frequency)

$\lambda = 8\,m$ (wavelength)

$\rho = 900\,kgm^{-3}$ (density of liquid)

Solution,

In liquid

$$v = \sqrt{\frac{E}{\rho}}$$

$$= \sqrt{\frac{k}{\rho}}$$

$$f \cdot \lambda = \sqrt{\frac{k}{\rho}}$$

squaring on both sides we get

$$(f \cdot \lambda)^2 = \left(\sqrt{\frac{k}{\rho}}\right)^2$$

$$f^2 \cdot \lambda^2 = \frac{k}{\rho}$$

$$k = \rho f^2 \cdot \lambda^2$$

$$= 900 \times (250)^2 \times 8^2$$

$$= 900 \times 62500 \times 64$$

$$= 3600000000$$

$$= 3.6 \times 10^9 \ N/m^{-2}$$

Therefore, the bulk modulus of a liquid is $3.6 \times 10^9\ N/m^{-2}$

- 18th poush, Sunday.

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* Laws of vibration of stretched string:-

For a stretched string, the frequency of transverse vibration is directly porportional to the square root of Tension 'T' and inversely proportional to the ~~mass~~ sq. rt. square root of mass per unit length 'm' and to the vibrating length 'l'.

Thus,

or, $f \propto \sqrt{T}$ — (i)

or, $f \propto \frac{1}{l}$ — (ii)

or, $f \propto \frac{1}{\sqrt{m}}$ — (iii)

combining eqn (i), (ii) and (iii) we get.

$$f \propto \frac{1}{l}\sqrt{\frac{T}{m}}$$

$$\therefore f = \frac{K}{l}\sqrt{\frac{T}{m}} \quad \text{— (iv)}$$

Thus, value of constant (k) = $\frac{1}{2}$ for fundamental frequency

therefore eqn (iv) becomes

$$\boxed{\therefore f = \frac{1}{2l}\sqrt{\frac{T}{m}}}$$

for n^{th} mode of frequency, $K = n/2$

$$\boxed{\therefore f_n = \frac{n}{2}\sqrt{\frac{T}{m}}}$$ (Numerical)

Determination of unknown frequency of sonometer:-

Fig (1) below shows the basic diagram of a sonometer.

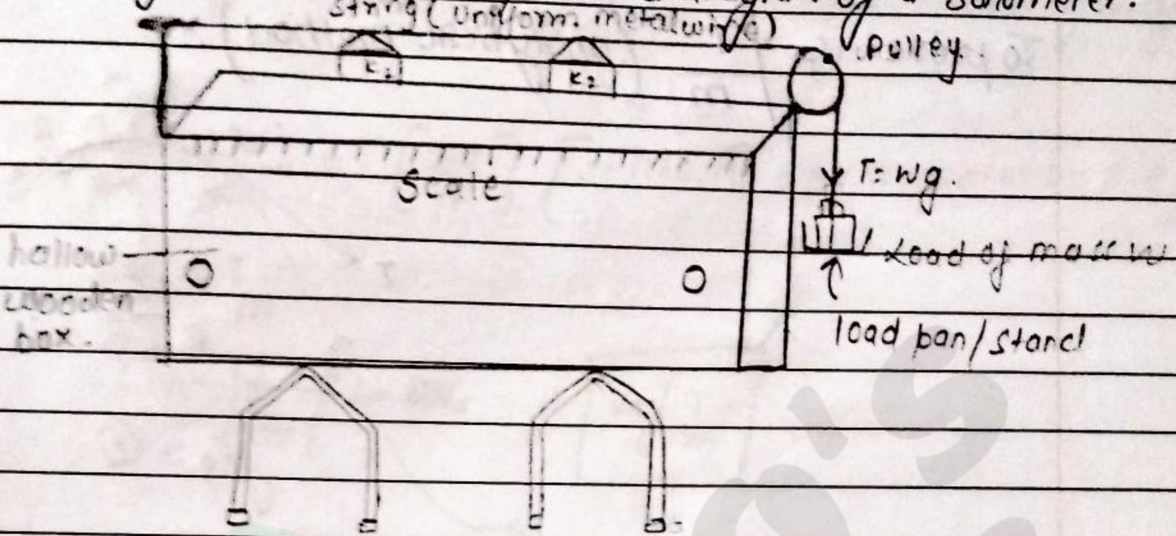

fig:- (1)

It has a hollow wooden box on which the uniform metal wire whose one end is fixed at the other end passes over a pulley. The free end of wire is connected to a pan to load for tension. Knife edges 'k_1' and 'k_2' can be slided on the upper face of the wooden box on scale.

The given turning fork of unknown frequency is vibrated and its ~~stem~~ ~~stamp~~ stem is placed on the upper phase of wooden box. By sliding the position of k_1 and k_2 the maximum vibration in the stretched wire is obtained. Now the distance (l) between k_1 and k_2 is noted. The radius 'r' of wire is measured with screw guaze. Now the frequency is calculated by using the formula:-

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\boxed{f = \frac{1}{2l}\sqrt{\frac{w \cdot g}{\pi r^2 \rho}}}$$ where ρ = density of the metal of wire

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w = Load plance on pan to apply tension.
g = acceleration due to gravity.

To prove $V = \sqrt{\frac{T}{m}}$ (analytical method) :-

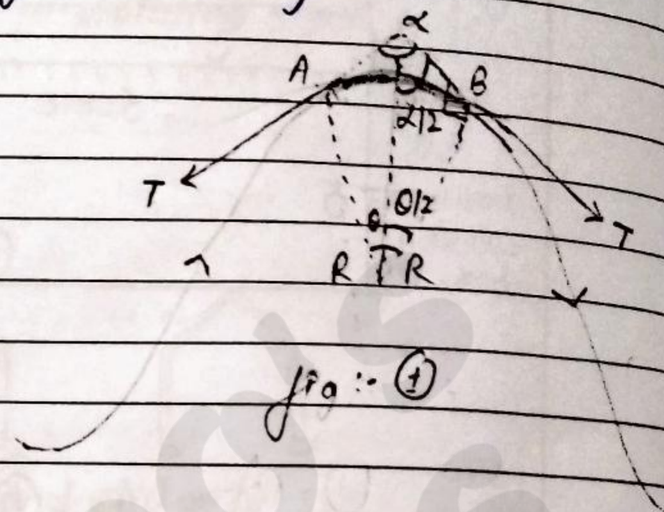

fig :- ①

fig ① shows the segment of a streatched string under tension T, carrying transverse wave with velocity v.

Let $AB = \Delta x$ is small element length of the string the resultant tension on this segment provides the centripetal force (f). Thus,

$$f = \sqrt{T^2 + T^2 + 2 \cdot T \cdot T \cos\alpha}$$

$$f = \sqrt{2T^2(1 + \cos\alpha)}$$

$$f = \sqrt{2T^2\left(1 + \cos\frac{2\alpha}{2}\right)}$$

$$f = \sqrt{2T^2\left[\cancel{1} + 2\cos^2\frac{\alpha}{2} - \cancel{1}\right]} \quad [\cos 2x = 2\cos^2 x - 1]$$

or, $f = 2T\cos\frac{\alpha}{2}$

$= 2T\cos(90^\circ - \theta/2)$ $[\because \theta/2 + \alpha/2 = 90^\circ]$

$= 2T\sin\theta/2$

$\simeq \cancel{2}T \cdot \frac{\theta}{\cancel{2}}$ $\left[\because \text{when } \theta \longrightarrow 0^\circ \text{ the } \sin\left(\frac{\theta}{2}\right) \simeq \theta/2\right]$

$= T \cdot \theta$

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$$\frac{(\text{mass of element}) \cdot v^2}{R} = T \cdot \theta$$

$$\therefore \frac{(m\Delta x) v^2}{R} = T \cdot \theta$$ (where, m = mass per unit length of string)

$$\therefore \frac{m\Delta x \cdot v^2}{R} = T\left[\frac{\Delta x}{R}\right]$$ [∴ Arc $\widehat{AB} = \Delta x =$ Radius $\theta = R \cdot \theta$]

$$V^2 = \frac{T}{m}$$

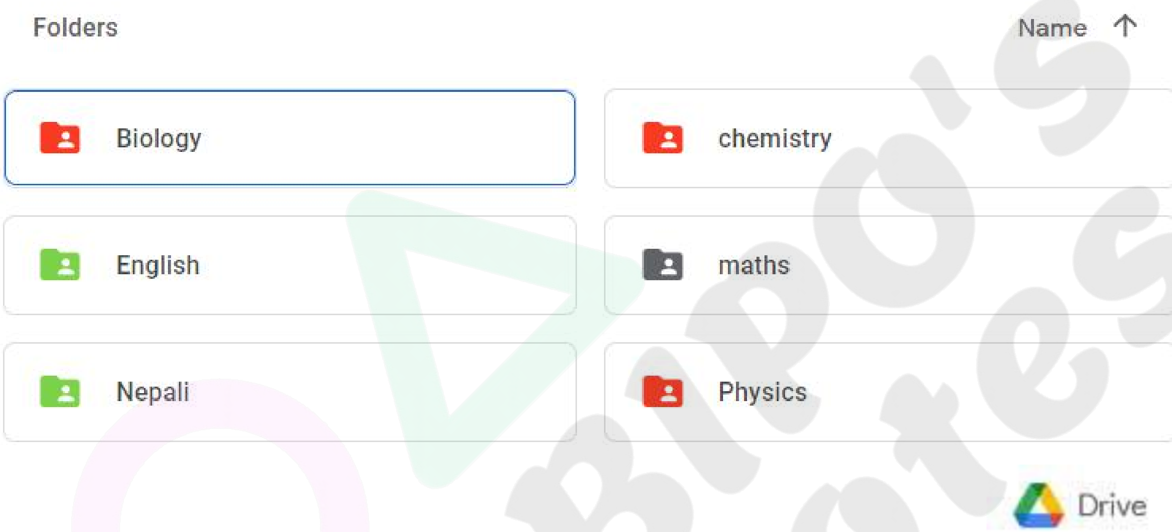
$\therefore$ ~~$v = \sqrt{\frac{T}{m}}$ { ∴ Arc~~ $\therefore \boxed{V = \sqrt{\frac{T}{m}}}$

* Verification of laws of vibration of stretched string:-

Bipin Khatri

(Bipo)

Class 12 complete notes and paper collection.



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