06/17 Sunday {chapter : 3}

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Quantization of Energy :-

* Bohr's theory of hydrogen atom:-

- Postulates:-

(1) Electron can revolve round the nucleus only in permitted circular orbits. Permitted orbits are those in which angular momentum is integral multiply of $\frac{h}{2\pi}$. Thus in nth permitted orbit,

$$\boxed{m v_n r_n = n \frac{h}{2\pi}} \quad \text{--- (1)}$$

(2) Electron gains or losses energy only when it jumps from one orbit to another orbit. The energy difference between these two orbits is equal to the energy of a photon i.e.

$$\boxed{E_{n_2} - E_{n_1} = hf} \quad \text{--- (2)}$$

Theory:-

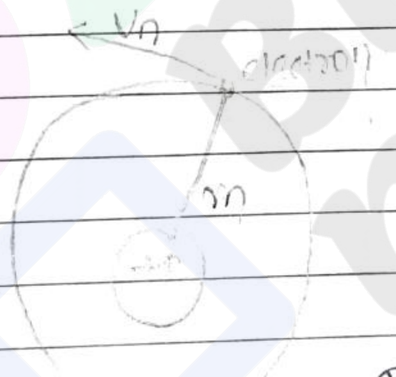

fig (1)

Let the electron of hydrogen atom moves in n^{th} permitted orbit of radius r_n with velocity v_n as in fig (1)

Then in the orbit,
Centripetal force = electron static force.

$$\frac{m v_n^2}{r_n} = \frac{1}{4\pi\epsilon_0} \frac{e \cdot e}{r_n^2} \quad \{\because |q_1| = |q_2| = e\}$$

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$$\boxed{\therefore v_n^2 = \frac{e^2}{4\pi\epsilon_0 m r_n}} \quad \text{— (3)}$$

from postulate (1),

$$\boxed{r_n = \frac{nh}{2\pi m v_n}} \quad \text{— (4)}$$

from eqn (4) and (3)

$$v_n^{\cancel{2}} = \frac{e^2}{4\pi\epsilon_0 \cancel{m}}\left(\frac{2\pi \cancel{m} \cancel{v_n}}{nh}\right)$$

$$\boxed{\therefore v_n = \frac{e^2}{2\epsilon_0 nh}} \quad \text{— (5)}$$

from eqn (5), eqn (4) gives

$$r_n = \frac{nh}{2\pi m}\left(\frac{2\epsilon_0 nh}{e^2}\right)$$

$$\boxed{\therefore r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}} \quad \text{— (6)}$$

putting the values of constants in eqn (6), we get.

$$r_n = (0.529 \times 10^{-10}) n^2$$

$$\therefore \boxed{r_1 = 0.529 \times 10^{-10} m \Rightarrow 0.529 A^\circ}$$

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→ Total energy of electron in the nth orbit is the sum of kinetic and electrostatic potential energy.

Thus,

$$E_n = \frac{1}{2} m v_n^2 + \frac{1}{4\pi\epsilon_0} \cdot \frac{e(-e)}{r_n}$$ [In case of scalar charge $e = -e$

$$= \frac{1}{2} \cancel{m} \left[\frac{e^2}{4\pi\epsilon_0 \cancel{m} r_n}\right] - \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r_n}$$ (from 3rd)

$$= \frac{e^2}{4\pi\epsilon_0 r_n}\left[\frac{1}{2} - 1\right]$$

$$= \frac{-e^2}{8\pi\epsilon_0 r_n}$$

$$= \frac{-e^2}{8\cancel{\pi}\epsilon_0}\left(\frac{\cancel{\pi} m e^2}{\epsilon_0 n^2 h^2}\right)$$ [value of r_n, from (6)]

$$\therefore \boxed{E_n = -\frac{me^4}{8\epsilon_0^2 n^2 h^2}} \quad \text{— (7)}$$

Putting values of constants in eqn (7), we get-

$m = 9.1 \times 10^{-34}$ kg

$e = 1.6 \times 10^{-19}$ C

$\epsilon_0 = 8.85 \times 10^{-12}$

$h = 6.625 \times 10^{-34}$ Js.

$$E_n = \frac{(-2.18 \times 10^{-18})}{n^2} \text{ Joules}$$

$$= \frac{-2.18 \times 10^{-18}}{n^2 (1.6 \times 10^{-19})} \text{ eV}$$ [$\because$ 1eV = 1.6×10^{-19} J]

$$\therefore \boxed{E_n = \frac{-13.6}{n^2}\ eV} \quad — 8$$

* Note:-

① The (−ve) sign in eqn (8), indicates that the upper orbits have more energy than the lower orbit.

$\boxed{\text{when} = \infty,\ \text{Energy} = 0}$ (i.e highest energy)

- Gap decreases: Energy increases as going to upper orbits.

② Also (−ve) sign indicates the attraction on electron by the proton in the nucleus.

* Formula for the wavelength of emitted spectral line:-

from postulate ②,

$$E_{n_2} - E_{n_1} = hf$$

or, $$-\frac{me^4}{8\epsilon_0^2 n_2^2 h^2} - \left(\frac{-me^4}{8\epsilon_0^2 n_1^2 h^2}\right) = hf$$

or, $$\frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{hc}{\lambda} \quad (\because c = f\lambda)$$

or, $$\frac{1}{\lambda} = \frac{me^4}{8\epsilon_0^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore \boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \quad — 9$$

where, $R = \frac{me^4}{8\epsilon_0^2 ch^3} = \boxed{1.097 \times 10^7\ m^{-1}}$ and 'R' is called

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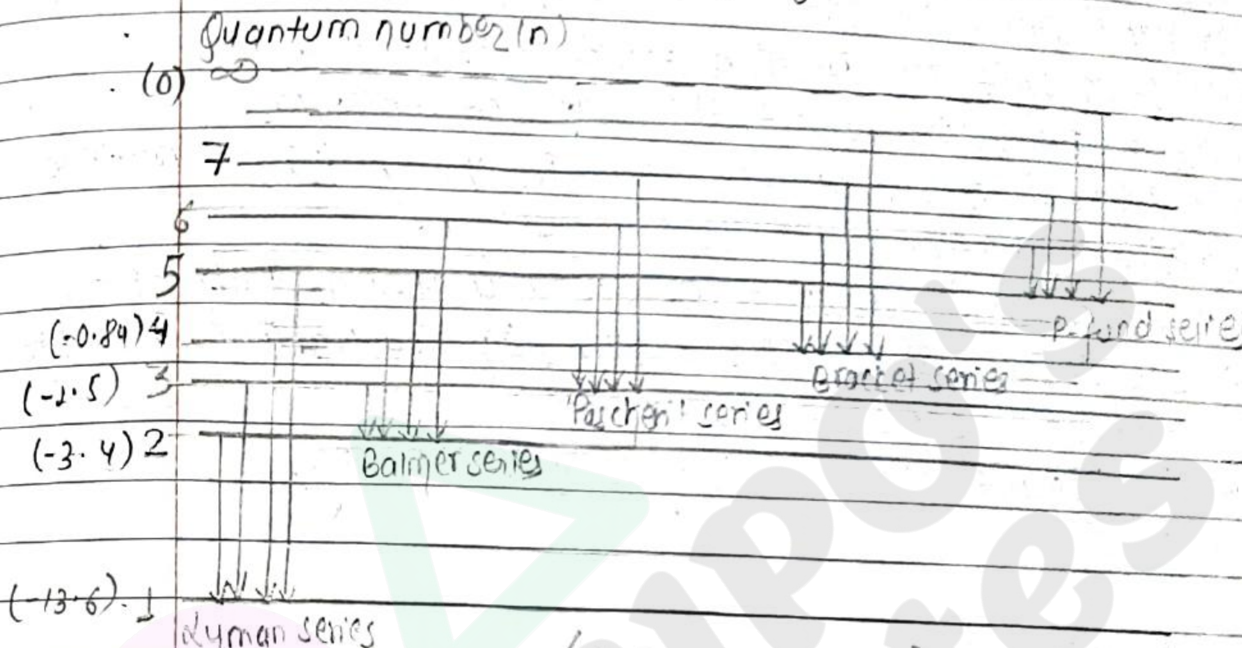
Rydberg constant.
Also, $\frac{1}{\lambda}$ is also known as wave number.

* Energy level diagram of Hydrogen Atom:-



(Emission spectra)

fig: 1

We know that, the wavelength of a spectral line of hydrogen atom is given by,

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)}$$ where, R = Rydberg constant. $(1.097 \times 10^{7} m^{-1})$

Corresponding to different values of n_1 and n_2 we get different wavelengths of photons (emitted or absorbed)
For example, the spectral lines for which
$n_1 = 1$; $n_2 = 2, 3, 4, 5 \ldots$ give Lyman series.

Similarly $n_1 = 2$; $n_2 = 3, 4, 5, 6 \ldots$ give Balmer series

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as shown in fig (i)

Similarly, Paschen, Bracket, P-fund series and so on.

* Limitations of Bohr's theory:- (Demerit's)

(i) This theory does not tell about the elliptical orbits.
(ii) It doesnot explain fine structure of spectral line.
(iii) It doesnot tell about the shape of molecule.
(iv) Bohr's theory could not explain the spectra of multi-electron system.
(v) It explains the spectrum of only H and H like atom.
(v) It doesnot explain the splitting of spectral lines in electric and magnetic fields
(vi) It doesnot follow de-Broglie & Heisenberg.

Merit's:-

(1) It explain line spectrum of H atom.
(2) It explains the stability of atom.
(3) It helps to calculate radius, velocity energy of monoelectron system like H atom, He^+ etc.

* Excitation potential and Excitation energy:-

Excitation energy :-

the excitation energy of an orbit (or state) of an atom is defined as the energy required to jump the electron from ground state to that orbit/state

For eg:- excitation energy of 3rd state will be

$= E_3 - E_1$

$= \frac{-13.6\ ev}{(3)^2} - \left[\frac{-13.6\ ev}{(1)^2}\right]$

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$= -1.51 + 13.60 ev$
$= 12.09 ev.$

Excitation potential is defined as the electric potential needed to accelerate the electron in other to accelerate the electron of the atom from ground state to the given state.

Thus in above example excitation potential of 3rd state of H atom will be

$$= \frac{(E_3 - E_1)}{e}$$

$$= \frac{12.09 ev}{e}$$

$$= 12.09 \text{ volt}$$

* Ionization potential :-

The minimum amount of energy required to completely free of an electron from its atom (i.e ionized) is called Ionization potential of that atom.

Thus for H atom

$$I.P = E_\infty - E_1$$

$$= -\frac{13.6 ev}{(\infty)^2} - \left[-\frac{13.6 ev}{(1)^2}\right]$$

$$= 0 + 13.6 ev$$

$$= 13.6 ev$$

$$\left[H + 13.6 ev \rightarrow H^+ + e^-\right]$$

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De-Broglie (hypothesis) theory:-

In 1924 Louis de-Broglie suggested the dual nature of matter. According to him, the moving particle sometimes behaves as a particle and sometimes as a wa[ve] associated with a moving particle. These waves are called the de-Broglie waves and the corresponding wavelength called (de-Broglie wavelength) is given by

$$\boxed{\lambda = \frac{h}{p}}$$; p = linear momentum = mv.

Proof: From planck's theory of radiation, energy of pho[ton] is

$$E = hf = \frac{hc}{\lambda} \quad -(1)$$

And from Einstein's mass energy relation

$$E = mc^2 = mc \cdot c = pc \quad -(2)$$

Equating (1) and (2) we get

$$\frac{hc}{\lambda} = p \cdot c$$

$$\boxed{\lambda = \frac{h}{p}}$$

$$\frac{hc}{\lambda} = pc$$

$$\lambda = \frac{h}{p}$$

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* de-Broglie wavelength in terms of kinetic energy:-

$$\text{kinetic energy} = E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{m}{m}\right)v^2$$

$$\therefore E_K = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

$$\therefore p = \sqrt{2mE_K} = \frac{h}{\lambda} \quad \left[\because \lambda = \frac{h}{p}\right]$$

$$\therefore \boxed{\lambda = \frac{h}{p}} = \frac{h}{\sqrt{2mE_K}}$$

-(1)

X-rays:- [Discovered by Roetgen in 1895 -1901 Nobel prize]

When fast moving electrons strike a metal surface of high melting point and high atomic number (like tungsten), beams of photons are emitted from the metal surface. These photon beams are called x-rays.

P.T.O

Production of X-rays:-

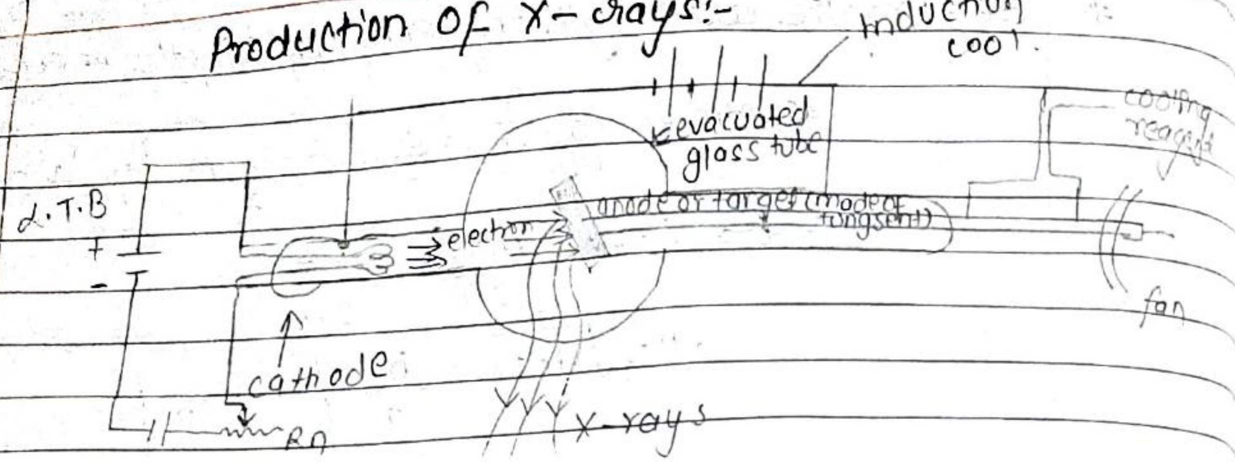

Fig (f) coolidge tube

fig(f) shows basic diagram of coolidge tube for the modern method of production of X-rays. Electrons emitted from filament (F) by thermionic emission and accelerated by very high P.D between anode and cathode (F) strike anode surface. X-rays are emitted from anode (or target). Only very small fraction of energy of electron striking anode is converted into X-rays while the remaining part is converted into heat due to which anode gets heated. In order to save anode from overheating. cooling arrangement (like liquid flow in spiral and fan)

* Quality of X-rays

The quality of X-rays depends on its frequency. The X-rays of max. frequency and min wavelength are emitted when total kinetic energy of electron striking anode is converted into X-rays

Then,

$e \cdot V = hf_{max} = \frac{hc}{\lambda_{min}}$; where $V = P.D$ {between anode and cathode}

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$$\therefore \lambda_{min} = \frac{hc}{eV}$$

or, $$\lambda_{min} = \frac{12400 \times 10^{-10}}{V} \text{ meter}$$

$$\boxed{\lambda_{min} = \frac{12400}{V} A^\circ}$$

* Propertise of X-rays:-

1. X-rays are electromagnetic waves moving with velocity of light in vaccum.
2. X-rays have short wavelength ($0.1 A^\circ$ to $100 A^\circ$)
3. X-rays ~~are not affected~~ deflected in electric and magnetic fields.
4. X-rays can pentrate blood, flesh but cannot penetrate thick metals, stones, bones etc.
5. X-rays affected photographic plates.
6. X-rays can eject photo electron from certain metals.
7. X-rays produce fluorescence on flurescent metals like ZnS.

* Application of X-rays:-

1. In medical science.
2. In indusky
3. In detective department
4. In engineering
5. In mint
6. In research.

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Bragg's law:-

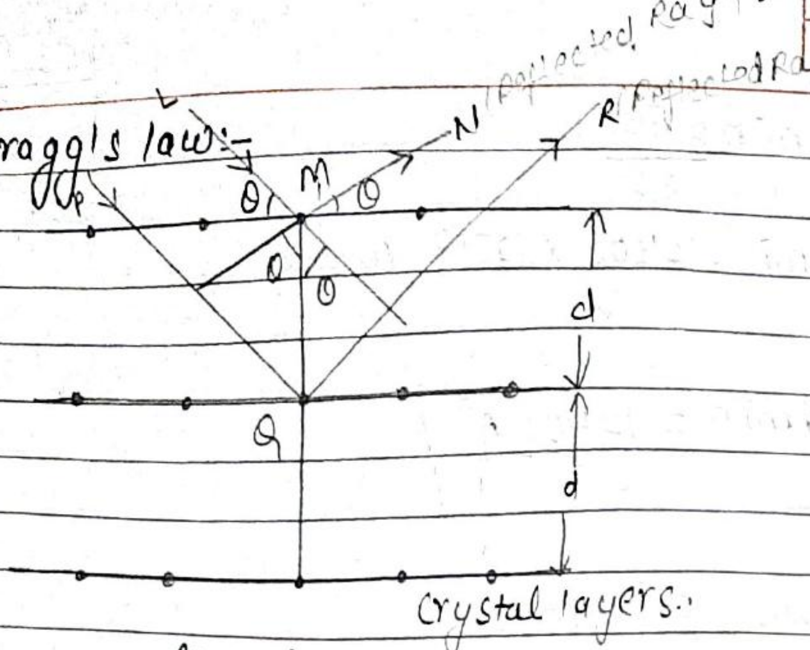

fig:-(2)

In 1913, father and son sir William Bragg and Sir lawrence Bragg worked out a mathematical relation to determine interatomic distance form X-ray diffraction pattern. This relation is called Bragg's law or Bragg's and equation is given by

$$2d\sin\theta = n\lambda$$

Bragg's law is possible because the inter-molecular spacing is of the order of wavelength of X-rays.

Proof of Bragg's law:-

In fig (2) a beam of X-ray of wavelength λ is incident at glancing angle (θ) and two rays LM and PQ are refled by two nearest layers of crystal separated by distance (d.)

The total path difference between upper and lower reflected rays is

$$HQ + QK = d\sin\theta + d\sin\theta = 2d\sin\theta.$$

If the path difference is integral multiple of wave length λ, then

$$\boxed{2d\sin\theta = n\lambda} \; ; n = 1, 2, 3$$

(order of reflection)

* Numericals [X-rays]

① An x-ray tube operates at 18 kV. Find the maximum speed of electron strike anode.

⇒ Soln;
Here,

$m = 9\times10^{-28}$ gm = mass of electron $= 9\times10^{-31}$ kg

$e = 4.8\times10^{-10}$ e.s.u $= 1.6\times10^{-19}$ columb

1 esu $= 3.33\times10^{-10}$ columb.

Now,

$eV = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{2eV}{m}}$

$= \sqrt{\frac{2\times1.6\times10^{-19}\times18000}{9\times10^{-31}}}$

$= \sqrt{\frac{57600\times10^{-19}}{9\times10^{-31}}}$

$= \sqrt{6400\times10^{-19+31}}$

$= \sqrt{6400\times10^{12}}$

$= \sqrt{6.4\times10^{15}}$

$\therefore v = 8\times10^{7}\ ms^{-1}$ #

② If the P.D across X-ray tube is 12.4 kV and current through it is 2 mA, calculate

(i) $\frac{N}{t}$ = Number of photon striking anode per second.

(ii) speed (iii) shortest wavelength (λ_{min})

⇒ Given,

$P.D(V) = 12.4\,kV \Rightarrow 12.4\times10^3$ volt.

Current $(I) = 2\,mA \Rightarrow 2\times10^{-3}$ Ampere.

Now,

(i) $I = \frac{q}{t} = \frac{N \cdot e}{t}$

or, $\frac{N}{t} = \frac{I}{e} = \frac{2\times10^{-3}}{1.6\times10^{-19}} \Rightarrow 1.25\times10^{16}$ electrons/sec

(ii) $eV = \frac{1}{2}mv^2$

$\therefore v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2.4\times10^{7}}{9.1\times10^{-31}}}$

$= \sqrt{\frac{39.68\times10^{-16}}{9.1\times10^{-31}}}$

$= \sqrt{4.3\times10^{15}}$

$\Rightarrow \boxed{6.5\times10^{7}}$ m/s #.

(iii) $\lambda_{min} = \frac{12400}{V}\,A^0 = \frac{12400}{12.4\times10^3} \Rightarrow 1\,A^0$ #.

(3) An X-ray tube operates at 13.6 KV. what is the frequency of the most energetic X-ray produce.

⇒ Given,

$V = 13.6\ KV \Rightarrow 13.6 \times 10^{3}$ volt.

we know,

$eV = hf_{max}$

$\therefore f_{max} = \frac{eV}{h} = \frac{1.6 \times 10^{-19} \times 13.6 \times 10^{3}}{6.625 \times 10^{-34}}$

$= \frac{21.76 \times 10^{-16}}{6.625 \times 10^{-34}}$

$= (3.28 \times 10^{18})\ Hz$ #.

(4) In X-ray tube, V = 20,000 volt. calculate (i) resultant energy of energy electron (ii) λ_{min} (iii) $h = 6.625 \times 10^{-34}$ JS

⇒ Given

$V = 20{,}000$ volt

(i) resultant energy of electron ⇒

$K.E = \frac{1}{2} mV^2 = e.V$

$= 1.6 \times 10^{-19} \times 20{,}000 \Rightarrow \boxed{3.2 \times 10^{-15}}$

(ii) $\lambda_{min} = \frac{12400}{V} A^0 = \frac{12400}{20{,}000} A^0$

$\Rightarrow 6.2 \times 10^{-1} A^0$

OR, $6.2 \times 10^{-1} \times 10^{-10}$ m

$\Rightarrow \boxed{6.2 \times 10^{-11} \text{ meter}}$

(4) The spacing of atomic number of crystal is 1.1×10^{-10} m where monochromatic X-rays are incident at glacing angle 50^0. The first order image is produced. Calculate wave length and glancing angle for second order.

⇒ Soln:
Here

$d = ?$
glancing angle for 2nd order $(\theta_2) = ?$

From Bragg's law,
∴ For nth order
$2d \sin \theta_n = n\lambda$.

∴ For first order
$d = 1.1 \times 10^{-10}$ m
$\theta_1 = 50^0$ for $n = 1$.
$\lambda = ?$

$2d \sin \theta_1 = n\lambda$
or, $2 \times 1.1 \times 10^{-10} \times \sin 50^0 = 1 \times \lambda$
or, $\lambda = 1.68 \times 10^{-10}$ meter

∴ For 2nd order
$2d \sin \theta_2 = 2\lambda$ [for $n = 2$ & $\lambda = 1.68 \times 10^{-10}$ m]
or, ~~$2 \times 1.1 \times 10^{-10} \times \sin$~~ 50^0
or, $\sin \theta_2 = \frac{\cancel{2}\lambda}{\cancel{2}d}$

or, $\theta_2 = \left(\frac{\lambda}{d}\right) = \sin^{-1}\left[\frac{1.68 \times \cancel{10^{-10}}}{1.1 \times \cancel{10^{-10}}}\right]$

or, $\theta_2 = \sin^{-1}(1.5)$

* Numericals.

(1) obtain the de Broglie wavelength of neutron of kinetic energy 150 e.v. (mass of neutron = 1.675×10^{-27} kg, planck's constant = 6.6×10^{-34} JS, 1eV = 1.6×10^{-19} J)

=) Soln;

For neutron

kinetic energy (E_k) = 150 eV => $150 \times 1.6 \times 10^{-19}$ J.

Mass (m) = 1.675×10^{-27} kg.

$h = 6.6 \times 10^{-34}$ Js.

de Broglie wave length (λ) = ?

we know,

$$\lambda = \frac{h}{\sqrt{2mE_k}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 150 \times 1.6 \times 10^{-19}}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{804 \times 10^{-46}}}$$

$$= \frac{6.6 \times 10^{-34}}{2.83 \times 10^{-22}}$$

$= (2.33 \times 10^{-12})$ meter #

Q) calculate the wavelength of an electron which has been accelerated through a potential difference of 200 V. Ta mass of the electrons as 9.1×10^{-31} kg and planck's constant as 6.6×10^{-34} Js.

⇒ Given.

Potential difference (V) = 200 Volt.
Mass of electron (m) = 9.1×10^{-31} kg.
$h = 6.6 \times 10^{-34}$ Js.
$e = 1.6 \times 10^{-19}$ coulomb.
$\lambda = ?$

we know,

$$\lambda = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{2 \cdot m \cdot (e \times V)}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 1.6 \times 10^{-19} \times 200}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{5824 \times 10^{-50}}}$$

$$= \frac{6.6 \times 10^{-34}}{7.63 \times 10^{-24}}$$

$$= 8.65 \times 10^{-1-34+24}.$$

$$\Rightarrow (8.65 \times 10^{-11}) \text{ meter}$$

(3) A hydrogenatom is in ground state. What is the quantium number to which it will be excited absorbing a photon of energy 12.75 ev?

⇒ Given

for H-atom in ground, $n_1 = 1$.

Quantum number $(n_2) = ?$ in which atom is excited by absorbing photon of energy 12.75 ev
i.e Excitation energy = 12.75 ev.

we know,
Excitation energy = $E_{n_2} - E_{n_1}$

$$\text{or, } 12.75\,ev = \frac{-13.6\,ev}{(n_2)^2} - \left[\frac{-13.6\,ev}{(1)^2}\right]$$

$$\text{or, } \frac{13.6\,ev}{(n_2)^2} = \frac{13.6\,ev}{1} - 12.75\,ev$$

$$\text{or, } \frac{13.6\,\cancel{ev}}{(n_2)^2} = 0.85\,\cancel{ev}$$

$$\text{or, } \frac{13.6}{0.85} = (n_2)^2$$

$$\text{or, } 16 = (n_2)^2$$

$$\text{or, } 4^{\cancel{2}} = (n_2)^{\cancel{2}}$$

$$\therefore \boxed{n_2 = 4}$$ #

Therefore, the quantum number is 4.

(4) Find the wavelength of second ~~time~~ line of Balmer series if the wavelength of its first line is 2.1×10^{-7} m.

⇒ Soln:
here

for Balmer series

$n_1 = 2$

$n_2 = 3, 4, 5, 6 \ldots$

∴ for first line of Balmer series ; $\lambda_1 = 2.1\times10^{-7}m$

$$\frac{1}{\lambda_1} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\text{or, } \frac{1}{\lambda_1} = R\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right]$$

$$\text{or, } \frac{1}{\lambda_1} = R\left[\frac{9-4}{36}\right]$$

$$\text{or, } \boxed{\frac{1}{\lambda_1} = \frac{R\times5}{36}} \text{ --- } (1)$$

∴ for 2nd line of Balmer series

$$\frac{1}{\lambda_2} = R\left[\frac{1}{(2)^2} - \frac{1}{(4)^2}\right]$$

$$\text{or, } \frac{1}{\lambda_2} = R\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$\text{or, } \frac{1}{\lambda_2} = R\left[\frac{4-1}{16}\right]$$

$$\text{or, } \boxed{\frac{1}{\lambda_2} = \frac{3}{16}R} \text{ --- } (2)$$

$\therefore$ Eqn $(1/2)$

$$\frac{\lambda_2}{\lambda_1} = \frac{(5/36)}{(3/16)}$$

or, $$\frac{\lambda_2}{\lambda_1} = \frac{5}{\cancel{36}_{9}} \times \frac{\cancel{16}^{4}}{3}$$

or, $$\frac{\lambda_2}{\lambda_1} = \frac{5}{9} \times \frac{4}{3}$$

or, $$\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

or, $$\cancel{\frac{\lambda_2}{\lambda_1}} \; \frac{\lambda_2}{2.1 \times 10^{-7}} = \frac{20}{27}$$

or, $$\lambda_2 = \frac{2.1 \times 10^{-7} \times 20}{27}$$

or, $$\lambda_2 = \frac{42 \times 10^{-7}}{27}$$

$\therefore$ $\lambda_2 = \boxed{1.56 \times 10^{-7}}$ meter #.

Therefore, the wavelength of second line of Balmer series is 1.56×10^{-7} m.

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(3) The spacing of atomic number of crystal is 1.1×10^{-10} m where monochromatic x-rays are incident at glancing angle 50°. the first order image is produced. calculate wave length and glancing angle for second order.

⇒ Soln;

here

From Bragg's law for nth order,

$2d \sin\theta_n = n\lambda$

∴ for first order,

$d = 1.1 \times 10^{-10}$ m

$\theta_1 = 50°$ for $n = 1$

$\lambda = ?$

$\lambda = ?$

glancing angle for 2nd order $(\theta_2) = ?$

$2d \sin\theta_1 = n\lambda$

or, $2 \times 1.1 \times 10^{-10} \times \sin 50° = 1 \times \lambda$

or, $\lambda = 1.68 \times 10^{-10}$ meter

∴ For 2nd order

$2d \sin\theta_2 = 2\lambda$ $\left[\begin{array}{l} \text{for } n = 2 \\ \lambda = 1.68 \times 10^{-10} \text{m} \end{array}\right]$

or, $\sin\theta_2 = \frac{\cancel{2}\lambda}{\cancel{2}d}$

or, $\theta_2 = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left[\frac{1.68 \times \cancel{10^{-10}}}{1.1 \times \cancel{10^{-10}}}\right]$

or, $\theta_2 = \sin^{-1}(1.5)$

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* An electron moving with velocity $10^7 ms^{-1}$ makes angles with uniform magnetic field 0.1 Tesla. By resolving velocity in perpendicular component. find out the distance between two turns of helical path. $\left(\frac{e}{m} = 1.8 \times 10^{11}\ C kg^{-1}\right)$

Here, $v\sin\theta$ is perpendicular to magnetic field B.

$$\therefore Be(v\sin\theta) = \frac{m(v\sin\theta)^2}{r}$$

radius of helical path $(r) = \frac{mv\sin\theta}{Be}$

Distance between two turns of helical path (called pitch) is given by,

$$x = \frac{2\pi mv\cos\theta}{Be}$$

$$x = \frac{2\pi v\cos\theta}{B\cdot\left(\frac{e}{m}\right)}$$

$$= \frac{2\pi \times 10^7 \times \cos 25^\circ}{0.1 \times (1.8 \times 10^{11})}$$

$$= 3.16 \times 10^{-3} \text{ meter}$$

$$\simeq 3.2 \times 10^{-3}\ m$$

$$= 3.2\ mm$$

Old is gold.

79. Obtain the de-Broglie wavelength of neutron of kinetic energy 150ev (mass of neutron = 1.675×10^{-27} kg) planck's constant = 6.6×10^{-34} Js, 1ev = 1.6×10^{-19} J)

⇒ Soln,

For neutron

From de-Broglie wavelength

wavelength (λ) = ?

Kinetic energy of neutron (K.E) = 150 ev

$= 150\times1.6\times10^{-19}$ J.

mass of neutron (m) = 1.675×10^{-27} kg

planck's constant (h) = 6.6×10^{-34} J/S.

From de-Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{h}{m\sqrt{\frac{2K.E}{m}}}$$

$$= \frac{6.6\times10^{-34}}{1.675\times10^{-27}\times1.69\times10^{5}}$$

$$= \frac{6.6\times10^{-34}}{2.83\times10^{-22}}$$

$$= 2.33\times10^{-12} m$$

[K.E = $\frac{1}{2}mv^2$]

$$v = \sqrt{\frac{2K.E}{m}}$$

$$= \sqrt{\frac{2\times150\times1.6\times10^{-19}}{1.675\times10^{-27}}}$$

$$= \sqrt{\frac{4.8\times10^{-17}}{1.675\times10^{-27}}}$$

$$= 1.69\times10^{5} m.$$

80) An x-ray tube operated at a dc-potential difference of 10 kV, produces heat at the target at the rate of 720 watt. Assuming 0.5% of the incident electrons is converted into X-radiation. Calculate the number of electrons striking per second at the target and velocity of the incident electrons.

⇒ Soln;

For x-ray tube

P.d between anode and cathod (V) = 10 kV

$= 10 \times 10^3$ V.

0.5% energy of cathode ray (i.e incident electron) is used in produced X-ray.

Heating power produced (P') = 720 watt.

(i) $\frac{N}{t} = ?$

(ii) Velocity of electron strikes on anode (v) = ?

Now,

$P' = (100 - 0.5)\%$ of P

$P' = \frac{99.5}{100} \times VI$ [P = VI]

$I = \frac{100 \times P'}{99.5 \times V}$

$= \frac{100 \times 720}{99.5 \times 10 \times 10^3}$

$= \frac{72000}{995000}$

$= 7.23 \times 10^{-2}$ A.

$I = \frac{q}{t}$

$$I = \frac{Ne}{t}$$

$$\therefore \frac{N}{t} = \frac{I}{e} = \frac{7.23\times10^{-2}}{1.6\times10^{-19}}$$

$$= 4.52\times10^{17}.$$

we know,

$$ev = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2ev}{m}}$$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times10\times10^{3}}{9.1\times10^{-31}}}$$

$$= \sqrt{\frac{3.2\times10^{-15}}{9.1\times10^{-31}}}$$

$$= \sqrt{3.51\times10^{-1}\times10^{16}}$$

$$= 5.92\times10^{7}\ m/s$$

81. calculate the wavelength of an electron which has been accelerated through a potential difference of 200 v. Take mass of the electron as 9.1×10^{-31} kg and planck's constant (6.5×10^{-34} Js)

⇒ Given,

wavelength of electron accelerated by volt (V) = 200

wavelength (λ) = ?

Plank's constant (h) = 6.5×10^{-34} Js.

mass of electron (m) = 9.1×10^{-31} kg.

we know,

$$eV = \frac{1}{2}mv^2$$

$$\text{or, } v = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times200}{9.1\times10^{-31}}}$$

$$= 8.38\times10^{6}.$$

Again,

$$\text{wave length } (\lambda) = \frac{h}{P} = \frac{h}{mv}$$

$$= \frac{6.5\times10^{-3}}{9.1\times10^{-31}\times8.38\times10^{6}}$$

$$= 8.6\times10^{-11}\ m.$$

$\therefore$ The wavelength of an electron is 8.6×10^{-11} m.

83. Determine the wavelength of a photon that has been accelerated through a potential difference of 20 kV. mass of electron = 9.1×10^{-31}

Soln;

$$\text{potential difference } (V) = 20\ kV$$

$$= 20\times10^{3}$$

we know,

$$eV = \frac{1}{2}mv^2$$

$$\text{or, } v = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times20\times10^{3}}{9.1\times10^{-31}}}$$

$$= 83.86\times10^{6}.$$

85 same as 83

Again,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 83.86 \times 10^{6}}$$

$$= 8.6 \times 10^{-12}\ m.$$

86. Calculate the de-Broglie wavelength of electron having kinetic energy of 400 eV.

→ Given,

$K.E = 400\ eV$

$= 400 \times 1.6 \times 10^{-19} J.$

We know,

$$K.E = \frac{1}{2} mv^2$$

$$\text{Or, } V = \sqrt{\frac{2\,K.E}{m}} = \sqrt{\frac{2 \times 400 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$\Rightarrow 11.85 \times 10^{6}$$

Again

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 11.85 \times 10^{6}}$$

$$= 6.12 \times 10^{-11}\ m.$$

88 same as 80

A X-ray tube works at a de-potential difference of 50 kV. Only 0.4% of the energy of cathode rays is converted into X-rays and heat is generated in the target at the rate of 600 watt. Estimate the current passed into the tube and the velocity of the electrons striking the target. (Mass of electron $= 9.1 \times 10^{-3}$ kg), charge of electron $= 1.6 \times 10^{-19}$ C

⇒ Given

for X-ray tube,

p.d. between anode and cathode $(V) = 50\,kV$

$= 50 \times 10^{3}\,V$.

0.4% energy of cathode ray (i.e. electron beam) is used in producing X-ray.

(i) Heating power produced $(P) = 600$ watts.

(ii) Current $(I) = ?$

(iii) Velocity of electron strikes on anode $(v) = ?$

Mass of electron $(m) = 9.1 \times 10^{-31}$ kg.

Charge of electron $(e) = 1.66 \times 10^{-19}$ C.

We know

$$P' = (100 - 0.4)\% \text{ of } P$$

$$\text{or, } P' = 99.6\% \text{ of } P$$

$$\text{or, } P' = \frac{99.6}{100} \times VI$$

$$\text{or, } I = \frac{100P'}{99.6V}$$

$$= \frac{100 \times 600}{99.6 \times 50 \times 10^{3}}$$

$$\Rightarrow 12.04 \times 10^{-3}\,A.$$

We know,

$$eV = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 50 \times 10^{3}}{9.1 \times 10^{-31}}}$$

$$\Rightarrow 1.3 \times 10^{8} \, m/s$$ #

(89) An X-ray spectrometer has a crystal of rock salt for which atomic spacing is 2.82 Å set an angle of 14^0 to the beam coming from a tube operated at a constantly increasing voltage. An intense first line appear when the voltage across the tube is 9045 V. calculate the value of Planck constant.

⇒ Soln

For X-ray in first order,

$n = 1$

Crystal spacing (or interatomic spacing) $= 2.82$ Å

$\Rightarrow 2.82 \times 10^{-10}$ m

$\theta = 14^0$

Volt (v) $= 9045$ V

Planck's constant $(h) = ?$

We have,

from Bragg's law,

$$2d \sin\theta = n\lambda$$

$$\text{or, } \lambda = \frac{2d \sin\theta}{n}$$

$$= \frac{2 \times 2.8 \times 10^{-10} \times \sin 14^0}{1}$$

$$\Rightarrow 1.35 \times 10^{-10}$$

Again

for X-ray photon,

$E = hf$

or, $h = \frac{E\lambda}{c}$

or, $h = \frac{eV \times \lambda}{c}$ $[E = e \cdot V]$

$= \frac{1.6 \times 10^{-19} \times 9045 \times 1.35 \times 10^{-10}}{3 \times 10^{8}}$

$\Rightarrow \boxed{6.5 \times 10^{-34}\ JS}$ #

(90) An X-ray tube works at a de-potential differe-nce of 50 kV and the current through the tube is 0.5 mA. find (i) the number of electrons hitting the target per second.

(ii) the energy falling on the target per second as kinetic energy of electrons.

(iii) the cut off wavelength of X-ray emitted.

The charge of electron $= 1.6 \times 10^{-19} C$, velocity of light $C = 3 \times 10^{8}\ ms^{-1}$, Planck's constant $=$ ~~6.62×10~~ $6.62 \times 10^{-34} J$

$\Rightarrow$ Soln In X-ray tube,

Volt $(V) = 50 \times 10^{3}$ Volt.

Current $(I) = 0.5\ mA = 0.5 \times 10^{-3} A$.

(i) Number of electron striking anode per sec $\left(\frac{N}{t}\right) = ?$

(ii) Energy falling per sec at anode $(E) = ?$

(iii) Cut off wavelength $(\lambda_{min}) = ?$

P.T.O.

Now

$$Current\,(I) = \frac{q}{t} = \frac{N \cdot e}{t}$$

$$or,\ \frac{N}{t} = \frac{I}{e} = \frac{0.5 \times 10^{-3}}{1.6 \times 10^{-19}}$$

$$\Rightarrow 3.125 \times 10^{15}\ num/sec.$$

(ii) $P = VI = 50 \times 10^{3} \times 0.5 \times 10^{-3}$

$$\Rightarrow 25\ watt$$

(iii) $$\lambda_{min} = \frac{12400\,A^{o}}{V} = \frac{12400 \times 10^{-10}}{50 \times 10^{3}}$$

$$\Rightarrow 248 \times 10^{-7}$$

(93) A circle ball is moving with a speed of 120 km/hr. What would be its de-Broglie Wavelength if its mass is 400 gm.

⇒ Given

for cricket ball,

Mass (m) = 400 gm = 0.4 kg.

$$Velocity\,(V) = 120\ km/hr = \frac{120 \times 10^{2}}{3600}\frac{m}{sec}$$

$$= \frac{10^{3}}{30}\ m/s.$$

Wavelength (λ) = ?

Now

$$\lambda = \frac{h}{P} = \frac{h}{mV} = \frac{6.625 \times 10^{-34}}{0.4 \times \left(\frac{10^{3}}{30}\right)}$$

$$= 4.96 \times 10^{-35}\ m.$$

Q4) X-rays are incident on the zinc sulphide crystal of crystal spacing 3.08×10^{-8} cm such that the first order reflection takes place at glancing angle 12°. Calculate the wavelength of X-rays and glancing angle for second order maximum.

⇒ Given

Crystal spacing $(d) = 3.08 \times 10^{-8}$ cm
$= 3.08 \times 10^{-10}$ m

(i) $\theta_1 = 120^\circ$ for $n = 1$ (for $n = 1$)

we know,

$2d \sin \theta_n = n\lambda$

or, $2d \sin \theta_1 = n \times \lambda$

or, $\lambda = \dfrac{2d \sin\theta_1}{n}$

$= \dfrac{2 \times 3.08 \times 10^{-10} \times \sin 12^\circ}{1}$

$= (1.28 \times 10^{-10})$ m

∴ wavelength of x-rays is found to be 1.28×10^{-10} m

Now,

for 2nd order, $n = 2$

$2d \sin\theta_2 = 2 \times \lambda$

or, $\sin\theta_2 = \dfrac{2\lambda}{2d}$

or, $\sin\theta_2 = \dfrac{1.28 \times \cancel{10^{-10}}}{3.08 \times \cancel{10^{-10}}}$

or, $\theta_2 = \sin^{-1}\left(\dfrac{1.28}{3.08}\right)$

∴ $\boxed{\theta_2 = 24.55^\circ}$

Hence, the glancing angle of 2nd order maximum is 24.55°.

(95) X-ray beam of wavelength 2.9 AU(A) is diffracted from the plane of cubic crystal. The first order diffraction is at formed at an angle of 35°. calculate the spaces between the planes.

⇒ Given,

For x-rays

$\lambda = 2.9 A^u = 2.9 \times 10^{-10} m$

for 1st order $n = 1$, $\theta_1 = 35^0$, $d = ?$

From Bragg's law,

$2d \sin \theta_n = n\lambda$.

$\therefore 2d \sin \theta_1 = 1\lambda$.

or, $d = \frac{\lambda}{2 \sin \theta_1} = \frac{2.9 \times 10^{-10}}{2 \times \sin 35^0}$

$= \frac{2.9 \times 10^{-10}}{2 \times 5.7 \times 10^{-1}}$

$= 2.5 \times 10^{-1-10+1}$

$= (2.5 \times 10^{1})$

(96) X-rays of wavelength 0.36 A° are diffracted by a Bragg's crystal spectograph at a glancing angle of 4.8°. find the spacing of the atomic planes in the crystal.

⇒ Given,

For x-rays,

Wavelength $(\lambda) = 0.36 A^0 = 0.36 \times 10^{-10} m$.

glancing angle $(\theta) = 4.8^0$

crystal spacing $(d) = ?$

we know,

from Bragg's law;

Let $n = 1$.

$2d \sin \theta = n\lambda$

or, $d = \frac{n\lambda}{2 \sin \theta}$

Teacher's Signature..............

$= \frac{1 \times 0.36 \times 10^{-10}}{2 \times \sin 4.8}$

$= \frac{1 \times 0.36 \times 10^{-10}}{2 \times 8.36 \times 10^{-2}}$

$= \frac{0.36 \times 10^{-10}}{16.72 \times 10^{-2}}$

$= 2.15 \times 10^{-2-10+2}$

$= (2.15 \times 10^{-10})$ meter #.

(Q7) An electron moving with velocity $10^7 ms^{-1}$ makes an angle with uniform magnetic field 0.1 Tesla. By resolving velocity in perpendicular components, find out the distance between two turns of helical path. $\left(\frac{e}{m} = 1.8 \times 10^{11} C\, kg^{-1}\right)$

⇒ soln;

Here

$V \sin\theta$ is perpendicular to magnetic field B.

$\therefore Be(V \sin\theta) = \frac{m(V \sin\theta)^2}{r}$

$\boxed{r = \frac{mv \sin\theta}{Be}}$ = radius of helical path.

distance between two turns of helical path is called pitch is given by

$$\boxed{x = \frac{2\pi m v \cos\theta}{Be}}$$

$$\therefore x = \frac{2\pi v \cos\theta}{B(e/m)} = \frac{2\pi \times 10^7 \times \cos 45^\circ}{0.2 \times (1.8 \times 10^{11})}$$

$= 3.16 \times 10^{-3}$ meter.

$\simeq 3.2 \times 10^{-3}$ m

$\simeq 3.2$ mm Ans.