Chapter 17: Differential Equations

Exercise 17.1

1. Solution

a. Given, $\frac{dy}{dx} = 4x$

Here, $\frac{dy}{dx}$ is the first order derivative, so its order is 1

Here, the power of $\frac{dy}{dx}$ is 1. So its degree is 1

b. Given, $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 2y = 0$

Here, $\frac{d^2y}{dx^2}$ is the second order derivative, so it's order is 2

Here, the power of y is 1. So, it's degree is 1.

c. Given, $\frac{d^2y}{dx^2} = xe^x$

Here, $\frac{d^2y}{dx^2}$ is the second order derivative. So, its' order is 2

Here, the power of x is 1, so its degree is 1.

d. Given, $x\frac{dy}{dx} + \frac{3}{\frac{dy}{dx}} = y^2$

Here, $\frac{dy}{dx}$ is the first order derivative, so it's order is 1.

Here, the power of y^2 is 2. So it's degree is 2.

e. $\frac{dy}{dx} = \sqrt{\frac{1 - x^2}{1 - y^2}}$

Here, $\frac{dy}{dx}$ is the first order derivative. So its order is 1.

Here, the power of $\frac{dy}{dx}$ is 2. So, its degree is 2.

f. Given, $\frac{dy}{dx} + 3y\left(\frac{d^2y}{dx^2}\right)^3 = 0$

Here, $\frac{d^2y}{dx^2}$ is the second order derivative. So, its order is 2.

Here, the power of $\frac{d^2y}{dx^2}$ is 3. So, its' degree is 3.

Exercise 17.2

1.

a. $\frac{dy}{dx} = \frac{x + 4}{y + 2}$

$(y + 2)\, dy = (x + 4)\, dx$

Integrating both sides

$\int (y + 2)\, dy = \int (x + 4)\, dx$

or, $\frac{y^2}{2} + 2y = \frac{x^2}{2} + 4x + c$

or, $y^2 + 4y = x^2 + 8x + c$

b. $x^2dx + y^2dy = 0$

Integrating

$\int x^2dx + \int y^2dy = \int 0$

$\frac{x^3}{3} + \frac{y^3}{3} = \frac{c}{3}$

$\therefore \quad x^3 + y^3 = c$

c. $y\frac{dy}{dx} = \cos x$

$ydy = \cos x\, dx$

Integrating both sides

$\int ydy = \int \cos x\, dx$

or, $\frac{y^2}{2} = \sin x + c'$ where c' is constant

$\therefore \quad y^2 = 2\sin x + 2c'$

$\therefore \quad y^2 = 2\sin x + c$

d. $\frac{dy}{dx} = e^{x+y}$

$\frac{dy}{dx} = e^x \cdot e^y$

$e^{-y}\, dy = e^x\, dx$

Integrating

$\int e^{-y}\, dy = \int e^x\, dx$

$-e^{-y} = e^x + c$

or, $\frac{-1}{e^y} = c^x + c$

or, $-1 = e^y\,(e^x + c)$

e. $e^{x-y}\, dx + e^{y-x}\, dy = 0$

or, $e^x . e^{-y}\, dx + e^y . e^{-x}\, dy = 0$

or, $\frac{e^x}{d^y}\, dx + \frac{e^y}{e^x}\, dy = 0$

$e^{2x}\, dx + e^{2y}\, dy = 0$

Integrating

$\int e^{2x}\, dx + \int e^{2y}\, dy = \int 0$

$\frac{e^{2x}}{2} + \frac{e^{2y}}{2} = \frac{c}{2}$

$\therefore \quad e^{2x} + e^{2y} = c$

f. $\frac{dy}{dx} = e^{x-y} + x^3e^{-y}$

$\frac{dy}{dx} = \frac{(e^x + x^3)}{e^y}$

$e^y\, dy = e^xdx + x^3dx$

Integrating

$$\int e^y dy = \int e^x dx + \int x^3 dx$$

$$e^y = e^x + \frac{x^4}{4} + c$$

2. Solution:

a. $(x + 2)\frac{dy}{dx} = y + 2$

or, $\left(\frac{1}{y+2}\right) dy = \frac{dy}{x+2}$

Integrating both sides

$$\int \frac{1}{x+2} dy = \int \frac{1}{x+2} dx$$

$\ln (y + 2) = \ln(x + 2) + \ln c$

$\ln (y + 2) = \ln\{c(x+2)\}$

$\therefore\quad y + 2 = c(x + 2)$

b. $x\frac{dy}{dx} + y - 1 = 0$

$x\frac{dy}{dx} = (1 - y)$

or, $\frac{1}{1-y} dy = \frac{dx}{x}$

Integrating

$-\ln (l - y) = \ln x + \ln c$

$\ln(1 - y)^{-1} = \ln cx$

$\therefore\quad cx = \frac{1}{1-y}$

$x(1 - y) = \frac{1}{c}$

$\therefore\quad x(1 - y) = c$

c. $\text{Cos}x \cdot \cos y \frac{dy}{dx} = -\sin x \cdot \sin y$

$\frac{\cos y}{\sin y} dy = -\frac{\sin x}{\cos x} dx$

$\cot y\, dy = -\tan x\, dx$

Integrating both sides

$$\int \cot y\, dy = -\int \tan x\, dx$$

$\log \sin y = \log \cos x + \log c$

$\sin y = c \cos x$

d. $\text{Sec}^2 x \cdot \tan y dx\ \ \sec^2 y \tan x\, dy = 0$

or, $\frac{\sec^2 x}{\tan x} = -\frac{\sec^2 y}{\tan y} dy$

Integrating both sides we get

$$\int \frac{\sec^2 x}{\tan x} dx + \int \frac{\sec^2 y}{\text{tax}y} dy = \int 0$$

$\ln(\tan x) + \ln(\tan y) = \ln c$

$\ln(\tan x \cdot \tan y) = \ln c$

$\therefore\quad \tan x \cdot \tan y = c$

e. $(e^y + 1) \cos 2dx + e^y \cdot \sin x\, dy = 0$

$\frac{\cos x\, dx}{\sin x} + \frac{e^y\, dx}{e^y + 1} = 0$

Integrating both sides

$\int \frac{\cos x}{\sin x} dx + \int \frac{e^y}{1+ey} dy = \int 0$

$\ln \sin x + \ln (e^y + 1) = \ln c$

$\ln \sin x(1 + e^y) = \ln c$

$\therefore \quad \sin x\,(1 + e^y) = c$

f. $(xy^2 + x)\, dx + (x^2y + y)\, dy = 0$

or, $x(1 + y^2)\, dx + y(1 + x^2)\, dy = 0$

or, $\frac{x\, dx}{1+x^2} + \frac{y}{1+y^2} dy = 0$

or, $\left(\frac{2x}{1+x^2}\right) dx + \left(\frac{2y}{1+y^2}\right) dy = 0$

Integrating both sides

$\int \frac{dx}{1+x^2} + \int \frac{2y}{1+y^2} dy = \int 0$

$\ln(1 + x^2) + \ln(1 + y^2) = \ln c$

$\therefore \quad \ln(1 + x^2)\,(1 + y^2) = \ln c$

$\therefore \quad (1 + x^2)\,(1 + y^2) = c$

$\ln(1+y) = x - \frac{x^2}{2} + c$

g. $\sqrt{1+x^2}\, dy + \sqrt{1 + y^2}\, dx = 0$

or, $\frac{dy}{\sqrt{1 + y^2}} + \frac{dx}{\sqrt{1 + x^2}} = 0$

Integrating

$\int \frac{1}{\sqrt{1 + y^2}} dy + \int \frac{1}{\sqrt{1 + x^2}} dx = \int 0$

or, $\ln (y + \sqrt{1+y^2}) + \ln(x + \sqrt{1+x^2}) = \ln c$

or, $\ln \{(x + \sqrt{1+x^2})\,(y + \sqrt{1+y^2})\} = \ln c$

or, $(x + \sqrt{1+x^2})\,(y + \sqrt{1+y^2}) = c$

h. $x\sqrt{1 - y^2}\, dx + y\sqrt{1 - x^2}\, dy = 0$

$\frac{x}{\sqrt{1 - x^2}} dx + \frac{y}{\sqrt{1 - y^2}} dy = 0$

Put $l–x^1 = u$ and $l–y^2 = v$

Then, $-2xcdx = du$, $-2ydy = dv$

So, $\frac{(-2x)dx}{\sqrt{1 - x^2}} + \frac{(-2y)\, dy}{\sqrt{1 - y^2}} = 0$

$\frac{dy}{\sqrt{u}} + \frac{dv}{\sqrt{v}} = 0$

$u^{-1/2}\, du + v^{-1/2}\, dv = 0$

Integrating

$2u^{1/2} + 2v^{1/2} = 2c$

$\therefore \quad \sqrt{u} + \sqrt{v} = c$

$\therefore \quad \sqrt{1 - x^2} + \sqrt{1 - y^2} = c$

i. $(1 - x^2)dy + xydx = x^2ydx$

or, $(1 - x^2)dy = xy\,(1 + y)dx$

or, $\dfrac{dy}{y(1+y)} = \dfrac{x\,dx}{1-x^2}$

or, $\left(\dfrac{1}{y} - \dfrac{1}{y+1}\right) dy = \dfrac{1}{-2}\left(\dfrac{-2x}{1-x^2}\right) dx$

Integrating

$\ln y - \ln(y+1) = -\dfrac{1}{2}\ln(1-x^2) + \ln c$

$\ln\left(\dfrac{y}{1+y}\right) = \ln(1-x^2)^{-1/2} + \ln c \Rightarrow \dfrac{y}{1+y} = c(1-x^2)^{-1/2}$

3. Solution:

a. $\sec^2 y\,(1+x^2)dy + 2x.\tan y\,dx = 0$

or, $\dfrac{\sec^2 y}{\tan y}\,dy + \dfrac{2x}{1+x^2}\,dy = 0$

Integrating, we get,

$\ln(\tan y) + \ln(1+x^2) = \ln c$

$(1+x^2)\tan y = c$

When $x = 1$, there $y = \dfrac{\pi}{4}$

$\therefore\quad (1+1)\tan\dfrac{\pi}{4} = c$

$2.1 = c$

$\therefore\quad c = 2$

b. $\cos y\,dy + \cos x\,\sin y\,dx = 0$

$\cos y\,dy = -\cos x.\sin y\,dx$

$\dfrac{\cos y}{\sin y}\,dy = -\cos x\,dx$

Integrating, we get

$\ln\sin y = -\sin x + c$

$\sin x + \ln\sin y = c$

When $x = \dfrac{\pi}{2}$ then $y = \dfrac{\pi}{2}$

then $c = 1$

$\therefore\quad \sin x + \ln\sin y = 1$

Exercise 17.3

1. Solution:

a. $x\dfrac{dy}{dx} = y + x$

$\dfrac{dy}{dx} = \dfrac{y}{x} + 1$ ... ... (i) It is a homogenous

Thus, differential equation

Put $y = vx$

Then $\dfrac{dy}{dx} = v + x\dfrac{dy}{dx}$

(i) becomes

$v + x\dfrac{dv}{dx} = v + 1$

$\therefore\quad x\dfrac{dv}{dx} = 1$

$dv = \dfrac{dx}{x}$

Integrating both sides

$s \int dv = \int \frac{1}{x} dx$

$v = \ln x + c$

$\frac{y}{x} = \ln x + c$

b. $\frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x^2}$

It is homogeneous differential equation.

Put $y = vx$

$\frac{dy}{dx} = v + x.\frac{dv}{dx}$

Then $v + x.\frac{dv}{dx} = v^2 + v$

$x\frac{dv}{dx} = v^2$

$v^{-2}\, dv = \frac{1}{x} dx$

Integrating both sides

$\int v^{-2} dv = \int \frac{1}{x} dx$

$-v^{-1} = \ln(cx)$

$-\frac{1}{v} \ln cx$

$-\frac{x}{y} = \ln(cx)$

c. $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ ... ... (i)

It is homogenous differential equation,

Put $y = vx$ then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

(i) becomes, $v + x\frac{dv}{dx} = v + \tan v$

$x\frac{dv}{dx} = \tan v$

$\cot v\, dv = \frac{1}{x} dx$

Integrating $\int \cot v\, dv = \int \frac{1}{x} dx$

$\log \sin v = \log cx$

$\sin\left(\frac{y}{x}\right) = cx$

d. $x\left(\frac{dy}{dx} + \tan\frac{y}{x}\right) = y$

or, $\frac{dy}{dx} + \tan\frac{y}{x} = \frac{y}{x}$ ... ... (i)

Put $y = vx$ then $\frac{dy}{dx} = v + x\,.\,\frac{dv}{dx}$

$v + x\frac{dv}{dx} + \tan v = v$

$x\dfrac{dv}{dx} = -\tan v$

$-\cot v\, dv = \dfrac{dx}{x}$

Integrating

$\int -\cot v\, dv = \int \dfrac{1}{x}\, dx$

$-\log \sin v = \log cx$

$\sin^{-1} v = cx$

$\sin^{-1}\left(\dfrac{y}{x}\right) = cx$

$\dfrac{1}{c} = x \sin\left(\dfrac{y}{x}\right)$

$\therefore \quad x \sin\left(\dfrac{y}{x}\right) = c$

e. $\dfrac{dy}{dx} - \dfrac{y}{x} - \sin\dfrac{y}{x} = 0$ ... ... (i)

Put $y = vx$ then $\dfrac{dy}{dx}\ v + x\dfrac{dv}{dx}$

(i) becomes,

$v + x\,.\,\dfrac{dv}{dx} - v - \sin v = 0$

$x\dfrac{dv}{dx} = \sin v$

$\dfrac{dv}{\sin v} = \dfrac{dx}{x}$

$\operatorname{cosec} v\, dv = \dfrac{1}{x}\, dx$

Integrating

$\int \operatorname{cosec} v\, dv = \int \dfrac{1}{x}\, dx$

$\log(\operatorname{cosec} v - \cot v) = \log x + \log c$

$\operatorname{cosec} v - \cot v = cx$

$\operatorname{cosec}\left(\dfrac{y}{x}\right) = \cot\left(\dfrac{y}{x}\right) = cx$

f. $\dfrac{dy}{dx} = \dfrac{y}{x} + \cos^2\left(\dfrac{y}{x}\right)$

Put $y = vx$

$\dfrac{dy}{dx} = v + x\,.\,\dfrac{dv}{dx}$

$v + x\,.\,\dfrac{dv}{dx} = v + \cos^2 v$

$x\dfrac{dv}{dx} = \cos^2 v$

$\sec^2 v\, dv = \dfrac{1}{x}\, dx$

Integrating

$\int \sec^2 v\, dx = \int \dfrac{1}{x}\, dx$

$\tan v = \ln(cx)$

$\tan\left(\frac{y}{x}\right) = \ln(cx)$

$\tan\left(\frac{y}{x}\right) = \ln(cx)$

2. Solution:

a. $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

$y^2 = (xy - x^2)\frac{dy}{dx}$

$\therefore \quad \frac{dy}{dx} = \frac{y^2}{xy - x^2}$ ... ... (i)

Put $y = vx$

Then, $v + x\frac{dv}{dx} = \frac{v^2x^2}{x^2v - x^2}$

$v + x\frac{dv}{dx} = \frac{v^2}{v-1}$

$x\frac{dv}{dx} = \frac{v^2}{v-1} - v$

$x\frac{dv}{dx} = \frac{v^2 - v^2 + v}{v-1}$

$x\frac{dv}{dx} = \frac{v}{v-1}$

$\frac{(v-1)}{v}dv = \frac{1}{x}dx$

Integrating

$v - \ln v = \ln x + \ln c$

$v = \ln(cxv)$

$\frac{y}{x} = \ln(cy)$

b. $x^2y\,dx - (x^3 + y^3)\,dy = c$

$\frac{dy}{dx} = \frac{x^2y}{x^3 + y^3}$ ... ... (i)

Put $y = vx$

$\frac{dy}{dx} = v + x.\frac{dv}{dx}$

(i) becomes

$v + x.\frac{dy}{dx} = \frac{v}{1 + v^3}$

$x\frac{dv}{dx} = \frac{v - v - v^4}{1 + v^3}$

$x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$

$\frac{(1+v^3)}{v^4}dv = -\frac{dx}{x}$

$v^{-4}dv + \frac{1}{v}dv = -\frac{1}{x}dx$

Integrating

$\frac{v^{-3}}{-3} + \ln v = -\ln x - \ln c$

$$\frac{1}{3v^3} = \ln(cxv)$$

$$\frac{x^3}{3y^3} = \ln(cy)$$

c. $(x^2 + y^2)\,dx - 2xy\,dy = 0$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy} \quad \ldots\ \ldots\ \ldots \text{(i)}$$

$$v + x\frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{2v}$$

$$\left(\frac{2v}{1-v^2}\right) dv = \frac{dx}{x}$$

$$-\ln(1-v^2) = \ln x + \ln c$$

$$\ln(1-v^2)^{-1} = \ln(cx)$$

$$(1 - v^2)^{-1} = cx$$

$$\frac{1}{1 - v^2} = cx$$

$$\frac{x^2}{x^2 - y^2} = cx$$

$$\frac{x}{x^2 - y^2} = c$$

$$\therefore \quad x^2 - y^2 = \frac{x}{c}$$

$$\therefore \quad x^2 - y^2 = cx$$

d. $(x + y)\,dx + (y - x)\,dy = 0$

$(y - x)\,dy = -(x + y)\,dx$

$$\therefore \quad \frac{dy}{dx} = -\frac{(x + y)}{y - x}$$

$$\therefore \quad \frac{dy}{dx} = \frac{x + y}{x - y} \quad \ldots\ \ldots\ \ldots \text{(i)}$$

Put $y = vx$, then $\dfrac{dy}{dx} = v + x.\dfrac{dv}{dx}$

Then (i) reduces to

$$v + x\,.\frac{dv}{dx} = \frac{x + vx}{x - vx}$$

$$v + x\frac{dv}{dx} = \frac{1 + v}{1 - v}$$

$$\text{or,}\quad x\frac{dv}{dx} = \frac{1 + v}{1 - v} - v$$

$$x\frac{dv}{dx} = \frac{1 + v - v + v^2}{1 - v}$$

$$x\frac{dv}{dx} = \frac{1 + v^2}{1-v}$$

$$\left(\frac{1 - v}{1 + v^2}\right) dv = \frac{dx}{x}$$

$$\frac{1}{1+v^2}\,dv - \frac{1}{2}\left(\frac{2v}{1 + v^2}\right) dv = \frac{1}{x}\,dx$$

Integrating

$$\tan^{-1} v = \frac{1}{2}\ln(1 + v^2) = \ln x + c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + \ln\sqrt{1 + v^2} + c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln\sqrt{x^2 + y^2} + c$$

e. $(x + y)^2\, dx = xy\, dy$

$$\frac{dy}{dx} = \frac{(x + y)^2}{xy}$$

$$v + x\frac{dv}{dx} = \frac{(x + vx)^2}{x^2 v}$$

$$v + x\frac{dv}{dx} = \frac{(1 + v2)^2}{v}$$

$$x\frac{dv}{dx} = \frac{(1 + v)^2 - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{1 + 2v}{v}$$

$$\frac{v}{1+2v}\, dv = \frac{dx}{x}$$

$$\frac{1}{2}\frac{(1 + 2v - 1)}{1 + 2v}\, dv = \frac{1}{x}\, dx$$

$$\frac{1}{2}\left(1 - \frac{1}{1+2v}\right) dv = \frac{1}{x}\, dx$$

$$\frac{1}{2}\, dv - \frac{1}{4}\left(\frac{2}{1+2v}\right) dv = \frac{1}{x}\, dx$$

Integrating

$$\frac{1}{2}v - \frac{1}{4}\log(1 + 2v) = \log(cx)$$

$$\frac{1}{2}\left(\frac{y}{x}\right) - \frac{1}{4}\log\left(1 + \frac{2y}{x}\right) = \log(cx)$$

f. $x^2\frac{dy}{dx} = \frac{y(x + y)}{2}$

$$\frac{dy}{dx} = \frac{y(x + y)}{2x^2} \quad \dots\dots\dots (i)$$

Put $y = vx$ then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

(i) becomes,

or, $v + x\,.\,\frac{dv}{dx} = \frac{vx(x + vx)}{2x^2}$

or, $v + x\,.\,\frac{dv}{dx} = \frac{v(1 + v)}{2}$

or, $x\frac{dv}{dx} = \frac{v + v^2}{2} - v$

or, $x\frac{dv}{dx} = \frac{v + v^2 - 2v}{2}$

or, $x\frac{dv}{dx} = \frac{v^2 - v}{2}$

$$\frac{1}{v(v - 1)}\, dv = \frac{dx}{2x}$$

$$\left(\frac{1}{v-1} - \frac{1}{v}\right) dv = \frac{1}{2}\frac{1}{x}\, dx$$

Integrating

$$\ln(v-1) - \ln(v) = \frac{1}{2}\ln x + \ln c$$

$$\ln\left(\frac{v-1}{v}\right) \ \ln(c\sqrt{x})$$

$$\therefore \quad \frac{v-1}{v} = c\sqrt{x}$$

$$1 - \frac{x}{y} = c\sqrt{x}$$

$$y - x = cy\sqrt{x}$$

3. Solution:

a. $\frac{dy}{dx} = \frac{y+1}{x+y+1}$

Put $y + 1 = vx$

Then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\,.\frac{dv}{dx} = \frac{vx}{x+vx}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v}$$

$$x\frac{dv}{dx} = \frac{v}{1+v} - v$$

$$x\frac{dv}{dx} = \frac{v - v - v^2}{1+v}$$

$$\frac{(1+v)}{v^2}\,dv = -\frac{dx}{x}$$

$$\left(v^{-2} + \frac{1}{v}\right)dv = -\frac{1}{x}\,dx$$

Integrating

$$-v^{-1} + \ln v = -\ln x - \ln c$$

$$\ln(cvx) = \frac{1}{v}$$

$$\ln(cx\,(y+1)) = \frac{x}{y+1}$$

b. $\frac{dy}{dx} = \frac{y+x+1}{x+1}$

Put $y = v(x+1)$

$$\frac{dy}{dx} = v + (x+1)\frac{dv}{dx}$$

So, $v + (x+1)\frac{dv}{dx} = v + 1$

$$(x+1)\frac{dv}{dx} = 1$$

$$dv = \frac{dx}{x+1}$$

Integrating

$$\int dv = \int\frac{1}{x+1}\,dx$$

$$v = \ln(x+1) + c$$

$$\frac{y}{x+1} = \ln(x+1) + c$$

$$\therefore \quad y = (x+1)\{\ln(x+1) + c\}$$

Exercise 17.4

1. Solution:

a. $xdx - ydy = 0$

Integrating both sides

$$\int xdx - \int ydy = \int 0$$

$$\frac{x^2}{2} - \frac{y^2}{2} = \frac{c}{2}$$

$$x^2 - y^2 = c$$

b. $xdy - ydx = 0$

$$\frac{xdy - ydx}{x^2} = 0$$

$$d\left(\frac{y}{x}\right) = 0$$

Integrating

$$\int d\left(\frac{y}{x}\right) = \int 0$$

$$\frac{y}{x} = c$$

$\therefore \quad y = cx$

c. $(x + y^2)dx = 2xydy$

$$\frac{xdy}{x^2} = \frac{2xy\,dy - y^2dx}{x^2}$$

$$\frac{1}{x}\,dx = \frac{2xy\,dy - y^2dx}{x^2}$$

$$\frac{1}{x}\,dx = d\left(\frac{y^2}{x}\right)$$

Integrating we get

$$\ln x = \frac{y^2}{x} + c$$

$$x\ln x = y^2 + cx$$

d. $ydx - \frac{x}{2}\,dy = 0$

$$ydx = \frac{x}{2}\,dy$$

or, $\frac{dx}{x} = \frac{dy}{y}$

$$\int \frac{dy}{y} = 2\int \frac{1}{x}\,dx$$

$$\ln y = 2\ln x + \ln c$$

$$\ln y = \ln x^2c$$

$$y = cx^2$$

e. $\frac{1}{x+1}\,dx + \frac{1}{y+1}\,dy = 0$

Integrating

$$\int \frac{1}{x+1}\,dx \neq \int \frac{1}{y+1}\,dy = \int 0$$

or, $\ln(x + 1) + \ln(y + 1) = \ln c$

or, $\ln(x + 1)(y + 1) = \ln c$

$\therefore \quad (x + 1)(y + 1) = c$

f. $\frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$

Integrating

$\int \frac{dy}{1+x^2} + \int \frac{dy}{1+y^2} = \int 0$

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}c$

or, $\tan^{-1}\left(\frac{x+y}{1+xy}\right) = \tan^{-1}c$

$\therefore \frac{x+y}{1-xy} = c$

$\therefore x + y = c(1 - xy)$

g. $\frac{x}{1+x^2}dx + \frac{y}{1+y^2}dy = 0$

or, $\frac{2x}{1+x^2}dx + \frac{2y}{1+y^2}dy = 0$

Integrating

$\ln(1 + x^2) + \ln(1 + y^2) = \ln c$

$\therefore \ln(1 + x^2)(1 + y^2) = \ln c$

$\therefore (1 + x^2)(1 + y^2) = c$

h. $(x + y)\,dy + (y - 2)\,dx = 0$

$xdy + ydy + ydx - xdx = 0$

$(xdy + ydx) + ydy - xdx = 0$

$d(xy) + ydy - xdx = 0$

Integrating

$\int d(xy) + \int ydy - \int xdx = \int 0$

$xy + \frac{y^2}{2} - \frac{x^2}{2} = \frac{c}{2}$

$2xy + y^2 - x^2 = c$

i. $2xy\,dx + (x^2 - y^2)\,dy = 0$

$2xydx + x^2dy - y^2dy = 0$

$d(x^2y) - y^2dy = 0$

Integrating, we get

$x^2y - \frac{y^3}{3} = c$

$3x^2y - y^3 = c$

j. $(x^2 + xy^2)\,dx + (x^2y + y^2)\,dy = 0$

or, $x^2dx + xy^2dx + x^2y\,dy + y^2dy$

or, $x^2dx + (xy^2dx + x^2ydy) + y^2dy$

or, $x^2dx + \frac{1}{2}d(x^2y^2) + y^2dy = 0$

Integrating

$\frac{x^3}{3} + \frac{1}{2}(x^2y^2) + \frac{y^3}{3} = c$

$2x^3 + 3x^2y^2 + 2y^3 = c$

2. Solution:

a. $\cos x.\cos y\,dy = \sin x.\sin y\,dx = 0$

$\cos x.\cos y dy = \sin x.\sin y\,dx$

$\frac{\cos y}{\sin y}dy = \frac{\sin x}{\cos x}dx$

$\frac{\cos y}{\sin y}dy - \frac{\sin x}{\cos x}dx = 0$

Integrating

$$\int \frac{\cos y}{\sin y} dy - \int \frac{\sin x}{\cos x} dx = \int 0$$

ln(siny) + ln(cosx) = lnc

ln(siny.cosx) = lnc

siny.cosx = c

b. sinx cosx dx – siny.cosy dy = 0

$\frac{1}{2}\sin 2x\, dx - \frac{1}{2}\sin 2y\, dy = 0$

or, sin2x dx – sin2ydy = 0

Integrating

$$\int \sin 2x\, dx - \int \sin 2y dy = \int 0$$

$$\frac{-\cos 2x}{2} + \frac{\cos 2x}{2} = \frac{c}{2}$$

$\therefore$ cos2y – cos2x = c

c. $\frac{dy}{dx} = \frac{1-\cos y}{1+\cos x}$

$\frac{dy}{dx} = \frac{2\sin^2 y/2}{2\cos^2 x/2}$

or, $\text{cosec}^2 \frac{y}{2} dy = \sec^2 \frac{x}{2} dx$

Integrating

$$\int \text{cosec}^2 \frac{y}{2} dy = \int \sec^2 \frac{x}{2} dx$$

or, $\frac{-\cot \frac{y}{2}}{\frac{1}{2}} = \frac{\tan \frac{x}{2}}{\frac{1}{2}} + \frac{c}{\frac{1}{2}}$

$\therefore$ $-\cot \frac{y}{2} = \tan \frac{x}{2}\, c$

d. $\frac{dy}{dx} = \frac{4x - y + 5}{x + 5y + 4}$

or, 4xdx – ydx + 5dx = xdy + 5ydy + 4dy

or, 4xdx + 5dx = (xdy + ydx) + 5ydy + 4dy

4xdx + 5dx = d(xy) + 5ydy + 4dy

Integrating both sides

$$\int 4xdx + 5\int dx = \int d(xy) + 5\int ydy + 4\int dy$$

$\frac{4x^2}{2} + 5x = xy + \frac{5y^2}{2} + 4y + c$

$4x^2 + 10x = 2xy + 5y^2 + 8y + c$

$4x^2 - 5y^2 + 10x - 8y - 2xy = c$

e. $\frac{dy}{dx} = \frac{x + y + 1}{2y - x + 2}$

or, 2ydy – xdy + 2dy = xdx + ydx + dy

2ydy + 2dy = xdx + ydx + xdy + dx

2ydy + 2dy = xdx + d(xy) + dx

Integrating

$\int 2ydy + \int 2dy = \int xdx + \int d(xy) + \int dx$

$y^2 + 2y = \frac{x^2}{2} + xy + x + c$

$2y^2 + 4y = x^2 + 2xy + 2x + c$

$2y^2 - x^2 - 2xy + 4y - 2x = c$

f. $\frac{dy}{dx} = \frac{x - 3(y+1)}{y + 3(x+1)}$

or, $ydy + 3(x + 1)\,dy = xdx - 3)(y + 1)\,dx$

or, $ydy + 3xdy + 3dy = xdx - 3ydx - 3dx$

or, $ydy + 3(xdy + ydx) + 3dy = xdx - 3dy$

$ydy + 3d(xy) + 3dy = (x - 3)\,dx$

Integrating we get

$\frac{y^2}{2} + 3xy + 3y = \frac{x^2}{2} - 3x + c$

$\therefore\ y^2 + 6xy + 6y = x^2 - 6x + 6$

g. $(\sin x.\tan y - 1)\,dx = \cos x.\sec^2 y\,dy = 0$

$\sin x.\tan y\,dx - \cos x.\sec^2 y\,dy = dx$

$-d(\cos x.\tan y) = dx$

$d(\cos x.\tan y) + dx = 0$

Integrating both sides

$\int d(\cos x.\tan y) + \int dx = \int 0$

$\cos x.\tan y + x = c$

Exercise 17.5

1. Solution:

a. Given, differential equation is $\frac{dy}{dx} + y = 1$ ... ... (i)

It is a linear differentiate equation of the type $\frac{dy}{dx} + py = Q$

Here, $p = 1$, $Q = 1$

$\int pdx = \int 1dx = x$

Integrating factor (I.F.) $= e^{spdx} = e^x$

Integrating equation (i) both sides by I.F.

$\left[\frac{dy}{dx} + y\right] e^x = 1 \times e^x$

Comparing both sides, we get $\int d(y.e^x) = \int e^x\,dx$

$\therefore\ y.e^x = e^x + c$

$\therefore\ y = 1 + ce^{-x}$

b. Given,

$\frac{dy}{dx} + y = e^x$ ... ... (i)

Comparing (i) with $\frac{dy}{dx} + py = Q$, we get

$p = 1$, $Q = e^x$

$\int pdx = \int 1dx = x$

Integrating factor (I.F.) is given by $e^{\int pdx}$

I.F. = e^x

Multiplying (i) both sides by I.F., we get

$$\left(\frac{dy}{dx} + y\right) e^x = e^x . e^x$$

or, $d(y.e^x) = e^{2x}$

Integrating both sides

$$\int d(y.e^x) = \int e^{2x}\, dx$$

$$y.e^x = \frac{e^{2x}}{2} + c$$

$$\therefore \quad y = \frac{e^x}{2} + ce^{-x}$$

c. $\frac{dy}{dx} + 2y = \frac{1}{2}(x^2 - x)$ ... ... (i)

Here, p = 2 and Q = $\frac{1}{2}(x^2 - x)$

$\int pdx = 2x$

I.F. = $e^{\int pdx} = e^{2x}$

Multiplying (i) both sides by I.F.

$$\left(\frac{dy}{dx} + 2y\right) e^{2x} = \frac{1}{2}(x^2 - x)e^{2x}$$

$$d(y.e^{2x})\, e^{2x} = \frac{1}{2}(x^2 - x)\, e^{2x}$$

Integrating both sides

$$\int d(y.e^{2x}) = \frac{1}{2}\int (x^2 - x)\, e^{2x}$$

$$y.e^{2x} = \frac{1}{2}\left[(x{+}{-}\,x)\frac{e^{2x}}{2} - \int (2x-1)\frac{e^{2x}}{2}\, dx\right]$$

$$y.e^{2x} = \frac{1}{2}(x^2 - x)\, e^{2x} - \frac{1}{4}\int (2x-1)\, e^{2x} dx$$

$$y.e^{2x} = \frac{1}{4}(x^2 - x)\, e^{2x} - \frac{1}{4}\left[(2x-1)\frac{e^{2x}}{2} - \int \frac{2e^{2x}}{2}\, dx\right]$$

$$y.e^{2x} = \frac{1}{4}(x^2 - x)\, e^{2x} - \frac{1}{8}(2x-1)\, e^{2x} + \frac{1}{8}e^{2x} + c$$

$$y.e^{2x} = \frac{1}{8}e^{2x}(2x^2 - 2x - 2x + 1 + 1) + c$$

$$\Rightarrow y.e^{2x} = \frac{1}{4}(x-1)^2\, e^{2x} + c$$

d. Sec3x $\frac{dy}{dx}$ – 2y sec3x = 1

$\therefore$ $\frac{dy}{dx} - 2y = \cos 3x$ ... ... (i) It is a linear differential equation of the form $\frac{dy}{dx} + py = Q$

Here, p = –2 and Q = cos3x

Now, $\int pdx = -\int 2dx = -2x$

I.F. = $e^{\int pdx} = e^{-2x}$

Multiplying (i) by I.F.

$\left(\frac{dy}{dx} - 2y\right) e^{-2x} = \cos 3x \,.\, e^{-2x}$

or, $d(y.e^{-2x}) = co3x \,.\, e^{-2x}$

Integrating both sides,

$\int d(y.e^{-2x}) = \int \cos 3x \,.\, e^{-2x}\, dx$

$y.e^{-2x} = \int \cos 3x.e^{-2x}\, dx$ ... ... (ii)

Let $I = \int \cos 3x.e^{-2x}\, dx$

or, $I = \cos 3x \int e^{-2x}\, dx - \int \left\{\frac{d}{dx}(\cos 3x) \,.\, \int e^{-2x}\, dx\right\} dx$

$= \frac{\cos 3x.\, e^{-2x}}{-2} - \int \frac{-3}{-2} \sin 3x \,.\, e^{-2x}\, dx)$

$I = -\frac{1}{2}\cos 3x \,.\, e^{-2x} - \frac{3}{5}\int \sin 3x.e^{-2x}\, dx$

or, $I = -\frac{1}{2}\cos 3x \,.\, e^{-2x} - \frac{3}{2}\left[\sin 3x \int e^{-2x}\, dx - \int\left\{\frac{d}{dx}(\sin 3x) \,.\, \int e^{-3x}\, dx\right\}\right]$

$I = -\frac{1}{2}\cos 3x \,.\, e^{-2x} - \frac{3}{2}\left[\sin 3x \,.\, \frac{e^{-2x}}{-2} - \int \frac{3}{-2}\cos 3x \,.\, e^{-2x}\, dx\right]$

$I = -\frac{1}{2}\cos 3x \,.\, e^{-2x} + \frac{3}{4}\sin 3x\, e^{-2x} - \frac{9}{4} I$

or, $\frac{13}{4} I = -\frac{1}{2}\cos 3x \,.\, e^{-2x} + \frac{3}{4}\sin 3x\, e^{-2x}$

$I = \frac{1}{13}(3\sin 3x - 2\cos 3x)\, e^{-2x}$

from (ii),

$y \,.\, e^{-2x} = \frac{1}{13}(3\sin 3x - 2\cos 3x)\, e^{-2x} + c$

$\therefore \quad y = \frac{1}{13}(3\sin 3x - 2\cos 3x) + ce^{2x}$

e. $\cos^2 x \frac{dy}{dx} + y = \tan x \Rightarrow \frac{dy}{dx} + \sec^2 x.y = \tan x \,.\, \sec^2 x$ ... ... (i)

$p = \sec^2 x$ and $Q = \sec^2 x.\tan x$

$\int p dx = \int \sec^2 x dx = \tan x$

I.F. $= e^{\int p dx} = e^{\tan x}$

Multiplying (i) by I.F. we get,

$d(y.\, e^{\tan x}) = \tan x.\, \sec^2 x.e^{\tan x}$

Integrating

$\int d(Iy.etanx) = \int e^{\tan x} \tan x \,.\, \sec^2 x\, dx$

$y.e^{\tan x} = \int e^u.udu$ where $u = \tan x$

$y.e^{\tan x} = u.e^u - e^u$

Integrating by parts

$y.e^{\tan x} = (u - 1)\, e^{u}$

$y.e^{\tan x} = (\tan x - 1)\, e^{\tan x} + c$

$\therefore \quad y = (\tan x - 1) + c.e^{-\tan x}$

f. $x\dfrac{dy}{dx} + 2y = x^2 \log x$

$\dfrac{dy}{dx} + \dfrac{2}{x} y = x \log x \; \ldots \ldots \ldots \text{(i)}$

Comparing (i) with $\dfrac{dy}{dx} + py = Q$, we get

$p = \dfrac{2}{x}$ and $Q = x \log x$

$\int p dx = \int \dfrac{2}{x} dx = 2 \ln x = \log x^2$

I.F. $= e^{\int p dx} = e^{\log x^2} = x^2$

Multiplying (i) by I.F.

$\left(\dfrac{dy}{dx} + \dfrac{2}{x} y\right) x^2 = x^3 \log x$

$d(y.x^2) = x^3.\log x$

Integrating both sites, we get

$\int d(y.x^2) = \int x^3 \log x \, dx$

$y.x^2 = \log x. \int x^3 dx - \int \left\{\dfrac{d}{dx}(\log x) . \int x^3 \, dx\right\} dx$

$y.x^2 = \log x . \dfrac{x^4}{4} - \int \dfrac{1}{x} . \dfrac{x^4}{4} dx$

$y.x^2 = \dfrac{x^4}{4} \log x - \dfrac{1}{4} s \int x^3 \, dx$

$y.x^2 = \dfrac{x^4}{4} \log x - \dfrac{1}{16} x^4 + c$

$\therefore \quad y = \dfrac{x^2}{4} \log x - \dfrac{x^2}{16} + cx^{-2}$

g. $x\dfrac{dy}{dx} - x = 1 + y$

$x\dfrac{dy}{dx} - y = 1 + x$

$\dfrac{dy}{dx} - \dfrac{1}{x} y = \left(\dfrac{1+x}{x}\right) \; \ldots \ldots \ldots \text{(i)}$

Comparing (i) with $\dfrac{dy}{dx} + py = Q$, we get

$p = -\dfrac{1}{x}$ and $Q = \dfrac{1 + x}{x}$

$\int p dx = \int -\dfrac{1}{x} dx = -\ln x = \ln x^{-1}$

I.F. $= e^{\int p dx} = e^{\int p dx} = e^{\ln x^{-1}} = x^{-1}$

Multiplying (i) both sides by I.F.

$\left(\dfrac{dy}{dx} - \dfrac{x}{1} y\right) x^{-1} = \left(\dfrac{1+x}{x}\right) x^{-1}$

$d(y.x^{-1}) = \frac{1 + x}{x^2}$

$d(y.x^{-1}) = x^{-2} + \frac{1}{x}$

Integrating both sides,

$\int (y.x^{-1}) = \int x^{-2}\,dx + \int \frac{1}{x}\,dx$

$y.x^{-1} = -x^{-1} + \ln x + c$

$y = -1 + x\ln x + cx$

h. $\frac{dy}{dx} + \frac{y}{x} = e^x$ ... ... (i)

Comparing (i) with $\frac{dy}{dx} + p.y = Q$ we get

$p = \frac{1}{x}$ and $Q = e^x$

$\int p\,dx = \ln x$

$I.F. = e^{\int p\,dx} = e^{\ln x} = x$

Multiplying (i) both sides by I.F.

$\left(\frac{dy}{dx} + \frac{y}{x}\right) x = x.e^x$

$d(y.x) = x.e^x$

Integrating both sides,

$\int d(y.x) = \int x.e^x\,dx$

$y.x = xe^x - e^x + c$

$y = \frac{(x-1)}{x} e^x + \frac{c}{x}$

i. $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1}x$

$\frac{dy}{dx} + \left(\frac{1}{1+x^2}\right) y = \frac{\tan^{-1}x}{(1 + x^2)}$ ... . (i)

Comparing (i) with $\frac{dy}{dx} + py = Q$, we get

$p = \frac{1}{1+x^2}$, $Q = \frac{\tan^{-1}x}{1+x^2}$

$\int p\,dx = \int \frac{1}{1+x^2}\,dx = \tan^{-1}x$

$I.F. = e^{\int p\,dx} = e^{\tan^{-1}x}$

Multiplying (i) by I.F. both sides

We get,

$d(y.e^{\tan^{-1}x}) = \frac{\tan^{-1}x}{1+x^2} . e^{\tan^{-1}x}$

Integrating both sides

$\int d(y.\,e^{\tan^{-1}x}) = \int e^{\tan^{-1}x}\frac{\tan^{-1}x}{1 + x^2}\,dx$

$y.e^{\tan^{-1}x} = \int e^{\tan^{-1}x}\frac{\tan^{-1}x}{1 + x^2}\,dx$

Put $e^{\tan^{-1}x} = u$ in RHS

Then $\frac{1}{1+x^2}dx = du$

Then,

$y.e^{\tan^{-1}x} = \int e^u.u\,du$

$y.e^{\tan^{-1}x} = \int u.e^u\,dx$

$y.\,e^{\tan^{-1}x} = u\int e^u\,du - \int\left\{\frac{du}{du}.\int e^u\,du\right\}du$

$y.\,e^{\tan^{-1}x} = u.e^u - \int 1.e^u\,du$

$y.\,e^{\tan^{-1}x} = ue^u = e^u + c$

$y.e^{\tan^{-1}x} = \tan^{-1}x\,e^{\tan^{-1}x} - e^{\tan^{-1}x} + c$

$y = \tan^{-1}x - 1 + \frac{c}{e^{\tan^{-1}x}}$

2. Solution:

a. $(1-x^2)\frac{dy}{dx} - x.y = 1$

$\therefore\ \frac{dy}{dX} + \left(\frac{-x}{1-x^2}\right)y = \frac{1}{1-x^2}$ ... ... (i)

Comparing (i) with $\frac{dy}{dx} + py = Q$, we get

$p = \frac{-x}{1-x^2}$ and $Q = \frac{1}{1-x^2}$

$\int pdx = \frac{1}{2}\int\frac{-2x}{1-x^2}dx = \frac{1}{2}\ln(1-x^2) = \ln\sqrt{1-x^2}$

I.F. $= e^{\int pdx} = e^{\ln\sqrt{(1-x^2)}} = \sqrt{1-x^2}$

Multiplying (i) both sides by I.F.

$\left[\frac{dy}{dx} + \left(\frac{-x}{1-x^2}\right)y\right]\sqrt{1-x^2} = \frac{1}{(1-x^2)}\sqrt{1-x^2}$

$d(y\,.\sqrt{1-x^2}) = \frac{1}{\sqrt{1-x^2}}$

Integrating

$\int d(y.\sqrt{1-x^2}) = \int\frac{1}{\sqrt{1-x^2}}dx$

$y.\sqrt{1-x^2} = \sin^{-1}x + c$

b. $\sec x\,.\frac{dy}{dx} - y = \sin x$

$\frac{dy}{dx} - \cos x\,.\,y = \sin x\,.\cos x$ ... ... (i)

Comparing (i) with $\frac{dy}{dx} + p.y = Q$, we get

$p = -\cos x$ and $Q = \sin x.\cos x$

$\int pdx = -\int\cos x dx = -\sin x$

I.F. $= e^{\int pdx} = e^{-\sin x}$

Multiplying (i) both sides by I.F.
$d(y.e^{\sin x}) = e^{-\sin x} . \sin x . \cos x\, dx$
Integrating

$\int d(y.e^{-\sin x}) = \int e^{-\sin x} . \sin x.\cos x\, dx$

$y.e^{-\sin x} = \int e^{-u} . u\, du$ where $\sin x = u$

$y.e^{-\sin x} = -u\, e^{-u} - e^{-u} + c$
$y.e^{-\sin x} = (-1-u)\, e^{-u} = + c$
$y.e^{-\sin x} = (-1 - \sin x)\, e^{-\sin x} + c$
$\therefore \quad y = (1 - \sin x) + c\, e^{\sin x}$
$\therefore \quad y + 1 + \sin x = c\, e^{\sin x}$

c. $\cos^2 x \dfrac{dy}{dx} + y = 1$

$\dfrac{dy}{dx} + \sec^2 x.y = \sec^2 x$ ... ... (i)

Comparing (i) with $\dfrac{dy}{dx} + p.y = Q$, we get

$\sec^2 x$ and $Q = \sec^2 x$

$\int pdx = \int \sec^2 x\, dx = \tan x$

Integrating factor (I.F.) $= e^{\int pdx} = e^{\tan x}$
Multiplying (i) both sides by I.F.

$\left(\dfrac{dy}{dx} + \sec^2 x.y\right) e^{\tan x} = \sec^2 x\, e^{\tan x}$
$d(ye^{\tan x}) = e^{\tan x} \sec^2 x$
Integrating both sides,

$y.e^{\tan x} = \int e^{\tan x} . \sec^2 x\, dx$

$y.e^{\tan x} = e^{\tan x} + c$
$y = 1 + c\, e^{-\tan x}$

$\text{Sin}x \dfrac{dy}{dx} + y.\cos x = \sin 2x$

$\dfrac{dy}{dx} + \cot x\, y = 2\cos x$ ... ... (i)

d. Here, $p = \cot x$ and $q = 2\cos x$

$\int pdx = \int \cot x\, dx = \log \sin x$

I.F. $= e^{\int pdx} = e^{\log \sin x} = \sin x$
Multiplying (i) by I.F. we get
$d(y.\sin x) = 2\cos x . \sin x$
Integrating

$\int d(y.\sin x) = \int \sin 2x\, dx$

$y.\sin x = -\dfrac{\cos 2x}{2} + c$

$y = -\dfrac{1}{2} \cos 2x . \operatorname{cosec} x + c.\operatorname{cosec} x$

3. Solution:

a. $(1+x)\dfrac{dy}{dx} - xy = 1 - x$

$\dfrac{dy}{dx} - \dfrac{x}{1+x}.Y = \dfrac{1-X}{1+X}$ ... ... (i)

$p = -\dfrac{x}{1+x}$ and $Q = \dfrac{1-x}{1+x}$

$\int pdx = -\int\dfrac{x}{x+1}dx = -\int\dfrac{x+1-1}{x+1}dx = -\int 1dx + \int\dfrac{1}{x+1} = -x + \ln(x+1)$

I.F. = $e^{\int pdx} = e^{-x+\ln(x+1)} = e^{-x}(x+1)$

Multiplying (i) by by I.F. we get

$d(y.e^{x}(x+1)) = (1-x)e^{-x}$

Integrating $\int d(y(x+1)e^{-x}) = \int e^{-x}(1-x)\,dx$

or, $y(x+1)e^{-x} = \dfrac{(1-x)e^{-x}}{-1} + e^{-x} + c$

or, $ye^{-x}(x+1) = -(1-x)e^{-x} + e^{-x} + c$

or, $ye^{-x}(x+1) = e^{-x}(1-1+x) + c$

or, $y(x+1)e^{-x} = ex^{-x} + c$

$\therefore$ $y(x+1) = x + ce^{x}$

b. $\dfrac{dy}{dx} + \dfrac{4x}{1+x^2}.y = -\dfrac{1}{(x^2+1)^2}$ ... ... (i)

Here, $p = \dfrac{4x}{x^2+1}$ and $Q = \dfrac{-1}{(x^2+1)^2}$

$\int pdx = 2\int\dfrac{2x}{x^2+1}dx = 2\ln(x^2+1) = \ln(x^2+1)^2$

I.F. $= e^{\int pdx} = (x^2+1)^2$

Multiplying (i) by I.F. we get

$d\{y.(x^2+1)^2\} = -1$

Integrating $\int d(y(x^2+1)^2) = -\int 1dx$

$y(x^2+1)^2 = -x + c$

c. $(x^2-1)\dfrac{dy}{dx} + 2xy = \dfrac{2}{x^2-1}$

$\therefore$ $\dfrac{dy}{dx} + \dfrac{2x}{2^2-1}y = \dfrac{2}{(x^2-1)}$ ... ... (i)

Comparing (i) with $\dfrac{dy}{dx} + p.y = Q$, we get

$p = \dfrac{2x}{x^2-1}$ and $Q = \dfrac{2}{(x^2-1)^2}$

Now, $\int pdx = \int\dfrac{2x}{x^2-1}dx = \ln(x^2-1)$

I.F. $= e^{\int pdx} = e^{\ln(x^2-1)} = (x^2-1)$

Multiplying (i) both sides by I.F.

$\left(\dfrac{dy}{dx} + \dfrac{2x}{x^2-1}y\right)(x^2-1) = \dfrac{2}{(x^2-1)^2}(x^2-1)$

$d\{y(x^2-1)\} = \dfrac{2}{x^2-1}$

Integrating

$\int d(y.(x^2 - 1)) = 2\int \frac{1}{x^2 - 1}\,dx$

$y.(x^2 - 1) = 2.\log\left(\frac{x-1}{x+1}\right) + c$

d. $\frac{dy}{dx} + \frac{y}{x.\ln x} = \frac{1}{x}$ ... ... (i)

Comparing (i) with $\frac{dy}{dx} + p.y = Q$

$p = \frac{1}{x.\ln x}$ and $Q = \frac{1}{x}$

$\int pdx = \int \frac{1}{x.\ln x}\,dx$

Put $\ln x = u$ then $\frac{1}{x}\,dx = du$

s $\int pdx = \int \frac{du}{u} = \ln u$

$I.F = e^{\int pdx} = e^{\ln u} = u = \ln x$

Multiplying (i) by I.F. we get

$d(y.\ln x) = \frac{1}{x}\ln x$

Integrating both sides

$\int d(y.\ln x) = \int (1,x)\ln x\,dx$

$y.\ln x = \int vdv$ where $\ln x = v$

$y.\ln x = \frac{v^2}{2} + c$

$y.\ln x = \frac{(\ln x)^2}{2} + c$

$\therefore \quad y = \frac{1}{2}\ln x + \frac{c}{\ln x}$

e. $\frac{dy}{dx} + \frac{y}{x} + y^2$

$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x.y} = 1$ ... ... (i)

Put $\frac{1}{y} = z$ then $-\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}$

(i) becomes

$-\frac{dz}{dx} + \frac{1}{x}.z = 1$

$\frac{dz}{dx} - \frac{1}{x}.z = -1$ ... ... (ii)

Comparing (ii) with $\frac{dy}{dx} + p.y = Q$ we get

$p = -\frac{1}{x}$ and $Q = -1$

$\int pdx = -\int \frac{1}{x}\,dx = -\ln x = \ln x^{-1}$

I.F. $= e^{\int pdx} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$

Multiplying (ii) by I.F.

$\left(\frac{dz}{dx} - \frac{1}{x} . z\right)\frac{1}{x} = -1 . \frac{1}{x}$

$d\left(z . \frac{1}{x}\right) = -\frac{1}{x}$

Integrating both sides, we get

$\int d\left(z . \frac{1}{x}\right) = -\frac{1}{x}$

Integrating both sides, we get

$\int d\left(z . \frac{1}{x}\right) = -\int\frac{1}{x} dx$

$z.\frac{1}{x} = -\ln x + c$

$\frac{1}{y} = -x\ln x + cx$

f. $\frac{dy}{dx} + xy = xy^3$

$\frac{1}{y^3}\frac{dy}{dx} + \frac{x}{y^2} = x$ ... ... (i)

Put $\frac{1}{y^2} = z$

Then $-\frac{1}{y^3}\frac{dy}{dx} = \frac{dz}{dx}$

(i) becomes

$\frac{dz}{dx} - xz = -x$ ... ... (ii)

Comparing (ii) with $\frac{dy}{dx} + p.y = Q$, we get

$p = -x$ and $Q = -x$

$\int pdx = -\int xdx = e - \frac{x^2}{2}$

I.F. $= e^{\int pdx} = e^{-\frac{x^2}{2}}$

Multiplying (ii) by I.F.

$d(z,e^{-x^2/2}) = -\,x.e^{-\frac{x^2}{2}}$

Integrating

$z.e^{-\frac{x^2}{2}} = \int -x . e^{-\frac{x^2}{2}} dx$

$z.e^{-\frac{x^2}{2}} = \int e^u\, du$ where $u = -\frac{x^2}{2}$

$\frac{1}{y^2}\; e^{-\frac{x^2}{2}} = e^{-\frac{x^2}{2}} + c$

$\frac{1}{y^2} = 1 + c\, e^{\frac{x^2}{2}}$

g. $(1 - x^2)\frac{dy}{dx} + x.y = xy^2$

$$\frac{1}{y^2}\frac{dy}{dx} + \left(\frac{x}{1-x^2}\right)\frac{1}{y} = \frac{x}{1-x^2} \text{ ... ... (i)}$$

Put $\frac{1}{y} = z$ then $\frac{1}{y^2}\frac{dy}{dx} = -\frac{dz}{dx}$

Then, $-\frac{dz}{dx} + \left(\frac{x}{1-x+}\right) . z = \frac{x}{1-x^2}$

$$\frac{dz}{dx} - \left(\frac{x}{1-x^2}\right) z = -\frac{x}{1-x^2} \text{ ... ... (ii)}$$

Here, $p = \frac{-x}{1-x^2}$ and $Q = \frac{-x}{1-x^2}$

$$\int pdx = \frac{1}{2}\int -\frac{2x}{1-x^2} = \frac{1}{2}\ln(1-x^2) = \ln\sqrt{1-x^2}$$

I.F. $= e^{\int pdx} = e^{\ln\sqrt{1-x^2}} = \sqrt{1-x^2}$

Multiplying (ii) by I.F. we get

$$d(z.\sqrt{1-x^2} = -\frac{x}{1-x^2}\sqrt{1-x^2}$$

Integrating

$$z\sqrt{1-x^2} = \int\frac{x}{\sqrt{1-x^2}}dx$$

$$z.\sqrt{1-x^2} = \frac{1}{2}\int\frac{du}{\sqrt{u}} \text{ where } 1-x^2 = u$$

$$\frac{1}{y}.\sqrt{1-x^2} = \sqrt{1-x^2} + c$$