Mathematics

MODEL QUESTIONS

Grade: 12
Full Marks: 75

Time: 3 hours

1

Group 'A' [1 × 11 = 11]

Rewrite the correct option in your answer sheet.

1. If 1, ω, ω^2 are the cube roots of unit then
   a. $\omega = \omega^2$ b. $\omega^2 = \omega^3$
   c. $1 + \omega + \omega^2 = 0$ d. $1 + \omega = \omega^2$
2. The number of ways that 7 beads of different colors can be strung together so as to form a necklace is
   a. 5040 b. 2520
   c. 720 d. 360
3. $\tan^{-1}\frac{5}{12}$ is equal to
   a. $\sin^{-1}\frac{12}{13}$ b. $\cos^{-1}\frac{12}{13}$
   c. $\sec^{-1}\frac{12}{13}$ d. $\operatorname{cosec}^{-1}\frac{12}{13}$
4. If $2\cos\theta + 1 = 0$ is the trigonometric equation of the locus related to the string attached in the wall of a hall then the general value for θ is
   a. $n\pi + (-1)^n\frac{2\pi}{3}$ for $n \in Z$
   b. $n\pi + \frac{2\pi}{3}$ for $n \in Z$
   c. $2n\pi \pm \frac{2\pi}{3}$ for $n \in Z$.
   d. $2n\pi + \frac{2\pi}{3}$ for $n \in Z$.
5. If $\vec{a} = 2\vec{i}$ and $\vec{b} = 3\vec{j}$ where, $\vec{i}$, $\vec{j}$ and $\vec{k}$ unit vectors along X, Y, Z- axes respectively, then the value $\vec{b} \times \vec{a}$ is equal to
   a. $-6\vec{k}$ b. $6\vec{k}$
   c. $6\vec{i}$ d. $6\vec{j}$
6. There is a large grassy area near the president house of Nepal. The area is the set of all points in a plane. The sum of whose distances from two fixed places (points) is constant. The conic section represented by the grassy area is...
   a. Circle b. Parabola
   c. Hyperbola d. Ellipse
7. Four unbiased coins are tossed successively. The mean and variance of the distribution differed by
   a. 1 b. 2
   c. 3 d. 4
8. The degree of the differential equation $\frac{d^3y}{dx^3} + 5\left(\frac{d^2y}{dx^2}\right)^2 + 4\left(\frac{dy}{dx}\right)^4 + 6 = 0$ is
   a. 1 b. 2
   c. 3 d. 4
9. According to L Hospital's rule the value of $\lim_{x \to 0}\frac{x^3}{4\sin x}$ is equal to
   a. $\frac{3}{4}$ b. 0 c. $\frac{1}{4}$ d. ∞
10. When Gauss forward elimination method is used for solving the equations $3x + 4y = 18$... (i) and $3y - x = 7$ ...(ii), we apply the operation like....
    a. $eq^n(i) + 4\ eq^n(ii)$ b. $eq^n(i) + 3\ eq^n(ii)$
    c. $eq^n(i) + eq^n(ii)$ d. $eq^n(ii) + 3\ eq^n(i)$
11. The amount of gravity exerted by the earth on the mass 10 kg ($g = 9.8\ ms^{-2}$) is ...
    a. 9.8 Joule b. 9.8 Newton
    c. 98 Joule d. 98 Newton

    OR

    For the quadratic function $f(Q) = aQ^2 + bQ + C$ for real numbers a, b, c and $a \neq 0$, the maximum value attained at
    a. $\left(\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$ b. $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$
    c. $\left(-\frac{b}{2a}, \frac{b^2 - 4ac}{4a}\right)$ d. $\left(\frac{b}{2a}, \frac{b^2 - 4ac}{4a}\right)$

Group 'B' [5 × 8 = 40]

12. The binomial expression for two algebraic terms a and x is given as $(a + x)^n$.
    a. Write the binomial theorem for any positive integer n in expansion form. [1]
    b. Write the general term of the expansion. [1]
    c. Write any one property of binomial coefficients. [1]
    d. Write the single term for $C(n, r) + C(n, r - 1)$. [1]
    e. How many terms are there in the expression? [1]
13. Given $n^4 < 10^n$ for a fixed positive integer $n \geq 2$, prove that $(n + 1)^4 < 10^{n+1}$ using principle of mathematical induction. [5]
14. a. Evaluate $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right)$ [3]
    b. Using vector method, find the area of the triangle with vertices A(1, 4, 6), B(– 2, 5, 1) and C(1, – 1, 1). [2]
15. The information given below relates to the advertisement and sales of a departmental store in lakhs of Nepalese rupees.

	Advertisement Expenditure (X)	Sales (Y)
Arithmetic Mean	20	100
Standard deviation	3	12
	Correlation coefficient between (X) and (Y) = 0.8	

    a. Find the two regression equations related to above data. 4
    b. What should be the advertisement expenditure if the department store wants to attain sales target of Rs. 200 lakhs. [1]
16. Suman and Nikita are studying about application of derivative and integration in a class. They ask each other the quiz questions as given below. On the basis of these

questions answer the following.

a. $f'(x)$ and $g'(x)$ are derivatives of the functions $f(x)$ and $g(x)$. What is the expression equal to $\lim_{x \to a} \frac{f(x)}{g(x)}$ according to L'Hospital's rule for form ∞/∞. [1]

b. State Rolle's Theorem. [1]

c. What is the expression equal to $\int \frac{1}{x^2 + a^2} dx$? [1]

d. What does 'C' represent in the expression $\int \frac{dx}{3 \sin x + 4 \cos x} = \frac{1}{5} \ln \left| \text{tax} \left(\frac{x}{2} + \frac{1}{2} \tan^{-1} \frac{4}{3}\right) \right| + C$?[1]

e. Write a difference between derivative and antiderivative. [1]

17. Integrate $\int \frac{1}{x^4 - 1} dx$ using the concept of partial fraction. Also give an example of proper rational fraction and improper rational fraction. [3 + 2]

18. Use simplex method and maximize: $Z(x, y) = x + y$ subject to constraints $2x + 3y \geq 22$, $2x + y \geq 14$, $x \geq 0$, $y \geq 0$. [5]

19. Write any one difference between like parallel forces and unlike parallel forces. A heavy uniform beam whose mass is 60 kg is suspended in a horizontal position by two vertical strings each of which can sustain a tension of 52.5 kg wt. How far from the centre of the beam must a body of mass 30 kg placed so that one of the strings may just break?[1 + 4]

OR

If the demand function $P = 85 - 4Q - Q^2$, find the consumer's surplus at demand 4 units and price 64 units. Also make a revenue function for demand equation $P = 20 + 5Q - Q^2$. Obtain the standard quadratic equation for marginal revenue. Q represents the number of units demands and P represent the price. [2 + 1 + 1 + 1]

Group 'C' [8 × 3 = 24]

20. A mixture is to be made of three foods, A, B and C which contain nutrients P, Q, R as shown in the table below. The quantity of P, Q, R is 45 units, 54 units and 45 units respectively.

Food	Units of nutrients per kg of the foods		
	P	Q	R
A	2	2	4
B	3	5	0
C	4	3	5

a. Express the information in equation form. [1]

b. Solve the equations using matrix. [5]

c. If the cost per kg of the foods A, B, C are Rs. 300, Rs. 240 and Rs. 180 respectively, find the total cost of the mixture by matrix method. [2]

21. A line makes an angle α, β, γ, δ with the four diagonals of a cube kept in a dining room.

a. Find the direction ratios of any two diagonals of the cube and express the diagonals in vector form [3]

b. Find the angle between any two diagonals of the cube.[2]

c. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$ [3]

22. A college hostel accommodating 1000 students; one of them came from abroad with infection of corona virus, then the hostel was isolated. If the rate at which the virus spreads is assumed to be proportional to the product of the number 'N' of infected students and number of non-infected students and the number of infected students is 50 after 4 days.

a. Express the above information in the form of differential equation. [2]

b. Solve the differential equation. [2]

c. Show that more than 95% students will be infected after 10 days. [4]

2

Group 'A' [1 × 11 = 11]

Rewrite the correct option in your answer sheet.

1. If ω is a complex cube root of unity, then the value of $(1 + \omega - \omega^2)(1 - \omega + \omega^2)$ is

a. ω b. ω^2
c. $1 + \omega$ d. 4

2. An equation $(m + 2)x^2 - 2(m + 4)x + (m + 7) = 0$ have equal roots. The value of m is

a. 2 b. −2
c. 7 d. −4

3. Solution of $\sin\left(2\sin^{-1}\frac{4}{5}\right)$ is

a. $\frac{4}{5}$ b. $\frac{24}{15}$
c. $\frac{25}{24}$ d. 1

4. If $\cos mx = \cos nx$, then the value of x is

a. $2n\pi$ b. $2n\pi \pm \frac{\pi}{3}$
c. $(4k - 1)\frac{\pi}{2(m \pm n)}, k = 0, \pm1, \pm2, \ldots$
d. $\frac{2k\pi}{m \pm n}, k = 0, \pm1, \pm2, \ldots$

5. The area of a parallelogram whose diagonals are the vectors $\vec{i} - 2\vec{k}$ and $4\vec{i} + 3\vec{j} + \vec{k}$ is

a. $5\sqrt{14}$ sq. units b. $\frac{3}{2}$ sq. units
c. $\frac{3}{2}\sqrt{14}$ sq. units d. $\sqrt{14}$ sq. units

6. The equation of a hyperbola in standard position satisfying transverse and conjugate axes are respectively 4 and 5 is

a. $\frac{x^2}{4} - \frac{4^2}{25} = 1$ b. $4x^2 - 7y^2 = 36$
c. $4x^2 + 7y^2 = 36$ d. $\frac{x^2}{4} - \frac{y^2}{5} = 1$

7. Four unbiased coins are tossed successively. The mean and variance of the distribution differed by

a. 1 b. 2
c. 3 d. 4

8. The points on the curve $x^2 + y^2 - 2x - 3 = 0$ where the tangents are parallel to the X-axis are

a. (1, 2), (1, −2) b. (1, 2), (1, 2)
c. (−1, 2), (1, −2) d. (1, 2), (1, 3)

9. The order and degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + 2y\left(\frac{d^2y}{dx^2}\right) = 0$ is

a. 2, 1 b. 1, 2
c. 1, 3 d. 3, 1

10. When Gauss forward elimination method is used for solving the equations
$3x + 4y = 18$ .... (i)
$3y - x = 7$ .... (ii)
we apply the operation
a. $eq^n(i) + 4\ eq^n(ii)$ b. $eq^n(i) + 3\ eq^n(ii)$
c. $eq^n(i) + eq^n(ii)$ d. $eq^n(ii) + 3\ eq^n(i)$

11. If the resultant of two like parallel forces acting at a distance of 3 m is 80 N at a distance of 75 cm from one of the forces, then the force is
a. 20 N b. 9.8 N
c. 60 N d. 40 N

OR

If profit function $(\pi) = Q^2 - 10Q + 9$, then the breakeven coin is
a. 9 or 10 b. 1 or 10
c. 1 or 9 d. 4 or 5

Group 'B' [5 × 8 = 40]

12. a. If the numerical coefficients in the second, third and fourth terms of the expansion of $(x + a)^n$ are 30, 375 and 2500 respectively, find the value of n. Let a, b, c and x be elements of a group G. [3]
b. Solve for x: $x^2 = a^2$ and $x^5 = e$. [2]

13. a. If $z = \cos\theta + i \sin\theta$, find the value of $z^n + \frac{1}{z^n}$ by using De Moivre's Theorem. [2]
b. Solve the system of equations by the row-equivalent method: $x + y + z = 6$, $x - y + z = 2$ and $x + y - z = 0$. [3]

14. a. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then show that: $x + y + z = xyz$ [3]
b. Find the eccentricity and the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$. [2]

15. From the following data

Age in years (X)	5	15	30	45	50	60
Weight in kg (Y)	10	35	50	65	55	45

compute the
a. correlation coefficient by Karl Pearson's method. [2]
b. line of regression for estimating X on Y and estimate the most probable age of the weight 37 kg. [3]

16. Evaluate:
a. $\int \frac{dx}{3 - 2x - x^2}$ [2]
b. $\int \frac{x^2}{(x^2 + 9)(x^2 + 4)} dx$ [3]

17. Solve $\frac{dy}{dx} + \frac{\cos x \sin y}{\cos y} = 0$. An equation reacting to the stability of an aeroplane is $\frac{dv}{dt} = g \cos \alpha - kv$, where v is the velocity and g, α, k are constants. Find an expression for velocity, if $v = 0$, when $t = 0$. [5]

18. Maximize $P = 25x + 45y$ subject to $x + 3y \le 21$, $2x + 3y \le 24$, $x, y \ge 0$ by using simplex method. [5]

19. a. Two unlike parallel forces, the greater of which is 75N, have a resultant 25N. Find the ratio of the distances of the resultant from the component forces. [2]
b. A projectile thrown from a point in a horizontal plane comes back to the plane in 4 sec. at a distance of 60 m in front of the point of projection. Find the velocity of projection. ($g = 10$ m/s²). [3]

OR

State the Hawkins-Simon conditions for the viability of the system. The demand and supply curves for an item are given by $P_d = 20 - 3Q - Q^2$ and $P_s = Q - 1$ respectively. Find the difference between consumer and producer surplus at the equilibrium price. [1 + 4]

Group 'C' [8 × 3 = 24]

20. a. In how many ways can the letters of the word "CALCULUS" be arranged so that the two L's do not come together? [3]
b. Sum to n terms of the series $1^2 + 3^2 + 5^2 + \ldots$ [3]
c. The sum of the roots of a quadratic equation is 4 and the sum of their squares is 14. Find the equation. [2]

21. a. Find the angle between the lines whose direction cosines are given by $l + m + n = 0$ and $2lm + 2ln - mn = 0$. [5]
b. Prove by the vector method: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. [3]

22. a. Find the derivative of ln sinx by using first principle. [4]
b. State the mean value theorem. Use it to verify for the function $f(x) = x^2 - 4$ in [2, 4]. [1 + 3]

3

Group 'A' [1 × 11 = 11]

Write the correct option in your answer sheet.

1. If 1, ω and ω are the cube roots of unity then $1 + |\omega| + \omega^2 = 1$
a. 0 b. 1
c. 2 d. 3

2. How many 3 digit even numbers can be formed from 0, 1, 2, 3, 4, 5, 6 with no repetition?
a. 90 b. 105
c. 120 d. None

3. $\tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right) =$
a. $\frac{\pi}{2} - \frac{x}{2}$ b. $\pi - \frac{x}{4}$
c. $\frac{\pi}{2} - x$ d. $\frac{\pi}{2} + x$

4. If $\tan x + \cot x = 2$ then $x =$
a. $n\pi + \frac{\pi}{4}$ b. $n\pi + \frac{\pi}{2}$
c. $n\pi + \frac{\pi}{3}$ d. $n\pi + \frac{\pi}{6}$

5. The area of triangle determined by the vector $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$ is
   a. 10.5 sq. units b. 15.5 sq. units
   c. 20.5 sq. units d. 40.5 sq. units
6. The locus of a point in a plane such that the difference of the distances from two fixed points is constant, is called
   a. a circle b. a parabola
   c. a hyperbola d. an ellipse
7. If $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$ then $P(A/B) =$
   a. $\frac{1}{4}$ b. $\frac{1}{2}$
   c. $\frac{1}{3}$ d. $\frac{1}{5}$
8. The degree of differential equation $\left(\frac{dy}{dx}\right)^3 + 3y\frac{d^2y}{dx^2} = 0$ is
   a. 0 b. 1
   c. 2 d. 3
9. The inlcinaiton with the x-axis of tangent of $x^2 + y^2 = 16$ at (0, 4) is
   a. $\frac{\pi}{2}$ b. 0
   c. $\frac{\pi}{4}$ d. $\frac{\pi}{3}$
10. In Gauss elimination method, the coefficient of the first variable in first equation must be
   a. zero b. non-zero
   c. negative d. positive
11. A projectile is fired horizontally from a wall of height 5 m from the level of the ground and reaches the ground at a horizontal distance of 1000m. What is the initial velocity? ($g = 10$ m/s²)
   a. 100 m/s b. 500 m/s
   c. 800 m/s d. 1000 m/s

   OR

   If the matrix of technical coefficients is $A = \begin{bmatrix} 0.5 & 0.25 \\ 0.2 & 0.6 \end{bmatrix}$ then Leontief matrix is
   a. $\begin{bmatrix} 0.5 & 0.25 \\ 0.2 & 0.6 \end{bmatrix}$ b. $\begin{bmatrix} 0.5 & 0.25 \\ 0.2 & 0.4 \end{bmatrix}$
   c. $\begin{bmatrix} 0.5 & -0.25 \\ -0.2 & 0.4 \end{bmatrix}$ d. $\begin{bmatrix} 0.5 & -0.75 \\ -0.8 & 0.6 \end{bmatrix}$

Group 'B' [5 × 8 = 40]

12. a. If the three consecutive coefficients in the expansion of $(1 + x)^n$ be 165, 330, 462, find n. [3]
    b. Solve: $3x + 6 = 5$ in $\mathbb{Z}_7$. [2]
13. a. If $z = \cos\theta + i\sin\theta$, find $z^n + \frac{1}{z^n}$ and $z^n - \frac{1}{z^n}$. [2]
    b. Solve the systems of equations by Cramer's rule.
    $x - 2y + 3z = 4$
    $2x + y - 4z = 3$
    $-3x + 4y - z = -2$ [3]
14. a. Prove that $\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5} = \frac{\pi}{4}$. [3]
    b. Find the equation of the ellipse whose major axis is twice the minor axis and which passes through the point (0, 1). [2]
15. In a partially destroyed laboratory record of an analysis of a correlation data the following results only are eligible:
    Variance of X = 9
    Regression equations, $4X - 5Y + 33 = 0$
    $20X - 9Y = 107$. [5]
    Find
    a. The mean value of X and Y.
    b. The standard deviation of Y.
    c. The coefficient of correlation between X and Y.
16. Evaluate the following integrals
    a. $\int\sqrt{\frac{1+x}{1-x}}\,dx$ [2]
    b. $\int\frac{1}{1 + \sin x + \cos x}\,dx$ [3]
17. Define linear and non-linear differential equation. Solve
    $\sin x\frac{dy}{dx} + \cos x \cdot y = x\sin x$ [5]
18. Use the simplex method to solve the following linear programming problems.
    Max. $P = 5x_1 + 3x_2$
    Constraints $2x_1 + x_2 \le 40$
    $x_1 + 2x_2 \le 50$
    $x_1, x_2 \ge 0$ [5]
19. a. Two like parallel forces of magnitudes P and Q are acting at the end points A and B of a rod of length AB. If two opposite forces each of magnitude S are added to P and Q, then prove that the line of action of the new resultant will be displaced through a distance $\frac{S \cdot AB}{P + Q}$. [3]
    b. Find the least velocity with which a stone must be thrown from one bank of a river of width 50 m, so as to strike the other bank. Find the greatest height attained by the stone. [$g = 10$ ms⁻²] [2]

    OR

    a. The following is the input/output table for two industries X and Y. The values are in million of rupees.

Producers	Users X	Users Y	Final Demand	Total Output
X	14	6	8	28
Y	7	18	11	36

    Determine the outputs if the final demand changes to 20 for X and 30 for Y. [3]
    b. Solve $Y_t + 0.3Y_{t-1} = 0$, given $Y_1 = 9$. State whether the solution is stable or not. [2]

Group 'C' [8 × 3 = 24]

20. a. A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them are relatives? [2]
    b. Prove by principle of mathematical induction. [3]
    $1\cdot3 + 3\cdot5 + 5\cdot7 + \cdots + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$
    c. If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$. [3]

21. a. Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to 3, – 1, 1 and – 3, 2, 4. [3]

b. Find the equation of the plane passing through (2, – 3, 1) and is perpendicular to the line joining the points (3, 4, – 1) and (2, – 1, 5). [2]

c. Using vector method, prove that: sin (A – B) = sin A cos B – cos A sin B. [3]

22. a. Find from first principle the derivative of x^x. [4]

b. State Rolle's theorem. Give its geometrical meaning. Verify Rolle's theorem for the function $f(x) = \sin x$ in $[0, \pi]$. [1+1+2]

4

Group 'A' [1 × 11 = 11]

Write the correct option in your answer sheet.

1. If $\frac{1+2i}{2+i} = r(\cos\theta + i\sin\theta)$, then r equals

a. 0 b. – 1
c. 1 d. 2

2. How many numbers are there between 99 and 1000 such that at least one of their digits is 7?

a. 251 b. 252
c. 253 d. 254

3. If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$, then x is

a. $\frac{1}{2}$ b. $\frac{\sqrt{3}}{2}$
c. $-\frac{1}{2}$ d. None of them

4. If $(1 + \tan\theta)(1 + \tan\phi) = 2$, then $\theta + \phi$ equals

a. $\frac{\pi}{6}$ b. $\frac{\pi}{4}$
c. $\frac{\pi}{3}$ d. $\frac{5\pi}{12}$

5. If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 4$ then $\vec{a} \cdot \vec{b}$ is equal to

a. $\sqrt{21}$ b. $\sqrt{12}$
c. $2\sqrt{21}$ d. $2\sqrt{12}$

6. Which of the following is not true for $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$?

a. The curve of the hyperbola is symmetric with respect to both the axes.
b. The eccentricity of the hyperbola is greater than unity.
c. $y = \pm\frac{b}{a}x$ are called the asymptotes of the hyperbola.
d. Equation of its directrices are $y = \pm\frac{a}{e}$.

7. The mean and variance of a binomial distribution are 12 and 9 respectively. Then number of trials (n) is

a. 48 b. 108
c. 12 d. 9

8. The differential equation $\frac{dy}{dx} + Py = Q$ is linear if

a. P is a function of y
b. Q is a function of y
c. P and Q both are functions of x and y both
d. P and Q both are constants or functions of x only

9. The value of $\lim_{x \to 1/2} \frac{\tan 3\pi x}{\sec \pi x}$ is

a. 3 b. $\frac{1}{3}$
c. 0 d. $\frac{\pi}{3}$

10. In Gauss elimination method, the coefficients of the variables of the equation a_{ij} where $i = j$ are known as

a. basic elements b. non-basic elements
c. pivot elements d. common elements

11. If two unlike parallel forces P and Q act at points 5 m apart. If the resultant force is 9N and acts at a distance of 10 m from the greater force P, then Q is equal to

a. 7N b. 6N
c. 18N d. 9N

OR

The order of the difference equation $Y_t - 0.7Y_{t-1} - 0.2Y_{t-2} + 300 = 0$ is

a. 1 b. 2
c. 3 d. 4

Group 'B' [5 × 8 = 40]

12. Prove that: $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!} = e - 1$.

13. State the principle of mathematical induction. Using it, prove that $n^2 > 2n + 1$ for all $n \geq 3$.

14. a. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{\pi}{2}$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

b. Find the equation of the locus of the set of all points, the sum of whose distances from (3, 0) to (9, 0) is 12. [3+2]

15. a. Number of hours studied and marks in mathematics are given as:

Hour Studied (X)	8	5	11	13	10	5	18	15	2	8
Marks in Mathematics (Y)	56	44	72	72	72	54	94	85	33	65

Calculate the rank correlation r between X and Y. [3+2]

b. An unbiased coin is tossed 10 times. What is the probability of getting 6 heads?

16. Prove that:

$$\int \frac{1}{3\sin x - 4\cos x}\,dx = \frac{1}{5}\ln\left|\frac{2\tan\frac{x}{2} - 1}{2\tan\frac{x}{2} + 4}\right| + C$$

17. A function $f(x)$ is defined as

$$f(x) = \begin{cases} 1 + \sin x & \text{for } 0 \leq x < \frac{\pi}{2} \\ 2 + \left(x - \frac{\pi}{2}\right)^2 & \text{for } \frac{\pi}{2} \leq x < \infty \end{cases}$$ Does $f'\left(\frac{\pi}{2}\right)$ exist?

18. Using the simplex method, maximize $z = 3x_1 + 4x_2$

subject to $x_1 + x_2 \leq 10$
$x_1 \leq 4,$
$x_2 \leq 8,$
$x_1, x_2 \geq 0.$

19. A stone is thrown with an initial velocity of 30 ms^{-1} at an angle of 30° above the horizontal. Find (a) the speed of the stone after 3 seconds of projection (b) the distance of the stone from its point of projection after 2 seconds of projection. [$g = 10\ ms^{-2}$]

OR

Given the technology matrix $A = \begin{bmatrix} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix}$

a. Test Hawkins-Simon Conditions for viability of the system.

b. If the demand vector $D = \begin{bmatrix} 2100 \\ 4200 \\ 6300 \end{bmatrix}$ then find the gross output vector X.

Group 'C' [8 × 3 = 24]

20. a. Determine the value of p for which one root of the equation $x^2 + px + 1 = 0$ is the square of the other. [3]

b. Let a and x be elements of a group G. Solve for x in terms of a if $x^2 = a^2$ and $x^6 = e$. [3]

c. In how many ways 4 letters from the word COLLEGE be selected such that 2 letters are alike and 2 letters are distinct? [2]

21. a. Find the lines whose direction cosines (l, m, n) satisfy the equations $l + m + n = 0$ and $2lm + 2ln - mn = 0$. Also find the angle between them. [5]

b. Prove by vector method that:
$\sin (A + B) = \sin A \cos B + \cos A \sin B$ [3]

22. a. Verify Lagrange's Mean Value Theorem for $f(x) = x(x - 2),\ x \in [1, 2]$ [3]

b. Reduce the differential equation $\frac{dy}{dx} + y = xy^2$ into linear form and solve it. [5]

5

Group 'A' [1 × 11 = 11]

Write the correct option in your answer sheet.

1. 5^{th} term from the end of the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^{12}$ is
   a. $-7920\ x^4$ b. $7920\ x^{-4}$
   c. $7920\ x^4$ d. $-7920\ x^{-4}$
2. Which of the following is not an Abelian group?
   a. set of real numbers under addition
   b. set of 2 × 2 matrices with non zero determinant under matrix multiplication
   c. set of integers under addition
   d. set of cube roots of unity under multiplication
3. The domain of $\sin^{-1} x$ is
   a. $(-\pi, \pi)$ b. $[-1, 1]$
   c. $(0, 2\pi)$ d. $(-\infty, \infty)$
4. The value of θ satisfying $\sin 7\theta = \sin 4\theta - \sin \theta\ (0 < \theta < \frac{\pi}{2})$ are
   a. $\frac{\pi}{9}, \frac{\pi}{4}$ b. $\frac{\pi}{3}, \frac{\pi}{9}$
   c. $\frac{\pi}{6}, \frac{\pi}{9}$ d. $\frac{\pi}{3}, \frac{\pi}{4}$
5. The area of parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is
   a. $|\vec{a} \times \vec{b}|$ b. $2|\vec{a} \times \vec{b}|$
   c. $\frac{1}{2}|\vec{a} \times \vec{b}|$ d. None of them
6. The projection of the line joining (3, 4, 5) and (4, 6, 3) on the joining (− 1, 2, 4) and (1, 0, 5) is
   a. $\frac{1}{2}$ b. $\frac{4}{3}$
   c. $\frac{5}{7}$ d. $\frac{2}{5}$
7. If a fair coin is tossed 10 times, then the probability of getting exactly 6 tails, is
   a. 0.2 b. 0.3
   c. 0.4 d. 0.8
8. If $y = f(x) = x^2 + 1$, then the actual change in y when $x = 1$ and $\Delta x = dx = 0.1$ is
   a. 0.21 b. 0.2
   c. 0.01 d. 0.5
9. The value of $\int \frac{1}{e^x + e^{-x}} dx$ is
   a. $\tan^{-1}(e^{-x}) + c$ b. $\tan^{-1}(e^x) + c$
   c. $-\tan^{-1}(e^x) + c$ d. $-\tan^{-1}(e^{-x}) + c$
10. In less than or equal to constraints, the non-negative variable that is used to balance both side is
    a. condition variable b. surplus variable
    c. slack variable d. solving variable
11. A particle is thrown with an initial velocity of 100 ms^{-1} at an angle of 30° above the horizontal. The time to attain the highest point is ($g = 10\ ms^{-2}$):
    a. 6 sec b. 7 sec
    c. 4 sec d. 5 sec

    OR

    If the demand and supply function is a market are $Q_d = 35 - 0.5P$ and $Q_s = -4 + 0.8P$ and the rate of adjustment of price when the market is out of equilibrium is $\frac{dP}{dt} = 0.25(Q_d - Q_s)$. Then the differential equation is
    a. $\frac{dP}{dt} = -0.335(P - 30)$ b. $\frac{dP}{dt} = -0.235(P - 30)$
    c. $\frac{dP}{dt} = -0.225(P - 30)$ d. $\frac{dP}{dt} = -0.325(P - 30)$

Group 'B' [5 × 8 = 40]

12. Distinguish between permutation and combination. In how many ways can the letters of the word "MONDAY" be arranged? How many of these arrangements do not begin with M? How many begin with M and do not end with Y?
13. The price of 3 commodiites X, Y and Z are rupees x, y and z per unit respectively. A purchases 4 units of Z and sells 3 units of X and 5 units of Y. B purchase 3 units of Y and sells 2 units of X and 1 unit of Z. C purchases 1 unit of X and sells 4 units of Y and 6 units of Z. In this process A, B and C earn Rs. 6,000; Rs. 5,000 and Rs. 13,000 respectively. Find the price per unit of 3 commodiites by using Cramer's rule. [8]
14. a. Solve $\sin \theta - \sqrt{3} \cos \theta = 2(-2\pi < \theta < 2\pi)$

b. The foot of the perpendicular drawn from (1, 2, 3) to the plane is (2, – 1, 2). Find the equation of plane. [3+2]

15. Height and weight of six persons is given below.

Height (Inches) (X)	62	72	70	60	67	70
Weight (Kg) (Y)	50	65	63	52	56	60

Write the equation for the prediction of weight for a given height. Find the correlation coefficient between X and Y.

16. a. Evaluate: $\int \sqrt{(x-\alpha)(\beta-x)}\,dx$

b. Prove that: $\int \text{cosec}\, x\, dx = \ln\left(\tan\frac{x}{2}\right) + c$ [3+2]

17. Using the definition of derivative, find the derivative of $\ln \cos^{-1}\sqrt{x}$.

18. Using simplex method, maximize $z = 14x_1 + 4x_2$ subject to

$2x_1 + x_2 \leq 3$

$x_1 - x_2 \leq 1$

$x_1, x_2 \geq 0$.

19. a. Find the mass of an object which on earth weighs 98 N. ($g = 9.8\ ms^{-2}$) [2]

b. P and Q are like parallel forces. If P is moved parallel to itself through a distance x, show that the resultant of P and Q moves a distance $\frac{P.x}{P+Q}$. [2+3]

OR

Consider a Lagged Keynesian macroeconomic national income model $Y_t = C_t + I_t$ where $C_t = 200 + 0.9Y_{t-1}$ and $I_t = 400$.

a. Write the national income equation as difference equation in Y_t.

b. Solve the difference equation. Hence describe the time path. Will the path stabilize?

c. If $Y_0 = 10{,}000$, find the particular solution.

Group 'C' [8 × 3 = 24]

20. a. Use De-Moivre's Theorem to find the square roots of $-2-2\sqrt{3}\,i$. [4]

b. Find the quadratic equation with rational coefficient whose one of the roots is $\frac{1}{5+3i}$. [2]

c. Find the sum to *n* terms of the series whose n^{th} term is $(2n-1)(n+2)$. [2]

21. a. Show that locus of a point which moves such that its sum of the distances from two fixed points is constant is ellipse. [3]

b. For what value of *k* makes the line joining points (1, 2, *k*) and (4, 5, 6) parallel to the line joining the points (– 4, 3, – 6) and (2, 9, 2)? [2]

c. If $\vec{a} = 3\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{b} = 2\vec{i} - 2\vec{j} + 4\vec{k}$, [3]

i. Find $\vec{a} \times \vec{b}$.

ii. Is $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$?

iii. Find a unit vector perpendicular to $\vec{a}$ and $\vec{b}$.

22. a. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-3x}$ for different values of A and B. [2]

b. Solve: $\frac{dy}{dx} + y\sec x = \tan x$ [3]

c. Find the angle of intersection of curves $xy = 6$ and $x^2y = 12$. [3]

6

Group 'A' [1 × 11 = 11]

Write the correct option in your answer sheet.

1. Which row operation is used to obtain the matrix $\begin{bmatrix} 1 & 2 & 3 & : & 4 \\ 2 & 1 & 2 & : & 5 \\ 0 & -1 & -4 & : & -6 \end{bmatrix}$ from the matrix $\begin{bmatrix} 1 & 2 & 3 & : & 4 \\ 2 & 1 & 2 & : & 5 \\ 3 & 2 & 1 & : & 3 \end{bmatrix}$?

a. $R_3 \to R_3 - 3R_1$ b. $R_3 \to R_3 - R_2 - R_1$

c. $R_3 \to R_3 - \frac{3}{2}R_2$ d. None

2. If *n* is any odd natural number, then $a^n + b^n$ is divisible by

a. $a+b$ b. $a-b$

c. a^2+b^2 d. a^3+b^3

3. The general solution of $3\,\text{cosec}^2 x - 4 = 0$ is

a. $n\pi + (-1)^n\frac{\pi}{3}$ b. $n\pi + \frac{\pi}{3}$

c. $n\pi \pm \frac{\pi}{3}$ d. $2n\pi \pm \frac{\pi}{3}$

4. If $\sin^{-1}\frac{4}{5} + \sec^{-1}\frac{5}{x} = \frac{\pi}{2}$ then x =

a. 3 b. 4

c. 5 d. 6

5. The sine of angle between the pair of vectors $3\vec{i} + \vec{j} + 2\vec{k}$ and $2\vec{i} - 2\vec{j} + 4\vec{k}$ is

a. $\frac{2}{\sqrt{7}}$ b. $\frac{\sqrt{2}}{5}$

c. $\frac{1}{3}$ d. $\frac{1}{\sqrt{2}}$

6. The locus of the centre of a circle which touches externally the given two circles is

a. circle b. parabola

c. ellipse d. hyperbola

7. The probability of hitting a target is 0.25. If 8 hitting are made, then the probability that none of them will hit the target is

a. 0.30 b. 0.40

c. 0.60 d. 0.80

8. $\int f'(x)\sqrt{f(x)}\,dx$ is given by

a. $\frac{2}{3}[f(x)]^{3/2} + c$ b. $\frac{3}{2}[f(x)]^{3/2} + c$

c. $\frac{3}{2}[f(x)]^{2/3} + c$ d. $-\frac{2}{3}[f(x)]^{3/2} + c$

9. The differential equation $\left(\frac{ds}{dt}\right)^2 + 3s\frac{d^2s}{dt^2} = 0$ is

a. second order, second degree, linear

b. second order, first degree, non linear

c. second order, first degree, linear

d. second order, second degree, non linear

10. In simplex method, the basic feasible solution must satisfy

a. negativity constraint b. non-negativity constraint

c. basic constraint d. non-basic constraint

11. Two like parallel forces of 5N and 15N act on a light rod at two points A and B respectively 6m apart. The distance of the resultant from the point A is

a. 1.5 m b. 2.5 m
c. 4.5 m d. 5.5 m

OR

The cobweb model basically analyze periodic fluctuation in price, demand and supply that oscillate about

a. maximum point b. minimum point
c. break even point d. equilibrium point

Group 'B' [5 × 8 = 40]

12. a. In how many ways can the letters of the word 'INTERVAL' be arranged so that: [3]
   i. all vowels are always together?
   ii. the relative positions of the vowels and consonants are not changed?

   b. Using De Moivre's theorem, find the cube roots of unity. [2]

13. a. Sum to n terms of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \cdots$. [3]

   b. Determine the value of m for which $3x^2 + 4mx + 2 = 0$ and $2x^2 + 3x - 2 = 0$ have a common root. [2]

14. a. Solve: $2\sin x \sin 3x = 1$. [3]

   b. Show that the line joining the points (1, 2, 3) and (4, 5, 7) is parallel to the line joining the points (– 4, 3, – 6) and (2, 9, 2). [2]

15. In a partially destroyed laboratory record of an analysis of a correlation data the following results only are eligible:

Variance of X = 9

Regression equations, $4X - 5Y + 33 = 0$
$20X - 9Y = 107$.

Find
a. The mean value of X and Y.
b. The standard deviation of Y.
c. The coefficient of correlation

16. Evaluate the followings integrals: $\int \frac{1}{(x-1)^2 (x-2)^3} dx$

17. State Lagrange's mean value theorem. Examine whether the function $f(x) = x^2 - 6x + 1$ satisfies Lagrange's mean value theorem. If it satisfies, then find the coordinates of the point at which the tangent is parallel to the chord joining the points A(1, – 4) and B(3, – 8).

18. Solve the system by partial pivoting method:

$$2x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 2x_2 + 3x_3 = 4$$
$$x_1 - x_2 + x_3 = 0$$

19. *A projectile is fired with a horizontal velocity of 30 ms⁻¹ from the top of a cliff 80 m high (a) How long will it take to strike the level ground at the base of the cliff? (b) How far from the foot of the cliff will it strike? (c) with what velocity will it strike the ground?*

OR

In a perfect competition, demand and supply functions of a commodity are given by $P_d = 40 - x^2$ and $P_s = 3x^2 + 8x + 8$. Find the consumer surplus and the producer surplus at the market equilibrium price.

Group 'C' [8 × 3 = 24]

20. a. State binomial theorem. If $C_0, C_1, C_2, \cdots, C_n$ be the binomial coefficients in the expansion of $(1 + x)^n$, prove that $C_0^2 + C_1^2 + C_2^2 + \cdots + C_n^2 = \frac{(2n)!}{(n!)^2}$. [4]

   b. Show that set of all positive rational numbers form a group under the composition defined by $a \circ b = \frac{ab}{2}$. [4]

21. a. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a triangle ABC, show that the vector area of the triangle is $\frac{1}{2}(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$. [3]

   b. A variable plane is at a constant distance p form the origin meets the axes of coordinates in A, B, C. Through A, B, C, planes are drawn parallel to the coordinate planes. Prove that the locus of their point of intersection is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$. [3]

   c. Find the equation of the ellipse in standard form, satisfying the following conditions foci at (0, ± 4), eccentricity $\frac{4}{5}$. [2]

22. a. Find the derivative of $\left(\sinh \frac{x}{a} + \cosh \frac{x}{a}\right)^{nx}$ [2]

   b. Define order and degree of a diferntial equation with examples. Solve $\frac{dy}{dx} = \frac{y}{x} - \sin^2 \frac{y}{x}$ [6]

7

Group 'A' [1 × 11 = 11]

Write the correct option in your answer sheet.

1. The value of $\frac{(\cos\theta + i\sin\theta)^4}{(\cos\theta - i\sin\theta)^5}$ is equal to

a. $\cos 9\theta + i\sin 9\theta$ b. $\cos\theta - i\sin\theta$
c. $\cos 5\theta - i\sin 5\theta$ d. $\cos 4\theta - i\sin 4\theta$

2. The number of ways in which the numbers on a clock face be arranged is

a. 12! b. 10!
c. 11! d. 12!

3. The principle value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is

a. $-\frac{2\pi}{3}$ b. $-\frac{\pi}{3}$
c. $\frac{4\pi}{3}$ d. $\frac{5\pi}{3}$

4. The general solution of $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ are

a. $2n\pi + \frac{\pi}{6}$ b. $2n\pi + \frac{11\pi}{6}$
c. $2n\pi + \frac{7\pi}{6}$ d. $2n\pi + \frac{\pi}{4}$

5. $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) =$

   a. $\vec{a} \times \vec{b}$ b. $\vec{b} \times \vec{c}$

   c. $\vec{c} \times \vec{a}$ d. 0

6. The equation of $\frac{x^2}{1-m} + \frac{y^2}{m-3} + 1 = 0$ represents an ellipse only if

   a. $m > 1$ b. $m < 3$

   c. $m = 0$ d. $1 < m < 3$

7. The correlation between X and Y is positive. Then the coefficient of correlation between – X and – Y is

   a. negative b. positive

   c. undetermined d. unity

8. The order and degree of the differential equation $\frac{d^3y}{dx^3} = \sqrt{5 + \left(\frac{dy}{dx}\right)^4}$, respectively, is

   a. 3 and 2 b. 3 and 4

   c. 1 and 4 d. 2 and 3

9. Which of the following statement is true?

   a. If a function is differentiable at a point, it is continuous at that point

   b. If a function is continuous at a point, it is differentiable at that point

   c. A function is differentiable if and only if it is continuous

   d. None of them

10. The system of equations $2x + y = 7$ and $x + 3y = 11$ is

    a. consistent and independent

    b. inconsistent and independent

    c. consistent and dependent

    d. inconsistent and dependent

11. A particle is thrown with an initial velocity of 100 ms^{-1} at an angle of 30° above the horizontal. The time to attain the highest point is (g = 10 ms^{-2}):

    a. 6 sec b. 7 sec

    c. 4 sec d. 5 sec

    OR

    If A is input-output coefficient matrix then I – A is called

    a. technology matrix b. Hawkins-Simon matrix

    c. input-output matrix d. Leontief matrix

Group 'B' [5 × 8 = 40]

12. Prove that: $\frac{5}{1 \cdot 2 \cdot 3} + \frac{7}{3 \cdot 4 \cdot 5} + \frac{9}{5 \cdot 6 \cdot 7} + \ldots \text{ to } \infty = -1 + 3 \log 2$.

13. a. Given the algebraic structure (G, *) with G = {1, ω, ω²} where ω represents the cube root of unity and * stands for the binary operation of ordinary multiplication of complex numbers, show that (G,*) is a group. [3]

    b. A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them are relatives? [2]

14. a. If $\cot^{-1} x + \cot^{-1} y + \cot^{-1} z = \pi$, prove that $xy + yz + zx = 1$. [3]

    b. Show that the plane $3x + 4y - 5z = 7$ is parallel to $6x + 8y - 10z = 15$ and perpendicular to $5x + 5y + 7z = 25$. [2]

15. The probability of a hitting a target is $\frac{1}{5}$. If 5 hitting are made then what is the probability

    a. of hitting the target at least thrice?

    b. of hitting the target at most twice?

    c. that the target is destroyed if three hitting are sufficient to destroy the target?

16. Evaluate: $\int \frac{\sin 2x}{(\sin x + \cos x)^2} dx$

17. Find from first principle the derivative of sin (ln x)

18. Using Simplex method, maximize $P = 50x_1 + 80x_2$ subject to $x_1 + 2x_2 \le 32$, $3x_1 + 4x_2 \le 84$, $x_1, x_2 \ge 0$.

19. Define parallel force. Two like parallel forces of magnitudes P and Q (P > Q) act at two points distance d apart. If P is reversed, show that the resultant is displaced through a distance $\frac{2PQ}{P^2 - Q^2} \cdot d$.

    OR

    The demand and supply functions in a competitive market are $Q_d = 500 - 5P$ and $Q_s = -40 + 20P$ respectively. The initial price P_0 is Rs. 100. Derive a function for the time path P and use it to predict price in time period 5 given that price adjusts is proportion to excess demand at the rate $\frac{dP}{dt} = 0.02(Q_d - Q_s)$. How many time periods would you have to wait for the price to drop by Rs. 40?

Group 'C' [8 × 3 = 24]

20. a. Using row-equivalent matrix method, solve [3]
    $x + 4y + z = 18$; $3x + 3y - 2z = 2$; $-4y + z = -7$.

    b. Prove by mathematical induction that,
    $1 + 2 + 3 + \cdots + n < (2n + 1)^2$. [3]

    c. If $ax^2 + bx + c = 0$ has its roots in the ratio 3:4, prove that $12b^2 = 49ac$. [2]

21. a. Show that the equation $9x^2 - 16y^2 + 18x + 32y - 151 = 0$ represents a hyperbola. Find its eccentricity, centre, foci and directrices. [4]

    b. Find the direction cosines (dc's) of a line whose direction ratios are 1, 2, 2. [1]

    c. Prove that the area of a parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a} \times \vec{b}|$. [3]

22. a. What do you mean by particular solution of a differential equation? Solve:
    $(x^3 + y^3)\,dy - x^2y\,dx = 0$, $y(0) = 1$ [5]

    b. Evaluate $\lim_{x \to 1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$, using L'Hospital's rule. [3]

■

Bipin Khatri

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Class 12 complete notes and paper collection.

Folders | Name ↑

- Biology
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