Chapter 20: Parallel Forces

Exercise 20.1

1. Find the resultant of two parallel forces 4 N and 6 N at a distance of 5 m, when they are (a) like parallel, (b) unlike parallel

Solution:

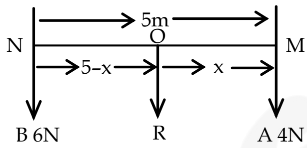
a. Let A and B be two parallel forces acting at points. M and N respectively.

The magnitude of the resultant is given by

R = A + B = 4N + 6N = 10N

The direction of the resultant is same as that of the two forces.

Let the position of the resultant R be at 0, at a at a distance x from M.



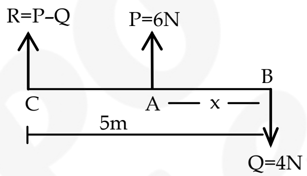
We have, $A \times Mo = B \times No$

or, $4 \times x = b\,(45 - x)$

or, $4x + 6x = 30 \Rightarrow x = 3m$

b. When they are unlike parallel

Let the resultant R of unlike parallel force P and Q then R = 6N – 4N = 2N



$$\frac{P}{AC} = \frac{Q}{AB} = \frac{R}{BC}$$

$$\frac{6}{5-x} = \frac{4}{x} = \frac{2}{5}$$

$$\frac{6}{5-x} = \frac{2}{5}$$

$30 = 10 - 2x$

$3x = -10m$

Also, $\frac{4}{x} = \frac{2}{5}$

or, $x = 10m$

$\therefore$ Resultant = 10m

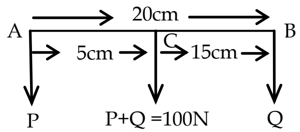
2. Find two like parallel forces at a distance of 20 cm equivalent to 100 N force, the line of action of one of them being at a distance of 5 cm from the given force.

Solution:

Suppose P and Q be two like parallel forces acting at the point A and B such that AB = 20cm. Then the line of action of their resultant is the force P + Q = 100N acting at the point C where AC = 5cm and BC = 15cm. By question, forces are parallel, so we have

$$\frac{P}{BC} = \frac{Q}{AC} = \frac{P+Q}{AB}$$

or, $\frac{P}{15} = \frac{Q}{5} = \frac{100}{20} = 5$



or, $\frac{P}{15} = 5$ or, $\frac{Q}{5} = 5$

or, $P = 15 \times 5 = 75N$ or, $Q = 5 \times 5$

or, $Q = 25N$

Hence, the required forces are 75N and 25N

3. Find two unlike parallel forces at a distance of 20 cm equivalent to 100 N force, the line of action of the greater of them being at a distance of 5 cm from the given force.

Solution:

Suppose P and Q be two unlike parallel forces (P > Q) acting at the point A and B such that AB = 20cm. Since forces are unlike, so that their resultant is the force P – Q = 100N acting at the point C, where AC = 5cm. Since forces are parallel. So, we have,

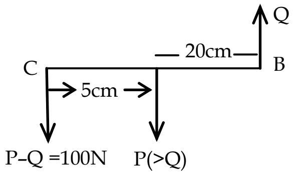
$\frac{P}{BC} = \frac{Q}{AC} = \frac{P-Q}{BC-AC} = \frac{P-Q}{AB} = \frac{100}{20} = 5$

$\frac{P}{25} = \frac{Q}{5} = 5$

$\therefore \frac{P}{25} = 5$ or, $\frac{Q}{5} = 5$

$\therefore P = 25 \times 5$ or, $Q = 5 \times 5$

$= 125N$ or, $Q = 25N$



Required forces are 125N and 25N.

4. The extremities of a straight bamboo pole 3 m long rests on two smooth pegs at A and B in the same horizontal line. A heavy load hangs form a point C on the pole. If AC = 3BC and the pressure at B is 140 N more than that at A, find the weight of the load.

Solution:

Let AB be the straight bamboo of 3m long. Let PN and (P + 140)N acting at A and B. Since the heavy load hangs at point C such that AC = 3BC. So, the line of action of the resultant acting at point C. Since forces are parallel,

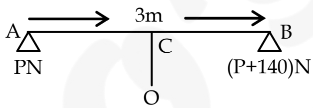
$\therefore \frac{P}{BC} = \frac{P+140}{AC} \Rightarrow \frac{P}{BC} = \frac{P+140}{3BC}$

$\Rightarrow 3P = P + 140$

Hence, P = 70N

Hence, the weight of the load = PV + (P + 140)N

= 70N + (70 + 140)N = 280N



5. A heavy uniform beam 5 m long is supported in a horizontal position by two props, one is at one end and the other is such that the beam projects 1.5 m beyond it. If the weight of the beam is 70 kg wt, find the pressures at the props.

Solution:

Let AB be the uniform beam AB = 5m

Let C be the midpoint of AB so that AC = CB = 2.5m

Let E and D be two props such that DB = 1.75m, CD = 2.5m – 1.75m = 0.75m, EC = 1.5m – 0.75m = 0.75m

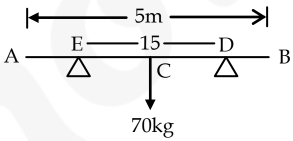
Let R_1 and R_2 be the reactions at E and F.

$\therefore \frac{R_1}{CD} = \frac{R_2}{EC} = \frac{70}{ED}$

$\Rightarrow \frac{R_1}{0.75} = \frac{R_2}{0.75} = \frac{70}{1.5}$

$\therefore R_1 = \frac{70 \times 0.75}{1.5} = 35$ kg

$R_2 = \frac{70 \times 0.75}{1.5} = 35$kg



6. P and Q are like parallel forces with the resultant R. If P is moved parallel to itself through a distance x, show that R is displaced by a distance $\frac{Px}{R}$.

Solution:

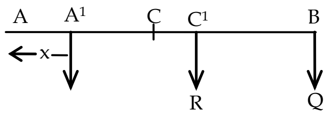
Let R be the resultant of two like parallel forces P and Q acting of A and B respectively. Suppose resultant R acts and P_1 then

$\frac{P}{CB} = \frac{Q}{AC} = \frac{R}{AB}$

from $\frac{P}{CB} = \frac{R}{AB} \Rightarrow CB = \frac{P.AB}{R}$ ... ... (i)

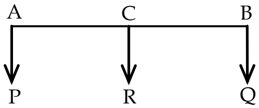
For the second case, let P acts at A^1 such that $AA^1 = x$ then the resultant displace from C to C^1.

So, $\frac{P}{C^1B} = \frac{Q}{A^1C^1} = \frac{R}{A^1B}$



from, $\frac{P}{C^1B} = \frac{R}{A^1B}$

or, $C^1B = \frac{P.\ A^1B}{R}$ ... ... (ii)



Now, $CC^1 = CB - C^1B$

$= \frac{P.AB}{R} - \frac{P.A^1B}{R}$ [$\because$ from (i) and (ii)]

$= \frac{P}{R}(AB - A^1B)$

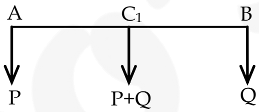
or, $CC^1 = \frac{PX}{R}$ Hence proved.

7. Two like parallel forces of magnitudes P and Q are acting at the end points A and B of a rod AB of length r. If two opposite forces each of magnitude F are added to P and Q, then prove that the line of action of the new resultant will move a distance $\frac{Fx}{P + Q}$.

Solution:

Suppose the two forces P and Q acting at A and B and let their resultant P+Q acting at C_1.

$\therefore \frac{P}{BC_1} = \frac{Q}{AC_1} = \frac{P + Q}{BC_1 + AC_1}$



or, $\frac{P}{BC_1} = \frac{P + Q}{AB}$

$\therefore BC_1 = P.\frac{AB}{P+Q}$

If the force P is moved parallel to itself through a distance x to D then the resultant act at C_2, where AD = x.

Then, $\frac{P}{BC_2} = \frac{Q}{DC_2} = \frac{P+Q}{BC_2+DC_2}$

or, $\frac{P}{BC_2} = \frac{P + Q}{BD}$

$\therefore BC_2 = \frac{P}{P+Q}.BD$

Now, the required distance which the resultant moves $C_1C_2 = BC_2 - BC_1$

$= \frac{P.BD}{P+Q} - \frac{P.AB}{P+Q}$

$= \frac{P}{P+Q}(BD - AB) = \frac{P}{P+Q}.AD = \frac{Px}{P+Q}$

8. Straight uniform rod is 3m long when a load of 5N is placed at one end it balances about a point 25cm from that end. Find the weight of the rod.

Solution:

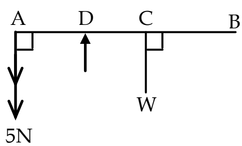
Let W be the weight of the rod AB, acting at centre point C of AB. A load 5N is placed at A, it balances 25cms from that end. i.e. AD = 25cms AB = 3m = 300 cms.

$\therefore AC = BC = \frac{300}{2} = 150$ cms

$\therefore DC = AC - AD$

= 150 cms – 25cms

= 125cms



Now, using the like parallel forces theorem.

$\frac{5}{DC} = \frac{W}{AD}$

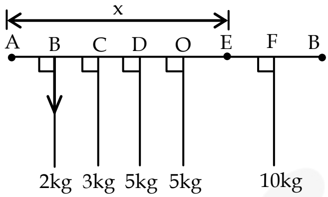
$\therefore W = \frac{5AD}{CD} = \frac{5 \times 25}{125} = 1N$

Exercise 20.2

1. Masses 2kg, 3kg, 5kg and 10kg are suspended from a uniform rod of length 8m, at a distance of 1m, 2m, 3m and 6m respectively from one end. If the mass of rod is 5kg, find the position of the point about which it will balance.

Solution:

Let AB be the rod whose length is 8m and 0 is the centre of the rod. So OA = OB = 4m.



Given, masses are shown in the figure. Let E be the required point such that AE = xm

So, taking moments about E, $2(x-1) + 3(x-2) + 5(x-3) + 5(x-4) = 10(6-x)$

or, $25x = 103$

$x = 4\frac{3}{25}$ m

i.e. The rod will balance about a point which is at a distance of $4\frac{3}{25}$ m from the end A.

2. Forces equal to 3, 4, 5 and 6N respectively act along the side of a square ABCD taken in order, find the magnitude, direction and line of action of their resultant.

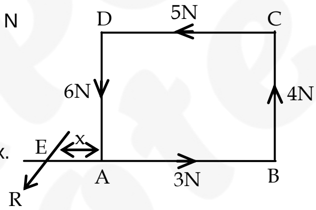
Solution:

Let a be the side of the square ABCD. Let the forces 3, 4, 5 and 6N act along AB, BC, CD and DA respectively. Resolving the forces along and perpendicular to AB, we have,

$x = 3 - 5 = -2$

$y = 4 - 6 = -2$

$\therefore$ The resultant $R = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$ N



If the resultant is inclined at angle θ with AB

$\text{Tan}\theta = \frac{y}{x} = \frac{-2}{-2} = 1 = \text{Tan}(180^o + 45^o)$

$\therefore \theta = 225^o$

Let the resultant R cut BA produced at E where AE = x.

Taking moment about E,

We have, $4 \times (a + x) + 5a - 6x = 0$

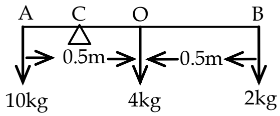
or, $x = \frac{9}{2}a$

3. Two weights of 10 kg and 2 kg hang from the ends of a uniform lever 10 m long and weighing 4 kg. Find the point in the lever about which it will balance.

Solution:

Let A and B be the end points of the lever, 0 be its midpoint and e be the point about which it will balance.

Let AC = x



Taking moment about C_1

$10x - 4(0.5 - x) - 2(1 - x) = 0$

or, $10x - 2 + 4x - 2 + 2x = 0$

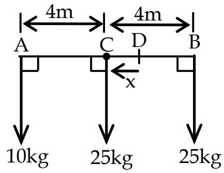
or, $16x = 4$

$x = \frac{1}{4} = 0.25$m

This means the lever will balance about the point 25cm from the 10kg end.

4. A uniform rod is of length 8m and weight 25kg and from its extremities are suspended weights 10kg and 25kg respectively. From what point must the rod be suspended so that it may remain in a horizontal position?

Solution: Let AB = 8m be a uniform rod whose weight 25kg act at C, (middle point of AB). Let weight 10kg and 25kg be suspended from A and B 10kg respectively. Also, suppose the rod be suspended at D so that the rod may rest horizontally such that CD = xm.



Taking moment about D,

$10 \times AD + 25 \times CD - 25 \times DB = 0$

or, $10 \times (4+x) + 25x = 5(4 - x)$

or, $8 + 2x + 5x = 20 - 5x \Rightarrow x = 1m$

So the point D must be a distance of (4 + 1) m. i.e. 5m from the end A.

5. ABCD is a square, along AB, CD, AD and DC equal forces, P act. Show that the magnitude of their resultant is equal to double of any components and acts along DC.

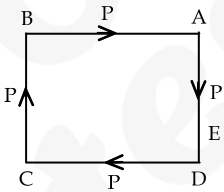
Solution:

Since, equal forces P act along the sides AB, CB, AD and DC of the square ABCD. Resolving the forces along and perpendicular to CD, we have,

$x = -P - P = -2P$, $y = -P + P = 0$

Let R be the resultant, then, $R = \sqrt{x^2 + y^2} = \sqrt{4p^2 + 0} = 2P$

Let θ be the angle made by R with CD. Then



$\text{Tan}\theta = \dfrac{x}{y} = \dfrac{0}{-2P} = 0 = \text{Tan}180^o$

$\therefore \quad \theta = 180^o$

Let the resultant cuts AD at E such that DE = x and CD = a

Taking moments about E,

$-p \times DE + p \times EA - p \times CD = 0$

or, $-p \times x + p \times (a - x) - p \times a = 0$

or, $-px + pa - px - pa = 0$

or, $-2px = 0$

$\therefore \quad x = 0$

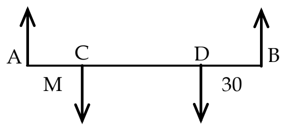
The resultant acts along DC. Hence proved.

6. A light rod of length 72 cm has equal weights attached to it, one at 12 cm from one end and the other at 30 cm from the other end; if it is supported by two vertical strings attached to its ends and if the strings cannot support a tension greater than the weight of 50 kg, what is the greatest magnitude of the equal weight?

Solution:

AB be a light rod of length 72cms. Let W be the equal weight suspended through C and D such that AC = 18cm,

BD =30cm



Now, taking moment about B, we get,

$50 \times AB = W \times CB + W \times DB$

or, $50 \times 72 = W \times 60 + W \times DB$

or, $50 \times 72 = W \times 60 + W \times 30 \quad [\because CB = 72 - 12 = 60]$

or, $W = \dfrac{50 \times 72}{90} = 40$

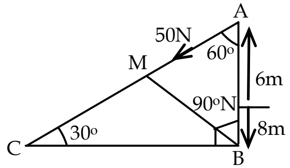
$W = 40$ kgs

7. A pole of length 6 m is placed with its end on a horizontal plane and is pulled by a string attached to its upper end, inclined at an angle of 30° to the horizon. If the tension of the string is equal to 50 N, find the horizontal force which applied at the midpoint of the pole keeps it in a vertical position.

Solution

Suppose AB = 6m, be the pole and its one end is placed in the horizontal line and its upper end situated by the string whose tension is 50N and making angle 30 with the horizontal line BC.

We draw perpendicular BM to the line of action of the tension 50N. Again the angle $\angle BAC = 60^o$. Let F be the horizontal force applied at the point N where BN = 8 metres. Now, taking moment about B, we get,



$50 \times BM = F \times BN$

or, $F = \dfrac{60 \times BM}{BN}$

$F = \dfrac{50 \times B \sin 60^o}{8} = \dfrac{50 \times \sqrt{3/2}}{8} = 5.4125N$

8. Forces equal to P, 2P, 3P act along the sides AB, BC, CA of an equilateral triangle. Find the magnitude and direction of the resultant. Also find where the line of action of the resultant meets BC.

Solution:

Since the force P, 2P, and 3P act along the sides AB, BC, CA of an equilateral triangle ABC. Resolving the forces along and perpendicular to BC, we have,

$x = 2p \cos 0^o + 3p \cos 120^o + p\cos(-120^o)$

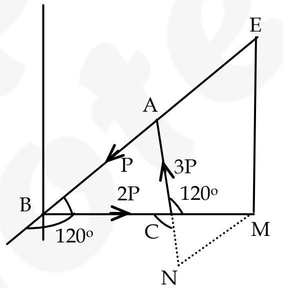
$= 2P - 3P\dfrac{1}{2} - \dfrac{P}{2} = 0$

$y = 2P \sin 0^o + 3P \sin/20^o + P\sin(-120^o)$

$= 0 + 3P\dfrac{\sqrt{3}}{2} - P\dfrac{\sqrt{3}}{2} = \dfrac{2P\sqrt{3}}{2} = P\sqrt{3}$

i. Magnitude of the resultant is given by,

$R = \sqrt{x^2 + y^2} = \sqrt{0 + p^2 . 3} = \sqrt{3}P$



ii. The direction of the resultant $\tan\theta = \dfrac{y}{x} = \dfrac{P\sqrt{3}}{0} = \infty = \text{Tan}90^o$

$\therefore \quad \theta = 90^o$

Direction of resultant is perpendicular to BC.

iii. The position of the line of action of the resultant

$P \times ME - 3P \times MN = 0$

$P(BC + x) \sin 60^o - 3P.CM \sin 60^o = 0$

or, $P(a + x) - 3P.x = 0$

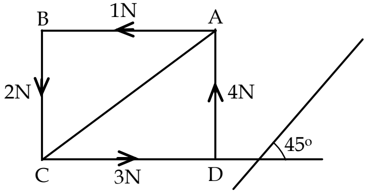
or, $x = \dfrac{a}{2}$ where, a = a side of equilateral triangle.

9. Forces equal to 1 N, 2 N, 3 N, 4 N act along the sides AB, BC, CD, DA, each equal to 2 units, of a square ABCD. Find the magnitude, direction and where the line of action of the resultant intersects C.

Solution:

Let the forces 1N, 2N, 3N and 4N act along the sides AB, BC, CD, DA respectively. Resolving the forces along and perpendicular to CD_1

We have,



$x = 3N - N = 2N$

$y = 4N - 2N = 2N$

Let R be the resultant, then,

$R = \sqrt{x^2 + y^2} = \sqrt{4N^2 + 4N^2} = 2\sqrt{2}N$

Let θ be the angle made by R with AB $\tan\theta$

$= \dfrac{y}{x} = \dfrac{2p}{2p} = 1$

$\theta = 45$

Hence the resultant is parallel to CA. Let the resultant cut CD produced at E, where DE = x.

Let CB = a

$\therefore$ $-4N \times DE + N\times DA + 2N\times CE = 0$

or, $-4x + 1 \times a + 2(a + x) = 0$

or, $-2x = -3a$

or, $x = \frac{3}{2}a$

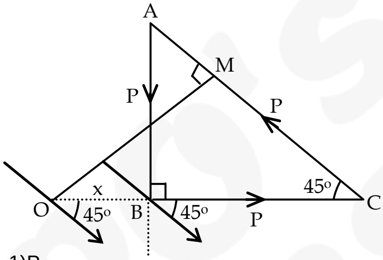
i.e. $DE = \frac{3}{2}CD$

10. Three forces each equal to P act along the sides of an isosceles triangle ABC, right angled at B, taken in order. Find the magnitude, direction and line of action of the resultant.

Solution:

Let the three forces each equal to P act along the sides BCV, CA and AB of an isosceles triangle ABC where $\angle ABC = 90^o$

Resolving the forces along and perpendicular to BC, we have,



$x = P\cos0^o + P\cos135^o + P\cos270^o$

$= P.1 + P\left(-\frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)P$

and, $y = P\sin0^o + P\sin135^o + P\sin270^o$

$= 0 + P.\frac{1}{\sqrt{2}} + P.(-1) = -\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)P$

The resultant $R = \sqrt{x^2 + y^2}$

$= \sqrt{\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)^2 P^2 + \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)^2 P^2} = (\sqrt{2}-1)P$

If the resultant is inclined at an angle θ with BC then $\tan\theta = \frac{y}{x} = -1$

$\therefore$ $\theta = 360^o - 45^o = 315^o$

The magnitude of the resultant is $(\sqrt{2}-1)P$ and its direction is parallel to AC.

Let BC = α. Let the resultant cut CB produced at 0 where BO = x

Now taking moment about 0, we have

$P.OM - P.OB = 0$

or, $P.(\alpha + x)\sin45^o - P.x = 0$

or, $\frac{(P\alpha + Px)}{\sqrt{2}} - Px = 0$

or, $\alpha + x - \sqrt{2}x = 0 \Rightarrow x = \frac{\alpha}{\sqrt{2}-1} = \frac{BC}{\sqrt{2}-1}$

$\therefore$ The resultant passes through 0, a point on the line CB produced where $BO = \frac{BC}{\sqrt{2}-1}$.

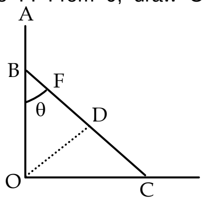
11. At what height from the base of a tree must the end of a rope be fixed so that a man on the ground, pulling at its other end with a given force, may have the greatest tendency to make the tree overturn?

Solution:

Let OB be the tree whose base is 0. Let BC, a rope of length ℓ be fixed on the tree at the point B.

Let a man at C, on the ground pull the rope with the force F. From 0, draw OD perpendicular to BC. Let $\angle OBC = \theta$.

Then the moment of force F about O.



$= F.OD$

$= F.OB\sin\theta$

$= F.BC.\sin\theta.\cos\theta$

$= \frac{1}{2}F.\ell.2\sin\theta.\cos\theta$

$= \frac{1}{2} F.\ell. \sin 2\theta$

The tendency to overturn the tree is maximum when the momentum is greatest. i.e. when $\sin 2\theta = 1 \Rightarrow \theta = 45^o$.

Now, $OB = BC \cdot \sin 45^o = \ell \cdot \frac{1}{\sqrt{2}} = \frac{\ell}{\sqrt{2}}$

$\therefore$ The rope must be fixed at a distance of $\frac{1}{\sqrt{2}}$ from the base on the tree.

12. Forces proportional to AB, BC and 2CA act along the sides of a triangle ABC taken in order. Show that the resultant is represented in magnitude and direction by CA and that its line of action meets BC at a point x where CX = BC.

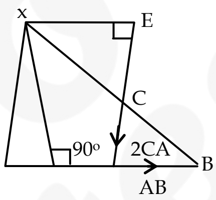
Solution:

Let ABC be a triangle and the forces proportional to AB, BC and 2CA act along the sides of triangle ABC taken in order.

Now, by triangle law of forces, the forces $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ are in equilibrium. Now the resultant of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ is $\overrightarrow{AC}$ along AC.

$\therefore$ The resultant of $\overrightarrow{AC}$ along AC and 2 along CA is

$2\overrightarrow{CA} + \overrightarrow{AC} = 2\overrightarrow{CA} - \overrightarrow{CA} = \overrightarrow{CA}$



Hence the resultant of the forces represent in magnitude and direction by CA.

Now, if the line of action of the resultant meets (produced) in x.

Taking moment about x, we get,

$BC \times O - 2AC \times EX + AB \times DX = 0$

or, $2CA \times EX = AB \times DX$

or, But $\sin c = \frac{EX}{CX}$ and $\sin B = \frac{DX}{BX}$ and $CA \sin C = AB \sin B$ ... ... (ii)

Hence, by (ii), equation (i) becomes

$2CA \times CX \sin C = AB \times B \times \sin B$

or, $2C \times (CA \sin C) = B \times (AB \sin B)$ [$\because$ by (ii)]

or, $2CX = BX$

Hence the resultant cuts BC in the ratio 2:1. So, we must have CX = BC. Hence proved.

13. The wire passing round a telegraph pole is horizontal and the two portions attached to the pole are inclined at angle 60^o to one another. The pole is supported by a wire attached to the middle point of the pole and inclined at 60^o to the horizontal. Show that the tension of this wire is $4\sqrt{3}$ times that of the telegraph wire.

Solution:

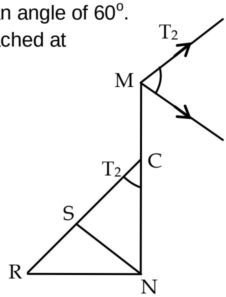
Suppose C is middle point of the telegraph wire MN.

Let the tension of the telegraph wire at M be T_1 and tension of the wire attached at C be T_2.

Since, the two portions attached to the pole are inclined at an angle of 60^o.

$\therefore$ The resultant tension of the two portions of the wire attached at

$M = 2 \times T_1 \cos \frac{60^o}{2}$ ... ... (i)



Again, draw perpendicular from N to RC at S.

Taking moment about N,

We get, $\left(2 \times T_1 \cos \frac{60^o}{2}\right) \times MN = T_2 \times SN$

But $SN = CN \sin 30^o = \frac{MN}{2} \sin 30^o$

Hence, equation (ii) becomes,

$2T_1 \cos30^o \times MN = T_2 . \frac{MN}{2} \sin30^o$

or, $2T_1 \frac{\sqrt{3}}{2} = T_2 \frac{1}{2} . \frac{1}{2}$

or, $T_1 \sqrt{3} = \frac{T_2}{4} \Rightarrow T_2 = 4\sqrt{3}\, T_1$

i.e. Tension of the wire attached at C is $4\sqrt{3}$ times that of the tension of the telegraph wire.

14. Forces forming a couple are 8N each and the arm of the couple is 5m. Find (a) force of an equivalent couple whose arm is 4m(b) the arm of an equivalent couple each of whose forces is 12N.

Solution: Since length of arm (p) = 5m

Force (f) = 8N

$\therefore$ Couple of moment = P×F = 5×8 = 40Nm.

a. Here moment of couple equivalent = 40Nm.
Arm of couple (P) = 4m
Force of couple (F) = ?
Since, couple of moment = F×P
or, 40 = F×4 $\Rightarrow$ F = 10N

b. Here, couple of moment equivalent = 40 Nm
Force of couple (F) = 12N
Arm of couple (P) = ?
Since, Couple of moment = P×F
or, 40 = P×12

$\therefore$ $P = \frac{40}{12} = \frac{10}{3} = 3\frac{1}{3}$ m

15. Given a couple of moment 20 Nm, (a) find the length of the arm if each force is 5 N, (b) find the magnitude of constituent force if the length of the arm is 2 m.

Solution:

The moment of the given couple = 20Nm.

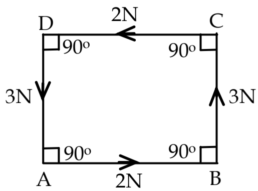
a. Let x be the arm of the couple each at whose force is 5N. So, its moment = 5 $\times$ x Nm.
Given, that, 5x = 20
x = 4m

b. Let f be the one force of couple whose arm is 2m so, its moment is = F $\times$ 2Nm
Given that, F×2 = 20
F = 10N

16. Forces of magnitude 2 N, 3 N, 2 N, and 3 N act along the sides of a square of each side 50 cm taken in order. Find the resultant.

Solution:

Let the forces 2, 3, 2, 3 Newton's act along the sides AB, BC, CD and DA respectively of the square ABCD each of length a metre.



Now, the force 2N and 2N Newton's acting along AB and CD forms a couple of moment

= 2×a = 2a Nm.

Again the force 3N and 3N act along BC and DA form a couple of moment

= 3 $\times$ a = 3a Nm

Since the both moments are in the same sense. Hence the four forces form a couple of moment

= (2a + 3a)

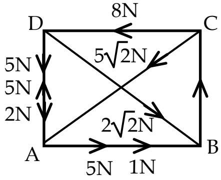
= 5a Nm where a is the size of the square.

17. ABCD is a square whose side is 2m, along AB, BC, CD and DA act forces equal to 1, 2, 8 and 5N and along AC and DB forces equal to $5\sqrt{2}$ and $2\sqrt{2}$ N. Show that they are

equivalent to a couple whose moment is equal to 16Nm.

Solution:

The components of the forces $5\sqrt{2}$N along AC are $5\sqrt{2}\cos 45^o$ along DC and $2\sqrt{2}\sin 45^o$ along DA. i.e. 2N along DC and 2N along DA.



Total force along AB = (5 + 1)N = 6N = 6N

Total force along CD = (8 – 2)N = 6N

Total force along DA = (5 – 5 + 2)N = 2N

The force 6N and 6N along AB and CD form a couple whose moment = 6×2 = 12 Nm

Again, the forces 2N and 2N along BC and DA form a couple whose moment

$= 2 \times 2 = 4$Nm

The resultant of two couples is a couple. Hence, the resultant of all forces is equivalent to a couple whose moment = (12 + 4) Nm

= 16Nm

18. ABCD is a square. Along the sides AB, BC, CD, DA act forces equal to 3 N, 3 N, 6 N and 2 N. Along the diagonals AC and DB act the forces equal to $\sqrt{2}$ N and $2\sqrt{2}$ N. Prove that the resultant is a couple. If each side of a square is 1m, what is the moment of the couple?

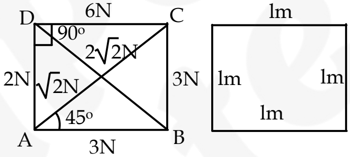
Solution

Here, the resolved part of the force $\sqrt{2}$ along AB = $\sqrt{2}\cos 45^o$

AD = $\sqrt{2}\sin 45^o$

Hence, total force along AB = $3 + \sqrt{2}\cos 45^o$

$= 3 + \sqrt{2} \times \dfrac{1}{\sqrt{2}} = 4$N



Total force along AD = AD = $2 + \sqrt{2}\sin 45^o$

$= 2 + \sqrt{2}\,\dfrac{1}{\sqrt{2}} = 3$N

Total force along BC = $3 + 2\sqrt{2}\sin 45^o = 3 + 2\sqrt{2}\,\dfrac{1}{\sqrt{2}} = 5$N

Total force along CD = $6 + 2\sqrt{2}\cos 45^o = 6 + 2\sqrt{2}\,\dfrac{1}{\sqrt{2}} = 8$N

Hence the forces 3N, 3N, 6N, 2N act long the sides AB, BC, CD, and DA. Hence, their resultant forces form couple (4, BC), (3, AB), (5, CD) and (8, AD)

Hence, their resultant is a couple.

Now, the moment of couple = $4 \times BC + 3\times AB$

$= 4\times1 + 3\times1 = 7$N

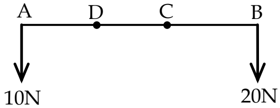
Also, the moment of couple = $5 \times CD + 8\times AD$

$= 5\times1 + 8\times1 = 13$N

19. Two like parallel forces of magnitude 10 N and 20 N are acting at two points of a rigid body and 3 m apart. If a couple of moment 12 Nm is combined with them, find the distance by which the resultant is displaced.

Solution:

Let A and B be the points where 10N and 20N forces act. Let C be the point from where the line of action passes. Let C be shifted to D after the couple of moment 12Nm is added.



Now, AC : AB = 20 : 10

or, $\dfrac{AC}{AC + CB} = \dfrac{20}{30}$

or, $AC = \dfrac{20}{30} \times 3 = 2$m

For a couple of moment 12Nm, if the arm is 3m, the constituent forces are of magnitude

4N force. So the force 4N will be added to 10N and subtracted from 20N (because of positive moment). So,

Now, DC = AC – AD = 2m – 2.6m = –0.6m = 0.6

20. ABCD is a rectangle with length AB = 30 cm and breadth BC = 20 cm. Forces of magnitude 4 N act along AB and CD and forces of magnitude 3 N act along AD and CB. Find the perpendicular distance between the resultant of 4 N and 3 N at A and that of those at C.

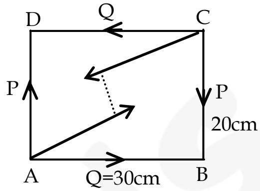
Solution:

Here, AB = CD = 30cms

BC = DA = 20 cms

Force, Q = 4N

P = 3N



The resultant of the forces P and Q at A.

$= \sqrt{P^2 + Q^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + b} = 5N$

Similarly, the resultant force at C,

$= \sqrt{P^2 + Q^2} = 5N$

These two resultant forces form a couple

Moment of these couple $= \sqrt{P^2 + Q^2}.d = 5d$

Where d = perpendicular distance between the line of two resultant forces.

Again the given forces form two couple. The moment of these couples.

$= Q.\ AD - P.AB = 4\times20 - 3\times30 = -10$

These two couples are equivalent to a single couple

$\therefore \quad \sqrt{P^2 + Q^2}.d = Q.\ AD - P.AB$

or, $5d = -10$

or, $d = -2$cms

$\therefore \quad d = 2$ cms (distance is never negative)