* Mean or average value of AC (over a half cycle) T/2.

$I = I_0 \sin \omega t$ be the ac passing through circuit in the given time. Small amount of charge dq send by ac in small time dt is $dq = I dt$ $\left[\because I = \frac{dq}{dt}\right]$

$$\therefore dq = I_0 \sin \omega t \, dt \quad \text{—(1)}$$

Now, the total charge 'q' send by ac over a half cycle can be obtained as:

$$q = \int_0^{T/2} dq$$

$$q = I_0 \int_0^{T/2} \sin \omega t \, dt$$

$$\text{or, } q = \frac{-I_0}{\omega} \left[\cos \omega t\right]_0^{T/2}$$

$$q = \frac{-I_0}{\omega} \left[\cos \frac{2\pi}{T} \times \frac{T}{2} - \cos 0\right]$$

$$q = -\frac{I_0}{2\pi/T} \left[\cos \pi - \cos 0\right]$$

$$q = \frac{-I_0 T}{2\pi} (-1-1)$$

$$q = \frac{-I_0 T}{2\pi} \times -2$$

$$\therefore q = \frac{I_0 T}{\pi}$$

$\therefore I_{av}$ be the mean or average value of ac,

$$I_{av} = \frac{\text{Charge}}{\text{time}} = q/_{T/2}$$

$$= \frac{I_0 T}{\pi} \times \frac{2}{T}$$

$$\therefore I_{av} = \frac{2}{\pi} I_0$$

$$\therefore I_{av} \approx 63\% \text{ of } I_0,$$

* Root mean square value of AC (I_{rms}):-

Let $I = I_0 \sin\omega t$ be the ac, passing through the circuit having resistance (R). Then, small amt. of Heat [dH] produce by ac in passing through the resistance in small time dt is given by,

$$dH_1 = I^2 R dt.$$

$$\text{or, } dH_1 = (I_0 \sin\omega t)^2 R dt.$$

$$\text{or, } dH_1 = I_0^2 R \sin^2\omega t\, dt.$$

$$\text{or, } dH_1 = I_0^2 R \left[\frac{1-\cos 2\omega t}{2}\right] dt$$

$$\text{or, } dH_1 = \frac{I_0^2 R}{2}\left[1\, dt - \cos 2\omega t\, dt\right] \text{ --- (1)}$$

Then total heat produced by ac over a complete cycle is

$$H = \int_0^T dH_1$$

$$\text{or, } H_1 = \frac{I_0^2 R}{2}\left[\int_0^T dt - \cancel{\int_0^T \cos 2\omega t\, dt}^{\,0}\right]$$

$$\text{or, } H_1 = \frac{I_0^2 R}{2}\left[t\right]_0^T$$

$$\text{or, } H_1 = \frac{I_0^2 R T}{2} \text{ --- (2)}$$

If I_{rms} be the root mean square value of ac then heat produced by I_{rms} in same resistance R in same time T is

$$H_2 = I_{rms} R T \text{ --- (3)}$$

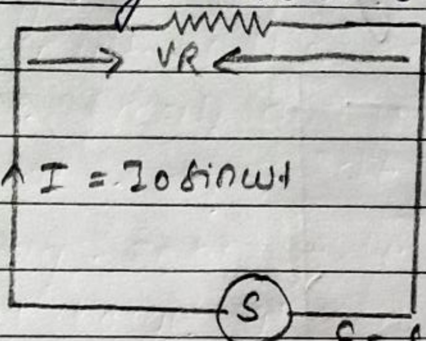
According to defination

$$H_2 = H_1$$

$$\text{or, } I_{rms} R T = \frac{I_0^2 R T}{2}$$

$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}} \approx 70.1\% I_0.$$

2 ~~ACth~~ # AC through resistor only (purely resistive circuit)



Let $I = I_0 \sin\omega t$ be the ac passing through the circuit containing a resistor of reactance (R) only. Let $E = E_0 \sin\omega t$ be the alternating source and V_R be p.d across R.

Here, only resistor is connected.

So, E = p.d across R i.e V_R.

$\therefore V_R = E$ —— eqn (i)

But $V_R = IR$

$= I_0 R \sin\omega t$. —— (ii)

from (i) and (ii)

$E = I_0 R \sin\omega t$

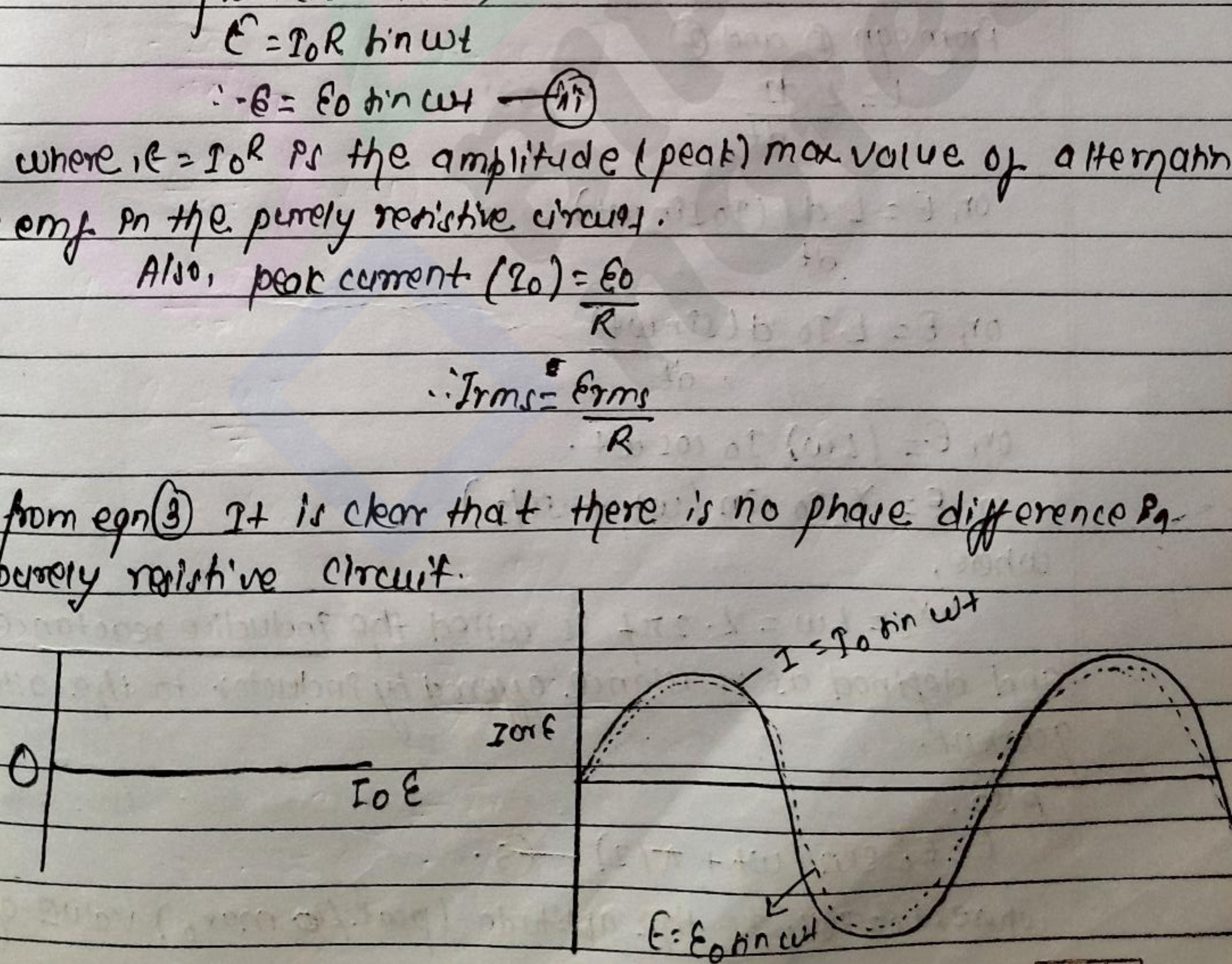
$\therefore E = E_0 \sin\omega t$ —— (iii)

where, $E = I_0 R$ is the amplitude (peak) max value of alternating emf in the purely resistive circuit.

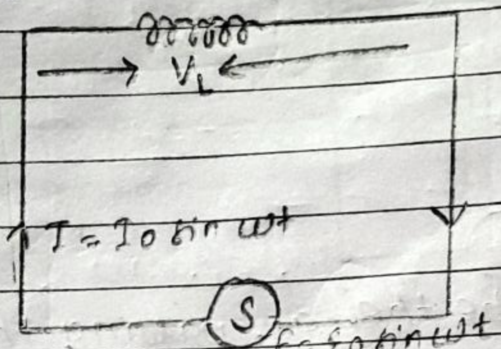
Also, peak current $(I_0) = \dfrac{E_0}{R}$

$$\therefore I_{rms} = \frac{E_{rms}}{R}$$

from eqn (3) It is clear that there is no phase difference in purely resistive circuit.



2. AC through Inductor only (Inductive circuit)



Let $I = I_0 \sin\omega t$ be the AC passing through the circuit containing a inductor of inductance L only. Let $E = E_0 \sin\omega t$ emf of alternating source and V_L be the P.d across 'L'.

So,

E = P.d across L i.e V_L

i.e $E = V_L$ —— (1)

But, $V_L = \frac{LdI}{dt}$ (In magnitude) —— (2)

from eqn (1) and (2)

$$E = L\frac{dI}{dt}$$

$$\text{or, } E = L\frac{d}{dt}(I_0 \sin\omega t)$$

$$\text{or, } E = L \cdot I_0 \frac{d(\sin\omega t)}{dt}$$

$$\text{or, } E = (L\omega) I_0 \cos\omega t.$$

$$\text{or, } E = X_L I_0 \sin(\omega t + \pi/2)$$

where,

$X_L = L\omega = L \cdot 2\pi f$ is called the inductive reactance and and defined as resistance offered by inductor to the alternating current.

Also,

$E = E_0 \sin(\omega t + \pi/2)$ —— (3).

where, $E_0 = I_0 X_L$ is the aplitude (peak/ max_0) value of alternating emf in the purely inductive circuit.

Also,

peak current $(I_0) = \dfrac{E_0}{X_L}$

$$\text{or, } \frac{I_0}{\sqrt{2}} = \cancel{E_0\sqrt{2}}\ \frac{E_0/\sqrt{2}}{X_L}$$

$$\therefore \boxed{I_{rms} = \frac{E_{rms}}{X_L}}$$

From eqn (3) it is clear that there is a phase difference of $\pi/2$ but alternating emf and alternating current in purely inductive circuit which is shown in phase diagram below.

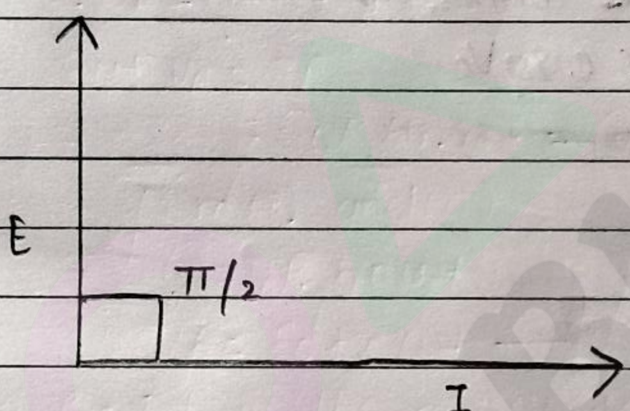
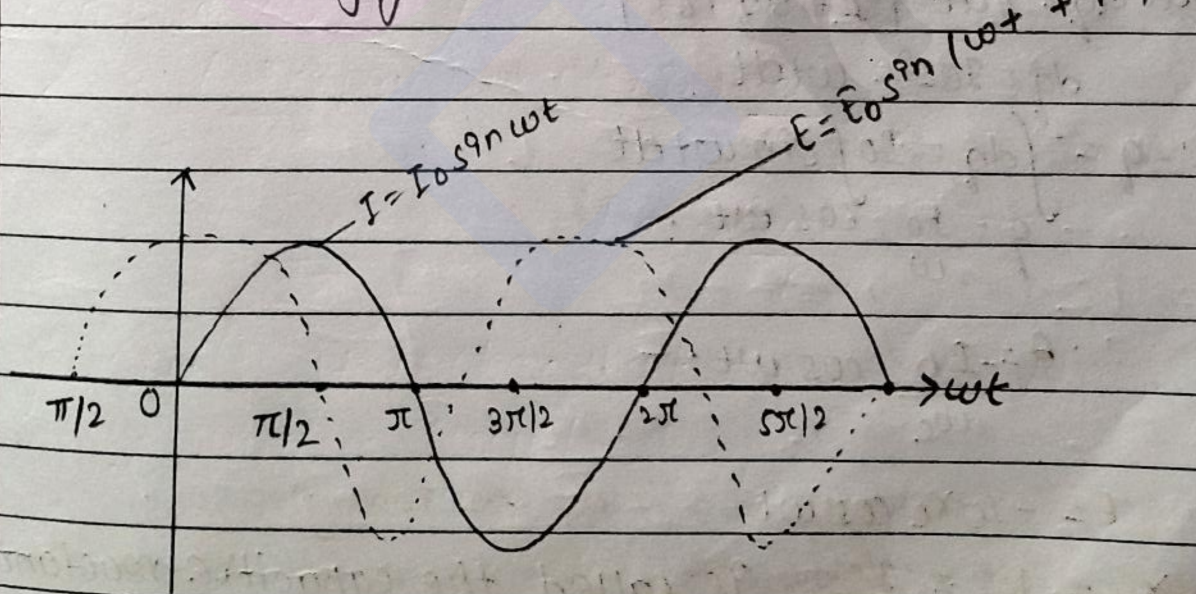
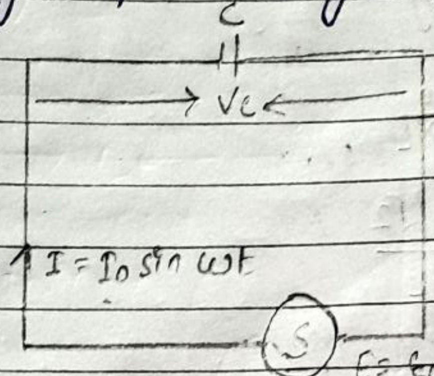


fig (a)



conclusion:- From above phase diagram it is cleared EMF leads the current by the phase angle $\pi/2$.

3. AC through capacitor only (capacitance circuit)



Let $I = I_0 \sin \omega t$ be the ac passing through the circuit containing a capacitor of capacitance only. Let $E = E_0 \sin \omega t$ emf of alternating source and V_C be the p.d across the c.

Here, only capacitor is connected,

so, E = p.d across c is V_C

i.e $E = V_C$ —— (1)

But $V_C = \frac{q}{C}$

$\therefore E = \frac{q}{C}$ —— (2)

But $dq = I dt$ $[I = dq/dt]$

$dq = I_0 \sin \omega t dt$

$\therefore q = \int dq = I_0 \int \sin \omega t dt$

$q = \frac{I_0}{\omega} \cos \omega t$.

$\therefore E = -\frac{I_0}{\omega C} \cos \omega t$.

$E = -I_0 X_C \cos \omega t$.

where $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ is called the capacitive reactant and defined as resistance offered by capacitor to the alternating current.

Also, $E = E_0 \sin(\omega t - \pi/2)$ — (3) where $E_0 = I_0 X_C$ is the amplitu del peak/ maximum value of alternating emf in the purely capacitor.

Also, peak current $(I_0) = \frac{E_0}{X_C}$.

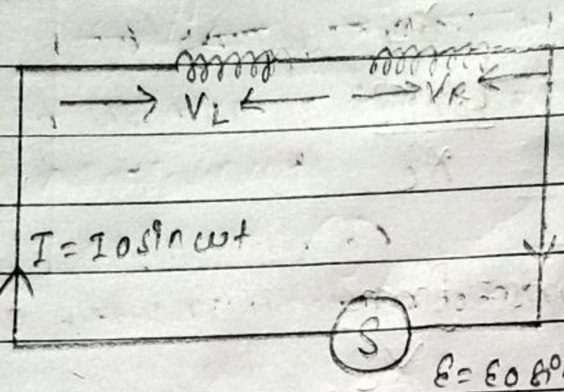
$$\text{or, } \frac{I_0}{\sqrt{2}} = \frac{E_0/\sqrt{2}}{X_C}$$

$$\therefore I_{rms} = \frac{E_{rms}}{X_C}$$

From eqn (3) it is clear that there is phase difference of $\pi/2$ between alternating emf & alternating current in purely capacitive circuit which is shown in phase diagram below.

From the above phase diagram it is clear that emf leads the current by the phase angle $\pi/2$.

(4) Ac through R and L in series. (RL circuit)



Let $I = I_0 \sin \omega t$ be the alternating current passing through the circuit containing R and L in series. Let V_L and V_R be the p.d across L and R respectively.

from fig $\boxed{E = V_R + V_L}$ — (1)

But $V_R = IR = I_0 R \sin \omega t$

$V_L = L \frac{dI}{dt}$

$V_L = \cancel{L \frac{d}{dt}}\ L \frac{d}{dt} (I_0 \sin \omega t)$

$V_L = I_0 (L\omega) \cos \omega t$.

or, $V_L = I_0 X_L \cos \omega t$.

where $X_L = L\omega$ is inductive reactance.

from eqn (1) $E = I_0 R \sin \omega t + I_0 X_L \cos \omega t$

Let $Z = \sqrt{R^2 + X_L^2}$ such that

$E = I_0 Z \left[\frac{R}{Z} \sin \omega t + \frac{X_L}{Z} \cos \omega t\right]$

Let $R/Z = \cos$ then $\sin \theta = \frac{X_L}{Z}$

$E = I_0 Z [\sin \omega t \cos \theta + \cos \omega t \sin \theta]$

$E = I_0 Z \sin (\omega t + \theta)$

or, $\boxed{E = E_0 \sin (\omega t + \theta)}$ — (2)

where $E_0 = I_0 Z$ is the peak or maximum value of alternating emf in R-L circuit.

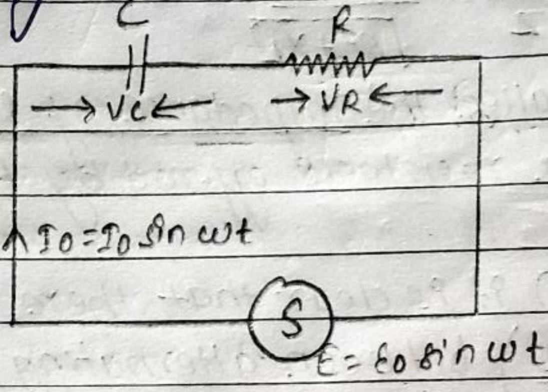
$\therefore$ Peak current $(I_0) = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + X_L^2}}$

where $Z = \sqrt{R^2 + X_L^2}$ is called the impedance of R-L circuit and which is defined as the resistance offered by R-L circuit to the alternating current.

From eqn (2) it is clear that there is a phase difference of $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$ between alternating current and alternating emf which is shown in fig below.

From above phase diagram it is clear that the emf leads the current by the phase angle $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$

(5) AC through R and C in series



Let $I = I_0 \sin \omega t$ be the ac passing through the circuit containing R and C in series. Let $E = E_0 \sin \omega t$ be the emf of alternating source and V_C and V_R be the pd across C and R respectively.

from fig,

$$\boxed{E = V_R + V_C} \quad \text{---} (1)$$

$$\text{But } V_R = IR = I_0 R \sin \omega t$$

$$V_C = \frac{q}{C}$$

$$\text{and } q = \int I dt$$

$$\text{or, } q = I_0 \int \sin \omega t \, dt$$

$$q = \frac{-I_0}{\omega} \cos \omega t$$

$$\therefore V_C = \frac{-I_0}{\omega C} \cos \omega t$$

$$\text{or, } V_C = -I_0 X_C \cos \omega t$$

where $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ is capacitive reactance.

from eqn (1)

$$E = I_0 R \sin \omega t - I_0 X_C \cos \omega t.$$

$$\text{or, } E = I_0 [R \sin \omega t - X_C \cos \omega t).$$

Let $Z = \sqrt{R^2 + X_C^2}$ such that,

$$E = I_0 Z \left[\frac{R}{Z}\sin\omega t - \frac{X_C}{Z}\cos\omega t\right]$$

$\therefore$ Let $\frac{R}{Z} = \cos\theta$ then $\sin\theta = \frac{X_C}{Z}$.

$$\therefore E = I_0 Z\,[\sin\omega t\cos\theta = \cos\omega t\sin\theta]$$

or, $E = I_0 Z \sin(\omega t - \theta)$

or, $\boxed{E = E_0 \sin(\omega t - \theta)}$ — (2)

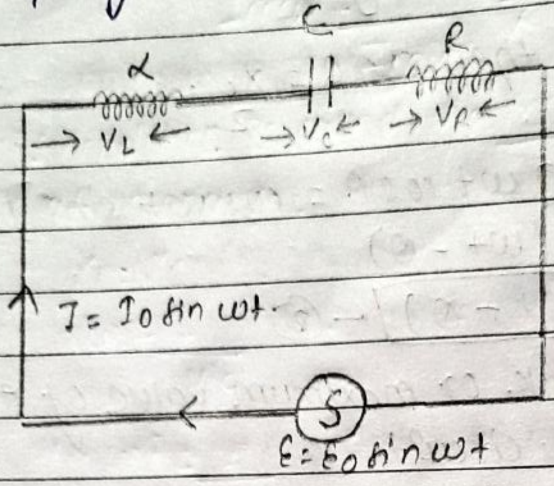
where $E_0 = I_0 Z$ is peak or maximum value of ~~alternating~~ alternating emf in RC circuit

$\therefore$ peak current $(I_0) = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + X_C^2}}$

where, $Z = \sqrt{R^2 + X_C^2}$ is called the impedance of emf of R-C circuit and define total resistance offered by R-C circuit to alternating current.

from eqn (2) it is clear that there is a phase difference of $\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$ between alternating emf and alternating current which is shown in phase diagram below:-

Imp
(6) AC through LCR circuit.



Let $I = I_0 \sin \omega t$ be the alternating current passing through the circuit containing L, C and R in series. Let $E = E_0 \sin \omega t$ be the emf of alternating source and V_L, V_C and V_R be the p.d across L, C and R respectively.

from fig,

$$\boxed{E = V_R + V_L + V_C} \quad \text{--- (1)}$$

$$V_L = L \frac{dI}{dt}$$

$$V_L = L I_0 \frac{d}{dt}(\sin \omega t)$$

$$V_L = (L\omega) I_0 \cos \omega t.$$

or, $V_L = X_L I_0 \cos \omega t$ where $X_L = L\omega$

Now,

$$V_C = \frac{q}{C}$$

$$q = \int I dt$$

$$q = I_0 \int \sin \omega t \, dt$$

$$q = \frac{-I_0}{\omega} \cos \omega t$$

$$V_C = \frac{-I_0}{\omega C} \cos \omega t$$

$V_C = -I_0 X_C \cos \omega t$

from eqn ①

$E = I_0 R \sin \omega t + X_L I_0 \cos \omega t - I_0 X_C \cos \omega t$

or, $E = I_0 [R \sin \omega t + (X_L - X_C) \cos \omega t]$

Let $Z = \sqrt{R^2 + (X_L - X_C)^2}$ such that

$E = I_0 Z [R/Z \sin \omega t + \left(\frac{X_L - X_C}{Z}\right) \cos \omega t]$

Let $\frac{R}{Z} = \cos \theta$ then $\frac{X_L - X_C}{Z} = \sin \theta$

$E = I_0 Z (\sin \omega t \cos \theta + \cos \omega t \sin \theta)$

$\boxed{E = I_0 Z \sin (\omega t + \theta)}$ — ②

where $E_0 = I_0 Z$ is the peak value of alternating emf in series is LCR circuit.

peak current $(I_0) = \frac{E_0}{Z}$

$$I_0 = \frac{E_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is called the impedeance of series LCR circuit and which is defined as total resistance offered by series LCR circuit to the alternating current.

from eqn ② it is clear that there is a phase difference of $\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$ between alternate current and alternating emf when ac is passing through series LCR circuit.

Here we discuss three different cases:-

(a) If $X_L > X_C$, then θ will be positive and circuit behaves as inductive circuit and emf leads the current.

(b) If $X_C > X_L$, then θ will be negative and circuit behaves as capacitive circuit and current leads the emf.

(c) If $X_L = X_C$ then impedance becomes minimize and $Z_{min} = R$ and

(a) Power consume in purely inductive circuit;-

we know in purely inductive circuit; $I = I_0 \sin\omega t$.

$$E = E_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Then instanteneous power developed in the coil.

$$P_{ins} = EI \quad (P = VI)$$

$$\text{or, } P_{ins} = I_0 E_0 \sin\omega t \sin(\omega t + \pi/2)$$

Then; small amount of workdone (dw) which appears in the forms of heat in small time dt is;

$$dw = P_{ins}\, dt \quad \left(P = \frac{dw}{dt}\right)$$

$$\text{or, } dw = I_0 E_0 \sin\omega t \cdot \sin\left(\omega t + \frac{\pi}{2}\right) dt$$

$$\text{or, } dw = I_0 E_0 \sin\omega t \cdot \cos\omega t\, dt.$$

$$\text{or, } dw = \frac{I_0 E_0}{2} (2 \sin\omega t) \cdot dt \quad \text{--- (1)}$$

Now, total amt of workdone to complete one cycle is.

$$w = \int_0^T dw = \frac{I_0 E_0}{2} \int_0^T \sin 2\omega t\, dt$$

$$\text{or, } w = \frac{I_0 E_0}{2} \times -\frac{1}{2\omega} [\cos 2\omega t]_0^T$$

$$\text{or, } w = \frac{-I_0 E_0}{4\omega} \left[\cos 2 \times \frac{2\pi}{T} \times T - \cos 0°\right]$$

$$\text{or, } w = -\frac{I_0 E_0}{4\omega} [\cos 4\pi - \cos 0°]$$

$$\text{or, } W = -\frac{I_0 E_0}{4\omega} [1 - 1]$$

$$\therefore W = 0$$

Now, avg. power consumed; $P_{av} = \frac{W}{T} = \frac{0}{T} = 0$

$\therefore P_{av} = 0.$

Hence, a purely inductive circuit consumed no power in ac circuit.

(b) Power consumed in purely capacitive circuit.

We know; in purely capacitive circuit.

$$I = I_0 \sin \omega t$$

$$E = E_0 \sin(\omega t - \pi/2)$$

$$E = -E_0 \cos \omega t.$$

Then instateneous power developed in the coil.

$$P_{ins} = EI \quad [P = VI]$$

$$\text{or, } P_{ins} = -I_0 E_0 \sin \omega t . \cos \omega t.$$

Then, small amt of work done (dw) which appears in the form of heat in small time dt is,

$$dw = P_{ins} \, dt \left[P = \frac{dw}{dt}\right]$$

$$\text{or, } dw = I_0 E_0 \sin \omega t \cdot \cos \omega t \cdot dt$$

$$\text{or, } dw = -\frac{I_0 E_0}{2} (2 \sin \omega t \cos \omega t) dt.$$

$$\text{or, } dw = -\frac{I_0 E_0}{2} \sin 2\omega t \, dt \quad \text{--- (1)}$$

Now,

total amt of workdone to complete one cycle is

$$W = \int_0^T dw$$

$$= -\frac{I_0 E_0}{2} \int_0^T \sin 2\omega t \, dt$$

$$\text{or, } W = -\frac{I_0 E_0}{2} \times \frac{1}{-2\omega} \left[\cos 2\omega t\right]_0^T$$

or, $W = +\frac{I_0E_0}{4\omega}\left[\cos\frac{2\times2\pi}{T}\times T - \cos 0°\right]$

or, $W = +\frac{I_0E_0}{4\omega}\left[\cancel{\cos 4\pi}\ \cos 4\pi - \cos 0\right]$

or $W = +\frac{I_0E_0}{4\omega}[1-1]$

$W = 0$

Now, avg. power consumed; $P_{av} = \frac{W}{T} = \frac{0}{T} = 0$

$\therefore P_{av} = 0.$

Hence, a purely capacitive circuit consumes no power in ac circuit.

(c) Power developed in R and L in series circuit:-

we know; in RL circuit

$I = I_0 \sin\omega t$ — (1)

$E = E_0 \sin(\omega t + \theta)$ — (2)

where $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$

Then, instantaneous power developed is

$P_{ins} = EI$

or $P_{ins} = I_0E_0 \sin\omega t \,(\sin(\omega t + \theta))$

Also, small amt of work done (dw) in small time dt is

$dW = P_{ins}\,dt$

$dw = I_0E_0 \sin\omega t, \sin(\omega t + \theta)\,dt.$

Now,

total work done (w) to complete one cycle is,

$w = \int_0^T dw$

$$W = I_0E_0\left[\int_0^T \sin\omega t\left[\sin\omega t\cos\theta + \cos\omega t \cdot \sin\theta\right]dt\right.$$

$$W = I_0E_0\left[\int_0^T \sin^2\omega t\, dt\cos\theta + \int_0^T \sin\omega t + \cos\omega t\, dt \sin\theta\right]$$

$$W = I_0E_0\left[\cos\theta\int_0^T\left(\frac{1-\cos 2\omega t}{2}\right)dt + \int_0^T \frac{1}{2}\left(2\sin\omega t\cos\omega t\, dt \sin\theta\right)\right]$$

$$W = I_0E_0\left(\cos\theta\left[\int_0^T dt - \int_0^T \underbrace{\cos 2\omega t\, dt}_{0} + \sin\theta\int_0^T \underbrace{\sin\omega t\, dt}_{0}\right]\right.$$

$$W = \frac{I_0E_0}{2}\cos\theta\left[t\right]_0^T$$

$$W = \frac{I_0E_0}{2}\cos\theta T \quad (3)$$

Now,

average power is,

$$P_{av} = \frac{W}{T} = \frac{I_0E_0}{2}\cos\theta\frac{T}{T}$$

$$P_{av} = \frac{I_0}{\sqrt{2}} \times \frac{E_0}{\sqrt{2}}\cos\theta \quad \text{when } \cos\theta = \frac{R}{\sqrt{R^2+X_L^2}}$$

$$P_{av} = I_{rms}E_{rms}\cos\theta$$

P_{av} = Apparent power × power factor.

(a) Power developed in R and C in series circuit:

we know,

In RC circuit

$$I = I_0 \sin \omega t \quad -(1)$$

$$E = E_0 \sin(\omega t - \theta) \quad -(2) \quad \text{where } \theta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

Thus instantaneous power developed is

$$P_{ins} = EI$$

$$\text{or, } P_{ins} = I_0 E_0 \sin \omega t \sin(\omega t - \theta)$$

Also small amount of work done (dw) in small time 't' is

$$dW = P_{ins}\, dt$$

$$dw = I_0 E_0 \sin \omega t \sin(\omega t - \theta)\, dt.$$

Now, total workdone (w) to complete one cycle is,

$$w = \int_0^T dw$$

$$w = I_0 E_0 \left[\int_0^T \sin \omega t \left[\sin \omega t \cos\theta - \cos \omega t \sin\theta\right] dt\right.$$

$$w = I_0 E_0 \left[\int_0^T \sin^2 \omega t\, dt \cos\theta - \int_0^T \sin \omega t\, \overset{\cos \omega t}{\cancel{\omega t}}\, dt \sin\theta\right]$$

$$w = I_0 E_0 \left[\cos\theta \int_0^T \frac{(1 - \cos 2\omega t)}{2} dt - \int_0^T \frac{1}{2}(2 \sin \omega t \cos \omega t\, \cancel{dt \sin\theta}\right]$$

$$w = \frac{I_0 E_0}{2}\left[\cos\theta \left\{\int_0^T dt - \overset{0}{\cancel{\int_0^T \cos 2\omega t\, dt}} - \sin\theta \overset{0}{\cancel{\int_0^T \sin 2\omega t\, dt}}\right]$$

$$w = \frac{I_0 E_0}{2} \cos \left[t\right]_0^T$$

$w = \frac{I_0 E_0}{2} \cos\theta T$ — (3)

Now average power is

$P_{av} = \frac{w}{T} = \frac{I_0 E_0}{2} \cos\theta \frac{T}{T}$

$P_{av} = \frac{I_0}{\sqrt{2}} \times \frac{E_0}{\sqrt{2}} \cos\theta$

$P_{av} = I_{rms} E_{rms} \cos\theta$; $\cos\theta = \frac{R}{\sqrt{R^2 + X_L^2}}$ is called power factor of R & C circuit.

$\therefore P_{av}$ = Apparent power x power factor.

* Power consumed in series LCR circuit.

we known in series LCR circuit,

$I = I_0 \sin\omega t$ — (1)

$E = E_0 \sin(\omega t + \theta)$ — (2) where $\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Then instanteneous power developed is

$P_{ins} = EI$

$= I_0 E_0 \sin\omega t \cdot \sin(\omega t + \theta)$

Now small amount of work done (dw) in small time dt is

$dw = P_{ins}\, dt.$

$dw = I_0 E_0 \sin\omega t \sin(\omega t + \theta) dt.$

Now, total work done (w) to complete one cycle is

$w = \int_0^T dw$

$w = I_0 E_0 \int_0^T \sin\omega t \{\sin\omega t \cos\theta + \cos\omega t \sin\theta\} dt$

$$W = I_0 E_0 \left[\cos\theta \int_0^T \sin^2 \omega t \, dt + \int_0^T \sin \omega t \cos \omega t \sin\theta dt\right]$$

$$W = I_0 E_0 \left[\cos\theta \int_0^T \frac{(1-\cos 2\omega t)}{2} dt + \frac{1}{2}\int_0^T (2 \sin \omega t + \cos \omega t) \sin\theta \, dt\right]$$

$$W = \frac{I_0 E_0}{2}\left[\cos\theta \left\{\int_0^T dt - \int_0^T \overset{0}{\cos 2\omega t \, dt}\right\} + \sin\theta \int_0^T \overset{0}{\sin 2\omega t \, dt}\right]$$

$$W = \frac{I_0 E_0}{2} \cos\theta \left[t\right]_0^T$$

$$W = \frac{I_0 E_0}{2} \cos\theta T \quad -(2)$$

Now average power

$$P_{av} = \frac{W}{T}$$

$$P_{av} = \frac{I_0 E_0}{2} \cos\theta$$

$$P_{av} = \frac{I_0}{\sqrt{2}} \times \frac{E_0}{\sqrt{2}} \times \cos\theta$$

$$P_{av} = I_{rms} E_{rms} \cos\theta$$

P_{av} = Apparent power x power factor

where $\cos\theta = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$ is called power factor of LCR circuit.

* Choke coil:- It is a device which is used to minimize the power developed in coil.

i.e. $\cos\phi \simeq 0$

[Short Question]

Q) Why choke coil is refered rather than resistor in an ac circuit?

Formula:-

① Power resistive circuit

$$\theta = 0^\circ$$

$$E_0 = I_0 R$$

$$I_0 = \frac{E_0}{R}$$

P.d across R

$$V_R = IR$$

② ~~Pow~~ Purely Inductive circuit.

$$\theta = \pi/2$$

$$X_L = L\omega = L 2\pi f = \frac{2\pi L}{T}$$

$$I_0 = \frac{E_0}{X_L}$$

$$I_{rms} = \frac{E_{rms}}{X_L}$$

[Pd across L]

$$V_L = I X_L$$

③ Purely capacitive circuit

$$\theta = \pi/2$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{T}{2\pi C}$$

$$I_0 = \frac{E_0}{X_C}$$

$$I_{rms} = \frac{E_{rms}}{X_C}$$

[Pd across C

$V_C = I X_C$]

(4) R-L circuit

Phase angle $\theta = \tan\left(\frac{X_L}{R}\right)$

$$\theta = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

L - impedance

$$Z = \sqrt{R^2 + X_L^2}$$

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + X_L^2}}$$

$$I_{rms} = \frac{E_{rms}}{\sqrt{R^2 + X_L^2}}$$

Power factor $(\cos\theta) = \frac{R}{Z}$

$$= \frac{R}{\sqrt{R^2 + X_L^2}}$$

P.d across R and L

$$V_{RL} = I\sqrt{R^2 + X_L^2}$$

$$V_{RL} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V_{RL} = \sqrt{V_R^2 + V_L^2}$$

(5) R-C circuit

Phase angle $(\theta) = \tan^{-1}\left(\frac{X_C}{R}\right)$

$$\theta = \tan^{-1}\left(\frac{1}{\omega C R}\right)$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + X_C^2}}$$

$$I_{rms} = \frac{E_{rms}}{\sqrt{R^2 + X_C^2}}$$

Power factor $(\cos\theta) = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}}$

P.d across R & C

$$V_{RC} = I\sqrt{R^2 + X_C^2}$$

Numerical:

1) Alternating voltage in an ac circuit is represented by $v = 100\sqrt{2}\sin 100\pi t$ volts. find its root mean square value and frequency.

⇒ Given,

$v = 100\sqrt{2}\sin 100\pi t$ volt.

comparing this eqn with $v = V_0 \sin \omega t$

$V_0 = 100\sqrt{2}$.

$$\therefore V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$= \frac{100\sqrt{2}}{\sqrt{2}}$$

$$= 100\ V.$$

Now,

~~ω = 100π~~ $\omega t = 100\pi t$

$\omega = 100\pi$

$2\pi f = 100\pi$, 50

$f = 50\ Hz$.

(2) A 1.5 μf capacitor is connected to 220 volt, 50 Hz source. Find the capacitive reactants and current (rms and peak) in the circuit. If the frequency is double what happens to the capacitive reactants and current?

⇒ Given,

Capacitor (C) = 1.5 μf $= 1.5 \times 10^{-6}$ F.

frequency (f) = 50 Hz.

(rms) E = 220 V.

Now,

capacitive reactance $(X_C) = \frac{1}{\omega C}$

$= \frac{1}{2\pi f C}$

$= \frac{1}{2 \times \pi \times 50 \times 1.5 \times 10^{-6}}$

$= \frac{1}{4.71 \times 10^{-04}}$

$= 2.12 \times 10^{-1} \times 10^{4}$

$= 2.12 \times 10^{3}$

Also,

$I = \frac{E}{X_C}$.

or, $\frac{I}{\sqrt{2}} = \frac{\frac{E}{\sqrt{2}}}{X_C}$

or, $I_{rms} = \frac{E_{rms}}{X_C} = \frac{220}{2.12 \times 10^{3}}$

Peak $(I_0) = \sqrt{2}\, I_{rms}$

$= \sqrt{2} \times 1.037 \times 10^{-1} = 1.467 \times 10^{-1}$ A.

when frequency is double

or $X_C' = \frac{1}{\omega' C}$

or, $X_C' = \frac{1}{2\pi f' C}$

or, $X_C' = \frac{1}{2\pi (2f) C}$ $\left[X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \right]$

or, $X_C' = \frac{X_C}{2}$

Also, $I = \frac{E}{X_C}$ or, $I \propto \frac{1}{X_C}$ or, $I \propto \frac{1}{\frac{1}{\omega C}}$ or, $I \propto \omega C$ or, $I \propto 2\pi f C$

or, $I \propto f$

$\therefore$ current will be double if frequency is increase.

(3) When an inductor (L) and resistor (R) in series are connected across 12 V, 50 Hz supply a current of 0.5 A. The current differ in phase from applied voltage by $\pi/3$ radian. Calculate value of R.

Given,

current $(I) = 0.5\ A$.

emf $(E) = 12\ V$.

Frequency $(f) = 50\ Hz$.

Angle $(\theta) = \pi/3$ radian.

Now,

$E = IR$.

or, $E = IZ$.

or $Z = \frac{E}{I}$

or, $\sqrt{R^2 + X_L^2} = \frac{12}{0.5} = 24$.

we know,

$$\cos\theta = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{Z}$$

or, $\cos\frac{\pi}{3} = \frac{R}{24}$

or, $\frac{1}{3} = \frac{R}{24}$

$\therefore R = 12\ \Omega$

The value of R is $12\ \Omega$.

(4) AC mains of 200 volts and 50 Hz is joined to a circuit containing an inductance of 100 mH and resistance 20 Ω in series. Calculated power consumed.

Given

emf (e) = 200 volts.

frequency (f) = 50 Hz.

Inductance (L) = 100 mH

$= 100 \times 10^{-3}$ H.

$R = 20\,\Omega$

Now,

$X_L = L\omega$

$= L 2\pi f$

$= 100 \times 10^{-3} \times 2\pi \times 50$

$= 31.41$

$E = IZ$

$e = I\sqrt{R^2 + X_L^2}$

$e = \sqrt{IR^2 + IX_L^2}$

$Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{20^2 + (31.41)^2}$

$= \sqrt{400 + 986.58}$

$= \sqrt{1386.58}$

$= 37.23.$

Again,

$I = \frac{e}{Z}$

$= \frac{200}{37.23} = 5.37.$

So,

Power consumed $= I^2 R$

$= (5.37)^2 \times 20$

$= 28.83 \times 20$

$= 576.78$ watt

Therefore, the power consumed is 576.78 watt.

(5) A coil of inductance and negligible resistance in series with resistance R. A supply voltage 40 V is connected to them if voltage across L is equal across R. calculate voltage across R and frequency of the supply.

Given [$L = 0.1$ H, $R = 40\,\Omega$]

Given,

In R-L circuit

Emf (E) = 40 volt.

Inductance (L) = 0.1 H.

Resistance (R) = $40\,\Omega$.

According to question

Pd across (R) $V_R = V_L$ (P.d across L)

or, $IR = IX_C$

$\therefore R = X_C$ ...... (i)

And, $I = \frac{E}{Z}$

or, $E = IZ$

or, $E = I\sqrt{R^2 + X_L^2}$

or, $E = \sqrt{(IR)^2 + (IX_L)^2}$

or, $E = \sqrt{V_R^2 + V_L^2}$

or, $E = \sqrt{2V_R^2}$

$\therefore V_R = \frac{E}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.284$ volts.

from eqn (i) ; $R = X_C$

$\Rightarrow R = L\omega$

$\Rightarrow R = L \cdot 2\pi f$

$\therefore f = \frac{R}{2\pi L}$

$= \frac{40}{2\pi \times 0.1}$

$= 63.67$ Hz

classmate

Imp *

6. A 60 V - 10 W lamp is to be run on 100 V, 60 Hz ac mains.
(i) calculate the inductance of the choke coil required.
(ii) If resistance is used instead of choke, what will be it value.

soln →

given →

$V = 60$ volt

Power $(P) = 10W$

EMF $(\varepsilon) = 100V$

frequency $(f) = 60 Hz$.

Now,

$P = I^2 R$ and, $R = \frac{V^2}{P} = \frac{60^2}{100} = 360\,\Omega$.

or, $P = I \cdot IR$

$P = IV$

or, $I = \frac{P}{V} = \frac{10}{60} = 0.167 A$.

$\therefore Z = \frac{\varepsilon}{I}$ $\left[\because I = \frac{\varepsilon}{Z}\right]$

or, $\sqrt{R^2 + X_L^2} = \frac{\varepsilon}{I}$

or, $R^2 + X_L^2 = \frac{\varepsilon^2}{I^2}$

or, $X_L^2 = \frac{\varepsilon^2}{I^2} - R^2$

or, $X_L = \sqrt{\frac{\varepsilon^2}{I^2} - R^2}$

or, $X_L = \sqrt{\frac{100^2}{0.167^2} - (360)^2}$

$= 480$

we know,

$X_C = 2\pi f L$

$\therefore L = \frac{X_C}{2\pi f} = \frac{480}{2\pi \times 60} = 1.27$ #

ii) If resistance (R') is used instead of choke. Then,

Total current = $\frac{\text{Total emf}}{\text{resistance}}$

$$or,\ I = \frac{E}{R+R'}$$

$$or,\ IR + R' = E$$

$$or,\ R' = \frac{E - IR}{I}$$

$$= \frac{100 - 1/6 \times 360}{1/6}$$

$$= \frac{40}{\frac{1}{6}}$$

$$= 240\,\Omega$$ #.

(7) An iron cored coil of 2H and 50 Ω resistor placed in series with a resistor of 450 Ω and 200 V, 50 Hz ac supply is connected across the arrangement. find.

(i) current flowing in the coil.

(ii) the voltage across the coil.

(iii) phase angle relative to the voltage supply.

⇒ Given,

Inductance (L) = 2H

Resistance of coil (R_C) = 50 Ω.

Resistor of resistance (R) = 450 Ω

emf (E) = 200 V.

frequency (f) = 50v.

Now,

$X_L = L\omega$

$= L2\pi f$

$= 2\times2\times\pi\times50 = 628.31\ \Omega$.

Now,

$$Z = \sqrt{(R+R_C)^2 + X_L^2}$$

$$= \sqrt{(50+450)^2 + (628.31)^2}$$

$$= 802.97.$$

we know,

$$\text{current}(I) = \frac{E}{Z} = \frac{200}{802.97} = 0.25\ A.$$

$$\text{voltage}(V) = IX_L$$

$$= 0.25\times628.31$$

$$= 157.07\ V$$

$$\text{phase angle}(\theta) = \tan^{-1}\left(\frac{X_L}{R+R_C}\right)$$

$$= \tan^{-1}\left(\frac{628.31}{500}\right)$$

$$= 51.48^\circ.$$

Imp

(8) A 50 V, 50 Hz ac supply is connected to a resistor of 40 Ω in series with a solenoid in inductance 0.2 H. The p.d across the end of the resistor is found to be 20 V. What is the resistance of the wire of the solenoid.

= Given,

emf (E) = 50 V

Frequency (f) = 50 Hz.

Resistor (R) = 40 Ω.

Inductance (L) = 0.2 H.

P.d across R (V_R) = 20 V.

Resistance of solenoid (R_L) = ?

we know,

$V_R = 20.$

or, $IR = 20$

or, $I = \frac{20}{R}$

$I = \frac{20}{40}$

$= \frac{1}{2}$ A.

Again,

$I = \frac{E}{Z}$

or, $I = \frac{E}{\sqrt{(R+R_L)^2+(X_L)^2}}$

or, $\frac{1}{2} = \frac{50}{\sqrt{(40+R_L)^2+(L2\pi f)^2}}$

or, $\sqrt{(40+R_L)^2+(0.2\times2\times\pi\times50)^2} = 100$

or, squaring on both sides we get.

or, $(40+R_L)^2 + 3947.84 = 10,000$

or, $(40+R_L)^2 = 6052.16$

or, $40+R_L = 77.79.$

$R_L = 77.79 - 40$

$\therefore R_L = 37.79\ \Omega$

9. An alternating voltage of 10 V and 5 kHz is applied to a resistor of resistance 4Ω in series with capacitor of capacitance i.e 10 MF. calculate P.d across resistor and capacitance. Also, calculate phase angle and power factor?

⇒ Given ⇒

classmate

Emf $(E) = 10\,V$

frequency $(f) = 5\times10^{3}\,Hz$

Resistance $(R) = 4\,\Omega$.

capacitance $(C) = 10\times10^{-6}\,F$.

$V_R = ?$

$V_C = ?$

Now,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi\times5\times10^{3}\times10\times10^{-6}} = \frac{1}{3.141\times10^{-1}} = 3.18$$

Also,

$$Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{4^2 + 3.18^2}$$

$$= 5.11$$

we know,

$$I = \frac{E}{Z} = \frac{10}{5.11} = 1.96\,A.$$

(i) $V_R = IR$

$$= 1.96\times4 = 7.83\,V.$$

(ii) P.d across capacitor $(V_C) = I X_C$

$$= 1.96\times3.18$$

$$= 6.23$$

phase angle $(\theta) = \tan^{-1}\left(\frac{X_C}{R}\right)$

$$= \tan^{-1}\left(\frac{3.18}{4}\right)$$

$$= 38.48^\circ.$$

Power factor $(\cos\theta) = R/Z = \frac{4}{5.11} = 0.78$.

10) If a supply voltage is 10 V and frequency 1 kHz and capacitance 2 MF. what value of R in circuit corrected in series in would allow a current of 0.1 A to flow.

Given,

voltage (V) = 10 V = emf

frequency (f) = 1 kHz = 1×10^3.

capacitance C(F) = 2 Mf = 2×10^{-6} F.

current (I) = 0.1 A.

Resistance (R) = ?

we know,

$$I = \frac{\varepsilon}{Z}$$

or, $I = \frac{\varepsilon}{\sqrt{X_C^2 + R^2}}$ where $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 1 \times 10^3 \times 2 \times 10^{-6}}$

$= 79.577$

or, $0.1 = \frac{10}{\sqrt{79.577^2 + R^2}}$

or, $\sqrt{79.577^2 + R^2} = 100$

or, $R^2 = 10,000 - 6332.49$.

$\therefore R = 60.05\ \Omega$ #.

(11) LCR alternating ~~circu~~ current series circuit of L = 2H, C = ~~1 mf~~ 1 μf and R = 100 Ω are connected in series with a source of frequency of 50 Hz. what is the phase difference betw een current and voltage.

Soln,

$X_L = L\omega = L 2\pi f$

$= 1 \times 2 \times \pi \times 50$

$= 314.15\ \Omega$

$$x_c = \frac{1}{\omega c}$$

$$= \frac{1}{2\pi f c} = \frac{1}{2\times\pi\times 50\times 1\times 10^{-6}} = 3183.09\,\Omega.$$

Again,

$$\theta = \tan^{-1}\left(\frac{x_L - x_C}{R}\right)$$

$$\theta = \tan^{-1}\left(\frac{3183.09 - 314.15}{100}\right)$$

$$= \tan^{-1}(-28.7)$$

$$= \cancel{-88^\circ} = 88^\circ$$

(12) In a series of LCR circuit, R = 25 Ω, L = 30 mH, C = 10 MH and these are connected to 240 V ac supplied. Calculate current in the circuit and voltage across capacitor.

⇒ Given →

Resistance $(R) = 25\,\Omega$.

Inductance $(L) = 30\text{ mH} = 30\times 10^{-3}$ H.

Capacitance $(C) = 10\,\mu F = 10\times 10^{-6}$ F.

EMF $(E) = 240$ V.

Frequency $(f) = 50$ Hz.

We know,

$$I = \frac{E}{Z} = \frac{E}{\sqrt{(X_L - X_C)^2 + R^2}}$$

where,

$$X_C = L\omega = L 2\pi f$$
$$= 2\times\pi\times 50\times 30\times 10^{-3}$$
$$= 9.42.$$

or,
$$I = \frac{240}{\sqrt{(X_L - X_C)^2 + R^2}}$$

$$I = \frac{240}{\sqrt{(9.42 - 318.30)^2 + 25^2}}$$

$$I = 0.77\ A.$$

$$X_C = \frac{1}{\omega c} = \frac{1}{2\times\pi\times 50\times 10\times 10^{-6}}$$
$$= 318.30.$$

P.d across capacitor (V_C) = IX_C

$= 0.77 \times 318.30$

$= \cancel{246.37\ V}$ 245.091 V.

(13) An inductor, resistor and capacitor are connected in series wires across the ac circuit. A voltameter read 6V when connected across inductor, 16 V resistor and 30 V across capacitor.

(i) What will be voltameter reading when place across series circuit.

(ii) What is power factor of circuit.

Given →

$V_L = 60\ V.$

$V_R = 16\ V.$

$V_C = 30\ V.$

$V_{LCR} = ?$

$\cos\theta = ?$

we know,

$$V_{LCR} = I \cdot Z$$
$$= I\sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(IR)^2 + (IX_L)^2 - (IX_C)^2}$$
$$= \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{16^2 + (60-30)^2}$$
$$= 34\ V.$$

$$\cos\theta = \frac{R}{Z}$$
$$= \frac{IR}{IZ} = \frac{V_R}{V_{LCR}}$$
$$= \frac{16}{34}$$
$$= 0.47\ \text{ন্ন}$$

For resonance condition.

1. Reactants are equal.
2. Minimum impedance.
3. Maximum current.
4. Natural frequency.
5.

(14) Calculate the reactance of an inductor L of inductor 100 mH capacitance (C) is 2 μf both at a frequency of 50 Hz. At what frequency their reactance are equal in magnitude.

Given.

Inductance $(L) = 100\ mH = 100 \times 10^{-3}\ H$.

Capacitance $(C) = 2\ \mu f = 2 \times 10^{-6}\ F$.

Frequency $(f) = 50\ H$.

Now

$$X_L = L\omega = L2\pi f = 100 \times 10^{-3} \times 2 \times \pi \times 50$$
$$= 31.47\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 2 \times 10^{-6}} = 159\ \Omega.$$

Also,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 2 \times 10^{-6}}} = 355.88\ Hz.$$

(15) A circuit consist of an inductor 200 mH and resistance 10 Ω in series with a variable capacitor and 0.1 V, 1 MH supply. calculate.

i) capacitance to give resonance.

ii) Quality factor of resonance.

Given

Inductance $(L) = 200\ mH = 200 \times 10^{-6}\ H$.

Resistance $(R) = 10\ \Omega$.

E.M.f (E) = 0.1 v.
frequency (f) = 1 mH = 1×10^{6} H.
Now

$$f = \frac{1}{2\pi\sqrt{LC}}$$

or, $f^2 = \frac{1}{4\pi^2 \times LC}$

or, $C = \frac{1}{4\pi^2 \times L \times f^2} = \frac{1}{4 \times \pi^2 \times 200 \times 10^{-6} \times 10^{12}} = 1.27 \times 10^{-10}$

Also,

Quality factor (Q) $= \frac{1}{2}\sqrt{\frac{L}{C}}$

$= \frac{1}{2}\sqrt{\frac{200 \times 10^{-6}}{1.27 \times 10^{-10}}}$

$= 628.31$

(16) A coil of inductance of 0.4 MH is connected a capacitance 400 PH. To what wave length is this circuit tuned.

Soln→

Given

Inductance (L) = 0.4 mH = 0.4×10^{-3} H
Capacitance (C) = 400 PH = 400×10^{-12} H.
we know,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times \pi\sqrt{0.4 \times 10^{-3} \times 400 \times 10^{-12}}}$$

$= 397.88 \times 10^{-3}$.

Also,
velocity (v) = f.λ

or, $\lambda = \frac{v}{f} = \frac{3 \times 10^8}{397.88 \times 10^3} = 753.90$ m

* Resonance condition:-

$X_L = X_C$

So, Z = impedence $= \sqrt{(X_L - X_C)^2 + R^2}$ since, $X_L = X_C$

$\therefore Z = R$

The minimus value of impedence $Z = R$.

Also,

$V_L = V_C \, [\because X_L = X_C]$

The current and potential are in same phase $\tan\phi = \frac{X_L - X_C}{R}$

$= 0/R$

$= 0.$

At Resonance, $X_L = X_C$

or, $\omega_0 L = \frac{1}{\omega_0 C}$

$\therefore \omega = \frac{1}{\sqrt{LC}}$

$\therefore$ Resonance frequency $(f_0) = \frac{1}{2\pi\sqrt{LC}}$

* Quality factor :- Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

* Choke coil:- It is a device which is used to minimize the power developed in coil. i.e $\cos\phi \approx 0$.

* Average value of current

$I_{avg} = \frac{\int i\,dt}{\int dt}$

→ Average current of full cycle is zero.

→ Average current of half cycle is $\frac{2I_0}{\pi}$ A.

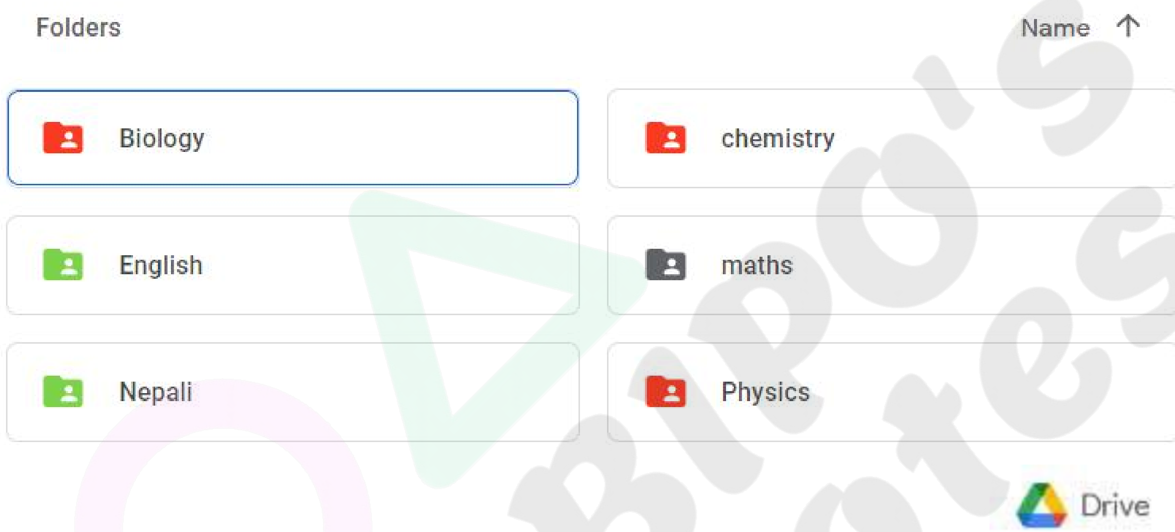
* RMS value:- Root mean square value.

$I_{rms} = \frac{I_0}{\sqrt{2}}$ → peak value.

Bipin Khatri

(Bipo)

Class 12 complete notes and paper collection.



Feedbacks:

admin@bipinkhatri.com.np | bipinkhatri.ram@gmail.com

Contact:

@im.bipo   www.bipinkhatri.com.np   @im.bipo