Chapter 19: System of Linear Equation

Exercise 19.1

1. a. Solution:

Given equations are

$4x + 5y = 12$ ... (i)

$3x + 2y = 9$ ... (ii)

Multiplying by 3 in (i) & 4 in eq. (ii) and subtracting eq. (ii) from eq. (i)

Forward elimination

$12x + 15y = 36$

$12x + 8y = 36$

$-\quad -\quad -$

$7y = 0$

$\therefore y = 0$

Backward substitution

Put the value of y in eq. (i), we get

$4x + 5 \times 0 = 12$

or, $4x = 12$

$x = 3$

$\therefore$ $x = 3, y = 0$

b. Solution:

Given equation are

$5x + 2y = 4$ ... (i)

$7x + 3y = 5$ ... (ii)

Multiplying by 7 in eq. (i) & 5 in eq. (ii) and subtracting eq. (ii) from eq. (i)

Forward elimination

$35x + 14y = 28$

$35x + 15y = 25$

$-\quad -\quad -$

$-y = 3$

$\therefore y = -3$

Backward substitution

$5x + 2y = 4$

or, $5x + 2 \times (-3) = 4$

or, $5x = 4 + 6$

or, $5x = 10$

$\therefore$ $x = 2$

Hence, the value of x & y are 2 and –3 respectively.

c. Solution:

$5x - 3y = 8$ ... (i)

$2x + 5y = 59$ ... (ii)

Forward elimination

Multiplying by 2 in eq. (ii) & 5 in eq. (ii) and subtracting eq. (ii) from eq. (i)

$10x - 6y = 16$

$10x + 25y = 295$

$-\quad -\quad -$

$-31y = -279$

$\therefore y = 9$

Backward substitution

Put the value of y in eq. (i)

$5x - 3 \times 9 = 8$

or, $5x = 8 + 27$

or, $5x = 35$

$\therefore$ $x = 7$

Hence, $x = 7$ & $y = 9$

d. Solution

$2x - 3y = 7$ ... (i)

$3x + y = 5$ ... (ii)

Forward elimination

Multiplying by 3 in eq. (i) & 2 in eq. (ii) and subtracting eq. (ii) from eq. (i)

$$\begin{array}{l} 6x - 9y = 21 \\ 6x + 2y = 10 \\ \underline{- \quad - \quad -} \\ -11y = 11 \end{array}$$

$\therefore$ $y = -1$

Backward substitution,

Put the value of y in eq. (ii)

or, $2x - 3y = 7$

or, $2x - 3 \times (-1) = 7$

or, $2x = 7 - 3$

$\therefore$ $x = 2$

Hence, the required value of x & y are 2 & –1 respectively.

2. a. Solution:

Here, given equation are

$5x - y + 4z = 5$ ... (i)

$2x + 3y + 5z = 2$ ... (ii)

$5x - 2y + 6z = -1$ ... (iii)

Multiplying by 2 in eq. (i) & 5 in eq. (ii) and subtracting eq. (ii) from eq. (i)

$$\begin{array}{l} 10x + 15y + 25z = 10 \\ 10x - 2y + 8z = 10 \\ \underline{- \quad - \quad - \quad -} \\ -17y + 17z = 0 \text{ ... (iv)} \end{array}$$

Again,

Subtracting eq. (iii) from eq. (i)

$$\begin{array}{l} 5x - y + 4z = 5 \\ 5x - 2y + 6z = -1 \\ \underline{- \quad + \quad - \quad +} \\ y - 2z = 6 \text{ ... (v)} \end{array}$$

Multiplying by 17 in eq. (v) & subtracting eq. (v) from eq. (iv)

$$\begin{array}{l} 17y + 17z = 0 \\ 17y - 34z = 102 \\ \underline{- \quad + \quad \quad -} \\ 51z = -102 \end{array}$$

$\therefore$ $z = -2$

The system of linear equations becomes

$5x - y + yz = 5$

$y - 2z = 6$

$2z = -2$

Put the value of z in eq. (v)

$y - 2 \times (-2) = 6$

or, $y = 6 - y$

$\therefore$ $y = 2$

Again, put the values of x & y in eq. (i)

$5x - y + yz = 5$

or, $5x - 2 + 4 \times (-2) = 5$

or, $5x = 5 + 7 + 8$
or, $5x = 15$
$\therefore$ $x = 3$
Hence, $x = 3$, $y = 2$, $z = -2$

b. Solution:
Here, given equations are
$x - y + 2z = 7$ ... (i)
$3x + 4y - 5z = -5$ ... (ii)
$2x - y + 3z = 12$ ... (iii)
Multiplying by 3 in eq. (i) & subtracting eq. (ii) from eq. (i)

$$\begin{array}{l} 3x - 3y + 6z = 21 \\ 3x + 4y - 5z = -5 \\ \underline{- \quad - \quad + \quad +} \\ -7y + 11z = 26 \text{ ... (iv)} \end{array}$$

Multiplying by 2 in eq. (i) and subtracting eq. (iii) from eq. (i)

$$\begin{array}{l} 2x - 2y + 4z = 14 \\ 2x - y + 3z = 12 \\ \underline{- \quad + \quad - \quad -} \\ -y + 2 = 2 \text{ ... (v)} \end{array}$$

Multiplying by 7 in eq. (v) and subtracting eq. (v) from eq. (iv)

$$\begin{array}{l} -7y + 11z = 26 \\ -7y + 7z = 14 \\ \underline{+ \quad - \quad -} \\ -y + z = 2 \text{ ... (iv)} \end{array}$$

Multiplying by 7 in eq. (v) and subtracting eq. (v) from eq. (iv)

$$\begin{array}{l} -7y + 11z = 26 \\ -7y + 7z = 14 \\ \underline{+ \quad - \quad -} \\ \text{or, } 4z = 12 \end{array}$$

$\therefore$ $z = 3$
The system of linear equations become
$x - y + 2z = 7$
$-y + z = 2$
$z = 3$
Put the value of z in eq. (v)
$-y + 3 = 2$
or, $-y = -3 + 2$
or, $y = 1$
Again,
Put the value of y & z in eq. (i)
$x - 1 + 2 \times 3 = 7$
or, $x = 7 - 6 + 1$
or, $x = 2$
Hence, $x = 2$, $y = 1$, $z = 3$.

c. We have,
$2x + 3y + 3z = 5$ ... (i)
$x - 2y + z = -4$ ...(ii)
$3x - y - 2z = 3$ ...(iii)
from (i) and (ii), we get
$7y + z = 13$ ...(iv)
from (ii) and (iii) we get
$5y - 5z = 15$
$\therefore$ $y - z = 3$ ...(v)

Adding (iv) and (v), we get

$8y = 16$

$\therefore \quad y = 2$

from (v), $2 - z = 3$

$\therefore \quad z = -1$

from (ii), $x - 4 - 1 = -4$

$\Rightarrow \quad x = -4 + 5 = 1$

Hence, $x = 1$, $y = 2$, $z = -1$

d. Solution:

Here, given equations are

$x + 2y + 3z = 14 \quad \text{... (i)}$

$3x + 4y + 2z = 17 \quad \text{... (ii)}$

$2x + 3y + z = 11 \quad \text{... (iii)}$

Multiplying by 3 in eq. (i) and subtracting eq. (ii) from eq. (i)

$$\begin{array}{l} 3x + 6y + 9z = 42 \\ 3x + 4y + 2z = 17 \\ - \quad - \quad - \quad - \\ \hline 2y + 7z = 25 \text{ ... (iv)} \end{array}$$

Multiplying by 2 in eq. (i) and subtracting eq. (iii) from (ii)

$$\begin{array}{l} 2x + 4y + 6z = 28 \\ 2x + 3y + z = 11 \\ - \quad - \quad - \quad - \\ \hline y + 5z = 17 \text{ ... (v)} \end{array}$$

Multiplying by 2 in eq. (v) & subtracting eq. (v) from (iv)

$$\begin{array}{l} 2y + 7z = 25 \\ 2y + 10z = 34 \\ - \quad - \quad + \\ \hline +3z = +9 \end{array}$$

$\therefore z = 3$

The system of linear equations becomes

$x + 2y + 3z = 14$

$y + 5z = 17$

$z = 3$

Put the value of z in eq. (v)

or, $y + 5 \times 3 = 17$

or, $y = 17 - 15 = 2$

$\therefore y = 2$

Put the value of x & y in eq. (i)

$x + 2y + 3z = 14$

or, $x + 2 \times 2 + 3 \times 3 = 14$

or, $x = 14 - 9 - 4$

or, $x = 14 - 13$

$\therefore \quad x = 1$

Hence, $x = 1$, $y = 2$ & $z = 3$

3. (a) Solution:

$x + 3y = 5 \quad \text{... (i)}$

$3x + y = 4 \quad \text{... (ii)}$

Multiplying by 3 in eq. (i) and subtracting eq. (ii) from eq. (i) Forward elimination

$$\begin{array}{l} 3x + 9y = 15 \\ 3x + y = 4 \\ - \quad - \quad - \\ \hline 8y = 11 \end{array}$$

$y = \frac{11}{8}$

Backward substitution

Put the value of y in eq. (i)

$x + \frac{3 \times 11}{8} = 5$

or, $x = 5 - \frac{33}{8}$

or, $x = \frac{7}{8}$

$\therefore \quad x = \frac{7}{8}, y = \frac{11}{8}$

It is consistent and has unique solution.

b. Solution:

Here,

$3x - 2y = 3 \quad \text{... (i)}$

$3x - 2y = 6 \quad \text{... (ii)}$

Subtracting eq. (ii) from eq. (i)

$$\begin{array}{l} 3x - 2y = 3 \\ 3x - 2y = 6 \\ - \quad + \quad - \\ \hline 0 = -3 \end{array}$$

Hence, it is inconsistent and has no solution.

c. **Solution:**

$-2x + 5y = 3 \quad \text{... (i)}$

$6x - 15y = -9 \quad \text{... (ii)}$

Multiplying by 3 in eq. (i) & adding eq. (i) and eq. (ii)

$$\begin{array}{l} -6x + 15y = 9 \\ 6x - 15y = -9 \\ - \quad + \qquad + \\ \hline 0 = 0 \end{array}$$

It is consistent having infinitely many solution.

d. **Solution:**

Given equations are:

$x - 2y - 5z = -12 \quad \text{... (i)}$

$2x - y = 7 \quad \text{... (ii)}$

$-4x + 5y + 6z = 1 \quad \text{... (iii)}$

Multiplying by 4 in eq. (i) and adding eq. (i) & eq. (iii)

$$\begin{array}{l} 4x - 8y - 20z = -48 \\ -4x + 5y + 6z = 1 \\ - \quad + \qquad + \\ \hline -3y - 14z = -47 \end{array}$$

or, $3y + 14z = 47 \quad \text{... (iv)}$

Multiplying by 2 in eq. (i) and subtracting eq. (ii) from eq. (i)

$$\begin{array}{l} 2x - 4y - 10z = -24 \\ 2x - y = 7 \\ - \quad + \quad - \\ \hline -3y - 10z = -31 \text{... (iv)} \end{array}$$

Adding eq. (iv) and eq. (v)

$$\begin{array}{l} 3y + 14z = 47 \\ -3y - 10z = -31 \\ \hline 4z = 16 \end{array}$$

$\therefore z = 4$

The system of linear equations becomes

$x - 2y - 5z = -12$
$3y + 14z = 47$
$z = 4$
Put the value of z in eq. (i)
$3y + 14 \times 4 = 47$
or, $3y = 47 - 56$
or, $3y = -9$
$\therefore$ $y = -3$

e. Here,
$2x - y + 4z = 4$ ... ... (i)
$x + 2y - 3z = 1$ ... ... (ii)
$3x + 3z = 6$ ... ... (iii)
Multiplying by 2 in equation (i) and subtracting equation (ii) from equation (i)
$2x - y + 4z = 4$
$2x + 4y - 6z = 2$
$- \quad - \quad + \quad -$
$-5y + 10z = 2$ ... ... (iv)
Multiplying by 3 in equation (i) and subtracting equation (iii) from equation (ii)
$3x + 6y - 92 = 3$
$3x + 3z = 6$
$- \quad - \quad -$
$6y - 12z = -3$
or, $2y - 4z = -1$ ... ... (v)
Multiplying by 2 in equation (iv) and adding equation (iv) and equation (v)
$-10y + 20z = 4$
$10y - 20z = -5$
$0 \; -1$
Here, $0 = -1$
It is inconsistent having no solution.

f. Here,
Given equations are
$x + 3y + 4z = 8$ ... ... (i)
$2x + y + 2z = 5$ ... ... (ii)
$5x + 2z = 7$ ... ... (iii)
Multiplying by 2 in equation (i) and subtracting equation (ii) from equation (i)
$2x + 6y + 8z = 16$
$2x + y + 22 = 5$
$- \quad - \quad - \quad -$
$5y + 6z = 11$ ... ... (iv)
Multiplying by 5 in equation (i) and subtracting equation (iii) from equation (i)
$5x + 15y + 20z = 40$
$5x + 0.y + 22 = 7$
$- \quad - \quad - \quad -$
$15y + 18z = 33$ ... ... (v)
Multiplying by 3 in equation (iv) and subtracting equation (v) from equation (iv)
$15y + 18z = 33$
$15y + 18z = 33$
$- \quad - \quad -$
$0 = 0$
It is consistent having infinitely many solution.

Exercise 19.2

1. Solution

a. Here,

$3x_1 + x_2 = 5$ ... ... (i)

$x_1 + 2x_2 = 5$ ... ... (ii)

from equation (i) and equation (ii)

$x_1 = \frac{5 - x_2}{3}$

$x_2 = \frac{5 - x_1}{2}$

Initially, $x_2 = 0$. Iteration – I

$x_1 = \frac{5}{3}$

$= 1.66$

$x_2 = \frac{5 - 1.66}{2}$

$= 1.67$

Iteration – II

$x_1 = \frac{5 - 1.67}{3}$

$= 1.11$

$x_2 = \frac{5 - 1.11}{2}$

$= 1.945$

Iteration – III

$x_1 = \frac{5 - 1.945}{3}$

$= 1.01$

$x_2 = \frac{5 - 1.01}{2}$

$= 1.995$

Iteration – IV

$x_1 = \frac{5 - 1.995}{3}$

$= 1.001$

$x_2 = \frac{5 - 1.001}{2}$

$= 1.9995$

From iteration – III and iteration – IV the value of x_1 and x_2 are nearly equal to 1 and 2.

$\therefore \quad x_1 = 1, x_2 = 2$

b. Here,

$2x_1 - x_2 = 8$ ... ... (i)

$3x_1 + 7x_2 = -5$ ... ... (ii)

from equation (i) and equation (ii)

$x_1 = \frac{8 + x_2}{2}$

$x_2 = \frac{-5 - 3x_1}{7}$

Initially, $x_2 = 0$

Iteration – I

$x_1 = \frac{8 + 0}{2}$

$= 4$

$x_2 = \frac{-5 - 3\times4}{7}$

$= \frac{-5 - 12}{7}$

$= -2.42$

Iteration – II

$x_1 = \frac{8 - 2.4^2}{2}$

$= 2.79$

$x_2 = \frac{-5 - 3\times2.79}{7}$

$= \frac{-5 - 8.37}{7}$

$= -1.91$

Iteration – III

$x_1 = \frac{8 - 1.91}{2}$

$= 3.045$

$x_2 = \frac{-5 - 3\times3.045}{7}$

$= \frac{-5 - 9.135}{7}$

Iteration – IV

$x_1 = \frac{8 - 2.01}{2}$

$= 2.995$

$x_2 = \frac{-5 - 3\times(+2.995)}{7}$

$= \frac{-5 - 8.985}{7}$

$= \frac{-14.135}{7}$

$= -2.01$

$= \frac{-13.985}{7}$

$= -1.99$

Iteration – V

$x_1 = \frac{8 - 1.99}{2}$

$= 3.005$

$x_2 = \frac{-5 - 3\times3.005}{7}$

$= \frac{-5 - 9.015}{7}$

$= -2.002$

Iteration – VI

$x_1 = \frac{8 - 2.002}{2}$

$= 2.999$

$x_2 = \frac{-5 - 3\times2.999}{7}$

$= \frac{-5 - 8.997}{7}$

$= 1.99$

From iteration – V and iteration – VI the value of x_1 and x_2 are nearly equation.

So, $x_1 = 3$, $x_2 = -2$

c. Here,

$3x_1 + x_2 = 5$ ... ... (i)

$x_1 - 3x_2 = 5$ ... ... (ii)

from equation (i) and equation (ii)

$x_1 = \frac{5 - x_2}{3}$

$x_2 = \frac{x_1 - 5}{3}$

Initially, $x_2 = 0$

Iteration – I

$x_1 = \frac{5 - 0}{3}$

$= 1.67$

$x_2 = \frac{1.67 - 5}{3}$

$= -1.11$

Iteration – II

$x_1 = \frac{5 + 1.11}{3}$

$= 2.03$

$x_2 = \frac{2.03 - 5}{3}$

$= -0.99$

Iteration – III

$x_1 = \frac{5 + 0.99}{3}$

$= 1.99$

$x_2 = \frac{1.99 - 5}{3}$

$= -1.00$

Iteration – IV

$x_1 = \frac{5 + 1.00}{3}$

$= 2$

$x_2 = \frac{2 - 5}{3}$

$= -1$

From iteration III and iteration IV the value of x_1 and x_2 are nearly equal so, $x_1 = 2$ and $x_2 = -1$.

d. Here,

$2x - 3y = 7$

$3x + y = 5$

or, $3x + y = 5$ ... (i)

or, $2x - 3y = 7$ ... (ii)

and do in similar way

$\because$The order of given equations are not diagonally dominant, so we should change to order.

2. Solution

a. Given equation are

$2x - y = 1$... ... ... (i)

$-x + 3y - z = 8$ ... ... (ii)

$y - 2z = 5$ ... ... (iii)

From equation (i) (ii) (iii),

$x = \frac{1 + y}{2}$

$y = \frac{8 + x + z}{3}$

$z = \frac{y - 5}{2}$

Initially, y = 0, z = 0

Iteration – I

$x = \frac{1}{2}$

$= 0.5$

$y = \frac{8 + 0.5 + 0}{3}$

$= 2.83$

$z = \frac{2.83 - 5}{2}$

$= -1.085$

Iteration – II

$x = \frac{1 + 2.83}{2}$

$= 1.915$

$y = \frac{8 + 1.915 - 1.085}{3}$

$= 2.943$

$z = \frac{2.943 - 5}{2}$

$= -1.0285$

Iteration – III

$x = \frac{1 + 2.943}{2}$, $y = \frac{8 + 1.975 - 1.0285}{3}$, $z = \frac{2.981.5}{2}$

$= 1.9715$ $= 2.981$ $= -1.0095$

From iteration II and III the value of x, y and z are nearly equal to 2, 3, and –1.

x = 2, y = 3 and z = –1

b. Here,

Given equations are

$3x + y - z = 2$ ... ... (i)

$2x - 5y + z = 20$ ... ... (ii)

$x - 3y - 8z = 3$ ... ... (iii)

From equation(i), (ii) and equation (iii)

$x = \frac{2 - y + z}{3}$

$y = \frac{2x + z - 20}{5}$

$z = \frac{x - 3y - 3}{8}$

Initially, y = 0, z = 0

Iteration – I

$x = \frac{2}{3} = 0.67$

$y = \frac{2\times0.67 + 0.20}{5} = -3.73$

$z = \frac{0.67 + 3\times3.73 - 3}{8} = -2.64$

Iteration – II

$x = \frac{2 + 3.73 - 2.64}{3}$

$= 1.03$

$y = \frac{2\times1.03 - 2.64 - 20}{5}$

$= \frac{2.06 - 22.64}{5}$

$= -4.116$

$z = \frac{1.03 + 3\times4.116 - 3}{8}$

$= 1.29$

Iteration – III

$x = \frac{2 + 4.116 + 1.29}{3}$

$= 2.46$

$y = \frac{2\times2.46 + 1.29 - 20}{5}$

$= -2.75$

$= \frac{2.46 + 3\times2.75 - 3}{8}$

$= 0.96$

Iteration – IV

Iteration – V

$x = \frac{2 + 2.75 + 0.96}{3}$

$= 1.9$

$y = \frac{2\times1.9 + 0.96 - 20}{5}$

$= -3.0$

$z = \frac{1.9 + 3\times3.0 - 3}{8}$

$= 0.98$

$x = \frac{2 + 3.0 + 0.98}{3}$

$= 1.99$

$y = \frac{2\times1.99 + 0.98 - 20}{5}$

$= -3.00$

$z = \frac{1.99 + 3\times3 - 3}{8}$

$= 0.99$

From iteration IV and V the value of x, y and z are nearly equal to 2, –3 and 1 respectively.

$x = 2, y = -3$ and $z = 1$

c. Here,

Given equations are

$5x_1 + 2x_2 + x_3 = 12$ ... ... (i)

$x_1 = 4x_2 + 2x_3 = 15$ ... ... (ii)

$x_1 = 2x_2 + 5x_3 = 20$ ... ... (iii)

from equation (i), (ii) and (iii)

$x_1 = \frac{12 - x_2 - x_3}{5}$

$x_2 = \frac{15 - 2x_3 - x_1}{4}$

$x_3 = \frac{20 - x_1 - 2x_2}{5}$

Initially, $x_2 = 0$, $x_3 = 0$

Iteration I

$x_1 = \frac{12}{5} = 2.4$

$x_2 = \frac{15 - 2\times0 - 2.4}{4} = 3.15$

$x_3 = \frac{20 - 2.4 - 2\times3.15}{5} = 2.26$

Iteration II

$x_1 = \frac{12 - 3.15 - 2.26}{5} = 1.31$

$x_2 = \frac{15 - 2\times2.26 - 1.31}{4} = 2.29$

$x_3 = \frac{20 - 1.31 - 2\times2.29}{5} = 2.82$

Iteration III

$x_1 = \frac{12 - 2.29 - 2.82}{5} = 1.37$

$x_2 = \frac{15 - 2\times2.82 - 1.37}{4} = 1.9$

$x_3 = \frac{20 - 1.37 - 2\times1.9}{5} = 2.9$

Iteration IV

$x_1 = \frac{12 - 1.9 - 2.9}{5} = 1.44$

$x_2 = \frac{15 - 2\times2.9 - 1.44}{4} = 1.94$

$x_3 = \frac{20 - 1.44 - 2\times1.94}{5} = 2.9$

Iteration V

$$x_1 = \frac{12 - 1.94 - 2.9}{5} = 1.4$$

$$x_2 = \frac{15 - 2\times2.9 - 1.44}{4} = 1.99$$

$$x_3 = \frac{20 - 1.4 - 2\times199}{5} = 2.9$$

From iteration IV and V, the value of x_1, x_3 and x_3 are nearly equal to 1, 2, 3 respectively.

Hence, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$

d. Here,

Given equation are

$x + 10y + z = 6$

$10x + y + z = 6$

$x + y + 10z = 6$

We can write the equations in

$10x + y + z = 6$ ... (i)

$x + 10y + z= 6$ ... (ii)

$x + y + 10z = 6$ ... (iii)

Order as the equations are not in diagonally dominant and do similarly.

e. Here,

$3x + y + z = 13$ ... (i)

$x - 4y - z = -14$ ... (ii)

$2x + 3y + 6z = 37$ ... (iii)

from equation (i), (ii) and (iii)

$$x = \frac{13 - y - z}{3}$$

$$y = \frac{x + 14 - z}{4}$$

$$z = \frac{37 - 2x - 3y}{6}$$

Initially, $y = 0$, $z = 0$

Iteration I

$x = 4.33$

$$y = \frac{4.33 + 14 - 0}{4} = 4.58$$

$$z = \frac{37 - 2\times4.33 - 3\times4.58}{6} = 2.43$$

Iteration II

$$x = \frac{13 - 4.58 - 2.43}{3} = 1.99$$

$$y = \frac{1.99 + 14 - 2.43}{4} = 3.39$$

$$z = \frac{37 - 2\times1.99 - 3\times3.39}{6} = 3.80$$

Iteration III

$$x = \frac{13 - 3.39 - 3.80}{3} = 1.9$$

$$y = \frac{1.9 + 14 - 3.80}{4} = 3.025$$

$$z = \frac{37 - 2\times1.9 - 3\times3.025}{6} = 4.0$$

Iteration IV

$x = \frac{13 - 3.025 - 4.0}{3} = 1.99$

$y = \frac{1.99 + 14 - 4.0}{4} = 2.9$

$z = \frac{37 - 2\times1.99 - 3\times2.9}{6} = 4.0$

From iteration III and IV the value of x, y and z are nearly equations.

So, x = 2, y = 3, z = 4

3. Solution

a. Given equations are

$3x + 1.52y = 1$ ... ... (i)

$2x + 1.02y = 1$ ... ... (ii)

Multiplying by 2 in equation (i) and 3 in equation (ii) and subtracting equation (ii) from equation (i)

$6x + 3.04y = 2$

$6x + 3.06y = 3$

$-\quad -\qquad -$

$-0.02y = -1$

$y = 50$

Put the value of y in equation (i)

$3x + 1.52\times50 = 1$

or, $3x - 1-76$

or, $3x = -75$

$\therefore$ $x = -25$

If the coefficient of y in equation (ii) is changed to 1.03. Then,

$3x + 1.52y = 1$ ... ... (iii)

$2x + 1.03y = 1$ ... ... (iv)

Multiplying by 2 in equation (iii) and 3 in equation (iv) and subtracting equation (iv) from equation (iii).

$6x + 3.04y = 2$

$6x + 3.09y = 3$

$-\quad -\qquad -$

$-0.05y = -1$

$\therefore$ $y = 20$

Put the value of y in equation (iii)

$3x + 1.52\times20 = 1$

or, $3x = 1 - 30.4$

or, $3x - 29.4$

$\therefore$ $x = -9.8$

It is observed that when a very small change in coefficient of y brings greater change in its solution.

So, $3\times(-9.8) + 1.52\times20 - 1 = 0$

$2\times(-9.8) + 1.02\times20 - 1 = -0.2$

which is very small, so it is ill conditioned.

Exercise 19.3

1. Solution

a. Let, $A = \begin{bmatrix} 3 & 2 \\ -1 & 6 \end{bmatrix}$

We can augment matrix A with unit matrix as,

$[A : I] = \begin{bmatrix} 3 & 2 & : & 1 & 0 \\ -1 & 6 & : & 0 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2/3 & : & 1/3 & 0 \\ -1 & 6 & : & 0 & 1 \end{bmatrix}$ $[\because R_1 \to \frac{1}{3} R_1]$

$\sim \begin{bmatrix} 1 & 2/3 & : & 1/3 & 0 \\ 0 & 20/3 & : & 1/3 & 1 \end{bmatrix}$ $[\because R_2 \to R_2 + R_1]$

$\sim \begin{bmatrix} 1 & 2/3 & : & 1/3 & 0 \\ 0 & 1 & : & 1/20 & 3/20 \end{bmatrix}$ $[\because R_2 \to \frac{3}{20} R_2]$

$\sim \begin{bmatrix} 1 & 0 & : & 3/10 & -1/10 \\ 0 & 1 & : & 1/20 & 3/20 \end{bmatrix}$

$\therefore \quad A^{-1} = \begin{bmatrix} \frac{3}{10} & \frac{-1}{10} \\ \frac{1}{20} & \frac{3}{20} \end{bmatrix}$

b. Here,

Let, $A = \begin{bmatrix} 2 & -1 \\ -3 & 3 \end{bmatrix}$

We can augment matrix A with unit matrix as

$[A : I] = \begin{bmatrix} 2 & -1 & : & 1 & 0 \\ -3 & 3 & : & 0 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & \frac{-1}{2} & : & \frac{1}{2} & 0 \\ -3 & 3 & : & 0 & 1 \end{bmatrix}$ $[\because R_1 \to \frac{1}{2} R_1]$

$\sim \begin{bmatrix} 1 & \frac{-1}{2} & : & \frac{1}{2} & 0 \\ 0 & \frac{3}{2} & : & \frac{3}{2} & 1 \end{bmatrix}$ $[\because R_2 \to R_2 + 3R_1]$

$\sim \begin{bmatrix} 1 & \frac{-1}{2} & : & \frac{1}{2} & 0 \\ 0 & 1 & : & 1 & \frac{2}{3} \end{bmatrix}$ $[\because R_2 \to \frac{2}{3} R_2]$

$\sim \begin{bmatrix} 1 & 0 & : & 1 & \frac{1}{3} \\ 0 & 1 & : & 1 & \frac{2}{3} \end{bmatrix}$ $[\because R_1 \to R_1 + \frac{1}{2} R_2]$

$\therefore \quad A^{-1} = \begin{bmatrix} 1 & \frac{1}{3} \\ 1 & \frac{2}{3} \end{bmatrix}$

c. Here,

Let, $A = \begin{bmatrix} 5 & 3 & -1 \\ 2 & 4 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

We can augment matrix with unit matrix as,

$[A : I] = \begin{bmatrix} 5 & 2 & -1 & : & 1 & 0 & 0 \\ 2 & 4 & 1 & : & 0 & 1 & 0 \\ 1 & 2 & 3 & : & 0 & 0 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & \frac{2}{5} & \frac{-1}{5} & : & \frac{1}{5} & 0 & 0 \\ 2 & 4 & 1 & : & 0 & 1 & 0 \\ 1 & 2 & 3 & : & 0 & 0 & 1 \end{bmatrix}$ $[\because R_1 \to \frac{1}{5} R_1]$

$$\sim \left[\begin{array}{ccc:ccc} 1 & \frac{2}{5} & \frac{-1}{5} & \frac{1}{5} & 0 & 0 \\ 0 & \frac{16}{5} & \frac{7}{5} & \frac{-2}{5} & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \quad [\because R_2 \to R_2 - 2R_1]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & \frac{2}{5} & \frac{-1}{5} & \frac{1}{5} & 0 & 0 \\ 0 & \frac{16}{5} & \frac{7}{5} & \frac{-2}{5} & 1 & 0 \\ 0 & \frac{8}{5} & \frac{16}{5} & \frac{-1}{5} & 0 & 1 \end{array}\right] \quad [\because R_3 \to R_3 - R_1]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & \frac{2}{5} & \frac{-1}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 1 & \frac{7}{16} & \frac{-1}{8} & \frac{5}{16} & 0 \\ 0 & \frac{8}{5} & \frac{16}{5} & \frac{-1}{5} & 0 & 1 \end{array}\right] \quad [\therefore R_2 \to \frac{6}{16} R_2]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & \frac{-3}{8} & \frac{2}{5} & \frac{-1}{8} & 0 \\ 0 & 1 & \frac{7}{16} & \frac{-1}{8} & \frac{5}{16} & 0 \\ 0 & \frac{8}{5} & \frac{16}{5} & \frac{-1}{5} & 0 & 1 \end{array}\right] \quad [\because R_1 \to R_1 - \frac{2}{5} R_2]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & \frac{-3}{8} & \frac{2}{5} & \frac{-1}{8} & 0 \\ 0 & 1 & \frac{7}{16} & \frac{-1}{8} & \frac{5}{16} & 0 \\ 0 & 0 & \frac{5}{2} & 0 & \frac{-1}{2} & 1 \end{array}\right] \quad [\because R_3 \to R_3 - \frac{8}{5} R_2]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & \frac{-3}{8} & \frac{2}{5} & \frac{-1}{8} & 0 \\ 0 & 1 & \frac{7}{16} & \frac{-1}{8} & \frac{5}{16} & 0 \\ 0 & 0 & 1 & 0 & \frac{-1}{5} & \frac{2}{5} \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & \frac{-3}{8} & \frac{2}{5} & \frac{-1}{8} & 0 \\ 0 & 1 & 0 & \frac{-1}{8} & \frac{2}{5} & \frac{-7}{40} \\ 0 & 0 & 1 & 0 & \frac{-1}{5} & \frac{2}{5} \end{array}\right] \quad [\because R_2 \to R_2 \to \frac{7}{16} R_3]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & \frac{2}{5} & \frac{-1}{5} & \frac{3}{20} \\ 0 & 1 & 0 & \frac{-1}{8} & \frac{2}{5} & \frac{-7}{40} \\ 0 & 0 & 1 & 0 & \frac{-1}{5} & \frac{2}{5} \end{array}\right] \quad [\because R_1 \to R_1 + \frac{3}{8} R_3]$$

$$A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{-1}{5} & \frac{3}{20} \\ \frac{-1}{8} & \frac{2}{5} & \frac{-7}{40} \\ 0 & \frac{-1}{5} & \frac{2}{5} \end{bmatrix}$$

d. Here,

Let, $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

We can augment matrix A with unit matrix as,

$$[A : I] = \begin{bmatrix} 1 & 2 & -2 & : & 1 & 0 & 0 \\ -1 & 3 & 0 & : & 0 & 1 & 0 \\ 0 & -2 & 1 & : & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -2 & : & 1 & 0 & 0 \\ 0 & 5 & -2 & : & 1 & 1 & 0 \\ 0 & -2 & 1 & : & 0 & 0 & 1 \end{bmatrix} \quad [\because R_2 \to R_2 + R_1]$$

$$\sim \begin{bmatrix} 1 & 2 & -2 & : & 1 & 0 & 0 \\ 0 & 1 & \frac{-2}{5} & : & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & -2 & 1 & : & 0 & 0 & 1 \end{bmatrix} \quad [\because R_2 \to \frac{1}{5} R_2]$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-6}{5} & : & \frac{3}{5} & \frac{-2}{5} & 0 \\ 0 & 1 & \frac{-2}{5} & : & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & -2 & 1 & : & 0 & 0 & 1 \end{bmatrix} \quad [\because R_1 \to R_1 - 2R_2]$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-6}{5} & : & \frac{3}{5} & \frac{-2}{5} & 0 \\ 0 & 1 & \frac{-2}{5} & : & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{5} & : & \frac{2}{5} & \frac{2}{5} & 1 \end{bmatrix} \quad [\because R_3 \to R_3 + 2R_2]$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-6}{5} & : & \frac{3}{5} & \frac{-2}{5} & 0 \\ 0 & 1 & \frac{-2}{5} & : & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & : & 2 & 2 & 5 \end{bmatrix} \quad [\because R_3 \to 5R_3]$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-6}{5} & : & \frac{3}{5} & \frac{-2}{5} & 0 \\ 0 & 1 & 0 & : & 1 & 1 & 2 \\ 0 & 0 & 1 & : & 2 & 2 & 5 \end{bmatrix} \quad [\because R_2 \to R_2 + \frac{2}{5} R_3]$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 3 & 2 & 6 \\ 0 & 1 & 0 & : & 1 & 1 & 2 \\ 0 & 0 & 1 & : & 2 & 2 & 5 \end{bmatrix} \quad [\because R_1 \to R_1 + \frac{6}{5} R_3]$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

2. Solution:

a. Here, Given equations can be written in matrix form as AX = B.

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 4 & -9 \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -2 \\ 1 \end{bmatrix}}_{B}$$

$X = A^{-1} B$

Matrix A can be augmented with unit matrix a

$$[A : I] = \begin{bmatrix} 2 & 3 & : & 1 & 0 \\ 4 & -9 & : & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & \frac{3}{2} & : & \frac{1}{2} & 0 \\ 4 & -9 & : & 0 & 1 \end{bmatrix} \qquad [\because R_1 \to \frac{1}{2} R_1]$$

$$\sim \begin{bmatrix} 1 & \frac{3}{2} & : & \frac{1}{2} & 0 \\ 0 & -15 & : & -2 & 1 \end{bmatrix} \qquad [\because R_2 \to R_2 - 4R_1]$$

$$\sim \begin{bmatrix} 1 & \frac{3}{2} & : & \frac{1}{2} & 0 \\ 0 & 1 & : & \frac{2}{15} & \frac{-1}{15} \end{bmatrix} \qquad [\because R_3 \to \frac{-1}{5} R_3]$$

$$\sim \begin{bmatrix} 1 & 0 & : & \frac{3}{10} & \frac{1}{10} \\ 0 & 1 & : & \frac{2}{15} & \frac{-1}{15} \end{bmatrix} \qquad [\because R_1 \to R]$$

$$\therefore \quad A^{-1} = \begin{bmatrix} \frac{3}{10} & \frac{1}{10} \\ \frac{2}{15} & \frac{-1}{15} \end{bmatrix}$$

Now, $x = A^{-1} B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{10} & \frac{1}{10} \\ \frac{2}{15} & \frac{-1}{15} \end{bmatrix} \times \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{10} \times (-2) + \frac{1}{10} \times 1 \\ \frac{2}{15} \times (-2) + \left(\frac{-1}{15}\right) \times 1 \end{bmatrix} = \begin{bmatrix} \frac{-6}{10} + \frac{1}{10} \\ \frac{-4}{15} - \frac{1}{15} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} \\ \frac{-1}{3} \end{bmatrix}$$

$\therefore x = \frac{-1}{2}, y = \frac{-1}{3}$

b. Here,

$4x + 5y = 9$

$5x - y = 4$

These equations can be written in matrix form as AX = B.

$$\underbrace{\begin{bmatrix} 4 & 5 \\ 5 & -1 \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 9 \\ 4 \end{bmatrix}}_{B}$$

$X = A^{-1} B. \to$ equation (i)

Matrix A can be augmented with unit matrix as,

$$[A : I] = \begin{bmatrix} 4 & 5 & : & 1 & 0 \\ 5 & -1 & : & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & \frac{5}{4} & : & \frac{1}{4} & 0 \\ 5 & -1 & : & 0 & 1 \end{bmatrix} \qquad [\because R_1 \to \frac{1}{4} R_1]$$

$$\sim \begin{bmatrix} 1 & \frac{5}{4} & : & \frac{1}{4} & 0 \\ 0 & \frac{-29}{4} & : & \frac{-5}{4} & 1 \end{bmatrix} \quad [\because R_2 \to R_2 - 5R_1]$$

$$\sim \begin{bmatrix} 1 & \frac{5}{4} & : & \frac{1}{4} & 0 \\ 0 & 1 & : & \frac{5}{29} & \frac{-4}{29} \end{bmatrix} \quad [\because R_3 \to \frac{-4}{29} R_3]$$

$$\sim \begin{bmatrix} 1 & 0 & : & \frac{1}{29} & \frac{5}{29} \\ 0 & 1 & : & \frac{5}{29} & \frac{-4}{29} \end{bmatrix} \quad [\because R_1 \to R_1 - \frac{5}{4} R_2]$$

from equation (i)

$X = A^{-1} B$

$$\text{or, } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{29} & \frac{5}{29} \\ \frac{5}{29} & \frac{-4}{29} \end{bmatrix} = \begin{bmatrix} \frac{9}{29} & + & \frac{20}{29} \\ \frac{45}{29} & - & \frac{16}{29} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\therefore \quad x = 1, y = 1$

c. Here,

The system of equations can be written in matrix form as AX = B.

$$\underset{A}{\begin{bmatrix} 1 & -2 \\ 3 & 7 \end{bmatrix}} \times \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} -7 \\ 5 \end{bmatrix}}$$

$x = A^{-1} B$

Matrix A can be augmented with unit matrix as,

$$[A : I] = \begin{bmatrix} 1 & -2 & : & 1 & 0 \\ 3 & 7 & : & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & : & 1 & 0 \\ 0 & 13 & : & -3 & 1 \end{bmatrix} \quad [\because R_2 \to R_2 - 3R_1]$$

$$\sim \begin{bmatrix} 1 & -2 & : & 1 & 0 \\ 0 & 1 & : & \frac{-3}{13} & \frac{1}{13} \end{bmatrix} \quad [\because R_2 \to \frac{1}{13} R_2]$$

$$\sim \begin{bmatrix} 1 & 0 & : & \frac{7}{13} & \frac{2}{13} \\ 0 & 1 & : & \frac{-3}{13} & \frac{1}{13} \end{bmatrix} \quad [\because R_2 \to R_1 + 2R_2]$$

Now,

$x = A^{-1} B$

$$\text{or, } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{13} & \frac{2}{13} \\ \frac{-3}{13} & \frac{1}{13} \end{bmatrix} \times \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{13} \times (-7) + \frac{2}{13} \times 5 \\ \frac{-3}{13} \times (-7) + \frac{1}{13} \times 5 \end{bmatrix} = \begin{bmatrix} \frac{-49 + 10}{13} \\ \frac{21 + 5}{13} \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$\therefore \quad x = -3, y = 2$

c, d, e & f are similar.