

# Simulation laboratory 3: Statistical analysis and bootstrapping

Rico Krueger

Transport and Mobility Laboratory  
School of Architecture, Civil and Environmental Engineering  
École Polytechnique Fédérale de Lausanne

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# Overview

## Objective:

- Analyse the results of a discrete events simulation.
- Evaluate the maximum queue length of a single road.

## Implementation:

- ① Recursive calculation of sample mean and variance
- ② Stopping criteria
- ③ Bootstrap mean square error of simulation metrics

1 Statistical analysis

2 Bootstrapping

3 My results

# Statistical analysis

## Jupyter notebook:

- 1 Implement your solution in the notebook **statistical\_analysis\_and\_bootstrapping.ipynb**.
- 2 Import your solution from the previous lab.

## TO DO:

- 1 Implement the function **moving\_mean\_var** for the recursive calculation of sample mean and variance.
- 2 Define a stopping criterion. Empirical consideration: choose a precision resulting in at least 100 simulation runs.
- 3 Statistically analyse the maximum queue length single road queue simulation implemented in the previous lab.
- 4 Plot sample mean and variance over the simulation runs.

1 Statistical analysis

2 Bootstrapping

3 My results

# Bootstrap mean square error

Calculate bootstrap MSE of parameter  $\theta$ , e.g.:

- **Mean** of the maximum queue length
- **95 percentile** of the maximum queue length
- **Worst case** of the maximum queue length

# Bootstrap mean square error

## TO DO:

- Implement the function **bootstrap** to calculate the bootstrap MSE of parameters of the maximum queue length.

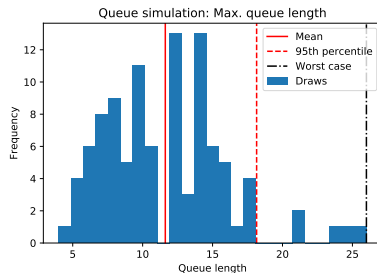
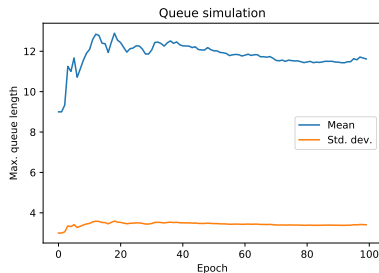
1 Statistical analysis

2 Bootstrapping

3 My results



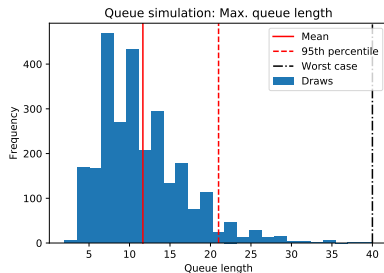
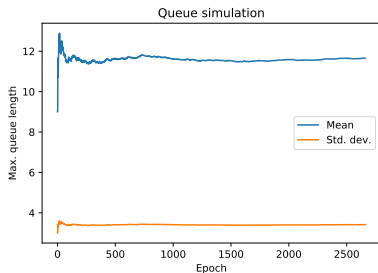
# Statistical indexes



Statistics of maximum queue length (stops when  $\sigma/\sqrt{n} < 0.5$ ):

- Mean = 11.6 (MSE = 0.193, BootstrapMSE = 0.181)
- 95 percentile = 18.1 (BootstrapMSE = 5.96)
- Worst = 26.0 (BootstrapMSE = 1.85)

# Statistical indexes



Statistics of maximum queue length (stops when  $\sigma/\sqrt{n} < 0.1$ ):

- Mean = 11.7 (MSE = 0.0100, BootstrapMSE = 0.00966)
- 95 percentile = 21.0 (BootstrapMSE = 0.463)
- Worst = 40.0 (BootstrapMSE = 4.31)