Error Analysis - 1

15/12/21

⊛ Characteristics of instrumentation and measurement system

1. Static error
2. Reproducibility and drift.
3. Repeatability
4. Range and Span
5. Accuracy and precision
6. Sensitivity
7. Non-linearity
8. Hysteresis
9. Dead Zone
10. Resolution and Threshold.

⊛ 1. True Value : Average of measured values.

2. Error : Difference b/w measured value and true value.

$$A_M - A_T$$

3. % Error : $\dfrac{\text{Error}}{\text{True value}} \times 100\,\%$

Q. A (0-150) V voltmeter has deviation of 1% of full scale value. If the voltage to be measured by the instrument is 75 V, calculate % error and range of measured value.

Ans. $SA = 1\%$ of 150

$= 1.5\ V$

True value $= 75\ V$

$$\%\ \text{error} = \frac{SA}{T.V} \times 100\% = \frac{1.5}{75} \times 100\% = 2\%$$

Range of measured value $= (75 \pm 1.5)\ V$

Q. A wattmeter of range 1000 W has an error of $\pm 1\%$ of f-s deflection. If the true power is 100 W, what would be the range of readings? Suppose error is specified as 1% of true values, what would be the range of the readings?

Ans. (i) $SA = 1\%$ of $1000 = 10\ W$

Range $= T.P \pm 10 = (100 \pm 10)\ W$

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(ii) $\delta A = 1\%$ of 100

$= 1\,W$

Range $= (100 \pm 1)\,W$

Q. The guarantee accuracy of a flow meter working on thermal principles is $\pm 3\%$ of f-s reading of $2.5 \times 10^{-6}\ m^3/s$. If the true value is $1.25 \times 10^{-6}\ m^3/s$. Calculate % error.

Ans. $\delta A = 3\%$ of 2.5×10^{-6}

$= \dfrac{7.5 \times 10^{-6}}{100}$

$\%\ \text{error} = \dfrac{0.075 \times 10^{-6}}{1.25 \times 10^{-6}} \times 100\% = 6\%$

Q. $R_1 = 10\Omega \pm 10\%$, $R_2 = 20\Omega \pm 5\%$, $R_3 = 100\,\Omega \pm 1\%$
Equivalent resistance in series ?

Ans. $\dfrac{\delta R_{eq}}{R_{eq}} = \dfrac{\delta R_1 + \delta R_2 + \delta R_3}{10 + 20 + 100} = \dfrac{1+1+1}{130} = 2.3\%$

$\therefore$ $R_{eq} = 130\Omega \pm 2.3\%$

Q. The resistance of a circuit is found by measuring current flowing and the power fed into the circuit. Find the limiting error in the measurement of resistances, when limiting errors in power and current are $\pm 1.5\%$ and $\pm 1\%$ respectively?

Ans. $\frac{\delta P}{P} = \pm 1.5\%$ $\qquad \frac{\delta I}{I} = \pm 1.0\%$

$P = I^2 R$

$\log P = \log (I^2 R)$

$\frac{dP}{P} = 2\frac{dI}{I} + \frac{dR}{R}$

$\Rightarrow \frac{dR}{R} = \frac{dP}{P} - 2\frac{dI}{I} = \pm 1.5 - 2(\pm 1\%)$

$= \pm 3.5\%$

Q. $\frac{\delta m}{m} = 2\%$ $\qquad \frac{\delta v}{v} = 3\%$ $\qquad E = \frac{1}{2}mv^2$

$\therefore \frac{\delta KE}{KE} = \frac{\delta m}{m} + 2\frac{\delta v}{v}$

$= 2 + (2 \times 3) = 8\%$

Error Analysis #2

16/12/21

Q. The current was measured during test as 30.4 A flowing in a resistance of $0.105\,\Omega$. Later it is discovered, that ammeter reading was low by 1.2 % and resistance reading was high by 0.3 %. The true power will be ______ ??

Ans. $I_m = 30.4\ A$ $\partial I / I = -1.2\,\%$

$R_m = 0.105\ \Omega$ $\partial R / R = +0.3\,\%$

$$\text{Power} = I^2 R$$

$$\frac{\partial P}{P} = 2\frac{\partial I}{I} + \frac{\partial R}{R} = 2(-1.2\,\%) + 0.3$$

$$= -2.1\,\% = -0.021$$

$$P_m = I_m^2 R_m = (30.4)^2 \times 0.105 = 97.03\ W$$

$$\frac{P_m - P}{P} = -0.021$$

$$\Rightarrow P_m = P(1 - 0.021)$$

$$\Rightarrow P = \frac{P_m}{1 - 0.021} = \frac{97.03}{0.979} = 99.11\ W$$

Q. The resistance of a wire is given by $R = \frac{4\rho l}{\pi d^2}$

ρ = resistivity (Ωm), l = length (m), d = diameter (m)

The error in measurement of each parameter ρ, l and d is $\pm 1\%$. Assuming that the errors are independent random variables, the % error in R is ___?

Ans. $\frac{\partial \rho}{\rho} = \frac{\partial l}{l} = \frac{\partial d}{d} = \pm 1\%$

$$\frac{\partial R}{R} = \frac{\partial P}{P} + \frac{\partial L}{L} - \frac{2\partial d}{d}$$

$$= \pm(1 + 1 - (\pm 1))$$

$$= \pm 4\%$$

Q. Resistances R_1 and R_2 have respectively nominal values of $10\,\Omega$ and $5\,\Omega$ and tolerances of $\pm 5\%$ and $\pm 10\%$. The range of values for parallel connection of R_1 and R_2 is ________ ?

Ans. $R_1 = 10\Omega \pm 0.5\% = 10 \pm \frac{5}{100} \times 10 = 10 \pm 0.5\ (9.5 - 10.5)$

$R_2 = 5\Omega \pm 10\% = 5 \pm \frac{10}{100} \times 5 = 5 \pm 0.5\ (4.5 - 5.5)$

Range :

$$= \left(\frac{9.5 \times 4.5}{9.5 + 4.5} \text{ ---- } \frac{10.5 \times 5.5}{10.5 + 5.5} \right)$$

$$= (3.053 \text{ ---- } 3.609)\ \Omega$$

Q. Given : $w = xy/z$. The variables x, y, z are measured with accuracy of 0.5%, 1% f/s and 1.5% of reading. The actual reading is 80, 20 and 50, with 100 being the full scale reading for all three. The maximum limiting error in measurement of w is ?

Ans. ~~Fs~~ F-s reading = 100

$x = 80$ $\quad\quad \partial x = 0.5 \text{ of } 80 = 0.4$

$y = 20$ $\quad\quad \partial y = 1 \text{ of } \cancel{20}\ f/s$

$\quad\quad\quad\quad = 1 \text{ of } 100 = 1$

$z = 50$ $\quad\quad \partial z = 1.5 \text{ of } 50 = 0.75$

$$\therefore \frac{\partial w}{w} = \frac{\partial x}{x} + \frac{\partial y}{y} + \frac{\partial z}{z}$$

$$= \pm\left(\frac{0.4}{80} + \frac{1}{20} + \frac{0.75}{50}\right)$$

$$= \pm\left(\frac{1}{200} + \frac{1}{20} + \frac{15}{1000}\right)$$

$$= \pm\ 0.07 \quad = \pm\ 7\% \text{ of reading.}$$

Q. When Wheatstone bridge shown in fig. is used to find R_x, the galvanometer G indicates 0 current. When $R_1 = 50\Omega$, $R_2 = 65\Omega$, $R_3 = 100\Omega$. If R_3 is known to be $\pm 5\%$ tolerance of its nominal value of 100Ω, the range of R_x will be :

Ans. $R_x = \frac{65}{50} \times 100 = 130\Omega$.

Now, $\partial R_x = 5\%$ of 130Ω

$= 6.5\Omega$

$\therefore$ Range of $R_x = 123.5\Omega$ to 136.5Ω

Q. A quantity x is calculated by using formula $x = (p-q)/r$. The measured values are $p = 9$, $q = 6$, $r = 0.5$. The absolute maximum error in the measurement of each of the quantities is ε. The absolute max. error in calculated value of x will be :

a) ε b) 3ε

c) 2ε d) 16ε

Ans. $\partial p = \partial q = \partial r = \varepsilon$

$$x = \frac{p-q}{r} = \frac{y}{r}$$

if $y = p - q$

$$\Rightarrow \frac{\partial y}{y} = \frac{\partial p}{y} - \frac{\partial q}{y}$$

$$= \frac{\partial p}{p-q} + \frac{\partial q}{p-q}$$

$$= \frac{\varepsilon}{3} + \frac{\varepsilon}{3}$$

$$\Rightarrow \frac{\partial y}{y} = \frac{2\varepsilon}{3}$$

$$\Rightarrow \frac{\partial x}{x} = \frac{\partial y}{y} + \frac{\partial r}{r}$$

$$\Rightarrow \frac{\partial x}{x} = \frac{2\varepsilon}{3} + \frac{\varepsilon}{0.5}$$

$$\Rightarrow \frac{\partial x}{x} = \frac{8\varepsilon}{3}$$

$$\Rightarrow \partial x = \frac{8\varepsilon}{3} \times (6)$$

$$= 16\varepsilon$$

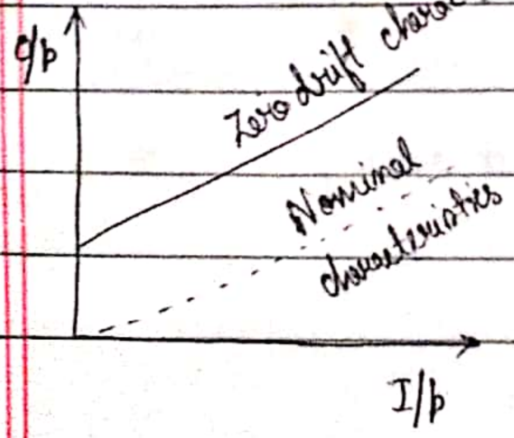
Error Analysis #3

17/12/21

Repeatability	Reproducibility
Variation that occurs when repeated measurements are made of the same item absolutely in identical conditions, i.e. same operator, same setup, same environmental condn.	Variation that results when different conditions are used to make measurements, i.e. different operators, different setup and different environmental conditions.

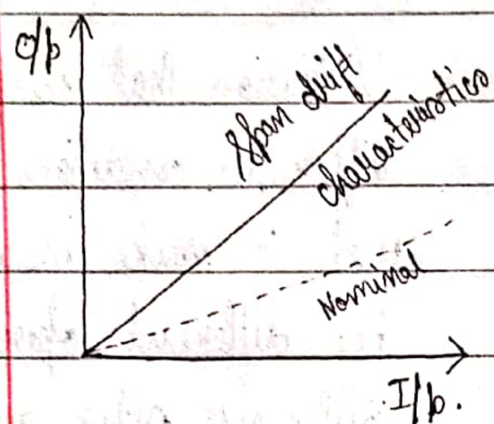
* **Drift** → Zero drift
  → Span drift / sensitivity drift
  → Zonal drift.

1. Zero drift



If the whole calibration gradually shifts due to slippage, permanent set or due to undue warming of electronic components.

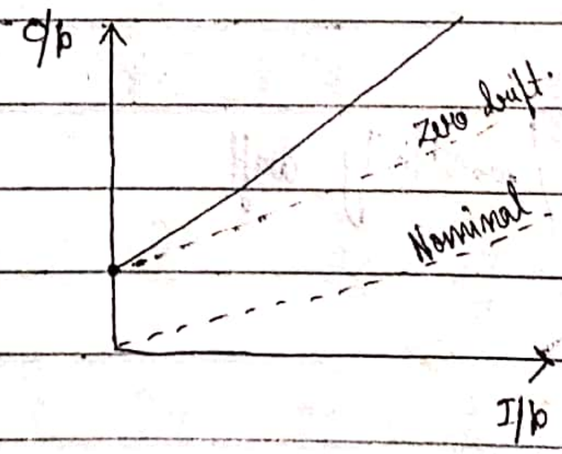
2. Span drift



If there is proportional change in the indication all along the upward scale.

3. Zonal drift

Zero drift + Span drift



Drift causes due to electric / magnetic fields thermal emf, change in temperature, mechanical vibrations, wear & tear, etc.

(*) Scale Range and Span

Range = X_{min} to X_{max}

Span = $X_{max} - X_{min}$

= diff. b/w maximum and minimum calibration.

(*) Accuracy and Precision

Accuracy reflects how close a measurement is, to a known or accepted value.

While precision reflects how reproducible the measurements are, even if they are far from the accepted value.

Measurement with both precision and accuracy are repeatable and very close to true values.

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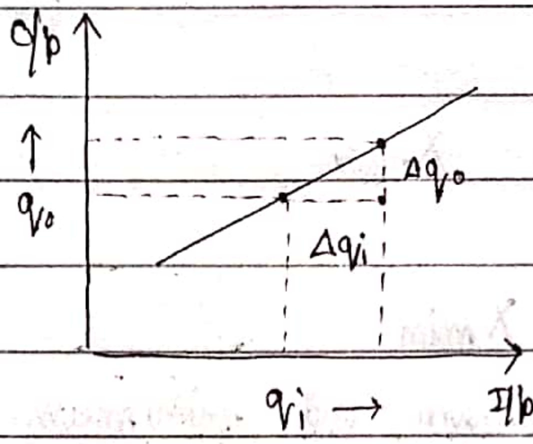
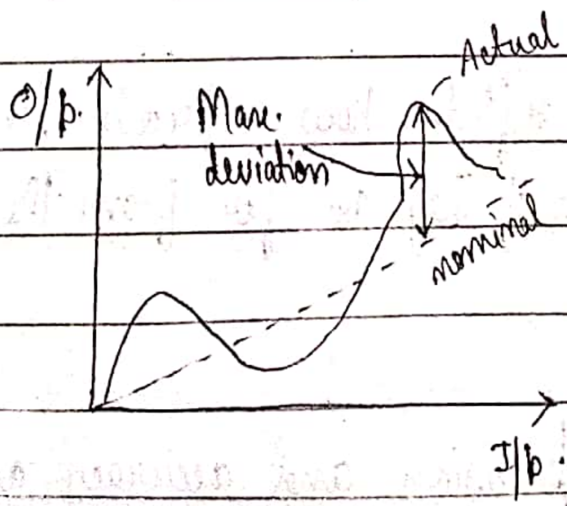
⊛ Sensitivity



$$\text{Sensitivity} = \frac{\text{change in o/p}}{\text{change in i/p}} = \frac{\Delta q_o}{\Delta q_i}$$

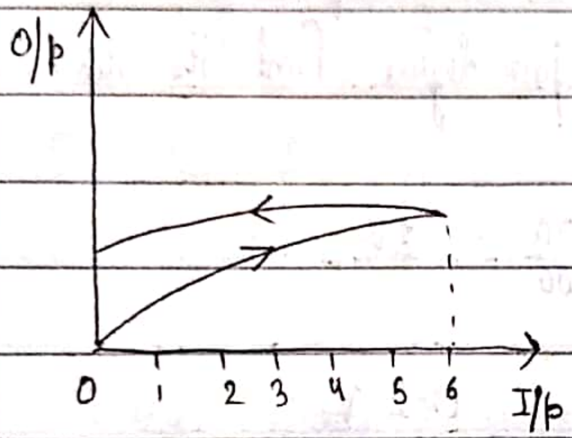
$$\text{Deflection factor} = \frac{1}{\text{sensitivity}}$$

⊛ Non-linearity



$$\text{Non-linearity} = \frac{\text{Max. deviation from nominal line}}{\text{full scale deflection}}$$

⊛ Hysteresis



It is the phenomenon, which gives diff. o/p effect when loading and unloading the system.

<u>Causes</u> : mechanical friction, elastic deformation, magnetic or thermal effects, rise and fall of temp.

⊛ Threshold and Resolution

Threshold : If the instrument i/p is increased very gradually from zero, then there will be some min. value below which no o/p change can be detected.
1st detectable o/p change → threshold

Resolution : It is the ability of the system to detect and indicate small changes in the characteristics of the measurement result.

Error Analysis #4

18/12/21

Q. A moving coil voltmeter (0-200 V) has uniform scale with 100 divisions and (1/10) th of a scale division can be estimated with fair degree. Find its resolution.

Ans. 1 scale division $= \frac{200}{100} = 2V$

Resolution $= \frac{1}{10} \times 2 = 0.2\ V$

Q. An instrument measures a range (0-250) N with a resolution of 0.1 % of f-s reading. Then determine the smallest change which can be measured ?

Ans. Resolution = smallest measurable change

$= 0.1\ \%$ of f-s reading

$= \frac{0.1}{100} \times 250 = 0.25\ N.$

Q. An ammeter has range of (0-30) A. The instrument has the following readings.

Current (A)	0	5	10	15	20	25
Ammeter readings	1	4	12	14	22	28

The non-linearity of instrument is 10 %.

Ans. $$\text{Non-linearity} = \frac{\text{Max. deviation}}{\text{Full scale value}} = \frac{28-25}{30} = \frac{3}{30} = 0.1$$

% Non-linearity = 10%.

Scanned with CamScanner

⊛ Types of Error

1. Gross error
2. Systematic error → instrumental error, environmental error, observational error.
3. Random error

⇒ **Gross Error** : Error occurs due to human mistakes in recording, reading and calculating results of measurements.

⇒ **Systematic Error** :

a) **Instrumental error** : Error occurs due to inherent shortcoming in instruments or misuse of instruments or loading effects.

Loading effects : A well caliberated voltmeter may give misleading voltage across high resistance and the same voltmeter when connected across low resistance gives dependable readings.

b) <u>Environmental error</u> : Errors occur due to conditions external to the measuring devices.
Maybe due to temperature, pressure, vibrations, ext. electric / magnetic field, dust, etc.

Arrangements should be done to keep conditions as constant as possible.

c) <u>Observational error</u> : Basically due to parallax error. When line of vision of observer is not exactly above the pointer.

⇒ <u>Random Error</u> : It is been consistently found that (Residual error) experimental results show variation from one reading to another even after all systematic errors are accounted for;

→ These errors are small, accidental and independent

→ The quantity being measured is affected by many happenings throughout the universe, that we are unaware. These happenings lumped together is residual error.

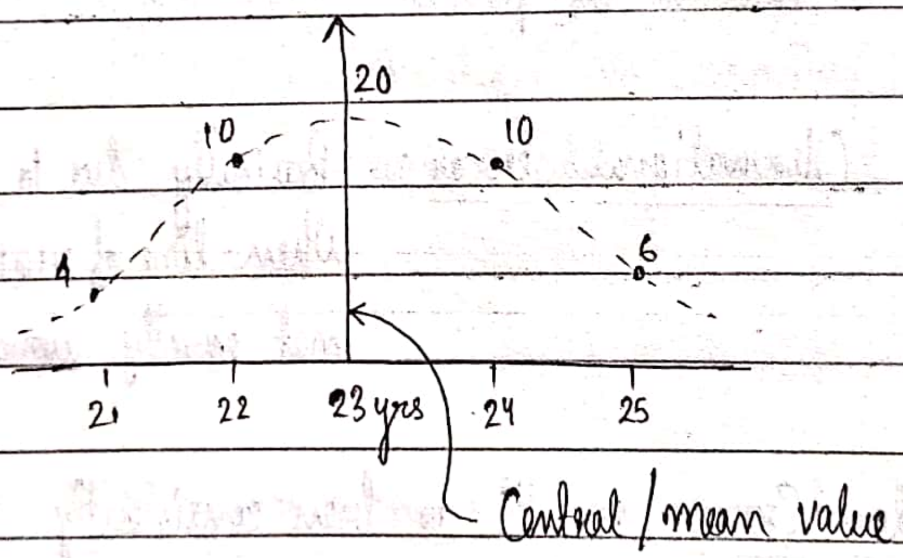
Error Analysis #5

19/12/21

Statistical Treatment of Data

No. of observed readings (n) = is known as frequency of occurrence

Frequency distribution curve



⇒ Arithmetic mean $\bar{x} = \dfrac{x_1 + x_2 + \dots + x_n}{n}$

⇒ Dispersion (Scatter, Spread) = The property which denotes the extent to which the values are dispersed about the central value

⇒ Deviation = It indicates departure of observed reading from arithmetic mean.

$d_1 = x_1 - \bar{x}$

$d_2 = x_2 - \bar{x}$ Algebric sum of all deviations $= 0$

$d_n = x_n - \bar{x}$

⇒ Average deviation $\bar{D} = \dfrac{|d_1| + |d_2| + \dots + |d_n|}{n}$

$= \sum_{n=1}^{n} \dfrac{|d_n|}{n}$

⇒ Standard deviation (Root Mean Square deviation)

$$SD = \sqrt{\frac{d_1^2 + d_2^2 + \dots + d_n^2}{n-1}} \quad ; \quad n < 20$$

$$SD = \sigma = \sqrt{\frac{d_1^2 + d_2^2 + \dots + d_n^2}{n}} \quad ; \quad n \geq 20$$

⇒ Variance $= (SD)^2 = \sum_{n=1}^{n} \dfrac{d_n^2}{n-1}$ or $\sum_{n=1}^{n} \dfrac{d_n^2}{n}$

(for $n < 20$) (for $n \geq 20$)

Q. Eight diff. students tuned in the circuit for resonance and value of resonance ~~[illegible]~~ freq. in kHz are:

412, 428, ~~408~~ 423, 415, 426, 411, 423, 416

Calculate: (i) Arithmetic mean

(ii) Average deviation

(iii) Standard deviation

(iv) Variance

Ans. (i) Mean $= \dfrac{412 + 428 + 423 + 415 + 426 + 411 + 423 + 416}{8}$

$= 419.25$ kHz.

(ii) Average deviation $= \dfrac{(7.25 + 8.75 + 3.75 + 4.25 + 6.75 + 8.25 + 3.75 + 3.25)}{8}$

$= 5.75$ kHz

(iii) Standard deviation $= 6.54$ kHz

(iv) Variance $= 42.77$

Q. The measurement of a source voltage are 5.9 V, 5.7 V and 6.1 V. The sample standard deviation is ?

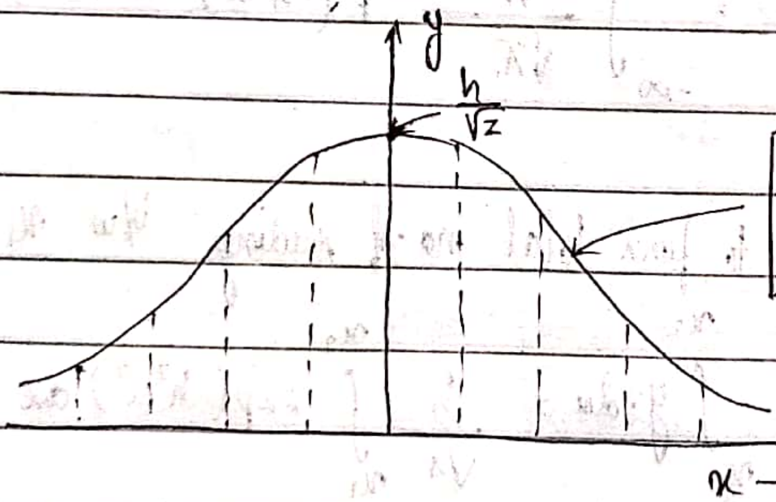
Ans. $d_1 = 0$

$d_2 = 0.2$

$d_3 = -0.2$

$$SD = \sqrt{\frac{0.04 + 0.04}{2}} = \sqrt{0.04} = 0.2V$$

⊛ Normal or Gaussian Curve of Error (Normal probability curve).



$$\boxed{y = \frac{h}{\sqrt{\pi}} \exp(-h^2x^2)}$$

h = precision index

π = pi

The law of probability states that the normal occurrence of deviation from the avg. value from an infinite no. of measurements is expressed as :

$$y = \frac{h}{\sqrt{\pi}} \exp(-h^2x^2)$$

x = mag. of dev. from avg.

y = no of readings at deviation x.

or $h = \frac{1}{\sigma\sqrt{2}}$

$\therefore \quad y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$

1. The curve is symmetrical about arithmetic mean val.
Area under curve is unity.

$$\int_{-\infty}^{\infty} y\, dx = \int_{-\infty}^{\infty} \frac{h}{\sqrt{\pi}} \exp(-h^2 x^2)\, dx = 1$$

2. If we need to find total no. of readings b/w x_1 and x_2

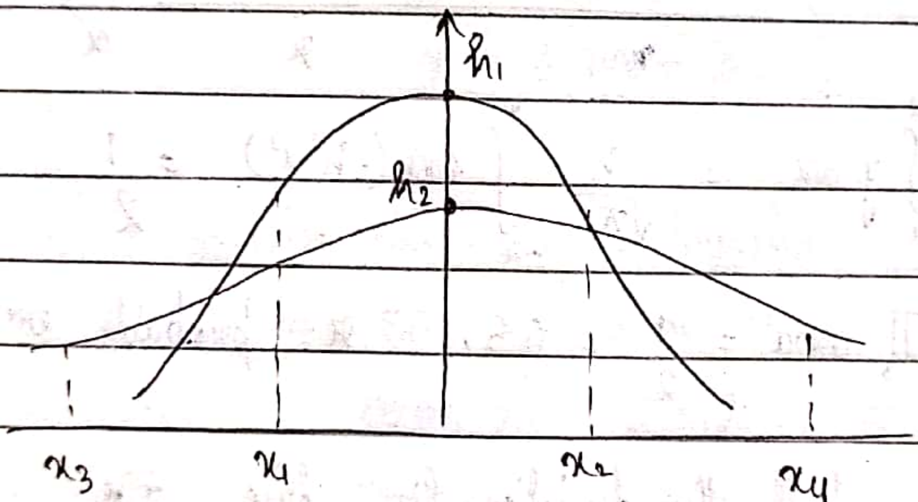
$$n_{1-2} = \int_{x_1}^{x_2} y \cdot dx = \frac{h}{\sqrt{\pi}} \int_{x_1}^{x_2} \exp(-h^2 x^2) \cdot dx$$

Probability of finding deviation b/w interval x_1 and x_2

3. Precision index :

$$\text{At } x = 0, \quad y = \frac{h}{\sqrt{\pi}}$$

For a given x, greater the dispersion, higher is the precision.



Curve 1 : High precision (and greater h)
Curve 2 : Lower precision (and Lesser h)

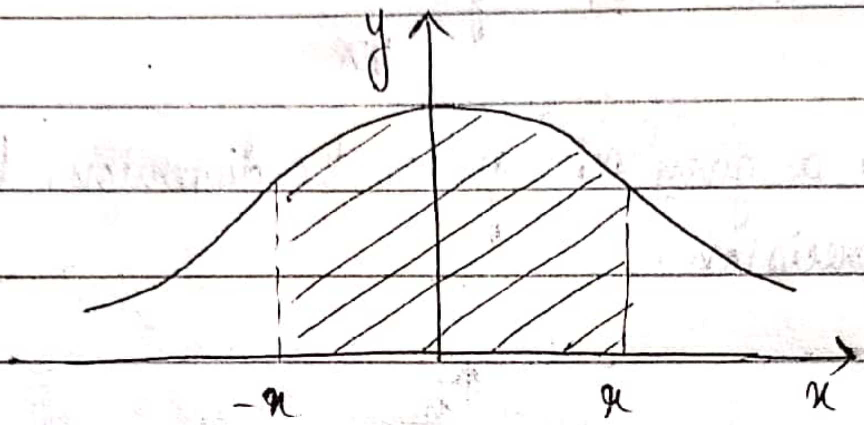
$P(x_1, x_2) \downarrow\downarrow \Rightarrow$ Precision $\uparrow\uparrow$

$\Rightarrow$ Dispersion $\downarrow\downarrow$

Error Analysis - #6

21/12/21

Probable error (r)



$$\int_{-r}^{r} y\,dx = \frac{h}{\sqrt{\pi}} \int_{-r}^{r} \exp(-h^2x^2) = \frac{1}{2}$$

If area $= \frac{1}{2} = 0.5$, $r =$ probable error.

i.e, half the deviation lies b/w $\pm r$.

$$\boxed{r = \frac{0.4769}{h}}$$

Average deviation of normal curve

$$\bar{D} = \int_{-\infty}^{\infty} |x|\, y\,dx = \frac{2h}{\sqrt{\pi}} \int_{0}^{\infty} \exp(-h^2x^2)\, x \cdot dx = \frac{1}{h\sqrt{\pi}}$$

$$\boxed{\bar{D} = \frac{r}{0.8453}}$$

(*) Standard deviation of normal curve

$$SD = \sigma = \sqrt{\int_{-\infty}^{\infty} x^2 y \cdot dx} = \frac{1}{\sqrt{2}h} = \frac{r}{0.6745}$$

$\therefore$ Probable error : $\left. \begin{array}{l} r = 0.6745\,\sigma \\ r = 0.8453\,\bar{D} \end{array} \right\}$

Q. We have a parallel ckt. with 2 branches. $I_1 = 100 \pm 2A$ and $I_2 = 200 \pm 5A$. Determine total current $I = I_1 + I_2$

a) Considering that errors in I_1 and I_2 are limiting

b) Considering that errors are SD.

Ans. a) $I = I_1 + I_2 = (300 \pm 7)A$

b) $I_1 = 100 \pm \underbrace{2A}_{\sigma I_1}$ $\qquad$ $I_2 = 200 + \underbrace{5A}_{\sigma I_2}$.

$$\sigma_I = \sqrt{\left(\frac{\delta I}{\partial I_1}\right)^2 \cdot \sigma_{I_1}^2 + \left(\frac{\delta I}{\delta I_2}\right)^2 \sigma_{I_2}^2}$$

$$= \sqrt{1^2 \cdot 2^2 + 1^2 \cdot 5^2} = \sqrt{29} = 5.38A.$$

$$I = (300 \pm 5.38)\,A.$$

Q. A resistance is determined by voltmeter - ammeter. The voltmeter reads 100 V with probable error ±12 V and ammeter reads 10 A with probable error ±2 A. Determine probable error in resistance.

Ans.

$$V = 100\,V \pm 12 \rightarrow r_v$$

$$A = 10A \pm 2 \rightarrow r_i$$

$$r_R = \sqrt{\left(\frac{\partial R}{\partial V}\right)^2 r_v^2 + \left(\frac{\partial R}{\partial I}\right)^2 r_i^2}$$

$$\frac{\partial R}{\partial V} = \frac{\partial}{\partial V}\left(\frac{V}{I}\right) = \frac{1}{I} = \frac{1}{10} = 0.1$$

$$\frac{\partial R}{\partial I} = \frac{\partial}{\partial I}\left(\frac{V}{I}\right) = V\left(\frac{-1}{I^2}\right) = \frac{-100}{10^2} = -1$$

$$\therefore r_R = \sqrt{(0.1)^2 \times 12^2 + (-1)^2 \times 2^2}$$

$$= \sqrt{1.44 + 4} = \sqrt{5.44}$$

$$= 2.33$$

$$\therefore R = (10 \pm 2.33)\,\Omega$$

Q. The law of deflection of galvanometer is $I = \frac{k\theta}{\cos\theta}$

I = current, k = constant, θ = deflection

If the angle of deflection θ is known to be within $\pm 0.1^\circ$ (standard deviation), of 15°, What is the % std. dev. of current I.

Ans. $\sigma_I = \sqrt{\left(\frac{\partial I}{\partial \theta}\right)^2 \sigma_\theta^2} = \frac{\partial I}{\partial \theta} \cdot \sigma_\theta$

$$\frac{\partial I}{\partial \theta} = k\left[\frac{1}{\cos\theta} + \frac{\theta \cdot \sin\theta}{\cos^2\theta}\right]$$

$$= k\left[\frac{1}{\cos 15^\circ} + \frac{15\pi}{180} \times \frac{\sin 15^\circ}{\cos^2 15}\right]$$

$$= 1.108\, k$$

$$\sigma_I = 1.108k \times \left(0.1 \times \frac{\pi}{180}\right) \text{ radian}$$

$$\therefore \text{ % std. dev} = \frac{\sigma I}{I} \times 100\,\% = \frac{1.108k\left(\frac{0.1\pi}{180}\right)}{k\left(\frac{15\pi}{180}\right)(\cos 15)^{-1}} \times 100\%$$

$$= 0.765\ \%$$

Q. A certain resistance has a voltage drop of 110.2 V and a current of 5.3 A. The uncertainty in the measurement are ±0.2 V and ±0.06 A. Calculate power dissipated in resistor and uncertainty in power.

Ans. $P = VI$

$V = 110.2 \pm 0.2$ — W_v

$I = 5.3 \pm 0.06$ — W_i

$\frac{\partial P}{\partial V} = I = 5.3$ $\quad$ $\frac{\partial P}{\partial I} = V = 110.2$

Uncertainty in power $= W_p = \sqrt{\left(\frac{\partial P}{\partial V}\right)^2 W_v^2 + \left(\frac{\partial P}{\partial I}\right)^2 W_i^2}$

$\Rightarrow W_p = \sqrt{5.3^2 \times 0.2^2 + 110.2^2 \times 0.06^2}$

$\Rightarrow W_p = 6.696$ W

Power dissipated $= (584 \pm 6.696)$ W

Q. Two resistors R_1 and R_2 are connected in series and then in parallel. The value of resistance are

$R_1 = 100 \pm 0.1\ \Omega$, $R_2 = 50 \pm 0.03\ \Omega$

Calculate uncertainty in combined resistance for both series and parallel arrangements.

(i) Series $= R = R_1 + R_2$

$$\frac{\partial R}{\partial R_1} = 1, \qquad \frac{\partial R}{\partial R_2} = 1$$

$$\omega_R = \sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 \cdot \omega_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 \omega_{R_2}^2} = \sqrt{0.1^2 + (0.03)^2}$$

$$\omega_R = \pm 0.1044\ \Omega.$$

(ii) Parallel $= R = \dfrac{R_1 R_2}{R_1 + R_2}$

$$\frac{\partial R}{\partial R_1} = \frac{(R_1+R_2) R_2 - R_1 R_2 (1+0)}{(R_1+R_2)^2}$$

$$= 0.111$$

$$\frac{\partial R}{\partial R_2} = \frac{(R_1+R_2) R_1 - R_1 R_2 (0+1)}{(R_1+R_2)^2} = 0.444$$

$$\therefore \omega_R = \sqrt{(0.111)^2 (0.1)^2 + (0.444)^2 (0.03)^2} = \pm 0.01734\ \Omega$$

Error Analysis #7

22/12/21

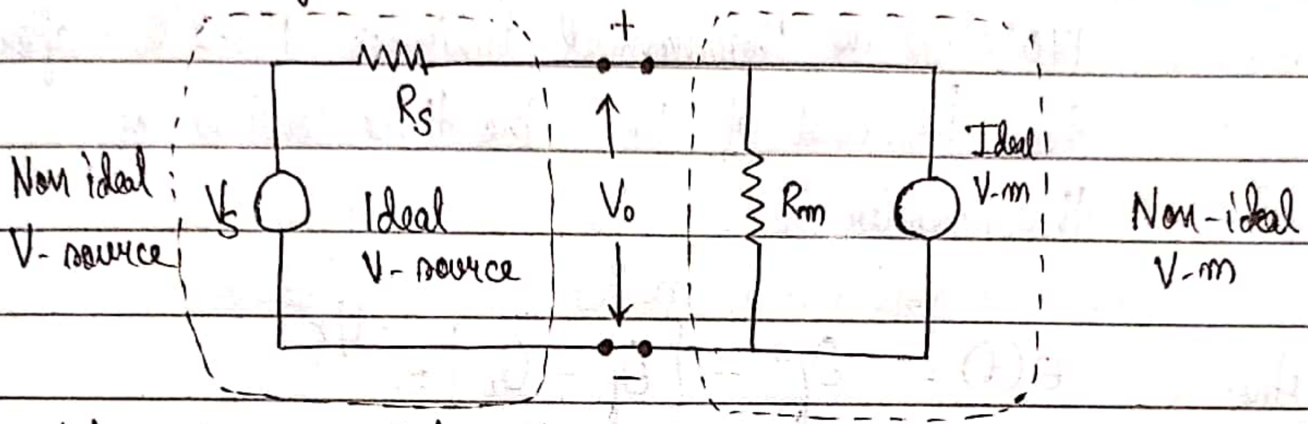
Q. A large no. of 230 Ω resistor are obtained by combining 120 Ω resistors with std. deviation 4 Ω and 110 Ω resistors with std. deviation of 3 Ω. The std. deviation of 230 Ω resistor will be ?

Ans.
$$R_1 = 120 \pm 4\,\Omega$$
$$R_2 = 110 \pm 3\,\Omega$$
$$R = R_1 + R_2$$
$$\frac{\partial R}{\partial R_1} = 1 \quad , \quad \frac{\partial R}{\partial R_2} = 1$$

$$\sigma_R = \sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 \sigma_1^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 \sigma_R^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= 5\,\Omega$$

$$\therefore \; R = (230 \pm 5)\,\Omega$$

Q. Consider a non ideal voltage source whose output voltage is measured by a non-ideal voltmeter as shown below :



Let V_e be the diff. b/w V_s and the measured voltage. V_e/V_s is a function of ____ ?

Ans. Using Voltage divide rule, $V_o = \left(\frac{R_m}{R_m + R_s}\right) V_s$

$$\therefore \quad V_e = V_s - V_o$$

$$= \left(1 - \frac{R_m}{R_m + R_s}\right) V_s$$

$$= \left(\frac{R_s + \cancel{R_m} - \cancel{R_m}}{R_m + R_s}\right) V_s = \left(\frac{R_s}{R_m + R_s}\right) V_s.$$

$$\therefore \quad V_e/V_s = \frac{R_s}{R_m + R_s} = \frac{1}{\left(\frac{R_m}{R_s}\right) + 1}$$

$\therefore$ V_e/V_s is a function of $\left(\frac{R_m}{R_s}\right)$.

Q. The thermometer is initially at a temperature of 70 °C and is suddenly placed in a liquid which is 170°C. The thermometer indicates 133.2°C after time interval of 3s. The time constant of thermometer is ____ s.

Ans. $\theta(t) = \theta_f - [\theta_f - \theta_i] e^{-t/\tau}$

$\Rightarrow 133.2 = 170 - 100 e^{-t/\tau}$

$\Rightarrow 100 e^{-t/\tau} = 36.8$

$\Rightarrow e^{-t/\tau} = 0.368$

$\Rightarrow \dfrac{-t}{\tau} = \ln(0.368)$

$\Rightarrow \dfrac{-t}{\tau} = -1$

$\Rightarrow \tau = t = \underline{3s}$ (Ans).

Q. A plot of land has measured dimensions of 50 x 150 m. The uncertainty in 50 m dimension is ± 0.01 m. Calculate uncertainty with which the 150 m dimension must be measured to ensure that the total uncertainty in the area is not greater than 150 % of that value it would have if 150 m dimension were exact.

Ans. Area = L × B = 150 × 50 = 7500 m^2

B = 50 ± 0.01 m $\qquad w_B = 0.01\ m$.

L = 150 ± w_L m.

w_A = 150 % of (1.5)

= 2.25 m^2

$$\frac{\partial A}{\partial B} = L = 150 \qquad \frac{\partial A}{\partial L} = B = 50$$

$$w_{area} = \sqrt{\left(\frac{\partial A}{\partial L}\right)^2 w_L^2 + \left(\frac{\partial A}{\partial B}\right)^2 w_B^2} = \sqrt{B^2 w_L^2 + L^2 w_B^2}$$

Now, $w_L = 0$

$\Rightarrow w_{area} = L w_B = 150 \times 0.01 = 1.5\ m^2$

Now, uncertainty in length is to be found out.

$$w_A = \sqrt{B^2 w_L^2 + L^2 w_B^2}$$

$$\Rightarrow 2.25 = \sqrt{50^2 w_L^2 + 150^2 w_B^2}$$

$$\Rightarrow 5.0625 = 2500 w_L^2 + 22500 \times (0.01)^2$$

$$\Rightarrow 5.0625 = 2500 w_L^2 + 2.25$$

$$\Rightarrow 2500 w_L^2 = 2.8125$$

$$\Rightarrow w_L^2 = 0.001125$$

$$\Rightarrow w_L = 0.0335 \text{ m}$$

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Q. A resistor has a nominal value of $10\,\Omega \pm 1\%$. A voltage is applied across the resistor and power consumed in the resistor is calculated as:

(i) $P = \dfrac{E^2}{R}$ (ii) $P = EI$

Calculate uncertainty in power determination in each case, when $E = 100 \pm 1\%$, $I = 10\,A \pm 1\%$

Ans. (i) $\omega_R = 0.01 \times 10 = 0.1\,\Omega$ $P = \dfrac{100^2}{10} = 10^3\,W$

$\omega_E = 0.01 \times 100 = 1\,V$

$\omega_I = 0.01 \times 10 = 0.1\,A$

$$\omega_P = \sqrt{\left(\frac{\partial P}{\partial E}\right)^2 \omega_E^2 + \left(\frac{\partial P}{\partial R}\right)^2 \omega_R^2}$$

$$= \sqrt{\left(\frac{2E}{R}\right)^2 \omega_E^2 + \left(\frac{-E^2}{R^2}\right)^2 \omega_R^2}$$

$$= \sqrt{\frac{4E^2}{R^2}\omega_E^2 + \frac{E^4}{R^4}\omega_R^2}$$

$$= \sqrt{\frac{4 \times 100^2 \times 1^2}{10^2} + \frac{100^4 \times (0.1)^2}{10^4}}$$

$$= 22.36\,W$$

$\therefore$ $P = 1000 \pm 2.236\%\ W$

(ii) $\frac{\partial P}{\partial E} = I$

$\frac{\partial P}{\partial I} = E$

$$\therefore \frac{W_P}{P} = \sqrt{\frac{\left(\frac{\partial P}{\partial E}\right)^2 W_E^2 + \left(\frac{\partial P}{\partial I}\right)^2 W_I^2}{P^2}} \times 100\%$$

$$= \sqrt{\frac{I^2 W_E^2 + E^2 W_I^2}{E^2 I^2}} \times 100\%$$

$$= \sqrt{\left(\frac{W_E}{E}\right)^2 + \left(\frac{W_I}{I}\right)^2} \times 100\%$$

$$= \sqrt{1 + 1} = \sqrt{2} = 1.414\%$$

$$\therefore \frac{W_P}{P} = 1.414\%$$

Q. The voltage and current drawn by a resistive load are measured with a 300 V voltmeter of accuracy 1 % of full scale and a 5A ammeter of accuracy 0.5 % of full scale. The readings obtained are 200 V and 2.5 A. The limiting error (in %) in computing load resistance is ____ ?

Ans. $\delta V = 1\%$ of $FSV = 3\ V$

$\delta I = 0.5\%$ of $FSV = 0.025\ A$

$V = 200\ V,\qquad I = 2.5\ A.$

$$\frac{\partial R}{R} = \frac{\partial V}{V} + \frac{\partial I}{I}$$

$$= \frac{3}{200} + \frac{0.025}{2.5} = 0.025$$

$$\Rightarrow \frac{\partial R}{R}\ (\text{in } \%) = \underline{2.5\ \%}$$