BACHELOR OF POWER ENGINEERING , 2021

NUCLEAR POWER GENERATION

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Reference books / materials

1. Introduction to Nuclear Engineering : Jhon R Lamarsh & Anthony J Baratta

1. Nuclear Reactor Engineering Reactor Design Basics : Samuel Glasstone & Alexander Sesonske

1. Introduction to Nuclear Engineering Richard Montgomery Stephenson

Material from open sources

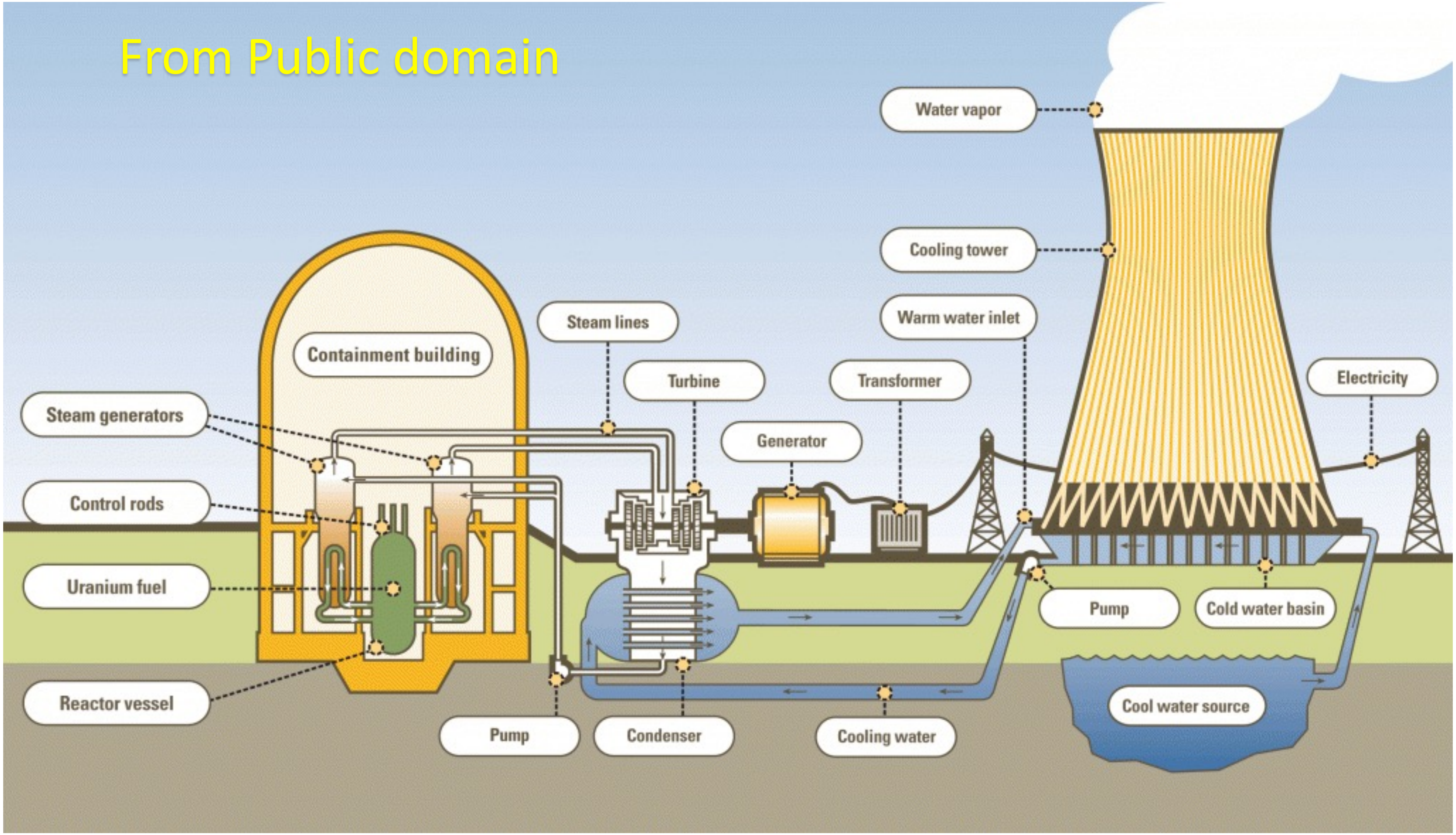
1. “Basic Physics of Nuclear Reactors”, Presentation by Prof Patrick Regan, University of Surrey

1. CANTECH knowledge repository

1. Lecture notes by Prof M Ragheb, University of Illinois, Urbana-Champaign.

What is Nuclear Power ?

- It is simply electricity made using a "nuclear process" *viz*. fission, and the material which undergoes fission is either Uranium and Plutonium.
- Underlying concerns, which may not be necessarily true.
  - Accidents
  - Waste disposal


From Public domain
Water vapor
Cooling tower
Warm water inlet
Steam lines
Containment building
Turbine
Transformer
Electricity
Steam generators
Generator
Control rods
Uranium fuel
Pump
Cold water basin
Reactor vessel
Cool water source
Pump
Condenser
Cooling water

Outline

What we plan to atleast read about

- Basic Nuclear Physics
- Introduction to the radiating world of “radiation”
- Neutron interactions
- Fission
- Reactors
- Allied topi

Nuclear World

Nucleus is the central, massive and dense object of an atom.

- Atoms ~ 10^{-10} m (°A) ; Nucleus ~ 10^{-15} m (Fm)
- Positively charged "protons" & electrically neutral "neutrons"
- Notation → ${}^{A}_{Z}X$

Mass and Energy

The back of the envelope principle of energetics of nuclear reaction is that "mass" and "energy" are the two sides of the same coin.

The units used for mass and energy are not our conventional units.

- Mass → Atomic Mass Unit (*amu*; *u*)
- Energy → Electron Volt (*eV*) {MeV, meV}

Mass is generally represented in Kg or grams.

We shall use a relative unit known as : Atomic Mass Unit (*amu*; *u*)

The mass of a neutral ^{12}C atom is taken precisely as 12 units

We know that 1 mole of a substance contains Avagrados number of atoms (6.023×10^{23}).
i.e one mole of a substance contains the same number of atoms / molecules.

$Hence$, 1 mole of ^{12}C contains 12 gm of the isotope, on the other hand
1 mole of natural O_2 contains $2 \times 15.99938 = 31.99876$ gm.

$\therefore$ 12 gm of ^{12}C has 6.023×10^{23} atoms.

$\therefore$ 1 atom of ^{12}C weighs $\frac{12 \times 10^{-3}}{6.023 \times 10^{23}}\ kg$

$$\therefore 1\ amu = \frac{12 \times 10^{-3}}{6.023 \times 10^{23}} \times \frac{1}{12} = 1.66053 \times 10^{-27}\ kg$$

The conventional units for energy are Joule or Erg.

kilo-**e**lectron-**V**olt, and is the unit of energy conventionally used in nuclear physics.

It corresponds to the energy gained by an electron when accelerated across a potential difference of 1 volt.

A charged particle "q", when accelerated across a potential difference of "V" volt, gains ebergy

$$energy = charge \times potential\ difference$$

$$1\ eV = (1.6 \times 10^{-19}) \times 1 \text{ Coulomb –Volt}$$
$$= 1.6 \times 10^{-19} \text{ Joule}$$

It is customary to use the following larger units

$$1\ keV = 10^3\ eV\ ;\ 1\ MeV = 10^6\ eV$$

Energy - Mass equivalence

"mass" and "energy" are the two sides of the same coin.

$$E = m_o \times c^2$$

Hence, mass can be converted into energy and vice versa.

Now, m_o , is referred to as "***rest mass***", *i.e* it represents the amount of energy we would have to spend to create a particle at rest, and the energy that would appear if a particle were to get annihilated.

Constituents of Nucleus

"protons (π)" and "neutrons (ν)" are the two constituents present inside the nucleus.

They are known as "nucleons"

They have nearly identical mass, but neutron is slightly heavier than the proton.

$$m_p = 1.00728\,u\,;\; m_n = 1.00867\,u\,;\; m_e = 5.5\times10^{-4}\,u$$

The neutron is not a stable particle *i.e* it is stable inside the nucleus, however a free neutron has a mean life of about 14 minutes and 42 seconds or a half life of about 10 minutes and 11 seconds.

Hence the rest mass of an electron $(m_e = 9.11 \times 10^{-31}\ kg\ )$is

$$E_o = \ m_0 \times c^2$$

$$= \ (9.11 \times 10^{-31}) \times (3 \times 10^8)^2$$

$$= 8.2 \times 10^{-14} \text{ Joule}$$

$$E_o \qquad = \ \frac{8.2 \times 10^{-14}}{1.6 \times 10^{-19}}$$

$$= 0.511 \text{ MeV}$$

From energy-mass equivalence, we have the energy associated with 1 u as

$$E_{1u} = 1.66053 \times 10^{-27} \times (3 \times 10^{8})^{2} \text{ Joule}$$

$$\therefore\ E_{1u} = 1.49447 \times 10^{-10} \text{ Joule}$$

$$\because\ 1\ eV = 1.6 \times 10^{-19} \text{ Joule} \Longrightarrow 1 \text{ Joule} = \frac{1}{1.6 \times 10^{-19}} \text{ eV}$$

$$\therefore\ E_{1u} = \frac{1.49447 \times 10^{-10}}{1.6 \times 10^{-19}} \approx 931 \times 10^{6} \text{MeV} = 931 \text{ MeV}$$

When a body is in motion, its mass increases relative to an observer at rest, according as

$$m = \frac{m_0}{\sqrt{1 - \ v^2/c^2}}$$

$$E_{KE} = total\ energy - rest\ mass\ energy\ = m\,c^2 - \ m_0\,c^2$$

$$E_{KE} = \frac{m_0}{\sqrt{1 - \ v^2/c^2}}\,c^2 - \ m_0\,c^2$$

$$= \ m_0\,c^2\ \left[\left[\frac{1}{\sqrt{1 - \ v^2/c^2}}\right] - 1\right]$$

It has been demonstrated that the conventional equation for Kinetic Energy is valid for most cases when

$$v \le 0.2\, c$$

$$v^2 \le 0.04\, c^2$$

$$\frac{1}{2}\,[v^2] \le \frac{1}{2} \times [0.04\, c^2]\frac{1}{2}\,[v^2] \le [0.02\, c^2]$$

$$\frac{1}{2}\, m\, [v^2] \le m_o\, [0.02\, c^2]$$

$$\frac{1}{2}\, m\, [v^2] \le m_o c^2 [0.02\,]$$

the rest mass of a neutron is almost 1.000 MeV, hence $0.02 \times 1\, MeV = 20\, MeV$.

In practice rarely do we come across neutrons having such large kinetic energies, hence for all practical purposes, we use the non-relativistic relation while dealing with neutrons

Molecular Weight

Elements are found to have isotopes (same proton number but different number of neutrons).

The atomic weight of the element is defined as the average atomic weight of the mixture.

Hence, if γ_i is the isotopic abundance of the i^{th} isotope of atomic weight M_i, then the atomic weight of the element is

$$M = \sum_i {(\gamma_i M_i)}/_{100}$$

The total mass of a molecule relative to the mass of a neutral ^{12}C atom is called the molecular weight.

For oxygen we have the following abundances

Isotope	Abundance (a/o)	Atomic weight
^{16}O	99.759	15.99492
^{17}O	0.037	16.99913
^{18}O	0.204	17.99916

Hence,

$$M\ (O) = \frac{1}{100}\,[(\gamma_{16} \times M_{16}) + (\gamma_{17} \times M_{17})] + (\gamma_{18} \times M_{18})$$

$$M\ (O) = \frac{1}{100}\,[(99.759 \times 15.999492) + (0.037 \times 16.99913)] + (0.204 \times 17.99916)$$

$$M\ (O) = 15.99938\ \text{(u)}$$

Gram atomic or gram molecular weights are defined as the amount of substance having a mass in grams equal to the atomic or molecular weight of the substance.

Thus one gram atomic weight or one mole of a ^{12}C is 12 *g* of this isotope.

1 mole of natural O_2 contains 2 ×15.99938 = 31.99876 gm.

10g of ^{12}C contain $\frac{10\ gm}{12\ gm} \times\ 6.022 \times 10^{23} = 5.018 \times 10^{23}$ atoms

18 gm of H_2O contain 6.022×10^{23} atoms.

50 gm of H_2O contain $\frac{50\ gm}{18\ gm} \times\ 6.022 \times 10^{23} = 1.672 \times 10^{24}$ atoms

50 gm of H_2O contain $\frac{50\ gm}{18\ gm} \times\ 1 = 2.77$ mole

Mass Defect & Binding Energy

It is observed that the mass of the nucleus is always less than the mass of its constituent nucleons. This difference in the mass is referred to as **Mass Defect** (Δm) and is customary expressed in *amu*

If Z number of protons and N number of neutrons combine to form a nucleus say X, then

$$Z\pi + N\upsilon \rightarrow X \; ; \qquad Z * m_p + N * m_n \rightarrow M_x \, ,$$

where : m_p, m_n, M_x : represent the mass of a proton, neutron and the atom in amu.

$$\therefore \; \Delta m = [mass\ of\ consituent\ nuceons] - [mass\ of\ the\ atom]$$

$$\therefore \Delta m = \left[\left(Z * m_p\right) + (N * m_n)\right] - [M_x]$$

Now, from the energy-mass equivalence, we shall have an energy equivalent of this mass defect. This is referred to as **Binding Energy**,

$$Binding\ Energy = \Delta m \times 931 \text{ MeV}$$

This represents the energy required to break / disassemble the nucleus into its independent constituent nucleons.

- He nucleus (2 protons and 2 neutrons), such that $M_{He} = 4.002603\ u$.

$$
\begin{aligned}
\delta m &= \left[(2 \times 1.00866) + (2 \times 1.00728)\right] - 4.002603 \\
&= 0.029277\ u \\
B.E &= 0.029277 \times 935 \text{ MeV} \\
&= 27.37 \text{ MeV}
\end{aligned}
$$

- Similarly for ^{12}C nucleus, with 6 protons and neutrons, we have

$$
\begin{aligned}
\delta m &= \left[(6 \times 1.00866) + (6 \times 1.00728)\right] - 12 \\
&= 0.09564\ u \\
B.E &= 0.09564 \times 935 \text{ MeV} \\
&= 89.42 \text{ MeV}
\end{aligned}
$$

- Similarly for ^{238}U nucleus, with 92 protons and 146 neutrons, such that $M_U = 238.050788\, u$, we have

$$
\begin{aligned}
\delta m &= \left[(146 \times 1.00866) + (92 \times 1.00728)\right] - 238.050788 \\
&= 1.882\ u \\
B.E &= 1.882 \times 935 \text{ MeV} \\
&= 1760.0 \text{ MeV}
\end{aligned}
$$

Another useful quantity is the **binding energy per nucleon**, this is the average energy needed to **"remove a single nucleon"** from the nucleus. It is defined as:

Binding Energy per nucleon = Binding Energy/ Nucleon Number

It is denoted by B.E/A, and this quantity represents the “stability” of the nucleus.

Higher the B.E/A, higher is its stability.

Elements with a high binding energy per nucleon are very difficult to break apart.

This is also referred to as Packing Fraction.

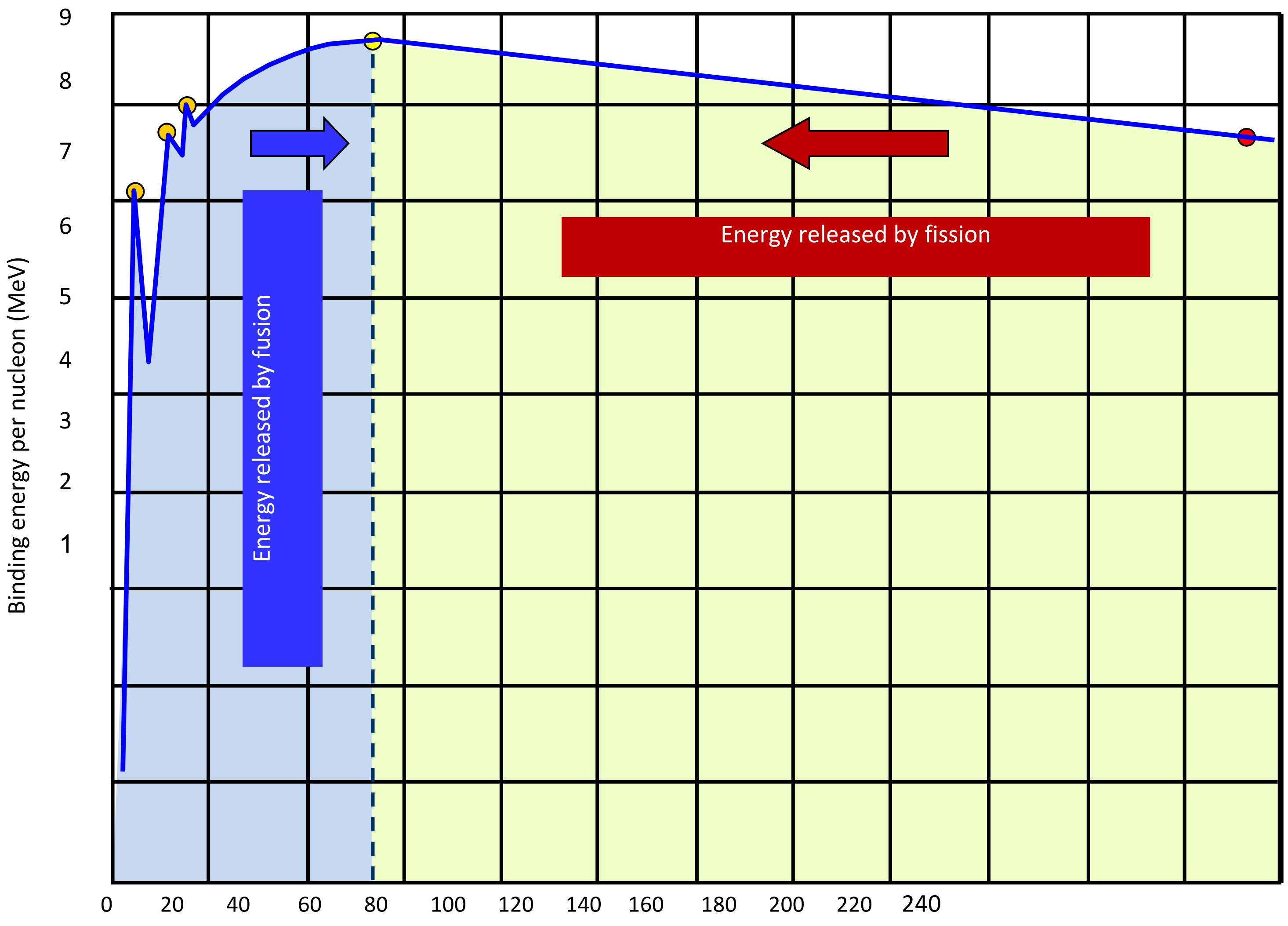


Figure 1

We observe that the value of $^{BE}/_{A}$ is fairly constant across the data on available nuclei. Of-fcourse the exception being very light nuclei. The value attains a maximum for ^{56}Fe nucleus.

Now, we observe that the difference in the ${}^{BE}/_{A}$ between heavy nuclei and the medium mass nuclei is about 1 ${}^{MeV}/_{A}$. Hence, if a heavy nucleus, were to undergo fission into two medium mass nuclei, this process would result in the liberation of 1 ${}^{MeV}/_{A}$ amount of energy. Now, say for example, we have a fission of a nucleus with $A = 240$ then, the fission process would liberate about

$$240(A) \times 1\,\frac{MeV}{A} \approx 240\ \text{MeV of energy}$$

Nuclear Radius

Now, if we were to assume that the nucleus is *spherical* in shape, a valid and realistic assumption, we find that

$$\begin{aligned} \text{radius} &\propto A^{1/3} \\ R &= R_o \times A^{1/3} \\ \text{where } R_o &= 1.2 \text{ fm} \end{aligned}$$

Consequently, the radii of the nuclei range from about 1.23 fm, for hydrogen to about 6 to 7 fm for heavier nuclei such as Pb.

Nuclear Density

Q value

Let us consider a nuclear reaction wherein the target (a), which is at rest (hence only has rest mass energy), and an energetic projectile (b), (which would have both kinetic as well as rest mass energy) fuse / combine to form two product nuclei $(c\ , d)$, $i.e$

$$a + b \to c + d$$

We now define a quantity known as the Q-value which is essentially related to / analogous the mass defect, and is defined as

$Q = [(m_a) + (m_b)] - [(m_c) + (m_d)]$, where the masses are in amu.

$$ {}_5\mathrm{B}^{10} + n \quad \longrightarrow \quad {}_3\mathrm{Li}^7 + \alpha $$

such that

$$
\begin{aligned}
B.E_B &= 64.750\ MeV \\
B.E_{Li} &= 39.245\ MeV \\
B.E_\alpha &= = 28.296\ MeV
\end{aligned}
$$

Then the Q value of the reaction is

$$
\begin{aligned}
Q &= [39.245 \ + \ 28.296] \ - \ 64.750 \\
&= 2.791\ MeV
\end{aligned}
$$

$$\alpha + {}_{7}\mathrm{N}^{14} \longrightarrow {}_{8}\mathrm{O}^{17} + p$$

such that

$$\begin{aligned}
M_\alpha &= 4.002603\ u \\
M_N &= 14.0031\ u \\
M_O &= = 16.9994\ u \\
M_p &= = 1.007825\ u
\end{aligned}$$

Then the Q value of the reaction is

$$\begin{aligned}
Q &= \big[(4.002603\ +\ 14.0031)\ -\ (1.007825\ +\ 16.9994)\big]u \times 931.5\frac{MeV}{u} \\
&= -1.418\ MeV
\end{aligned}$$

Now let us consider the (dT) fusion reaction, where we have a deuterium and a triton (isotopes of Hydrogen) fuse /combine to form an alpha particle plus a neutron :

$$_1D^2 + {}_1T^3 \rightarrow {}_0n^1 + {}_2He^4$$

Now,

$Q = [(2.014102) + (3.016049)] - [(1.008665) + (4.002604)]$

$= 0.018882\, u$

$$\therefore Q = 0.018882 \times 931 = 17.58 \text{ MeV}$$

Now, let us consider the neutron induced fission of a heavy nucleus (such nuclei are referred to as “fissile” nuclei).

We know that the absorption of a neutron by a fissile nucleus results in the formation of two medium mass nuclei, along with the emission of 2 energetic neutrons.

$$_{0}n^{1} + {}_{92}U^{235} \rightarrow {}_{36}Kr^{97} + {}_{56}Ba^{137} + 2\,{}_{0}n^{1}$$

Now the Q value for this reaction is obtained as

$$Q = [(m_n) + (M_U)] - [(M_{kr}) + (M_{Ba}) + (2\,m_n)]$$

$$Q = [(1.00867) + (235.04390)] - [(96.92120) + (136.90610) + (2\times1.00867)]$$

$$Q = 0.20821\,u = 0.020821 \times 931\ \text{MeV} = 193.94\ \text{MeV}$$

Radioactivity

It is that process wherein a nucleus attempts to attain stability by emission of either α or β particles or γ rays.

Radiation is the propagation of energy either in particulate or wave form.

Radioactivity is a "statistical / probabilistic" process. Since, it is a statistical process,

Law of radioactive decay states that the probability for decay at any given instant depends on (is proportional to) the number of radioactive atoms present at that given instant.

If at a given instant t, N, represents the number of radioactive atoms present, then the rate of decay (probability for decay) is

$$\frac{dN}{dt} \propto -N$$

$$\frac{dN}{dt} = -\lambda N$$

$$N = N_o e^{-\lambda t}$$

We now, define a quantity, known as the **“half-life $T_{1/2}$”,**

Time interval in which the half the number of radioactive nuclei may have decayed,

$$\frac{N_o}{2} = N_o e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = \frac{1}{e^{\lambda T_{1/2}}}$$

$$\ln 2 = e^{\lambda T_{1/2}}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{T_{1/2}}$$

We then define a term known as the “Mean (Average) life”, which provides the average life expectancy of the atoms of a radioactive species

$$\tau \times \lambda = 1 \leftrightarrow$$

$$\tau = \frac{1}{\lambda} = \frac{1}{0.693/T_{1/2}}$$

$$\tau = \frac{T_{1/2}}{0.693} = 1.141 \times T_{1/2}$$

We define a term known as **"Activity",** which provides us with a measure of the intensity of the radioactivity (radioactive transformations), and it is defined as

$$\frac{dN}{dt} = -\lambda N$$

Where the left hand side represents "Activity".

The units for this are "counts(**d**isintegrations) **p**er **s**econd", and ignoring the negative sign , which has no numerical significance, we have

$$A(t) = \lambda N(t) = \lambda \times N_o e^{-\lambda t} = A_o e^{-\lambda t}$$

The conventional units for Activity is Curie, where

$$1\ Curie = 3 \times 10^{10}\ dps$$

Alpha Decay

$$_{Z}X_{N}^{A} \longrightarrow {}_{Z-2}Y_{N-2}^{A-4} + {}_{2}He_{2}^{4}$$

$$_{Z}X_{N}^{A} \longrightarrow {}_{Z-2}Y_{N-2}^{A-4} + \alpha$$

Since, the parent was initially at rest, the conservation of momentum demands,

$$m_\alpha * v_\alpha = M_d * V_d \Rightarrow V_d = \frac{m_\alpha}{M_d} * v_\alpha = \frac{4}{A-4} * v_\alpha$$

Hence, the recoil energy of the daughter nucleus, following the alpha emission would be,

$$E_{recoil} = \frac{1}{2} * M_d * V_d{}^2 = \frac{1}{2} * M_d * \left[\frac{m_\alpha}{M_d}\right]^2 * v_\alpha{}^2$$

$$= \frac{1}{2} * m_\alpha * v_\alpha{}^2 * \frac{m_\alpha}{M_d} = E_\alpha * \frac{m_\alpha}{M_d}$$

Beta Decay

$$
\begin{aligned}
{}_0\mathrm{n}_1^1 &\longrightarrow {}_1\mathrm{p}_0^1 + e^- + \bar{\nu} \\
{}_\mathrm{Z}\mathrm{X}_\mathrm{N}^\mathrm{A} &\longrightarrow {}_{\mathrm{Z}+1}\mathrm{Y}_{\mathrm{N}-1}^\mathrm{A} + e^- + \bar{\nu} \\
{}_1\mathrm{p}_0^1 &\longrightarrow {}_0\mathrm{n}_1^1 + e^+ + \nu \\
{}_\mathrm{Z}\mathrm{X}_\mathrm{N}^\mathrm{A} &\longrightarrow {}_{\mathrm{Z}-1}\mathrm{Y}_{\mathrm{N}+1}^\mathrm{A} + e^+ + \nu
\end{aligned}
$$

Gamma Decay

Gamma rays are nuclear in origin, and are electromagnetic rays, travelling with speed of light, and can propagate through vacuum as well. If we were to assume a gamma ray having energy $E_\gamma = 439\ keV$, (this gamma ray de-excites two levels, which are an energy difference of 439 keV) , we know that

$$E_\gamma = h\,\upsilon = h\frac{c}{\lambda}$$

We know that the momentum of the gamma ray $\left(p_\gamma\right)$, is related to it's energy $\left(E_\gamma\right)$, as

$$p_\gamma = \frac{E_\gamma}{c}$$

Since, the parent nucleus was initially at rest, conservation laws demand that ,

$$P_d = p_\gamma$$

$$T_d = \frac{{P_d}^2}{2 \times M_d} = \frac{{P_\gamma}^2}{2 \times M_d} = \frac{{E_\gamma}^2}{2 \times M_d \times c^2}$$

Where, T_d, is the recoil energy of the daughter nucleus.

Atom Density

If the target material is an element of atomic/molecular weight M, then 1 mole (weight in grams) of the substance would contain $N_A = 6.02 \times 10^{23}$ number of atoms. We then define a quantity known as the atom density (N), also referred to as "number density" as the number of atoms per cubic meter, i.e

$$N = \frac{\rho\, N_A}{M}$$, where ρ is the density in gm/cc

For example, if the density of Na is $\rho_{Na} = 0.97\ gm/cc$, and its atomic weight is 22.990 then the atom (number) density is

$$N = \frac{\rho\, N_A}{M} = \frac{(0.97\ gm/cc) \times \left(6.022 \times 10^{23}\ atoms/mol\right)}{22.90\ gm\ /mol} = 0.0254 \times 10^{24}\ atoms/cc$$