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Evaluation of resistance–temperature calibration equations for NTC thermistors

Chiachung Chen *

Department of Bio-Industrial Mechatronics Engineering, National ChungHsing University, 250 Kuokuang Rd., Taichung 40227, Taiwan

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ABSTRACT

The characteristics of thermistors for temperature measurement are the high sensitivity to yield a high resolution and the high accuracy in the narrow temperature range. This performance of this sensing element is affected by its calibration equation. Seven calibration equations were selected to evaluate the fitting agreement of the resistance–temperature data of four types of thermistors in this study. The parameters of these calibration equations are estimated using least squares method. The performance of these equations was compared with several statistics. The results of this study indicated that the basic equation and Steinhart and Hart equations were not the adequate calibration equation for four table data of thermistor. The Hoge-3 equation, $1/T = A_0 + A_1\ell nR_T + A_2(\ell nR_T)^2 + A_3(\ell nR_T)^3 + A_4(\ell nR_T)^4$, was the best equation for seven calibration equations. The form of this equation was a 4th-degree polynomial equation. It was easy to be incorporated into IC circuit to serve as a calculation equation to transform the measured resistance into the temperature value. The nonlinear least squares method and the criteria for comparison also could be applied to evaluate the fitting ability of calibration equations for other thermistors.

1. Introduction

The thermistor is a kind of semiconductor. The resistance is changed with temperature. The resistance can be detected directly and simplicity by a digital multimeter. After the resistance is determined over a range of temperatures, the relationship between resistance of thermistor and temperature can be established and be called as the calibration equation. This calibration equation was then further applied to transform the measure resistance value of the thermistor into the temperature. The resistance of some types of thermistors is decrease with the increase of temperature, so they are called as negative temperature coefficient (NTC) thermistors. The advantages of thermistors for temperature measurement are the high sensitivity to yield a high resolution and the high resistivity permits small mass units with fast response [1]. This temperature measuring element is suitable for biomedical application because of its accuracy in the narrow temperature range. Beside of temperature measurement, thermistor can serve as the device used in temperature control and compensation [1].

The performance of the thermistor for temperature measurement is affected by the calibration equation. A simple and popular equation to express the relationship between the resistance and temperature is called as the Basic equation. This equation is a two-parameter exponential equation [1–3].

The determination of the parameters of the exponential equation had been introduced detailed [2]. However, the nonlinearity of the errors could be found as the measured temperature was calibrated with a standard platinum resistance thermometer.

In order to reduce the nonlinear errors of thermistor, the parameters of Basic equation are assumed as the function of linear relationship with temperature [4]. The authors suggested that these linear equations could be incorporated into a resistive linearizing circuit to improve the performance of temperature measurement.

* Corresponding author. Tel.: +886 4 22857562; fax: +886 4 22857135.
E-mail address: ccchen@dragon.nchu.edu.tw

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The three-term Steinhart and Hart equation is the other model widely used for thermistor [5]. The procedure for calculating Steinhart–Hart constants have been described detailed [6]. On the document, the typical errors was within the range of ±1.5 °C on the temperature ranged from −20 to 50 °C. However, the definition of the errors and accuracy were not described in this document. This Steinhart–Hart equation was adopted to improve the precision measurement of absolute temperatures [7]. If the nonlinear characteristics of the thermistor can be modeled, the author proposed that the accuracy of the temperature can be improved better than 10 mK.

Alexander and Mac Quarre [8] selected the Steinhart and Hart equation and presented the procedure to calculate the negative temperature coefficient (NTC) of the thermistors in order to improve its accuracy.

More calibration equations were evaluated by Hoge [9] to model the relationship between resistance and temperature for six types of thermistor. The criteria for evaluation were the quantitative value of root-mean-square deviation (RMSD) and residual plots. The results indicated that in the higher power terms of independent variable could improve the accuracy. However, the author found more precision computer programs were required to determine the constants of equation.

Ilic et al. [10] proposed two new equations with two-parameter and three-parameter for the calibration equations of thermistors. Two curves were compared with the Steinhart and Hart equation using the table data of resistance for four types of negative temperature coefficient resistors (NTCRS). They found the two-parameter equation was the best equation. However, the form of this equation was very complex.

In this study, seven calibration equations were selected to evaluate the fitting agreement of the resistance–temperature data of four types of thermistors. The parameters of these calibration equations are estimated using least squares method. The performance of these equations was compared with several statistics.

2. Calibration equations and data analysis

2.1. Calibration equations

On this section, seven calibration equations are proposed to describe the relationship between the resistance of thermistors and temperature.

(a) the Basic equation
The Basic equation is also called as the two-term exponential equation

$$R_T = R_{T0}\exp\left(B\left(\frac{1}{T}-\frac{1}{T_0}\right)\right) \tag{1}$$

where R_T is the resistance of thermistor at temperature T (K), R_{T0} is the resistance of thermistor at the nominal operating temperature at T_0, B is the coefficient.

Eq. (1) can be expressed as follows:

$$1/T = \frac{a}{1 + b\ell nR_T} \tag{2}$$

where a and b are constants.

(b) The Steinhart and Hart equation

$$1/T = A_0 + A_1\ell nR_T + A_3(\ell nR_T)^3 \tag{3}$$

where A_0, A_1, A_3 are constants.

Five equations are modified from the Hoge's equation [8].

(c) Hoge-1 equation

$$1/T = A_0 + A_1\ell nR_T + A_2(\ell nR_T)^2 \tag{4}$$

(d) Hoge-2 equation

$$1/T = A_0 + A_1\ell nR_T + A_2(\ell nR_T)^2 + A_3(\ell nR_T)^3 \tag{5}$$

(e) Hoge-3 equation

$$1/T = A_0 + A_1\ell nR_T + A_2(\ell nR_T)^2 + A_3(\ell nR_T)^3 + A_4(\ell nR_T)^4 \tag{6}$$

where A_0, A_1, A_2, A_3 and $A_4$are constants.

(f) Hoge-4 equation

$$1/T = A_0 + A_1\ell nR_T + A_2(\ell nR_T)^2 + A_5/\ell nR_T \tag{7}$$

where A_5 is constant.

(g) Hoge-5 equation

$$\ell nR_T = b_o + b_1/T + b_2 \tag{8}$$

where b_0, b_1 and b_2 are constants

Eq. (8) can be expressed as:

$$1/T = \frac{C_1 + C_2\ell nR}{1 + C_3\ell nR} \tag{9}$$

where C_1, C_2 and C_3 are constants

2.2. Resistance–temperature data of thermistors

Four table data of thermistors were selected to evaluate the fitting ability of these calibration equations in this study.

(a) ATP A1004-C_3 thermistor [11]
(b) FE UUA41J1 thermistor [12]
(c) YSI L100 thermistor [13]
(d) YSI L300 thermistor [13]

The table of resistance vs. temperature was established by manufacturers. The experimental procedure of the measurement and the traceable standard were described as follows [11–14]. The calibrations were executed by comparing with the reading of an ITS-90 standard platinum resistance thermometer at interval temperature over the range from −50 to 100 °C. The standard platinum resistance thermometer was calibrated on the International Temperature Scale of 1990 and by a limited number of fixed-points. The thermistor resistance was detected by the $8_{1/2}$ digit voltmeter that calibrated with a NIST calibrated 10 V reference source.

The distribution between resistance and temperature of four types of thermistor are presented in Figs. 1 and 2.

2.3. Data analysis

The software, SigmaPlot Ver.10.0, is used to estimate the parameters of seven calibration equations with nonlin-

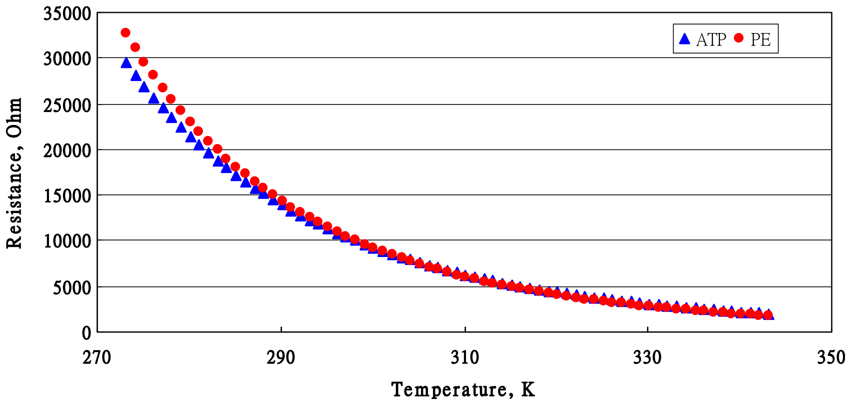

Fig. 1. The distribution between resistance and temperature of ATP and FE thermistors.

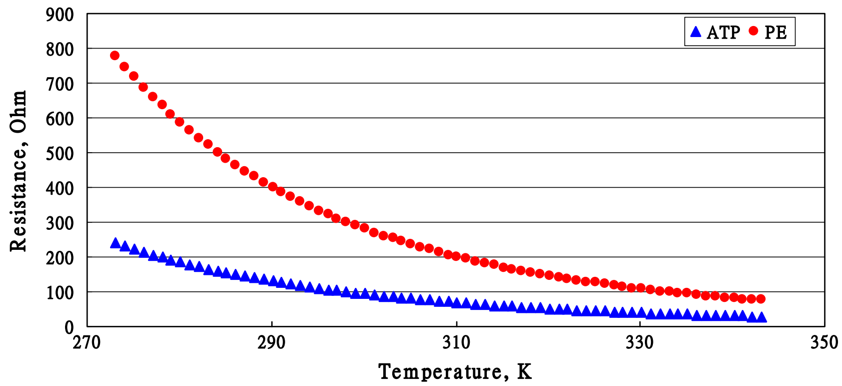

Fig. 2. The distribution between resistance and temperature of YSI L100 and L300 thermistors.

ear regression analysis technique. In these equations, T is the dependent variable and $\ell n R_T$ is the independent variable.

Residual plots were applied as the qualitative criterion to assess the adequateness of models. For an adequate model, data distribution of residual plots should indicate a horizontal band. If the residual plots indicated a systematic clear pattern, the model cannot be accepted [15].

The quantitative criteria were defined as follows:

$$e_i = y_i - \hat{y}_i \tag{10}$$

where e_i is the error of calibration equation, y_i is the dependent variable and $\hat{y}_i$ is the predicted values of calibration equation.

Two statistics, $e_{\max}$ and $e_{\min}$ are adopted as the quantitative criteria of equations. The $e_{\max}$ is the maximum e_i value and the $e_{\min}$ is the minimum e_i value.

The $|e|_{\text{ave}}$ value was used to evaluate the accuracy of equations. The smaller of the $|e|_{\text{ave}}$ value, the better accuracy of the equation

$$|e|_{\text{ave}} = \frac{\sum |e_i|}{n} \tag{11}$$

where $|e_i|$ is the absolute e value of e_i, n is the number of data.

2.4. Measurement uncertainty

The value was used to evaluate the precision of equations and can be recognized as the measurement uncertainty if these calibration equations [16,17]. According to the ISO GUM [16], the uncertainty of measurement was evaluated by a “TYPE A” or “TYPE B” method. The TYPE A evaluation is the method by the statistical analysis of observation. The uncertainty from a calibration equation can be calculated from the standard deviation of the calibration equation [17,18]

$$e_{\text{std}} = \left(\frac{(e_i - \overline{e_i})^2}{n-1}\right)^{0.5} \tag{12}$$

where $\overline{e_i}$ is the average of $e_{i,}$ the numerical value is zero for $\overline{e_i}$, Eq. (12) can then be expressed as:

$$e_{\text{std}} = \left(\frac{(e_i)^2}{n-1}\right)^{0.5} \tag{13}$$

3. Comparison of seven calibration equations

The estimated parameters of seven calibration equations for four types of thermistor are listed in Table 1.

Table 1
Estimated parameters of seven equations for different thermistors.

Model	Parameters	ATP	FE	L100	L300
S&H	A_0	1.0280735×10^{-3}	1.129389×10	1.7683020×10	1.5710287×10
	A_1	2.3929453×10^{-4}	2.340446×10^{-4}	3.4139643×10^{-4}	3.0874798×10^{-4}
	A_3	1.5595241×10^{-7}	8.8144335×10^{-8}	1.3728081×10^{-7}	1.1773643×10^{-7}
Basic	a	273.6	275.4	187.3	207.5
	b	0.07551	0.07104	0.06539	0.06622
Hoge-5	b_0	910.8	849.8	562.3	629.9
	b_1	−1.209	−7.225	−2.809	−3.657
	b_2	0.1825	0.1765	0.1830	0.1828
Hoge-1	A_0	1.1373852×10^{-3}	1.190788×10^{-3}	1.7793959×10^{-3}	1.5896719×10^{-3}
	A_1	2.0220202×10^{-4}	2.13149×10^{-4}	3.3360157×10^{-4}	2.9831049×10^{-4}
	A_2	4.1756279×10^{-6}	2.3572×10^{-6}	1.8027935×10^{-6}	1.9291623×10^{-6}
Hoge-2	A_0	1.0252550×10^{-3}	1.173298×10^{-3}	1.7461085×10^{-3}	1.5517946×10^{-3}
	A_1	2.4025026×10^{-4}	2.1910452×10^{-4}	3.5690864×10^{-4}	3.1948686×10^{-4}
	A_2	$-1.0751465 \times 10^{-7}$	1.684947×10^{-6}	$-3.5688474 \times 10^{-6}$	$-1.9794181 \times 10^{-6}$
	A_3	1.5996514×10^{-7}	2.5152643×10^{-8}	4.0764907×10^{-7}	2.3821308×10^{-7}
Hoge-3	A_0	1.0039146×10^{-3}	1.0895097×10^{-3}	1.5649303×10^{-3}	1.4770887×10^{-3}
	A_1	2.4990558×10^{-4}	2.5714877×10^{-4}	5.2614421×10^{-4}	3.7519417×10^{-4}
	A_2	$-1.7401482 \times 10^{-6}$	$-4.7668339 \times 10^{-6}$	$-6.2317477 \times 10^{-5}$	-1.745008×10^{-5}
	A_3	2.8224660×10^{-7}	5.0949247×10^{-7}	9.3914016×10^{-6}	2.1347327×10^{-6}
	A_4	$-3.4230052 \times 10^{-9}$	$-1.3580963 \times 10^{-8}$	$-5.1070466 \times 10^{-7}$	-8.660045×10^{-8}
Hoge-4	A_0	1.4728413×10^{-3}	1.2450539×10^{-3}	1.8876073×10^{-3}	1.7050084×10^{-3}
	A_1	1.6436231×10^{-4}	2.0701382×10^{-4}	3.0852681×10^{-4}	2.7693434×10^{-4}
	A_2	5.5917055×10^{-6}	2.5881766×10^{-6}	3.7157450×10^{-6}	3.2372542×10^{-6}
	A_5	$-9.8659807 \times 10^{-4}$	$-1.5891328 \times 10^{-4}$	$-1.5371607 \times 10^{-4}$	$-2.0544677 \times 10^{-4}$

The quantitative criteria of these calibration equations are listed in Tables 2 and 3.

3.1. ATP thermistor

For the table data of the ATP thermistor, the $|E_{ave}|$ value could serve as the index to evaluate the accuracy of the measurement performance. The E_{std} is the measurement uncertainty to compare the precision. From the numerical values of Tables 2 and 3, the Basic equation had the largest values of e_{max}, e_{min}, $|E|_{ave}$ and E_{std} values. The residual plots show the systematic residual pattern for the errors distribution (Fig. 3). This result indicated that this equation was not an adequate model for the ATP thermistor.

The clean systematic pattern of residual plots was also found for the Hoge-1 and Hoge-5 equations (Figs. 4 and 5). Besides the Basic equation, the $|E|_{ave}$ value of these two equations was 5.2444 and 3.2572, respectively. However, the $|E|_{ave}$ of the S&H, Hoge-2, Hoge-3 and Hoge-4 equations were 1.3725, 1.3639, 1.3662 and 1.4201, respectively. The random distribution of residual plots was found for these four equations (Figs. 6 and 7).

The E_{std} values listed in Table 3 for the Hoge-1 and Hoge-5 equations were 0.7452 and 0.4761, respectively. The E_{std} values for the S&H, Hoge-2, Hoge-3 and Hoge-4 equations were 0.2629, 0.2303, 0.2290 and 0.2333, respectively.

Table 2
Criteria for the evaluation of seven calibration equations for different thermistors.

Mode1	Criteria	ATP	FE	L100	L300
Basic	E_{min}	−449.122	−295.65	−133.46	−160.578
	E_{max}	239.972	165.46	82.081	88.1010
	$\|E\|_{ave}$	180.707	120.585	50.399	61.734
S&H	E_{min}	−6.945	−9.5440	−18.1377	−15.245
	E_{max}	3.5095	11.8705	43.2650	31.463
	$\|E\|_{ave}$	1.3725	3.8644	14.8987	5.2913
Hoge-1	E_{min}	−17.0761	−9.7868	−74.5271	−38.9853
	E_{max}	12.0127	11.9859	45.1879	18.4999
	$\|E\|_{ave}$	5.2444	3.1443	15.5192	6.7545
Hoge-2	E_{min}	−3.6004	−8.4708	−54.0757	−23.2459
	E_{max}	7.1863	10.8339	48.6401	15.2780
	$\|E\|_{ave}$	1.3639	2.9304	14.6577	5.0756
Hoge-3	E_{min}	−3.7109	−8.5207	−41.846	−19.5735
	E_{max}	7.2740	10.6318	54.107	16.2196
	$\|E\|_{ave}$	1.3662	2.9801	14.0645	4.9214
Hoge-4	E_{min}	−4.1559	−8.7960	−49.0148	−20.9324
	E_{max}	7.6117	10.7381	50.7247	15.4552
	$\|E\|_{ave}$	1.4201	3.0906	14.4431	4.9796
Hoge-5	E_{min}	−7.7501	−11.0121	−45.3319	−17.8754
	E_{max}	10.7921	8.3075	72.4456	36.8038
	$\|E\|_{ave}$	3.2572	3.0164	15.4558	6.6564

Table 3
Measurement uncertainty of seven calibration equations for different thermistors.

Mode1	ATP	FE	L100	L300
Basic	22.131	16.771	7.0651	8.6298
S&H	0.2629	0.5756	2.4853	0.9291
Hoge-1	0.7452	0.4891	2.5903	1.1098
Hoge-2	0.2303	0.4636	2.3731	0.8471
Hoge-3	0.2290	0.4575	2.2531	0.8328
Hoge-4	0.2333	0.4772	2.3281	0.8359
Hoge-5	0.4760	0.4708	2.6081	1.8024

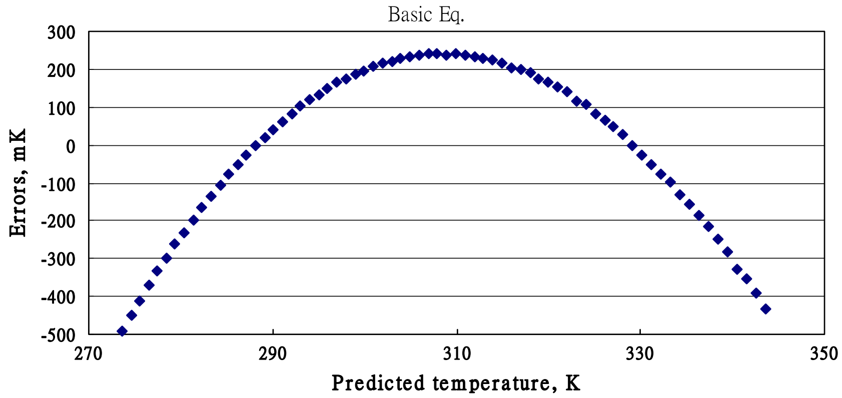

Fig. 3. The residual plots of the basic equation for the ATP thermistor.

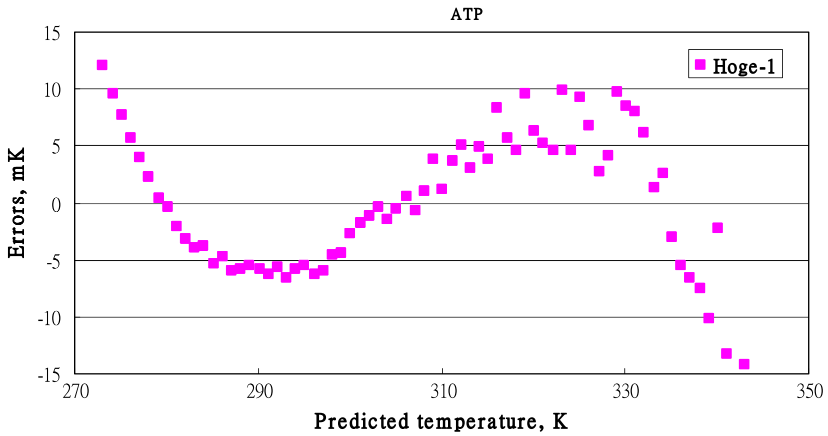

Fig. 4. The residual plots of the Hoge-1 equation for the ATP thermistor.

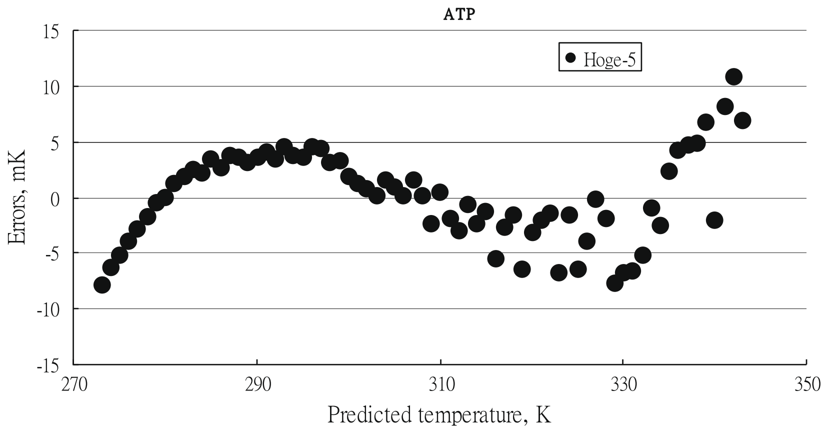

Fig. 5. The residual plots of the Hoge-5 equation for the ATP thermistor.

From the above discussion, the S&H, Hoge-2, Hoge-3 and Hoge-4 equations could be recognized as the adequate calibrations equation for the ATP thermistor. The Hoge-3 equation had the smallest $|E|_{ave}$ and E_{std} values. It is the best equation for the ATP thermistor by comparing these criteria.

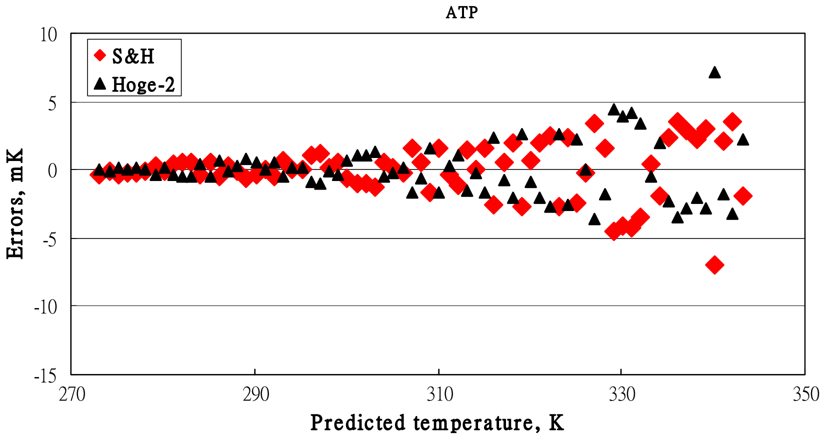

Fig. 6. The residual plots of the S&H and Hoge-2 equations for the ATP thermistor.

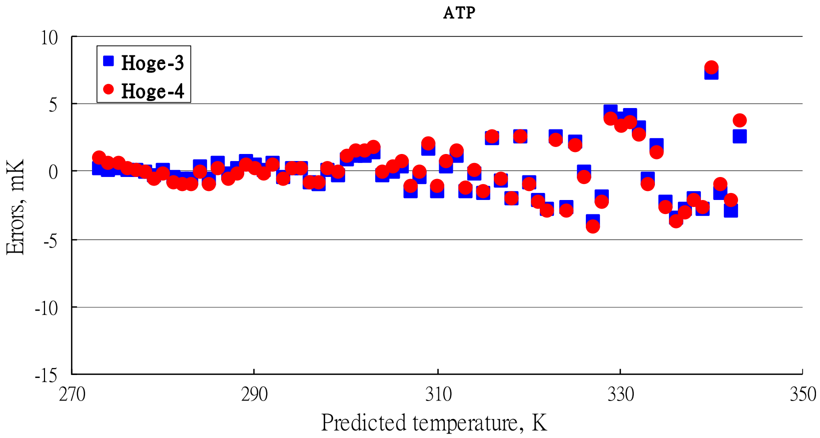

Fig. 7. The residual plots of the Hoge-3 and Hoge-4 equations for the ATP thermistor.

3.2. FE thermistor

The results of residual plots indicated that the Basic, the S&H and the Hoge-5 calibration equations had the systematic residual pattern.

The other equation revealed the uniform distribution of residuals. The results indicated the adequate of these equations.

The $|E|_{ave}$ value of the S&H equation was 3.8644. The $|E|_{ave}$ values for the Hoge-1, Hoge-2, Hoge-3, Hoge-4 and Hoge-5 equations were 3.1443, 2.9304, 2.9801, 3.0966 and 3.0164, respectively. The Hoge-2 equation had the smallest $|E|_{ave}$ value.

The E_{std} value of the S&H equation was 0.5756. The E_{std} values for the Hoge-1, Hoge-2, Hoge-3, Hoge-4 and Hoge-5 equations were 0.4891, 0.4636, 0.4575, 0.4772 and 0.4708, respectively. The Hoge-3 equation had the smallest value for the other equations.

From the above discussion, the Hoge-2 and Hoge-3 equations were the adequate calibration equations for the FE thermistors by comparing two criteria.

3.3. L100 thermistor

The residual plots of seven calibration equations for the L100 thermistor are presented in Figs. 8–14. Only the Basic equation had the systematic residual pattern. The other equations all had the random distribution of residual plots. At the higher range of temperature, the residuals plot showed a funnel-type distribution of residuals. The errors distribution became diversity. That is, these calibration equations had the better precision performance within the range from 270 to 315 K.

From the quantitative criteria in Tables 2 and 3, the Hoge-3 equation had the smallest $|E|_{ave}$ and E_{std} values. However, the accuracy and precision of the YSI L100 thermistor that calculated by this equation was not as good as the performance of the ATP and the FE thermistors.

3.4. L300 thermistor

The distribution patterns of residual plots of these calibration equations for the YSI L300 thermistor were similar

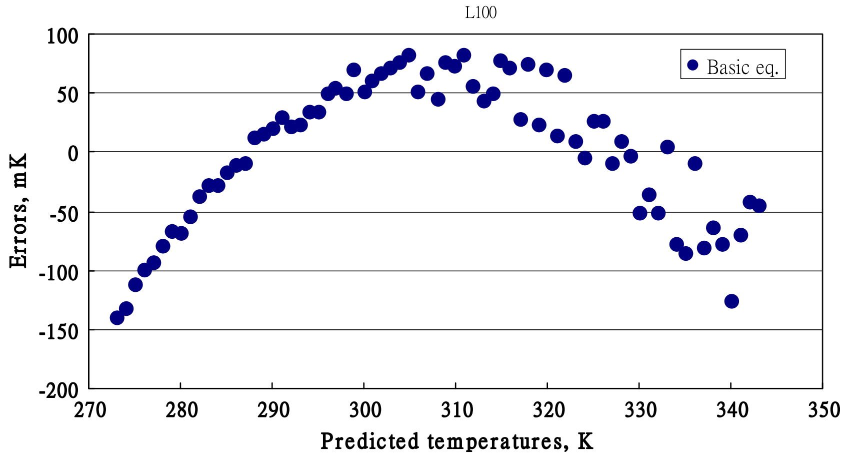

Fig. 8. The residual plots of the basic equation for the L100 thermistor.

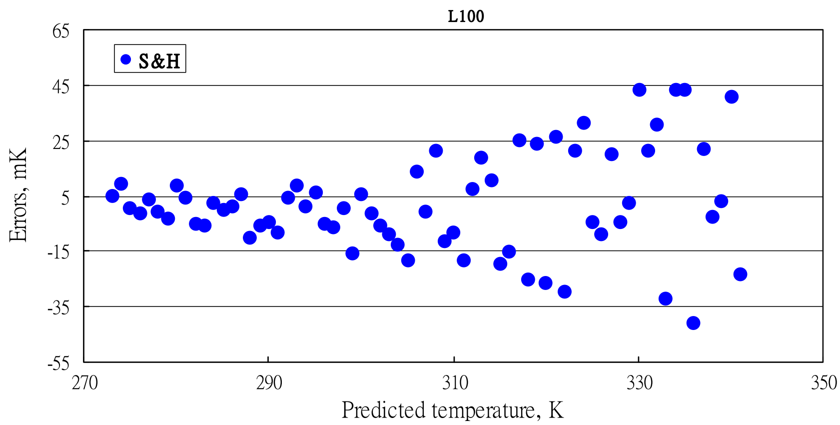

Fig. 9. The residual plots of the S&H equation for the L100 thermistor.

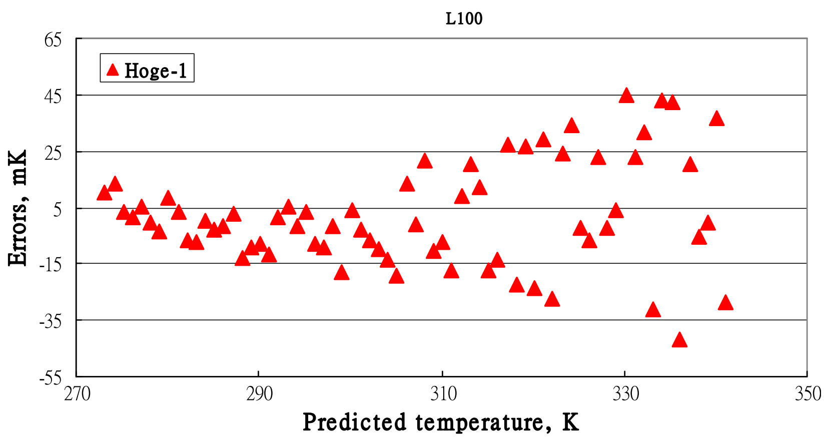

Fig. 10. The residual plots of the Hoge-1 equation for the L100 thermistor.

as that of YSI L100 thermistors. Only the Basic equation had the systematic residual pattern. Comparing the quantitative criteria for other six calibration equations, the Hoge-3 equation had the smallest $|E|_{ave}$ and E_{std} value. This equation was the best calibration equation for all seven calibration equations by comparing these criteria.

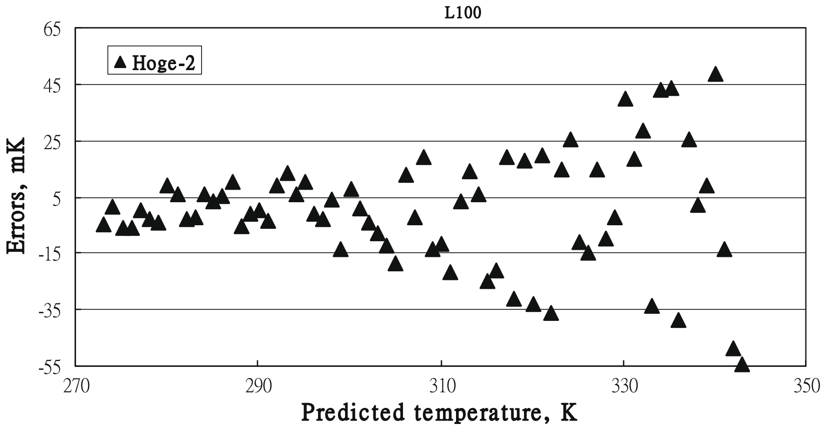

Fig. 11. The residual plots of the Hoge-2 equation for the L100 thermistor.

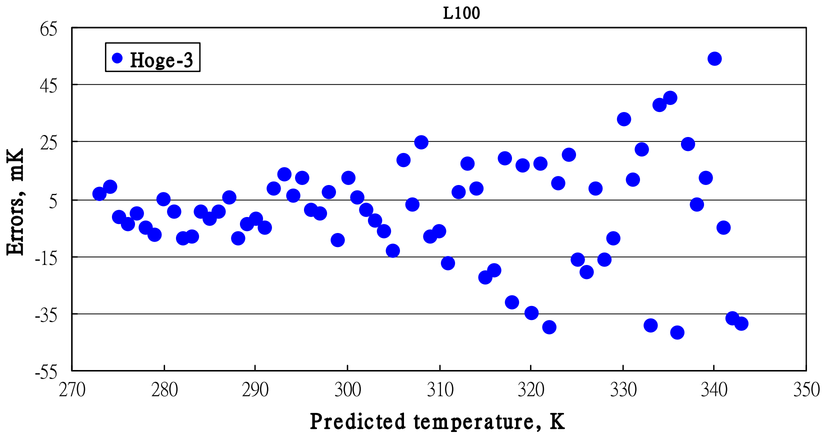

Fig. 12. The residual plots of the Hoge-3 equation for the L100 thermistor.

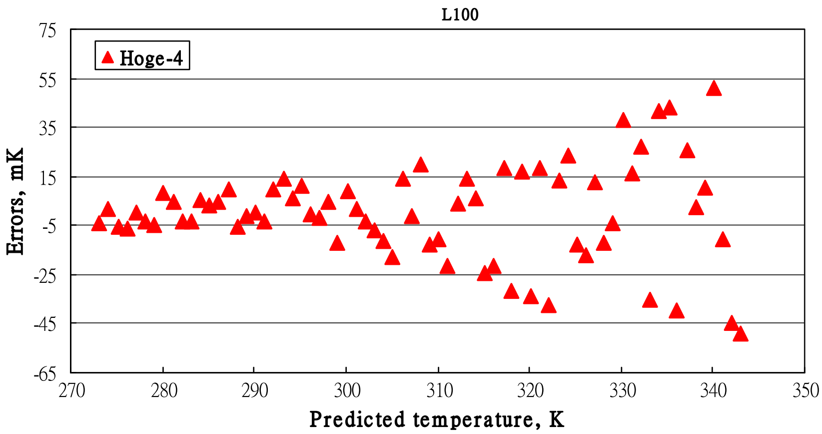

Fig. 13. The residual plots of the Hoge-4 equation for the L100 thermistor.

4. Conclusion

Four types of thermistor, table temperature–resistance data in the range from 0 to 70 °C, were selected to evaluate the fitting agreement of seven calibration equations. The parameters of these equations were established using the nonlinear least squares method. The qualitative criterion was the distribution pattern of residual plots. The quanti-

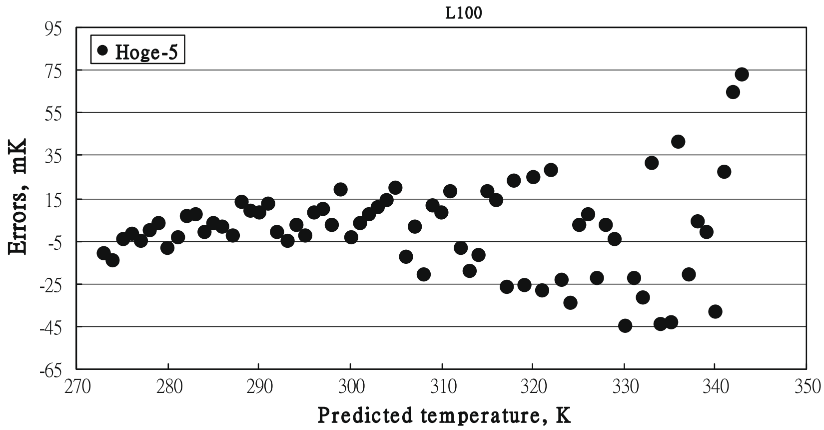

Fig. 14. The residual plots of the Hoge-5 equation for the L100 thermistor.

tative criteria included the average of the absolute errors and standard deviation of the errors for these calibration equations.

The results of this study indicated that the popular equation, the Basic equation and Steinhart and Hart equations were not the adequate calibration equation for all four table data of thermistor. The Hoge-3 equation,

$1/T = A_0 + A_1\ell nR_T + A_2(\ell nR_T)^2 + A_3(\ell nR_T)^3 + A_4(\ell nR_T)^4$, was the best equation for seven equations. The form of this equation was a 4th-degree polynomial equation. It was easy to be incorporated into IC circuit served as a calculation equation to transform the measured resistance into the temperature value. The average value of the absolute errors was less than 15 mK for the YSI L100 thermistor, less than 5 mK for the YSI L300 thermistor, less than 3.0 mK for the FE thermistor and no more than 1.5 mK for the ATP thermistor. The standard deviations of the errors were within ± 1.0 mK except the YSI L100 thermistor. The nonlinear least squares method and the criteria for comparison also could be applied to evaluate the fitting ability of calibration equations for other thermistors.

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