Vehicle Routing Problem with Profit

It is a maximization problem where The aim is to visit nodes maximize the sum of collected profits under specific circumstances while respecting a given vehicle time limit. For example, assume there are two sets of customers, Ones are frequent customers that are mandatory for delivery, and another set is the non-frequent potential customers with known and estimated profits. Both sets have known demands and service requirements over a planning horizon of multiple days. The objective is to determine vehicles' routes that maximize the net profit, while satisfying vehicles' capacity, route duration and consistency constraints. Vehicles are required to start and end at the same depot.

Parameter:-

No. of vehicle K= 6 (vehicle_count)

Total time duration for each route = 10 (T_{max})

No. of demand points V = 9 (customer_count)

Profit associated with vertex 'i'= p_i

Decision Variable:-

X_{ij}^k = Binary variable equal to 1 if **arc (i, j) ∈ A** is traversed by vehicle route k ∈ K, 0 otherwise.

Objective Function:- Maximize $\sum_{k=0}^{K} \sum_{i,j=0}^{A} Xij * p_i$

Constraints:-

$$\sum_{k \in K} \sum_{i \in V, i \neq j} x_{ij}^k = 1 \qquad \forall j \in V \setminus \{0\}$$

$$\sum_{j \in V \setminus \{0\}} x_{0j}^k = 1 \qquad \forall k \in K$$

$$\sum_{i \in V, i \neq j} x_{ij}^k - \sum_{i \in V} x_{ji}^k = 0 \qquad \forall j \in V, \forall k \in K$$

Time constraint for each node :- $\sum_{i,j\in A}^{V} \mathrm{t}\,ij * \mathrm{X}ij \leq \mathrm{T}max$

$$\sum_{k\in K}\sum_{(i,j)\in S, i\neq j} x_{ij}^{k} \leq |S| - 1 \qquad S \subseteq V \setminus \{0\}$$

$$x_{ij}^{k} \in \{0,1\} \qquad \forall k \in K, \forall (i,j) \in E$$

1st constraint means “only one visit per vehicle per customer’s location”. 2nd constraint means “depart from depot”. 3rd constraint means “the number of vehicles coming in and out of a customer’s location is the same”. 4th constraint limits the max. time for each route. 5th constraint for “removal of subtours” and 6th constraint for Binary decision variable.