



# Non-Local Mean

A. Buades, B. Coll and J. -. Morel, "A non-local algorithm for image denoising"

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# **Problem Definition**

# ■ Problem Definition

## What is Noise

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- Unwanted information which deteriorates image quality
- Due to illumination, temperature, or signal processing disturbances etc.

# Problem Definition

## Common Noises

- **Salt and Pepper Noise**
  - **Sparse white and black pixels**
  - **Caused by camera heat, dust etc.**



Image Source:  
[https://en.wikipedia.org/wiki/Image\\_noise#/media/File:Noise\\_salt\\_and\\_pepper.png](https://en.wikipedia.org/wiki/Image_noise#/media/File:Noise_salt_and_pepper.png)

# Problem Definition

## Common Noises

- **Gaussian Noise**
  - Gaussian distributed
  - Good model of noise in many images
  - Poor lighting, heat, transmissions etc.



Image Source:

[https://www.researchgate.net/figure/Noisy-image-Gaussian-noise-with-mean-and-variance-0005\\_fig2\\_252066070](https://www.researchgate.net/figure/Noisy-image-Gaussian-noise-with-mean-and-variance-0005_fig2_252066070)

# Problem Definition

## Image Denoising

### Goal

Recover the original image from noisy image.

$$v = D_h v + n(D_h, v)$$

- $v$  observed values
- $D_h$  filter,  $D_h v \rightarrow \text{true values as } D_h \rightarrow \text{perfect filter}$
- $n(D_h, v)$  noise perturbation
  - Ideally would be gaussian white noise

# Previous Work



# Previous Work

## Local Smoothing Filters

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- Collect information from adjacent pixels to smooth out disturbance.

## Denoising by Averaging

# Local Smoothing Filters

## Gaussian Filtering

- Optimal in flat parts of the image
- Blurred around edges and texture

## Anisotropic Filtering

- The straight edges are well restored
- Flat and textured regions are degraded

## Neighborhood Filtering

- Not robust when images is very noisy

# Motivation

## Local Filters Disadvantages

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- **Visual Quality:**

- **Either edge or texture details are degraded**

- **Method Noise:**

- **Smooths out even when the image has not much noise**

**What if we consider non locally?**

# Non-Local Mean

## Non-Local Mean

- Denoising with **weighted** average across **all pixels**.

### Definition

$$NL[v](i) = \sum_{j \in I} w(i, j) v(j)$$

- $v = \{v(i) \mid i \in I\}$  the input, a noisy image
- $NL[v](i)$  the estimated value
- $w(i, j)$  weight of **similarity** between pixel  $i$  and  $j$

# Weight of Similarity Motivation

- want to smooth out patch  $p$
- Weights of  $q1$  and  $q2$  should have value larger than  $q3$ , since they are **similar**

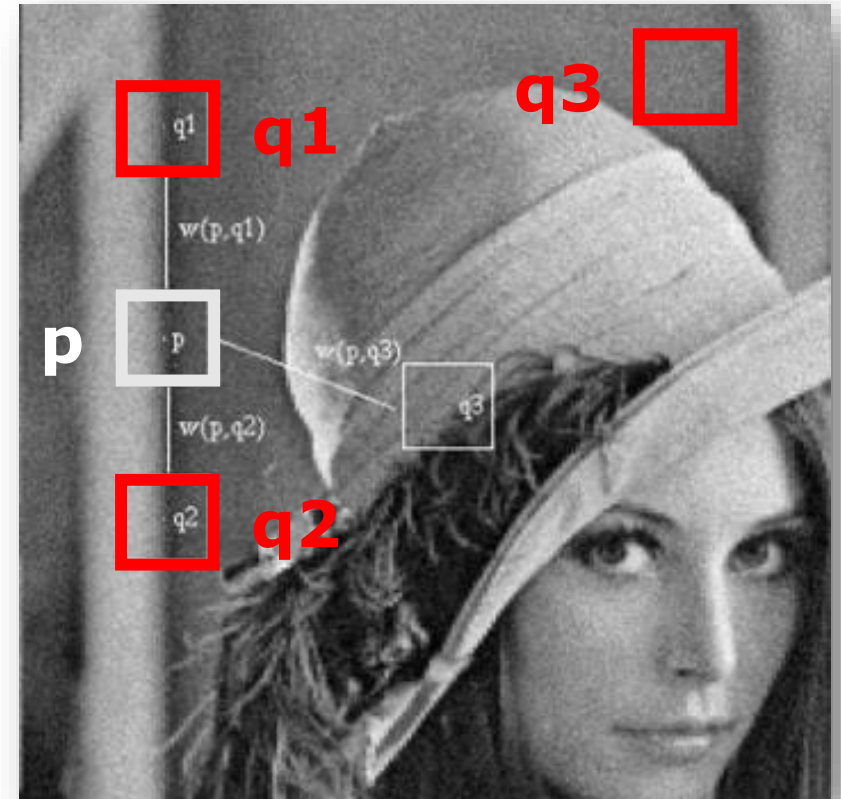


Image Source: A. Buades, B. Coll and J. -. Morel, "A non-local algorithm for image denoising"

## ■ Similarity Defined

■ Q: When are two pixels  $i, j$  considered “similar” ?

■ A: By euclidean distance of intensity gray level

$$\left\| v(N_i) - v(N_j) \right\|_{2,a}^2$$

■  $N_i$  a patch centered at pixel  $i$

■  $v(N_i)$  intensity gray level vector

■  $a > 0$  the standard deviation of the Gaussian kernel

# Weight of Similarity Motivation

- Also want to respect texture
- $q1$ ,  $q3$  same intensity, but  $q1$  preserves textures
- Pixels nearby have similar texture thus should be weighted more

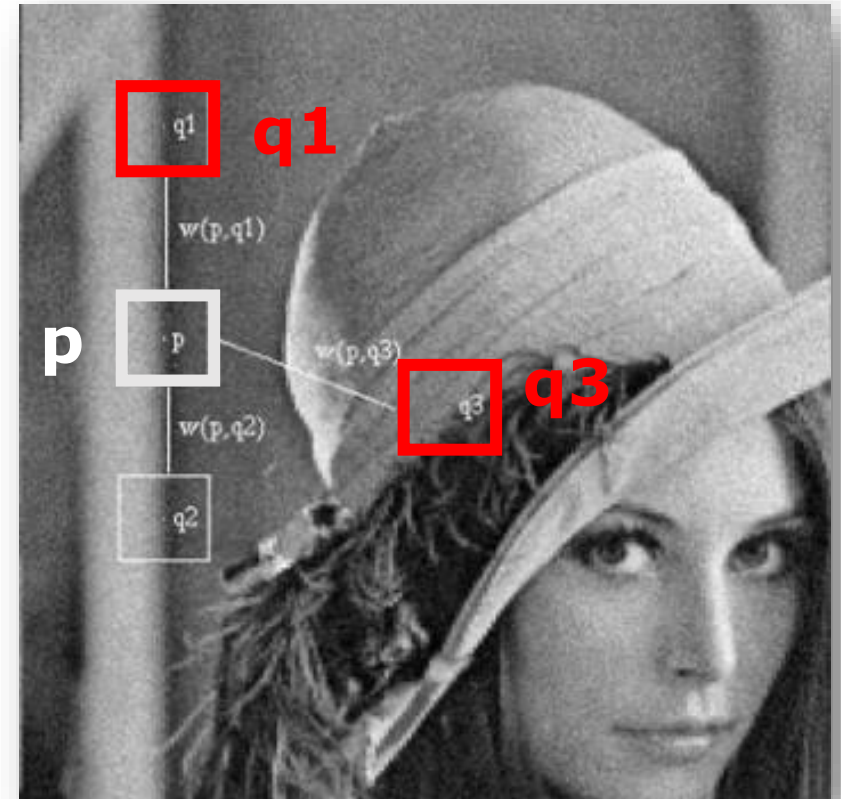
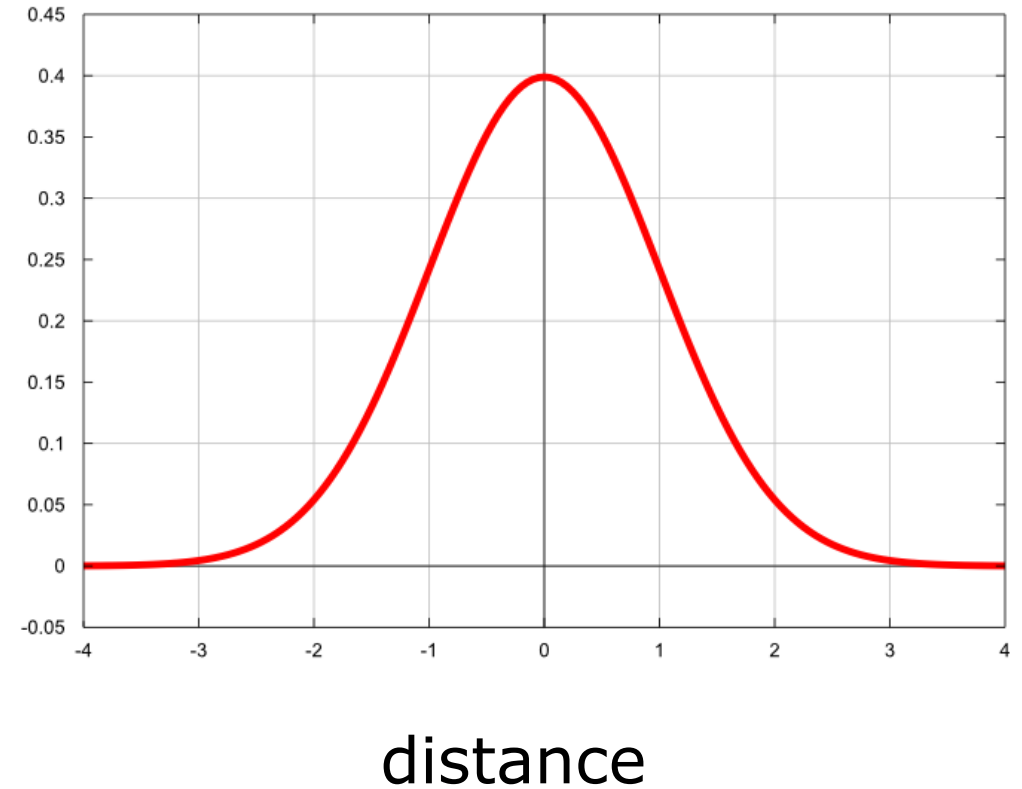


Image Source: A. Buades, B. Coll and J. -. Morel, "A non-local algorithm for image denoising"



# Weight of Similarity Motivation

- Gaussian weighting
- Centered at pixel  $i$       weighting
- As the distance increase,  
weighting decrease



## ■ Weights Defined

- Patches close together with a similar grey level have larger weights.

$$w(i, j) = \frac{1}{Z(i)} e^{-\frac{\|v(N_i) - v(N_j)\|_{2,a}^2}{h^2}}$$

- $Z(i) = \sum_{j \in I} e^{-\frac{\|v(N_i) - v(N_j)\|_{2,a}^2}{h^2}}$ , the normalizing constant
- $h$ , decay of exponential function, the degree of filtering

## Weights Defined

- Patches with a similar grey level have larger weights

$$w(i, j) = \frac{1}{Z(i)} e^{-\frac{\|v(N_i) - v(N_j)\|_{2,a}^2}{h^2}}$$

### Properties

- $0 \leq w(i, j) \leq 1$
- $\sum_{j \in I} w(i, j) = 1$
- $E \left[ \|v(N_i) - v(N_j)\|_{2,a}^2 \right] = \|u(N_i) - u(N_j)\|_{2,a}^2 + 2\sigma^2$

## Non-Local Mean

### Putting together

$$NL[v](i) = \sum_{j \in I} \frac{1}{Z(i)} e^{-\frac{\|v(N_i) - v(N_j)\|_{2,a}^2}{h^2}} v(j)$$

- $v = \{v(i) \mid i \in I\}$  the input, a noisy image
- $NL[v](i)$  the estimated value

# Implementation

# Implementation

## Code

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- **GitHub:** <https://github.com/surprise777/CSC320>
- **Complexity:**  $O(wmN^2)$ 
  - $w$  search window size, e.g.  $w = 441$  for  $21 \times 21$  pixels windows
  - $m$  patch size, e.g.  $m = 49$  for  $7 \times 7$  patch
  - $N^2$  number of pixels

# Comparisons

# Comparisons – Visual Quality

**Preserves more detail on edge and texture**

- 1) Noisy Image
- 2) Gauss Filtering
- 3) Anisotropic Filter
- 4) Total Variation
- 5) Neighborhood Filtering
- 6) Non-Local Mean

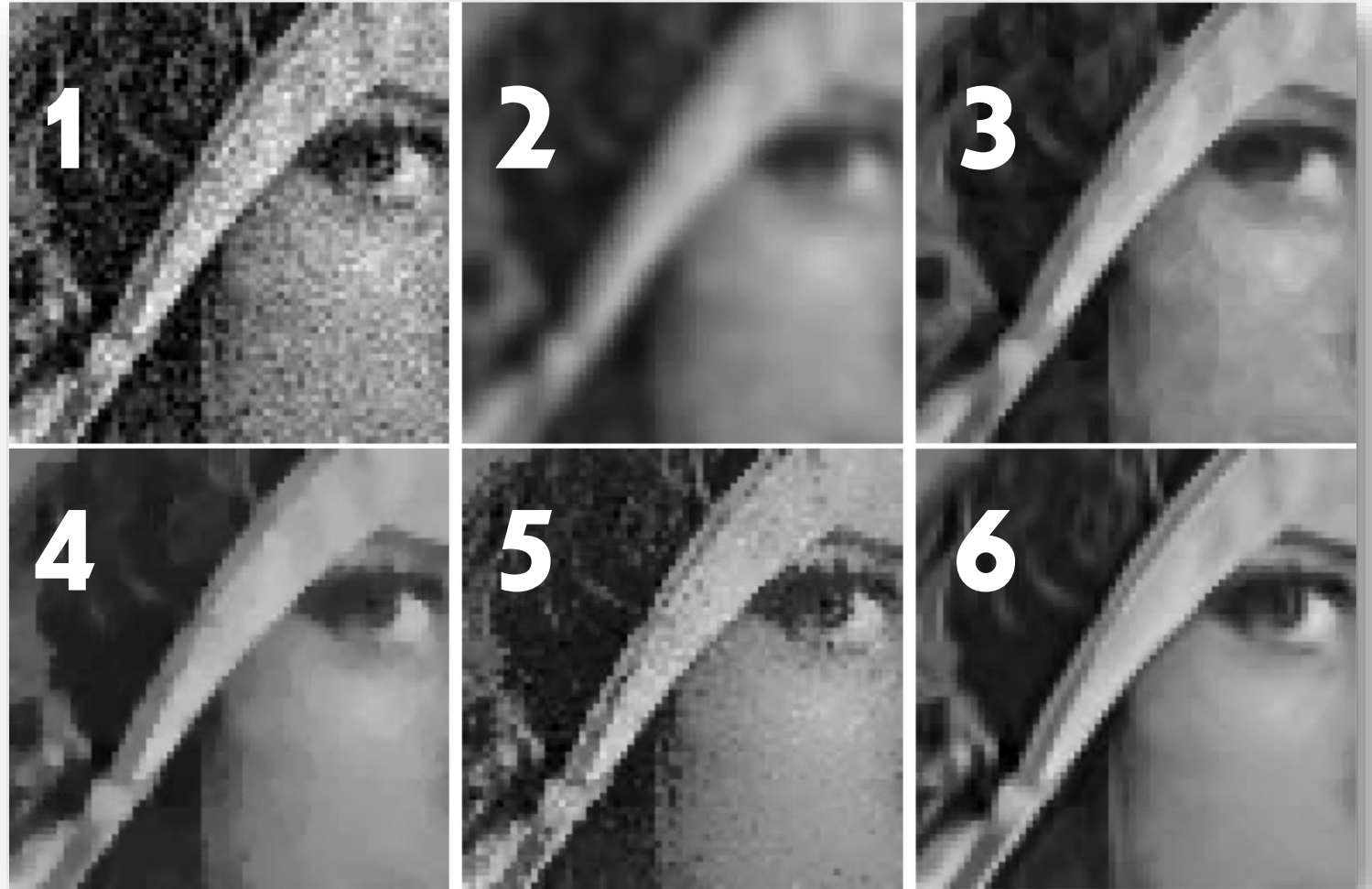


Image Source: A. Buades, B. Coll and J. -. Morel, "A non-local algorithm for image denoising"



# Comparisons – Method Noise

**Method noise looks like Gaussian white noise**

- 1) Noisy Image
- 2) Gauss Filtering
- 3) Anisotropic Filter
- 4) Total Variation
- 5) Neighborhood Filtering
- 6) Non-Local Mean

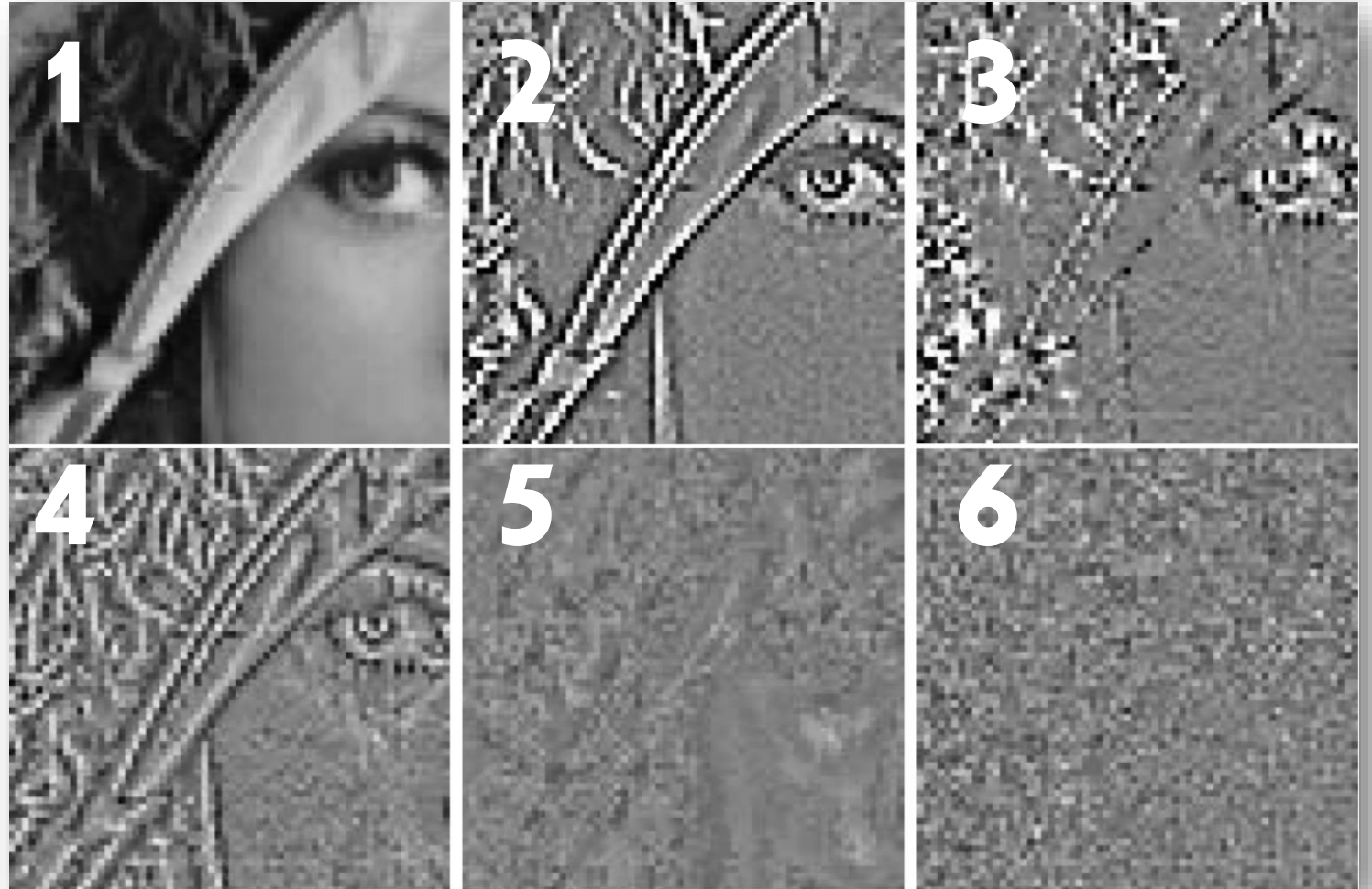


Image Source: A. Buades, B. Coll and J. -. Morel, "A non-local algorithm for image denoising"

# **Limitations & Further Work**

# Limitations & Further Works

## 01 Efficiency Concern

- Cannot be used on device that requires instant result e.g. phone
- When number of patches is too large or the image is too noisy, will require infeasible time to detect similarities
- Further work on improving runtime:
  - Thaipanich T, Oh BT, Wu PH, Xu DR, Kuo CCJ (2010) Improved image denoising with adaptive nonlocal means (ANL-means) algorithm.
  - Karnaukhov, V.N., Mozerov, M.G. Fast Non-Local Mean Filter Algorithm Based on Recursive Calculation of Similarity Weights.

# Limitations & Further Work

## 02 Types of Noise to Filter

- Original algorithm designed toward additive noise e.g. Gaussian noise, but not so much in multiplicative noise e.g. Speckle noise
  - Speckle noise often appear in Ultrasonic Imaging
- Further work on extension toward multiplicative noise:
  - Teuber T., Lang A. (2012) Nonlocal Filters for Removing Multiplicative Noise.
  - Radlak K., Smolka B. (2014) Adaptive Non-local Means Filtering for Speckle Noise Reduction

**THANKS**