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Assignment 3

Solution (1)

- Singularity is the configuration of robotic arms where it looses its one or more degrees of freedom. At singular configuration and near singular configuration many uncertain or abrupt change in joint variables, joint torque can be observed. Singularity is sort of unstable configuration.

- By equating the determinant of J^TJ (J is jacobian matrix) to zero, one can easily obtain the singular configurations. At the boundaries of workspace, we always have singularities and it is trivial singularity. However, singularities may exists in between the workspace also.

- For a perticular (given) configuration, we may determine the value of $\det(J^TJ)$ and if it comes out to near zero or approches to zero then we can say that given configuration is near to singular configuration.

Solution (2) Reading and Review of DH parameters is done.

Solution (3) Python code is attached with this PDF file.

Solution (4) Code developed in problem (3) is used and results obtained for both RRP Stanford manipulator and RRP SCARA manipulator matches with the textbook results.

Solution (5)

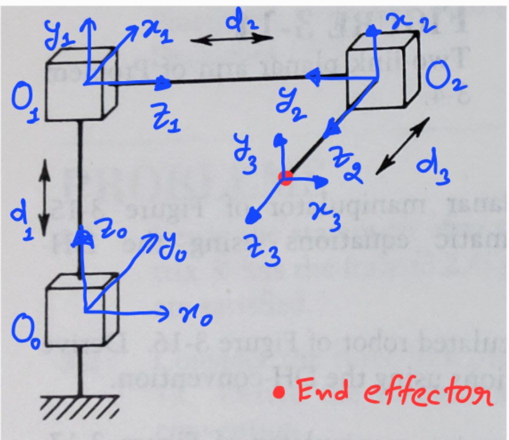


FIGURE 3-17
Three-link cartesian robot.

- Refrence frames drawn according to DH convention.
- d_1, d_2 and d_3 are joint variables for prismatic joints.
- DH parameter

link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	$\pi/2$
2	0	$-\pi/2$	d_2	$\pi/2$
3	0	0	d_3	$-\pi/2$

Now, we know that

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i}C_{\alpha_i} & S_{\theta_i}S_{\alpha_i} & a_iC_{\theta_i} \\ S_{\theta_i} & C_{\theta_i}C_{\alpha_i} & -C_{\theta_i}S_{\alpha_i} & a_iS_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

On substituting DH parameters for link 1, 2 and 3, we get

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, $T_0^1 = A_1$, $T_0^2 = A_1A_2$, $T_0^3 = A_1A_2A_3$

In problem only forward kinematic equations is asked. So, we only need to calculate T_0^3.

$$T_0^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orientation of end effector w.r.t. base frame $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

and position of end effector w.r.t. base frame $= \begin{bmatrix} d_2 \\ -d_3 \\ d_1 \end{bmatrix}$

- Moreover, Jacobian matrix and velocity vector for end effector is also obtained through code developed. (see python code)
- Results obtained through hand calculation matches with results obtained through python code.

Solution 6

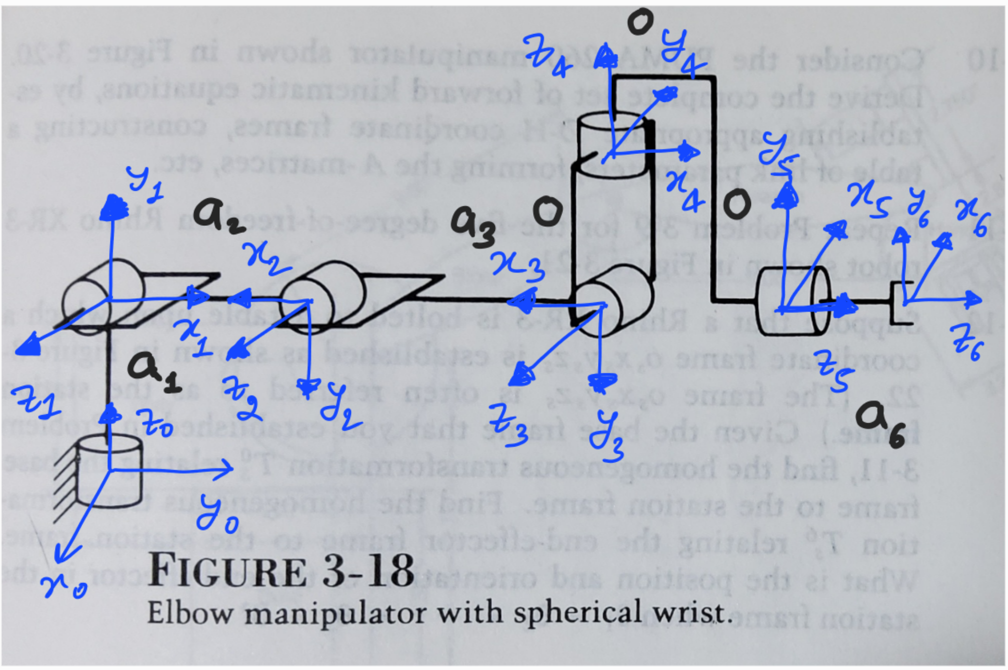


FIGURE 3-18
Elbow manipulator with spherical wrist.

- Refrence frame drawn according to DH convention
- a_1, a_2, a_3 and a_6 are link lengths.
- There are zero link lengths at spherical joints.
- $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ and θ_6 are joint variables.
- DH parameters

link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	a_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	$-\pi/2$	0	θ_4
5	0	$\pi/2$	0	θ_5
6	0	0	a_6	θ_6

Now, we know that

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- On substituting DH parameters for all the links we get A_1, A_2, A_3, A_4, A_5 & A_6 and then we calculate T_0^6 i.e. $A_1 A_2 A_3 A_4 A_5 A_6$.
- On performing calculations we get,

$$A_1 = \begin{bmatrix} C_{\theta_1} & 0 & S_{\theta_1} & 0 \\ S_{\theta_1} & 0 & -C_{\theta_1} & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & a_2 C_{\theta_2} \\ S_{\theta_2} & C_{\theta_2} & 0 & a_2 S_{\theta_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} C_{\theta_3} & -S_{\theta_3} & 0 & a_3 C_{\theta_3} \\ S_{\theta_3} & C_{\theta_3} & 0 & a_3 S_{\theta_3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} C_{\theta_4} & 0 & -S_{\theta_4} & C_{\theta_4} \\ S_{\theta_4} & 0 & C_{\theta_4} & S_{\theta_4} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C_{\theta_5} & 0 & S_{\theta_5} & 0 \\ S_{\theta_5} & 0 & -C_{\theta_5} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} C_{\theta_6} & -S_{\theta_6} & 0 & 0 \\ S_{\theta_6} & C_{\theta_6} & 0 & 0 \\ 0 & 0 & 1 & a_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

On multiplying these A_i in sequential order one may easily get T_0^6. (Done in python code)

Solution (7)

(i) Directly drive (2R Manipulator):

In this type of drive arrangement motors for each joint is mounted on the joint itself. This drive arrangement increases the weight also, along with this it takes a lot of space at each joint (circular housing need to be provide at each joint to hold the motor). More singular configurations in this drive as compare to others.

(ii) Remotely driven (2R Manipulator):

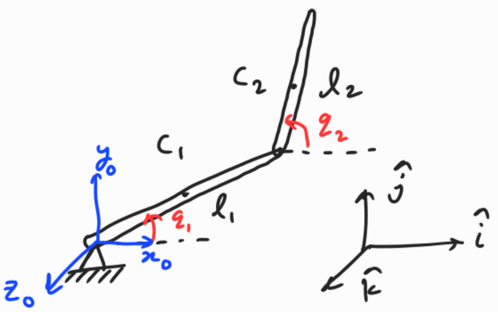
In this type of configuration motors are placed away from the joints, generally at the origin of the base inertial frame. It reduces the additional wt. of the motors from the link. Althoug it get difficult to use this drive as gear ratio increases. Rotation of the link (2) remains completely independent of the rotation of link (1). Less coriolis forces are also an advantage.

(iii) 5 bar link parallelogram (2R Manipulator):

Actually it contains 4 bars only but in the theory of mechanism it is a convention to count the ground as an additional link. Hence name is 5 bar link parallelogram. It forms close kinematic chain. The main advantage of this configuration is that, we get decoupled dynamic equilibrium equations, which are easy to solve. This type of configuration provide large workspace and majorly used in industries/assembly lines.

Solution (8) Remotely driven 2R Manipulator:

Dynamic equilibrium equation



Note: q_1 & q_2 are absolute angles.

- c_1 is center of mass (CM) of link (1)
- c_2 is CM of link (2)
- $\dot{q}_1$ and $\dot{q}_2$ are joint velocities.

location of $C_1 = \begin{bmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} \end{bmatrix} = \begin{bmatrix} \frac{l_1 \cos q_1}{2} \\ \frac{l_1 \sin q_1}{2} \\ 0 \end{bmatrix} \Rightarrow \quad v_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 & 0 \\ \frac{l_1}{2}\cos q_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$

location of $C_2 = \begin{bmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} \end{bmatrix} = \begin{bmatrix} l_1 c q_1 + \frac{l_2}{2} c q_2 \\ l_1 s q_1 + \frac{l_2}{2} s q_2 \\ 0 \end{bmatrix} \Rightarrow \quad v_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1 \cos q_1 & \frac{l_2}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$

For angular velocities of c_1 & c_2 we can straightforwardly write

$$\omega_1 = \dot{q}_1 \hat{k} \quad , \quad \omega_2 = \dot{q}_2 \hat{k}$$

Hence, kinetic energy of the manipulator equals

$$K = \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

where,

$$D(q) = \begin{bmatrix} m_1 \frac{l_1^2}{4} + m_2 l_2^2 + I_1 & m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

for calculating christoffel symbols $c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$

$$c_{111} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_1}\right] = 0$$

$$c_{121} = c_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = 0$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = -m_2 l_1 \frac{l_2}{2}\sin(q_2 - q_1)$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = m_2 l_1 \frac{l_2}{2}\sin(q_2 - q_1)$$

$$c_{212} = c_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Potential energy of manipulator system

$$V = m_1 g \frac{l_1}{2}\sin(q_1) + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2}\sin q_2\right)$$

Now $\phi_1 = \frac{\partial V}{\partial q_1} = m_1 g \frac{l_1}{2}\cos q_1 + m_2 g l_1 \cos q_1 = \left(\frac{m_1}{2} + m_2\right) g l_1 \cos(q_1)$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 g \frac{l_2}{2}\cos(q_2)$$

So, dynamic equilibrium equations are

$$\boxed{\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_2^2 + \phi_1 &= \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 &= \tau_2 \end{aligned}}$$

Solution (9) Review done!

Solution (10) Derivation of dynamic equilibrium equation, given $D(q)$ & $V(q)$

Let's consider kinetic energy (K) and potential energy (V)

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q} = \frac{1}{2}\sum_{i,j}^{n} d_{ij}(q)\dot{q}_i\dot{q}_j$$

Now, Euler-Langrange equation of system can be derived as follows.

$$L = K - V = \frac{1}{2}\sum_{i,j}^{n} d_{ij}(q)\dot{q}_i\dot{q}_j - V(q)$$

dynamic equations are given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \tau_k$ ——— (1)

let's compute $\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q)\dot{q}_j$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_j d_{kj}(q)\ddot{q}_j + \sum_j \frac{d}{dt} d_{kj}(q)\dot{q}_j$$

$$= \sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j}\frac{\partial d_{kj}}{\partial q_i}\dot{q}_i\dot{q}_j \quad \text{——— (2)}$$

and $\frac{\partial L}{\partial q_k} = \frac{1}{2}\sum_{i,j}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j - \frac{\partial V}{\partial q_k}$ ——— (3)

On substituting eq (2) & (3) in eqⁿ (1) we get,

$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i\dot{q}_j - \frac{\partial V}{\partial q_k} = \tau_k$$

since, $D(q)$ is positive definite and symmetric matrix, we have $d_{ij} = d_{ji}$

$$\Rightarrow \sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j}\frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k$$

$$\Rightarrow \boxed{\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j} c_{ijk}\dot{q}_i\dot{q}_j + \phi_k = \tau_k \quad , k = 1, 2, \ldots\ldots n}$$

where, $c_{ijk} = \frac{1}{2}\left\{\frac{\partial d_{ki}}{\partial q_j} + \frac{\partial d_{kj}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k}\right\}$ and $\phi_k = \frac{\partial V}{\partial q_k}$

Solution (11) Python code is attached with this file.

Link:
https://colab.research.google.com/drive/1xCryY38wYTZFyiqqvkhUJZ-Fl3TuZXzi?usp=sharing