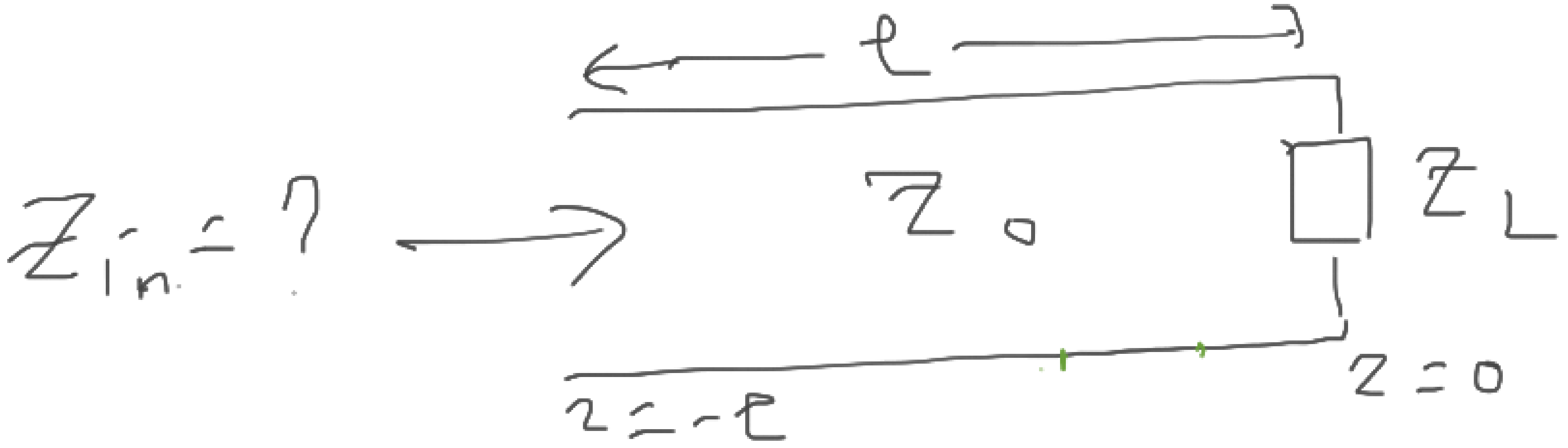
$\boxed{08/04/2021}$

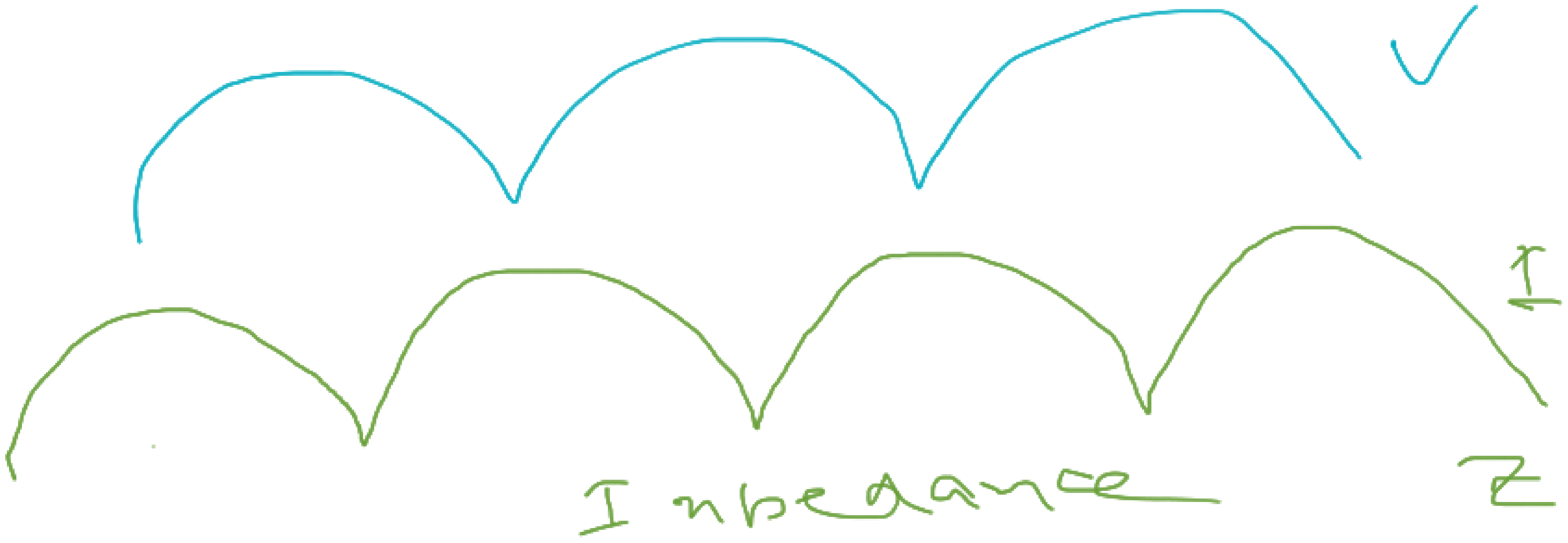
Input Impedance of a lossless transmission line (Z_{in})

Capital Z → impedance

Small z → distance / length

(3)





with $\gamma = j\beta$ for lossless line (ie, $\alpha = 0$),
we have

(9) : $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$

(10) : $I(z) = \dfrac{V_0^+}{Z_0} e^{-j\beta z} - \dfrac{V_0^-}{Z_0} e^{+j\beta z}$

$\left[\because \dfrac{V_0^+}{I_0^+} = Z_0 = -\dfrac{V_0^-}{I_0^-} \right]$

At distance $z = -\ell$ from load,
input impedance is given by

$$Z_{in}(z = -\ell) = \frac{V(z=-\ell)}{I(z=-\ell)}$$

$$Z_{in}(z=-l) = Z_0 \cdot \frac{V_0^+ e^{j\beta l} + V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l} - V_0^- e^{-j\beta l}}$$

(substituted $z = -l$ in (9) & (10) & divide]

$$= Z_0 \cdot \frac{\cancel{V_0^+}\left(e^{j\beta l} + \frac{V_0^-}{V_0^+} e^{-j\beta l}\right)}{\cancel{V_0^+}\left(e^{j\beta l} - \frac{V_0^-}{V_0^+} e^{-j\beta l}\right)}$$

$$= Z_0 \cdot \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}}$$

$$\left[\because \Gamma = \frac{V_0^-}{V_0^+}\right]$$

$$\Rightarrow Z_{in} = Z_0 \frac{e^{j\beta l} + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta l}}{e^{j\beta l} - \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{+j\beta l}}$$

$$\left[\because \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}\right]$$

$$\Rightarrow Z_{in} = Z_0 \cdot \frac{(Z_L + Z_0) e^{j\beta l} + (Z_L - Z_0) e^{-j\beta l}}{(Z_L + Z_0) e^{j\beta l} - (Z_L - Z_0) e^{-j\beta l}}$$

$$\Rightarrow Z_{in} = Z_0 \frac{Z_L (e^{j\beta l} + e^{-j\beta l}) + Z_0 (e^{j\beta l} - e^{-j\beta l})}{Z_L (e^{j\beta l} - e^{-j\beta l}) + Z_0 (e^{j\beta l} + e^{-j\beta l})}$$

Recall $e^{jx} = \cos x + j\sin x$ (trigonometric)

$\Rightarrow e^{j\beta l} = \cos(\beta l) + j\sin(\beta l)$

& $e^{-j\beta l} = \cos(\beta l) - j\sin(\beta l)$

Using above,

$$Z_{in} = Z_0 \frac{Z_L \cos(\beta l) + jZ_0 \sin(\beta l)}{jZ_L \sin(\beta l) + Z_0 \cos(\beta l)}$$

$$\Rightarrow Z_{in} = Z_0 \frac{Z_L + jZ_0 \frac{\sin(\beta l)}{\cos(\beta l)}}{Z_0 + jZ_L \frac{\sin(\beta l)}{\cos(\beta l)}}$$

$$\Rightarrow \boxed{\underset{(\Omega)}{Z_{in}} = Z_0 \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}}$$

$\beta = \frac{2\pi}{\lambda}$

for a lossless line ($\alpha = 0$ $\Rightarrow \gamma = \alpha + j\beta = j\beta$)

If line is not lossless, i.e., $\alpha \neq 0$ & $\gamma = \alpha + j\beta$

$$\underset{(\Omega)}{Z_{in}} = Z_0 \frac{Z_L + Z_0\tanh(\gamma l)}{Z_0 + Z_L\tanh(\gamma l)}$$

hyperbolic tan function

Recall (trigonometry)

hyperbolic function

$$\sinh(\beta l) = j\sin(\beta l)$$

$$\cosh(\beta l) = \cos(\beta l)$$

$$\tanh(j\beta l) = j\tan(\beta l)$$

Leave this slide (for lossy line)

$$e^{\pm\gamma l} = \cosh(\gamma l) \pm \sinh(\gamma l)$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$

$\tanh(\gamma l) = \tan(j\beta l)$ for lossless line ($\alpha = 0$) ie, $\gamma = j\beta$

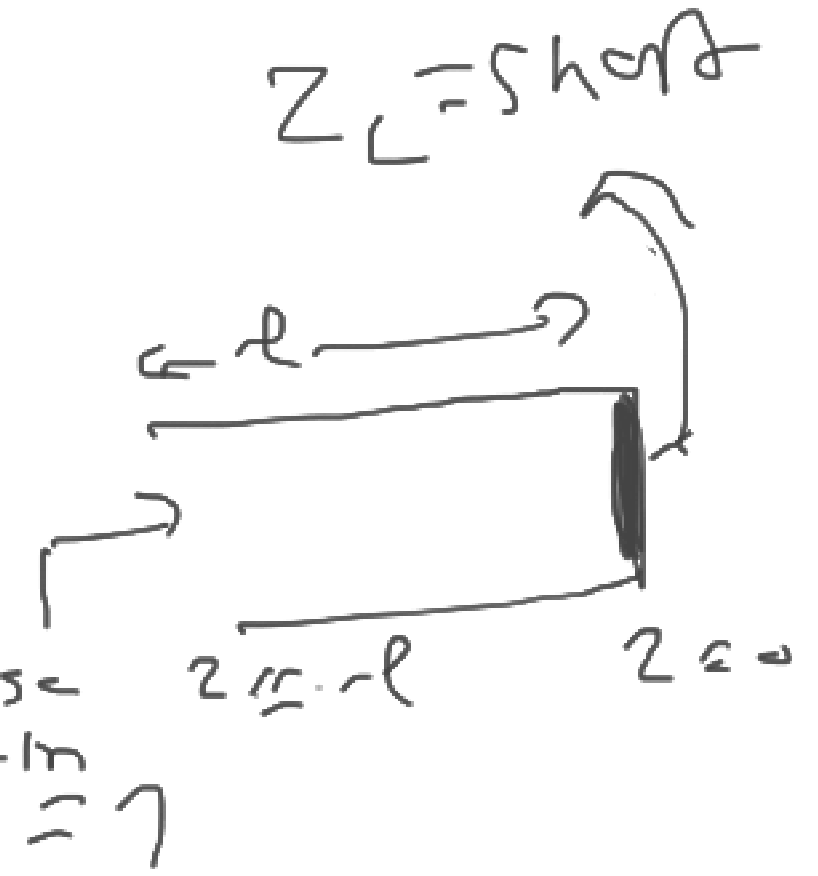
$$Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

$$Z_{in}(\Omega) = Z_0 \cdot \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \longleftarrow \text{A}$$

for lossless line (i.e, $\alpha = 0 \Rightarrow \gamma = j\beta$)

Z_{in} depends on Z_L & l

$\boxed{Z_{in} \text{ dependency on } Z_L}$



Case-I : $Z_L = 0$ (Short circuit)

$\therefore$ A : $Z_{in}^{sc} = Z_0 \dfrac{jZ_0 \tan\beta l}{Z_0}$

input impedance of a short circuit (s.c.) line $\Rightarrow$ $\boxed{Z_{in}^{sc} = jZ_0 \tan\beta l}$

no real part $\longleftarrow$ purely reactive line

$$Z_{in}^{sc} = jZ_0 \tan(\beta l)$$

At $l = 0$, $Z_{in}^{sc} = 0$ $\quad [\because \tan \beta l = 0]$

At $l = \frac{\lambda}{4}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$

$$\Rightarrow \tan \beta l = \tan\left(\frac{\pi}{2}\right) = \infty$$

$$\Rightarrow Z_{in}^{sc} = \infty$$

At $l = \frac{\lambda}{2}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$

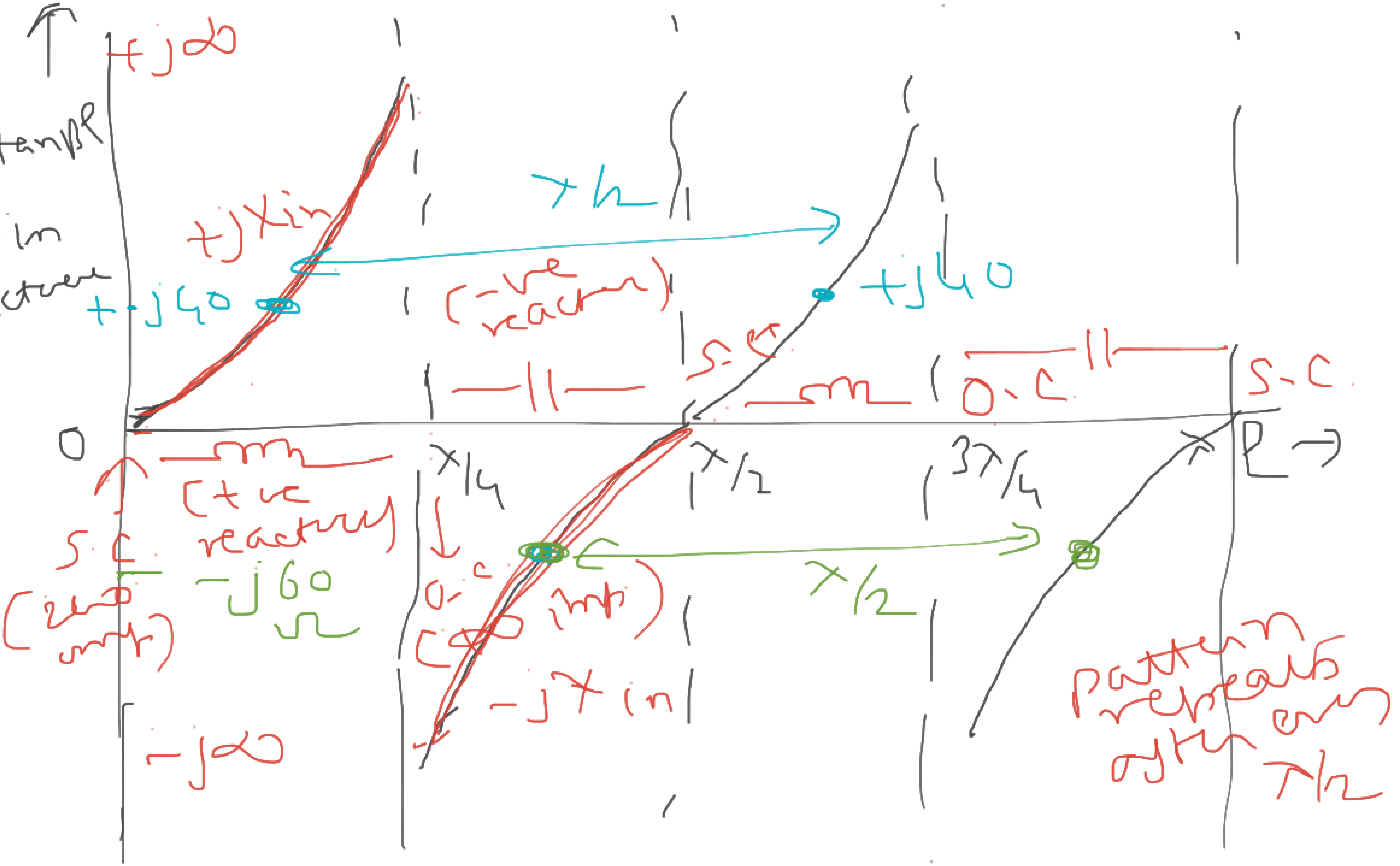
$$\Rightarrow \tan \beta l = \tan(\pi) = 0$$

$$\Rightarrow Z_{in}^{sc} = 0$$

$Z_{in}^{sc} = jZ_0 \tan\beta l$

$= jX_{in}$

X = reactance



Conclusion

1) An inductor of any value (from $j0$ to $+j\infty\,\Omega$) could be created using a short-circuited line (i.e, line terminated in $Z_L = 0$) provided that line length is between

0 to $\lambda/4$ or

$\frac{\lambda}{2}$ to $3\lambda/4$, or

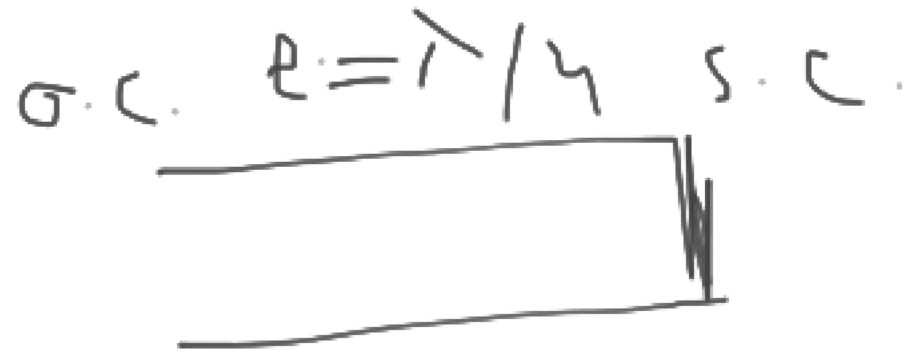
$\left(+\frac{n\lambda}{2}\right)$, $n = 0, 1, 2, \ldots$ multiples of $\lambda/2$

– . – –

2) S.C. $\frac{\lambda}{4}$ line acts as an inductor (RF choke)

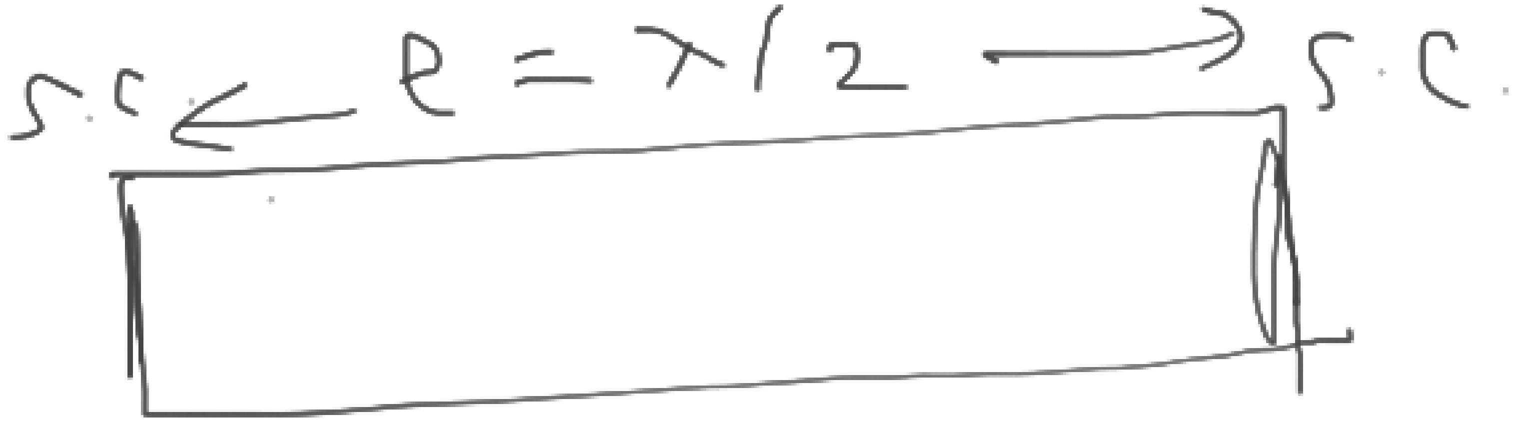
3) S.C $\ell = \lambda/4$ O.C

O.C $\lambda/4$ S.C



4) S.C $\lambda/2$ S.C

O.C $\lambda/2$ O.C



3) C or L $\lambda/2$ C or L