25/03/2021

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$ (m^{-1})

$\gamma = \alpha + j\beta$

α (dB/m or Np/m), β (rad/m)

$Z_0 = \sqrt{\dfrac{(R+j\omega L)}{(G+j\omega C)}}$ (Ω)

$v_p = \dfrac{\omega}{\beta}$, $\omega = 2\pi f$, $\beta = \dfrac{2\pi}{\lambda}$

case-I : Lossless line (Ideal) $\rightarrow$ $R=0,\ G=0$

$\therefore \gamma = \alpha + j\beta = j\omega\sqrt{LC} = 0 + j\omega\sqrt{LC}$

$\Rightarrow \boxed{\alpha = 0}$; $\boxed{\beta = \omega\sqrt{LC}}$ (linear fun. of ω)

$\therefore \boxed{v_p = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}}}$

$Z_0 = R_0 + jX_0$

$\boxed{R_0 = \sqrt{\dfrac{L}{C}},\quad X_0 = 0}$

Case-2 : Low loss line : $\boxed{\begin{array}{c} R << \omega L \\ \& \; G << \omega C \end{array}}$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right) \cdot j\omega L \cdot j\omega C}$$

$$= j\omega\sqrt{LC}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{1/2}$$

$$= j\omega\sqrt{LC}\left[1+\frac{R}{2j\omega L}\right]\left[1+\frac{G}{2j\omega C}\right]$$

$$\left[\because (1+x)^n = 1+nx+ --- \right]$$

$$\Rightarrow \gamma = \alpha + j\beta = j\omega\sqrt{LC}\left[1 + \frac{1}{2j\omega}\left(\frac{R}{L} + \frac{G}{C}\right)\right]$$

$$\cancel{-\frac{RG}{\omega^2 LC}} \rightarrow \text{neglect because } R \ll \omega L,\ G \ll \omega C$$

$$= j\omega\sqrt{LC} + \frac{\cancel{j\omega}\sqrt{LC}}{2\cancel{j\omega}}\frac{R}{L} + \frac{\cancel{j\omega}\sqrt{LC}}{2\cancel{j\omega}}\cdot\frac{G}{C}$$

$$\gamma = \alpha + j\beta = j\underbrace{\omega\sqrt{LC}}_{\beta} + \underbrace{\frac{1}{2}R\sqrt{\frac{C}{L}} + \frac{1}{2}G\sqrt{\frac{L}{C}}}_{\alpha}$$

$\beta \rightarrow$ linear fun. of freq

α (attenuat constant)

$$\boxed{v_p = \frac{\omega}{\beta} = 1/\sqrt{LC}}$$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{\frac{R}{j\omega L} + 1}{\frac{G}{j\omega C} + 1} \cdot \frac{\cancel{j}\omega L}{\cancel{j}\omega C}}$$

$$= \sqrt{\frac{L}{C}} \left(1 + \frac{R}{j\omega L}\right)^{1/2} \left(1 + \frac{G}{j\omega C}\right)^{-1/2}$$

$$Z_0 = \sqrt{\frac{L}{C}} \left[1 + \frac{1}{2j\omega}\left(\frac{R}{L} - \frac{G}{C}\right)\right]$$

$[(1+x)^n = 1 + nx + \cdots]$ neglecting terms $\frac{-RG}{4j\omega^2 LC}$ $\because R \ll \omega L$, $G \ll \omega C$

$$Z_0 = \sqrt{\frac{L}{C}}\left[1 + \frac{1}{2j\omega}\left(\frac{R}{L} - \frac{G}{C}\right)\right]$$

$$= R_0 + jX_0$$

$$\Rightarrow R_0 = \sqrt{\frac{L}{C}}$$

$$\& \; jX_0 = \sqrt{\frac{L}{C}} \cdot \frac{1}{2j\omega}\left(\frac{R}{L} - \frac{G}{C}\right) \approx 0$$

$$\left[\because R << \omega L \;\&\; G << \omega C\right]$$

case-3 : Distortionless line

losses of electric field = losses of magnetic field

$$\Rightarrow \quad RC = GL$$

$$\Rightarrow \quad \frac{R}{L} = \frac{G}{C} \quad \Rightarrow \quad G = \frac{RC}{L}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(R + j\omega L)\left(\frac{RC}{L} + j\omega C\right)}$$

$$= \sqrt{\frac{R^2 C}{L} - \omega^2 LC + j\omega RC + j\omega CR}$$

$$\gamma = \alpha + j\beta = \sqrt{\frac{R^2C}{L} - \omega^2 LC + 2j\omega CR}$$

$$= \sqrt{\frac{C}{L}\left(R^2 - \omega^2 L^2 + 2j\omega RL\right)}$$

$$= \sqrt{\frac{C}{L}\left(R + j\omega L\right)^2}$$

$$\gamma = \alpha + j\beta = \sqrt{\frac{C}{L}}\left(R + j\omega L\right)$$

$$\Rightarrow \boxed{\alpha = R\sqrt{\frac{C}{L}} = R\sqrt{G}} \quad \left[\because G = \frac{RC}{L}\right]$$

$$\boxed{\beta = \omega\sqrt{LC}} \qquad \boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}}$$

$$Z_0 = \underbrace{R_0 + jX_0}_{\downarrow \text{ reactance}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

(+ve : → inductive) → $+jX_0$

(−ve : → capacitive) → $-jX_0$

$$Z_0 = \sqrt{\frac{R\left(1+\frac{j\omega L}{R}\right)}{G\left(1+\frac{j\omega C}{G}\right)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$$\left[\because G = \frac{RC}{L}\right]$$

$\Rightarrow Z_0 = R_0 + jX_0$

$$\Rightarrow \boxed{R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}} \quad \text{and} \quad X_0 = 0}$$

Q: A 50 Ω distortionless line has attenuation of 0.01 dB/m.
The line has capacitance of 0.1 nF/m $\rightarrow$ (C)

→ Find R, L & G of line
→ " vel. of wave propagation
→ " loss of in 100 m long

Soln: $Z_0 = 50\,\Omega$, $C = 0.1\,nF/m = 10^{-10}\,F/m$

For distortionless line,

$$\frac{R}{L} = \frac{G}{C} \quad \text{--- (1)}$$

$$R_0 = \sqrt{\frac{L}{C}} \quad \text{--- (2)}$$

$$\alpha = R\sqrt{\frac{C}{L}} \quad \text{--- (3)}$$

3 eqns
3 unknowns (R, L, G)

7

② : $\sqrt{\frac{L}{C}} = 50 \; (= Z_0)$

③ : $R = \alpha \sqrt{\frac{L}{C}} = 0.057 \; \Omega/m$

② : $L = C R_0^2 = 0.25 \times 10^{-6} \; H/m$
$= 0.25 \; \mu H/m$

① : $G = \frac{RC}{L} = 22.8 \times 10^{-6} \; S/m$
$= 22.8 \; \mu S/m$
$(S = mho = ohm^{-1})$

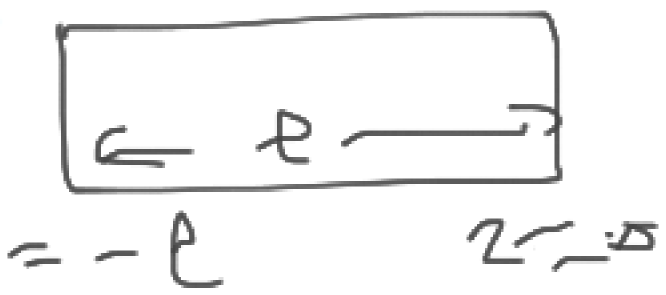
$\rightarrow v_p = \frac{1}{\sqrt{LC}} = 2 \times 10^{8} \; m/sec$

$\rightarrow \alpha = 0.01 \frac{dB}{m} \times 100 \, m = \underline{\underline{1 \, dB}}$

Voltage Reflection Coefficient (Γ or ρ)

→ Lossline assumed ($\alpha = 0$)

$\gamma = \alpha + j\beta \Rightarrow \gamma = j\beta$



Recall

(9): $V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$

(10): $I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$

$$= \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$\left(\because \gamma = j\beta \;;\; \frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-} \right)$$

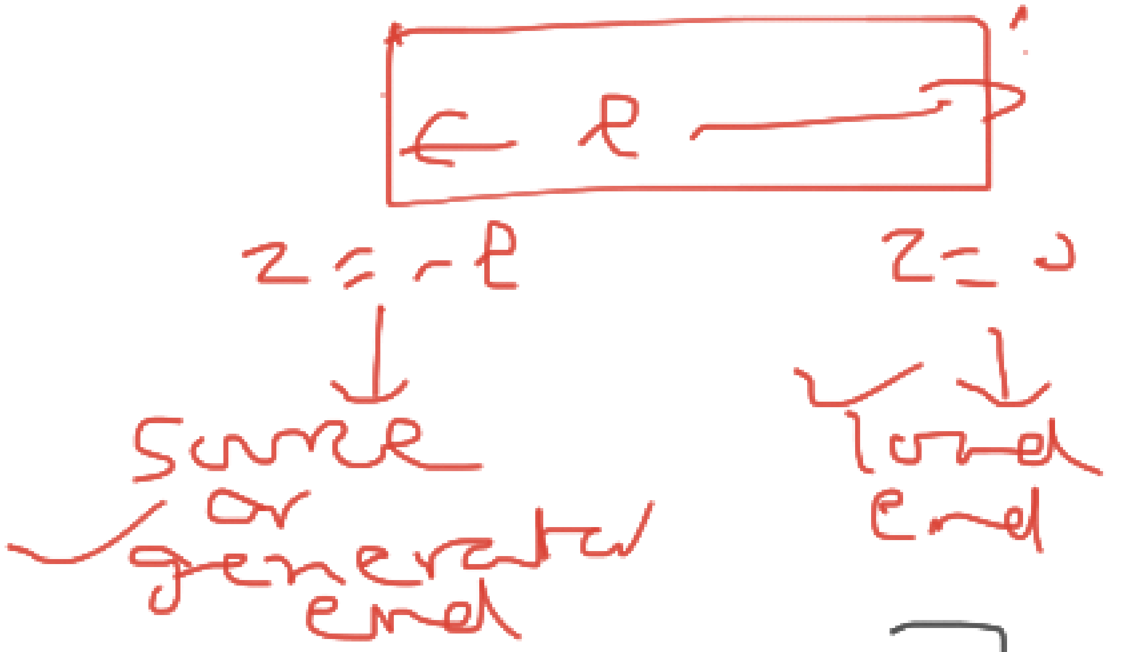
$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$

For lossless line ($\alpha = 0 \Rightarrow \gamma = j\beta$)

(9): $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$ — (A)

(10): $I(z) = \dfrac{V_0^+}{Z_0} e^{-j\beta z} - \dfrac{V_0^-}{Z_0} e^{j\beta z}$ — (B)

$$Z_L = \frac{V_L}{I_L}$$



Voltage V_L is voltage at load end (at $z = 0$)

$\therefore$ (A): $V_L = V(z=0) = V_0^+ + V_0^-$ $[\because e^0 = 1]$

& (B): $I_L = I(z=0) = \dfrac{V_0^+}{Z_0} - \dfrac{V_0^-}{Z_0} = \dfrac{1}{Z_0}\left[V_0^+ - V_0^-\right]$

$$\therefore Z_L = \frac{V_L}{I_L} = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0$$

$$\Rightarrow V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+$$

$$\Rightarrow \boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$ (dimensionless qty)

voltage reflection coefficient

(preferred over $\frac{I_0^-}{I_0^+}$)

→ ratio of amplitude (voltage) of forward travelling wave to that of reflected (or backward travelling wave)

Since, $\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-}$ from (13)

$$\Rightarrow \boxed{\frac{I_0^-}{I_0^+} = \frac{-V_0^-}{V_0^+} = -\Gamma}$$

current reflection coefficient

Since, Z_L is a complex qty,
$Z_0 = 50 + j0$

Γ is a complex qty, it can be written as

$$\Gamma = |\Gamma| e^{j\theta_r} = \underbrace{|\Gamma|}_{\text{magnitude of reflection coefficient}} \underbrace{\angle \theta_r}_{\text{angle of reflection coefficient}}$$

Case - I : Matched line (perfect match)

$$\boxed{Z_L = Z_0}$$ → desirable

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$ (no reflection)

Case - II : Short circuited line (shorted line)

extreme

$$\boxed{Z_L = 0}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 = 1\angle\pi = 1\angle 180^\circ$$

→ 100% reflection
& reversal of phase
of reflected wave

Case-III : Open-circuited line

extreme

$$\boxed{Z_L = \infty}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 1 = 1\angle 0^\circ$$

100% reflection with no phase change of reflected wave

In general,

$$\boxed{-1 < \Gamma < 1}$$

or $\boxed{|\Gamma| \text{ is between } \underset{(0\%)}{0} \text{ \& } \underset{(100\%)}{1}}$