23/05/2021 contd....

$$-\frac{dV(z)}{dz} = (R+j\omega L).I(z) \quad \text{—} ⑤$$

&

$$-\frac{dI(z)}{dz} = (G+j\omega C).V(z) \quad \text{—} ⑥$$

These are time harmonic trans. line eqns.

Here, V & I are coupled.

Eqns. ⑤ & ⑥ can be combined to solve for V(z) & I(z), i.e., how voltage (V) & current (I) vary along trans. line of length z.

To separate (decouple) $V(z)$ & $I(z)$:

Take $\frac{d}{dz}$ of one eqn, say (5) & substitute into another, (5), we get

(6): $$-\frac{d^2 I(z)}{dz^2} = (G+j\omega C)\frac{dV(z)}{dz}$$

$$\Rightarrow \frac{dV(z)}{dz} = -\frac{d^2 I(z)/dz^2}{(G+j\omega C)}$$

$$\Rightarrow -(R+j\omega L)\, I(z) = \frac{-d^2 I(z)/dz^2}{(G+j\omega C)}$$

[substituting (5)]

$$\Rightarrow \frac{d^2 I(z)}{dz^2} - (R+j\omega L)(G+j\omega C) I(z) = 0$$

$$\Rightarrow \boxed{\frac{d^2 I(z)}{dz^2} - \gamma^2 I(z) = 0} \quad \text{--- (7)}$$

Similarly,

$$\boxed{\frac{d^2 V(z)}{dz^2} - \gamma^2 V(z) = 0} \quad \text{--- (8)}$$

obtained by taking d/dz of (5) & substituting in (6)

where $\gamma^2 = (R+j\omega L)(G+j\omega C)$

$$\Rightarrow \boxed{\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}} \quad \text{A}$$

$\omega = 2\pi f$

Propagation constant (unit: m^{-1})

It is a complex qty of the form

$$\gamma = \alpha + j\beta$$

attenuation constant → real part

(Np/m)

or dB/m

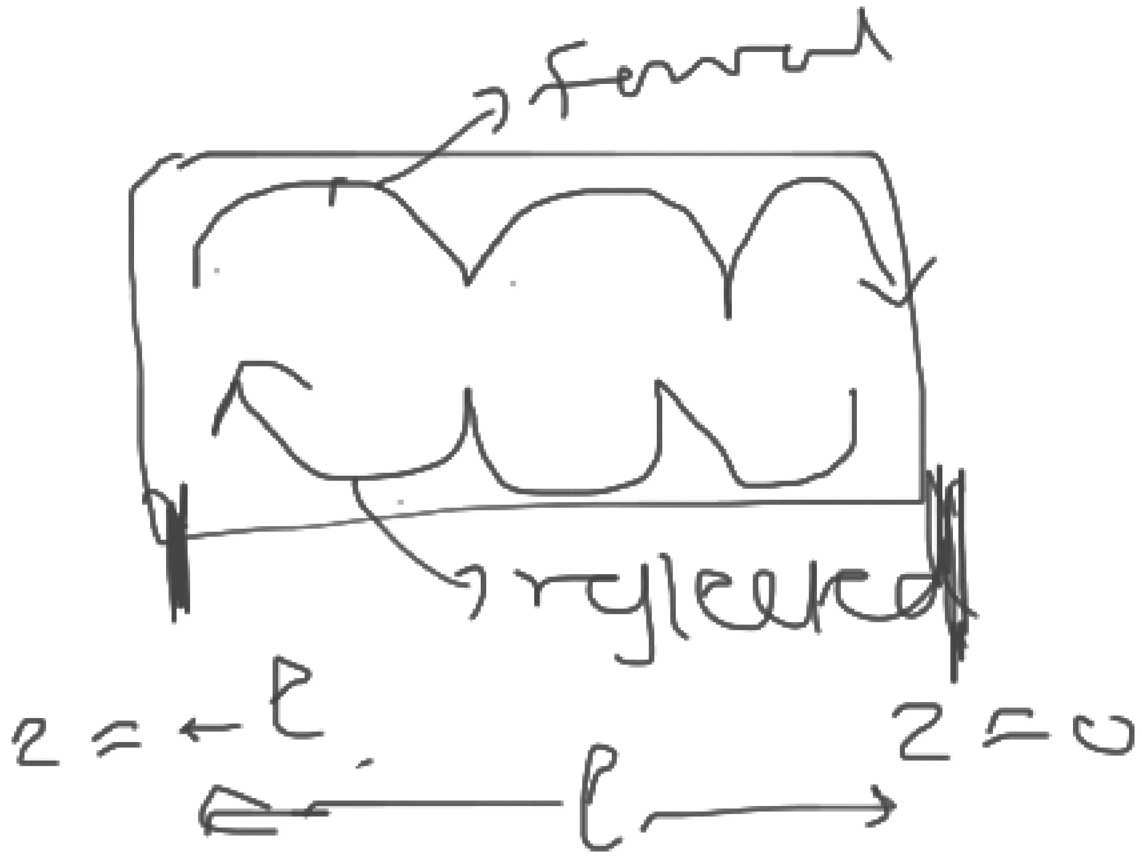
Phase constant

(rad/m)

To solve for $V(z)$ & $I(z)$ from (8) & (7) respectively, we assume that (8) & (7) have solutions of the form :-

$$V(z) = V^+(z) + V^-(z) \quad \Leftarrow \text{superposition}$$

$$\Rightarrow \boxed{V(z) = \underbrace{V_0^+ e^{-\gamma z}}_{\substack{\text{forward travelling wave} \\ \text{(incident)} \\ (+)}} + \underbrace{V_0^- e^{\gamma z}}_{\substack{\text{backward/reverse travelling wave} \\ \text{(reflected)} \\ [-]}}} \quad \text{--- (9)}$$



where $e^{-\gamma z}$ represent forward $(+z)$ travelling wave

& $e^{+\gamma z}$ " backward $(-z)$ travelling wave

& $I(z) = I^{+}(z) + I^{-}(z)$

$$\Rightarrow \boxed{I(z) = I_0^{+} e^{-\gamma z} + I_0^{-} e^{+\gamma z}} \quad (i)$$

(9) :- $V(z) = V_0^{+} e^{-\gamma z} + V_0^{-} e^{\gamma z}$

$$\Rightarrow \frac{dV(z)}{dz} = \frac{d}{dz}\left[V_0^{+} e^{-\gamma z} + V_0^{-} e^{\gamma z}\right]$$

$$\Rightarrow \frac{dV(z)}{dz} = \gamma\left[-V_0^{+} e^{-\gamma z} + V_0^{-} e^{\gamma z}\right] \quad -(i)$$

$\therefore$ (5) : $-\gamma\left[-V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}\right]$

$= (R+j\omega L)\, I(z)$

[from (i)]

$\Rightarrow I(z) = \dfrac{\gamma}{(R+j\omega L)}\left[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right]$ — (ii)

Compare (ii) with (1),

$\dfrac{V_0^+}{I_0^+} = Z_0 = \dfrac{-V_0^-}{I_0^-}$, where $Z_0 = \dfrac{(R+j\omega L)}{\gamma}$ — (iii)

$$\Rightarrow Z_0 = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}} \quad [\text{form } A]$$

$$\Rightarrow \boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}} \quad (\Omega) \; B$$

$\Omega \to$ Ohms

Characteristic impedance

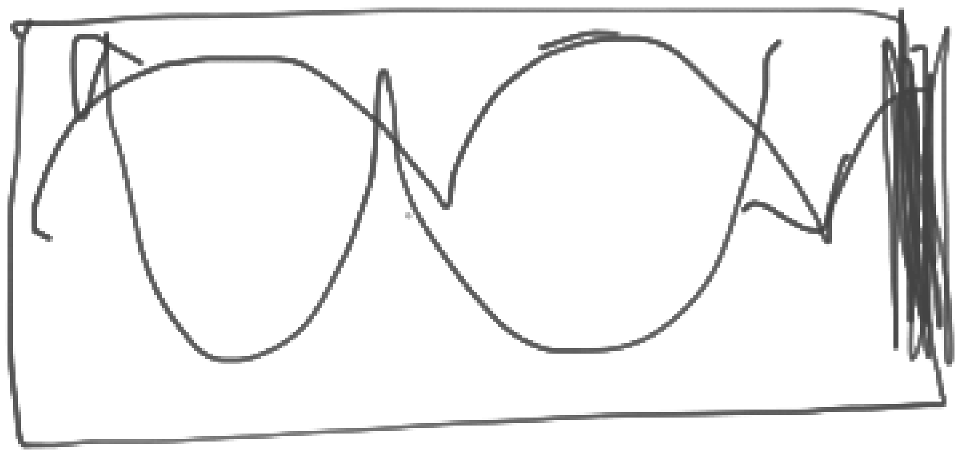
$\hookrightarrow$ natural impedance of a trans line if it were infinitely long owing to distributed capacitance, inductance as voltage & current waves "propagate" along its length with a velocity $v \left(= c/\sqrt{\epsilon_r}\right)$

$c = 3 \times 10^8$ m/s

(ii), Z_0 is defined as $\frac{V_0^+}{I_0^+}$ (or $-\frac{V_0^-}{I_0^-}$)

ratio of forward wave voltage amplitude to
" " current "

or = " " backward " voltage " to
" " current "



If $R \ll j\omega L$, & $G \ll j\omega C$

then

B : $\boxed{Z_0 = \sqrt{\dfrac{L}{C}}}$

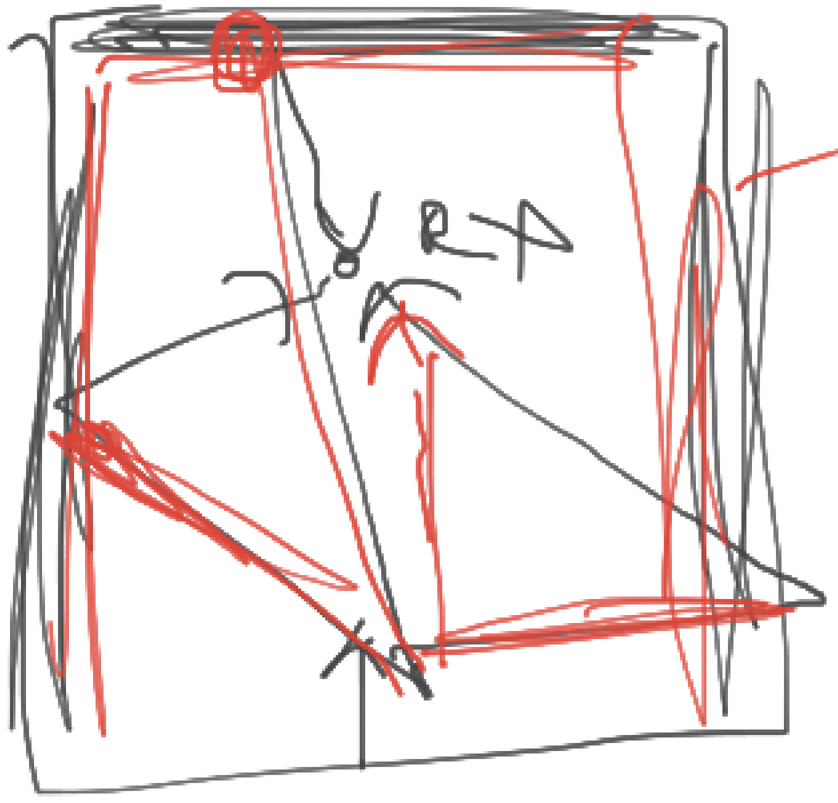
In,

B , typically, characteristic impedance

$Z_0 = 50\,\Omega$ (satellite, defence, radar RF circuits)

or

$75\,\Omega$ (for broadcasting industry)



absorbers

Z_0 = matched load

perfect matching

⇓ impedances are same

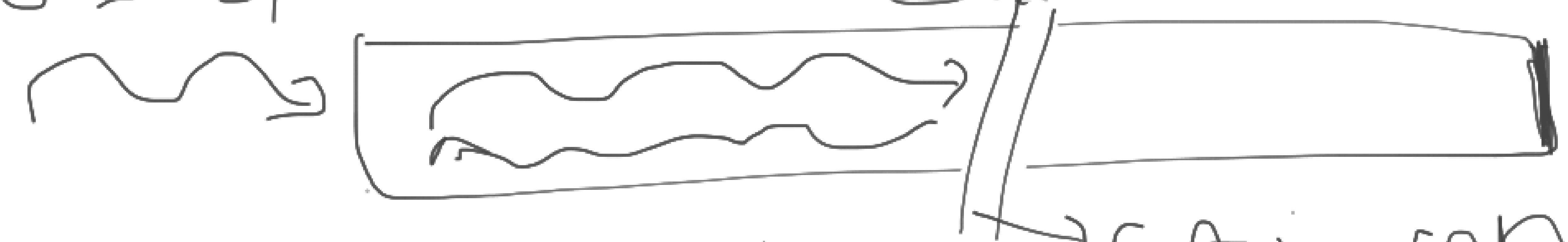
TDR (Time Domain Reflectometer)

in time

measures reflected signal

$v = c/\sqrt{\varepsilon_r}$

cable



3 (load matched) $50\,\Omega$

1 (open circuit)

Cut in cable

$d = v\Delta t$

2 Short circuit (discontinuous)

$\gamma = \alpha + j\beta$

α = attenuation constant

Np (neper)/m

dB (decibels)/m

$$\boxed{1\ Np = 8.686\ dB}$$

β = phase constant

= rad/m

$\beta = \frac{2\pi}{\lambda}$

λ = wavelength

$\lambda = c/f$

v_p (phase velocity)

→ vel of em signal in medium

$\boxed{v_p = \omega/\beta} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$ m/s in vacuum