06/April/2021

Standing Waves (contd.)

Recall

$$|V(z)| = |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta z + \theta_r)\right]^{1/2}$$

Maxima & Minima are formed due to — (C)
Superposition of two waves —

- forward travelling or incident wave

and

- backward travelling or reflected wave

Aim: To find amplitudes & positions of these maxima & minima along trans. line

Amplitude of maxima (peak value) correspond to $\cos(2\beta z + \theta_r) = 1$ in (C)

$$\therefore (C): \quad |V(z)|_{max} = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma|(1)\right]^{1/2}$$

$$= |V_0^+| \left[(1 + |\Gamma|)^2\right]^{1/2}$$

$$\boxed{|V(z)|_{max} = |V_0^+| \left[1 + |\Gamma|\right]} \quad \text{(D)}$$

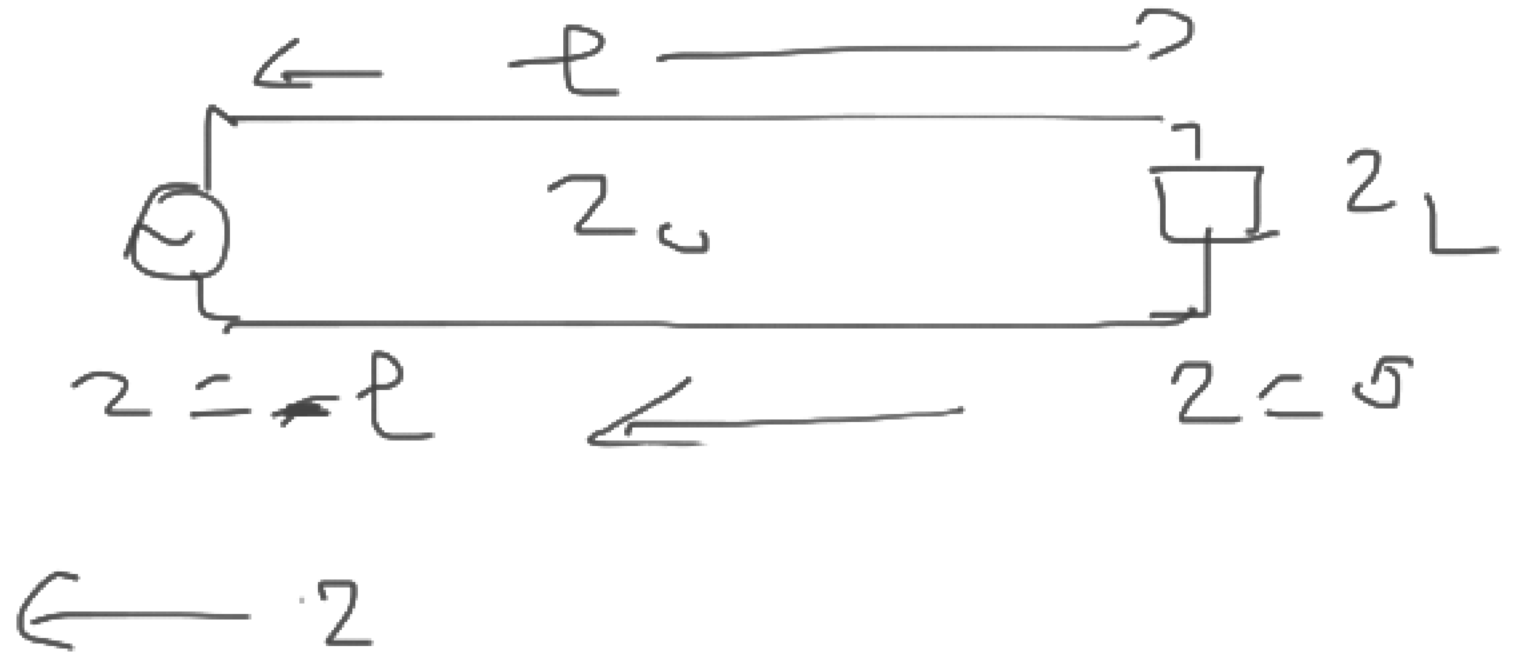
Sim, minima correspond to $\cos(2\beta z + \theta_r) = -1$ in (C)

$$\therefore (C): \quad |V(z)|_{min} = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma|(-1)\right]^{1/2}$$

$$= |V_0^+| \left[(1 - |\Gamma|)^2\right]^{1/2}$$

$$\boxed{|V(z)|_{min} = |V_0^+| \left[1 - |\Gamma|\right]} \quad \text{(E)}$$

Positions of Maxima & Minima along trans line



The max. value of standing wave pattern of $|V(z)|$ given by (e) corresponds to position on line at which incident & reflected waves are in phase, i.e., from (C)

$$(2\beta z + \theta_r) = 2n\pi,$$

$$n = 0, 1, 2, - - -$$

$\Rightarrow (2\beta z + \theta_r) = 2n\pi$, $n = 0, 1, 2, \text{—}$

$\Rightarrow (-2\beta \ell_{max} + \theta_r) = -2n\pi$, $z = -\ell_{max}$

$\Rightarrow -2\beta \ell_{max} = -2n\pi - \theta_r$

$\Rightarrow \ell_{max} = \dfrac{\theta_r + 2n\pi}{2\beta}$

$\Rightarrow \boxed{\ell_{max} = \dfrac{\theta_r \lambda}{4\pi} + \dfrac{n\lambda}{2}}$ $\left[\because \beta = \dfrac{2\pi}{\lambda}\right]$

First max occurs at

$\ell_{max_1} = \dfrac{\theta_r \lambda}{4\pi}$ for $n = 0$

Second max occurs at

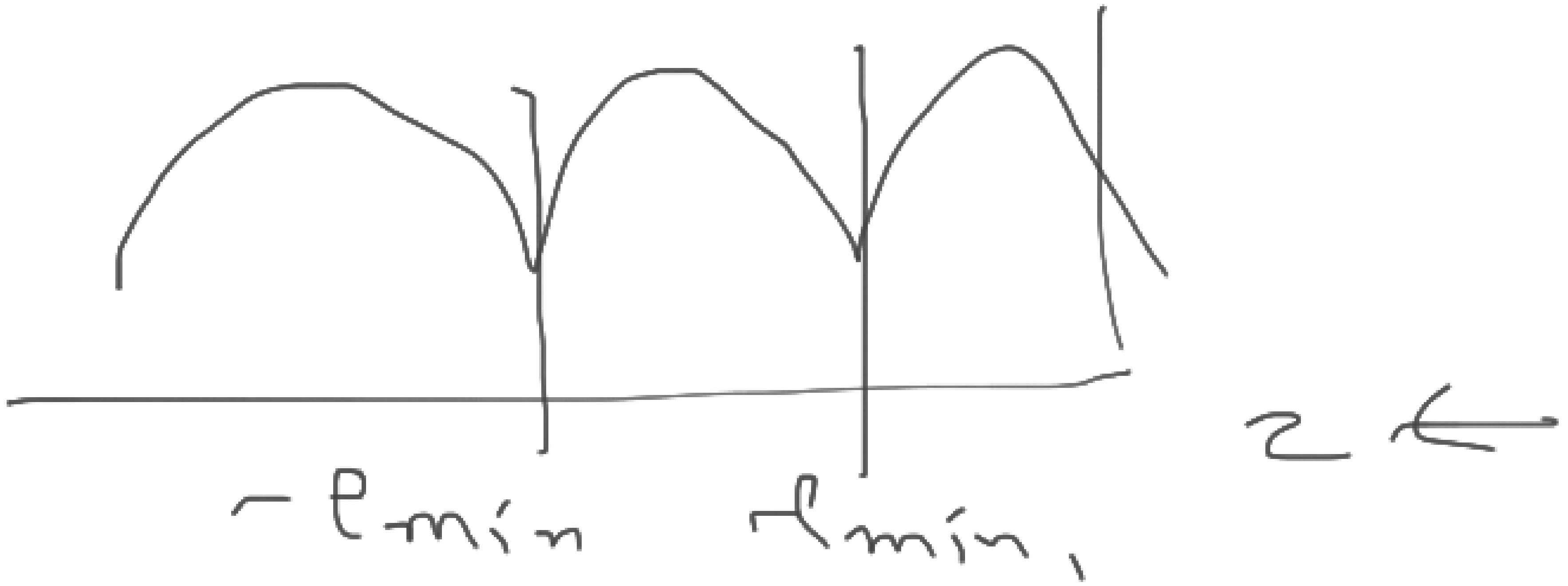
$\ell_{max_2} = \dfrac{\theta_r \lambda}{4\pi} + \dfrac{\lambda}{2}$ for $n = 1, \text{— — —}$

It implies that distance between two successive maximas is $\lambda/2$

The minimas of standing wave pattern of $V(z)$ given by (c) correspond to points on line at which incident & reflected waves are out of phase, i.e., in (c),

$$(2\beta z + \theta_r) = (2n+1)\pi, \quad n = 0, 1, 2, \ldots$$

$$\Rightarrow -2\beta \ell_{min} + \theta_r = -(2n+1)\pi$$



$$\Rightarrow \ell_{min} = \frac{\theta_r + (2n+1)\pi}{2\beta}$$

$$= \frac{\theta_r + (2n+1)\pi}{2\left(\frac{2\pi}{\lambda}\right)} \quad \left[\because \beta = \frac{2\pi}{\lambda}\right]$$

First min will occur at $n=0$

$$\ell_{min_1} = \frac{\theta_r \lambda}{4\pi} + \frac{\pi}{4\pi/\lambda}$$

$$= \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{4}$$

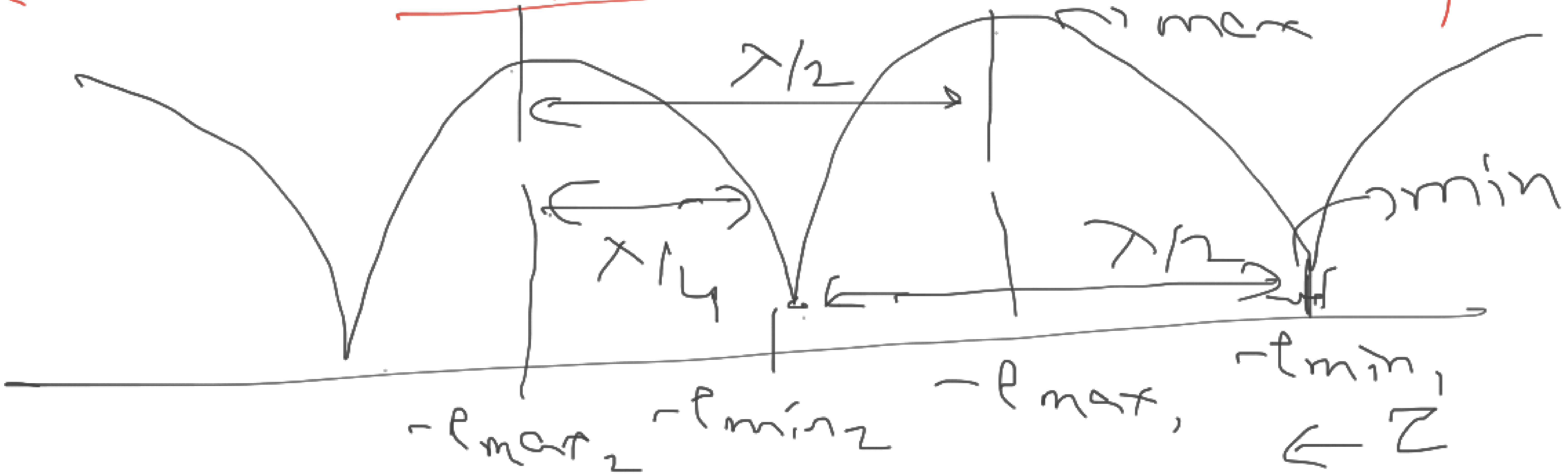
Second minimum will occur at $n=1$

$$\ell_{min_2} = \frac{\theta_r \lambda}{4\pi} + \frac{3\lambda}{4}, \; - - -$$

It implies that distance between two successive minimas $= \frac{\lambda}{2}$

(in like distance between two successive maximas $= \lambda/2$)

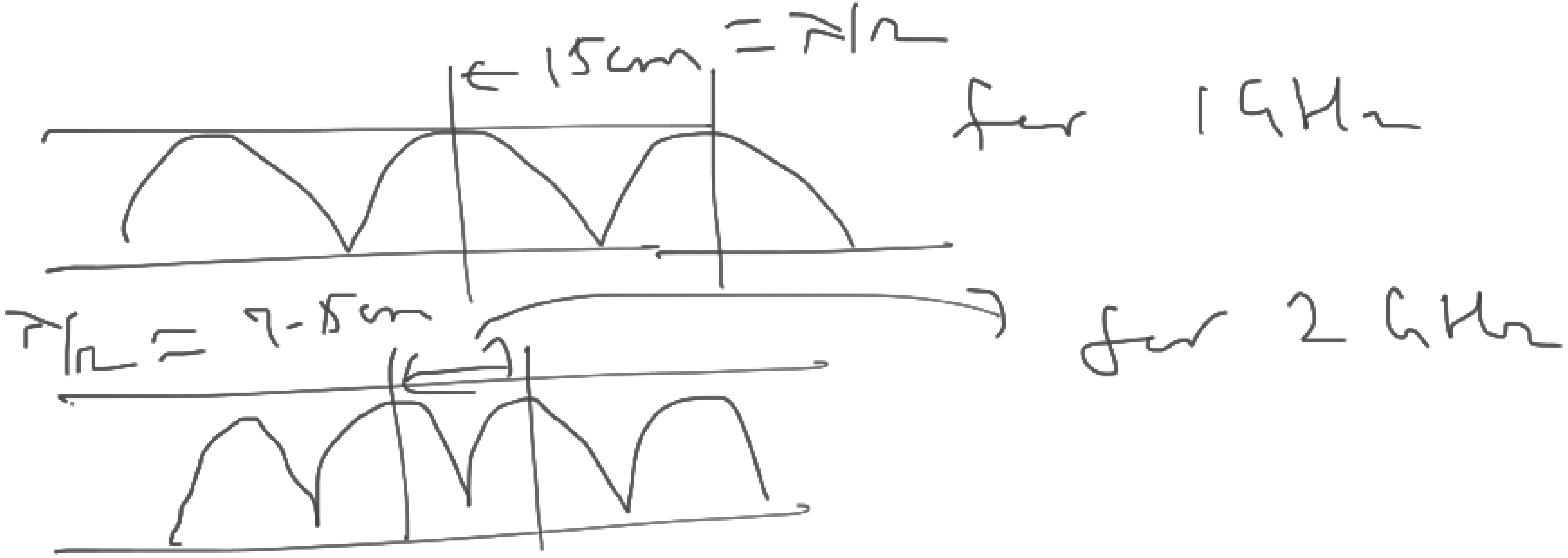
$\Rightarrow$ Distance between a maximum and a subsequent minimum $= \lambda/4$



* For a particular load, position of maxima & minima are always fixed. → waves appear to be standing → standing waves

* $f = 1\,GHz \Rightarrow \lambda = 30\,cm \Rightarrow \lambda/2 = 15\,cm$

$f = 2\,GHz \Rightarrow \lambda = 15\,cm \Rightarrow \lambda/2 = 7.5\,cm$



(*) → Pattern (voltage or current) of standing wave repeats after every $\lambda/2$

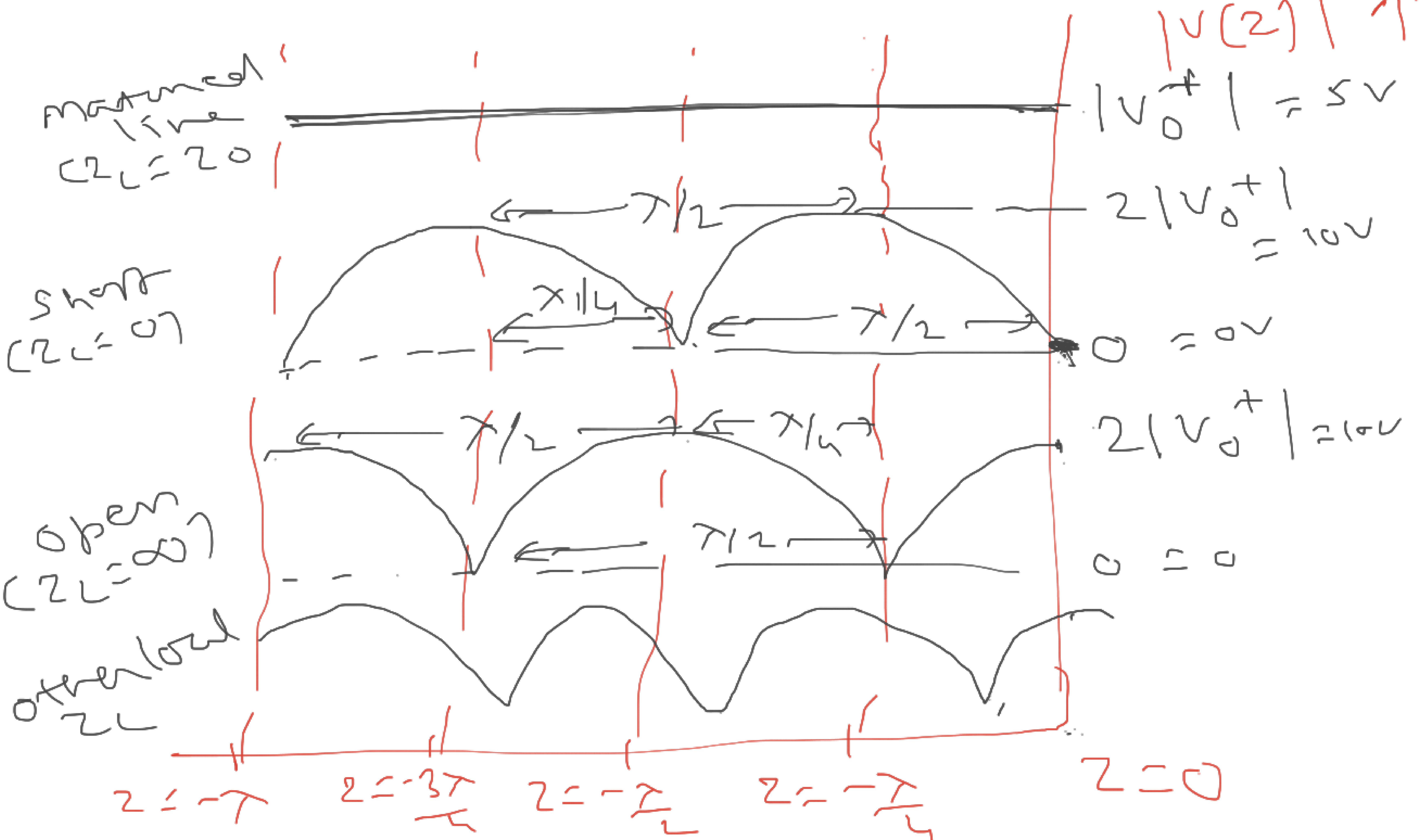
<u>cases</u>: 1) matched load (or perfect match)
ie, $Z_L = Z_0 \Rightarrow |\Gamma| = 0$

2) Short, ie, $Z_L = 0 \Rightarrow |\Gamma| = 1$, $\Gamma = -1 = 1\angle 180^\circ$

3) Open, ie, $Z_L = \infty \Rightarrow |\Gamma| = 1$, $\Gamma = 1$

Substituting for $|\Gamma|$, in $|V(z)|_{max}$ (D) &

$|V(z)|_{min}$ (E)



matched line
($Z_L = Z_0$)
$|V_0^+| = 5V$
$T/2$
$2|V_0^+| = 10V$
short ($Z_L = 0$)
$T/4$
$T/2$
$0 = 0V$
$T/2$
$T/4$
$2|V_0^+| = 10V$
open ($Z_L = \infty$)
$T/2$
$0 = 0$
other load Z_L
$|V(z)|$
$z = -T$
$z = -\frac{3T}{4}$
$z = -\frac{T}{2}$
$z = -\frac{T}{4}$
$z = 0$

Example

$$Z_0 = 50\,\Omega \quad , \quad \underline{Z_L = 75\,\Omega}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75-50}{75+50} = \frac{25}{125} = 0.2$$

$$|\Gamma| = 0.2$$

$$|V(z)|_{max} = \overbrace{|V_0^+|}^{5\,V}\left[1 + \overbrace{|\Gamma|}^{0.2}\right]$$

$$= 5 \times 1.2 = 6.0\ V$$

$$|V(z)|_{min} = |V_0^+|\left[1 - |\Gamma|\right]$$

$$= 5 \times 0.8 = 4.0\ V$$

VSWR (Voltage Standing Wave Ratio)

or SWR or 'S'

$$\boxed{VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

M.L. $\Rightarrow |\Gamma| = 0$, $VSWR = 1$ ideal case

Short $\Rightarrow |\Gamma| = 1$, $VSWR = \infty$

Open $\Rightarrow |\Gamma| = 1$, $VSWR = \infty$

$$\Rightarrow \boxed{1 \le VSWR \le \infty}$$

ideal case

$$\boxed{VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|V_{inc}| + |V_{ref}|}{|V_{inc}| - |V_{ref}|}}$$

Let $V_{inc} = 5\,V$

Let $V_{ref} = 2\,V$

$$\Rightarrow VSWR = \frac{|5| + |2|}{|5| - |2|} = \frac{7}{3} = 2.333$$

(between 1 and ∞)

$\boxed{\text{Return loss}}$ (dB)

$$dB = 20\log\frac{V_{out}}{V_{in}} = 10\log\frac{P_{out}}{P_{in}}$$

$$= \left[\because P = \frac{V^2}{R}\right]$$

VSWR

or $|\Gamma| \to$ voltage ref. coeff

$$\boxed{\text{Return loss (R.L.)} = -20\log(|\Gamma|)}$$

If $|\Gamma| = 0.25$,

$$\text{Return loss} = -20\log(0.25)$$

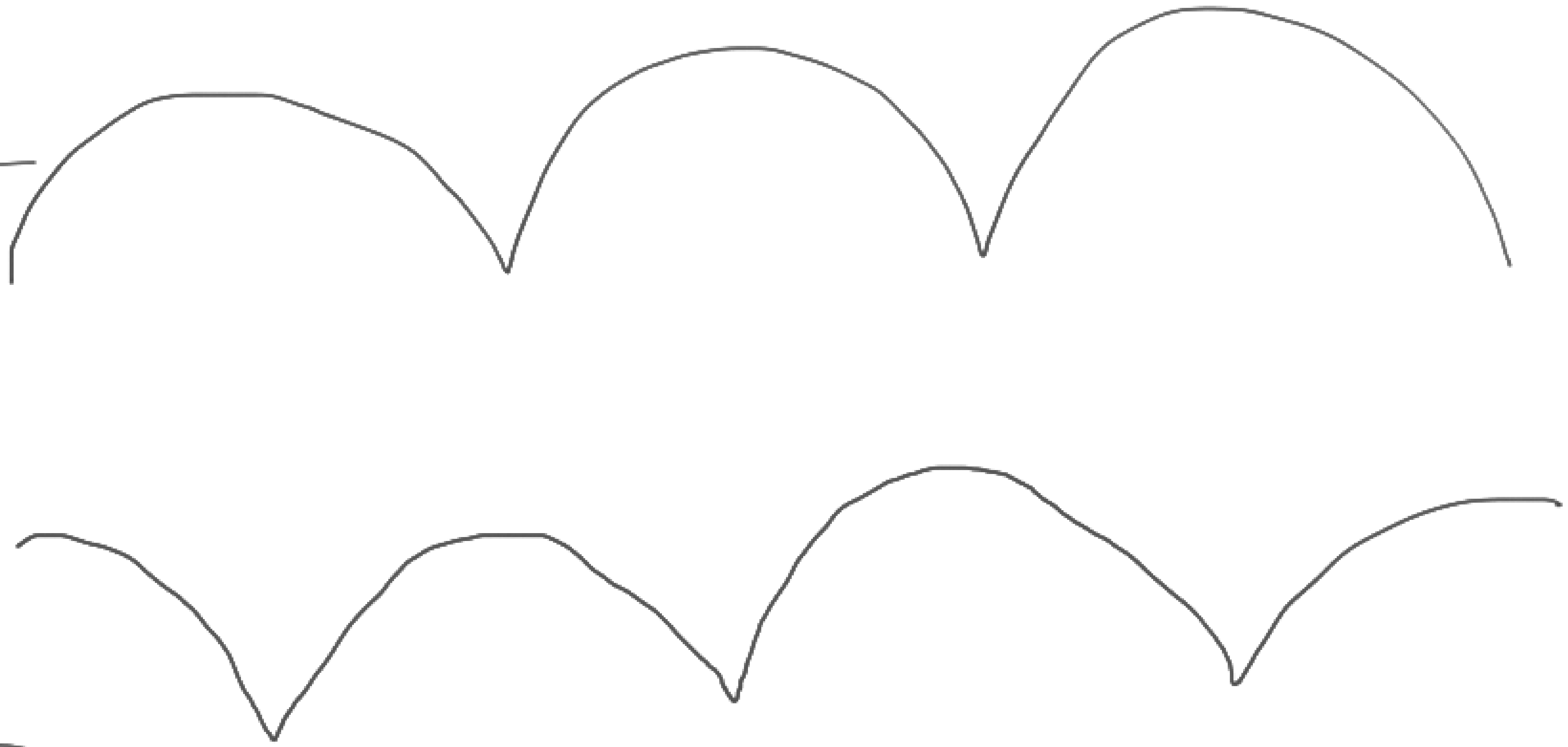
$$= \underline{\qquad\qquad} \text{ dB}$$

Summary

Termination of line	Z_L	$\lvert\Gamma\rvert$	Γ	VSWR
Matched or perfect (Ideal)	Z_0	0	0	1
Short	0	1	-1	∞
Open	∞	1	1	∞
any other load	...	...	...	...

voltage

current



(Standing wave)

Voltage maximum $\Leftrightarrow$ current minimum

current maximum $\Leftrightarrow$ voltage minimum

$\rightarrow \gamma = \alpha + j\beta$

γ: prop. constant (m^{-1})

α: attn constant (Np/m or dB/m)

β: phase constant (rad/m), $\beta = \frac{2\pi}{\lambda}$

$\Rightarrow \Gamma$, VSWR, R.L.

$\Rightarrow Z_0$

$\Rightarrow$ lossless ($\alpha = 0$), lowloss & distortionless ($R \ll \omega L$, $G \ll \omega C$) ($RL = GC$)

$\Rightarrow v_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{\omega}{\beta}$