01 | 04 | 2021

Recall : Voltage reflection coefficient (Γ or ρ)

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Z_L is complex qty
$\therefore \Gamma$ " " "

$$\frac{I_0^-}{I_0^+} = -\frac{V_0^-}{V_0^+} = -\Gamma$$

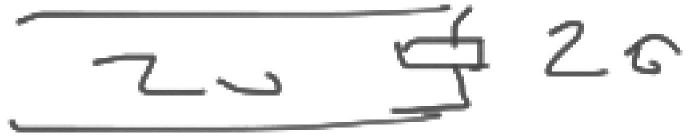
change of phase by 180°

$$\boxed{Z_L = Z_0\left(\frac{1+\Gamma}{1-\Gamma}\right)}$$

1. Matched load (perfect matched)

$$Z_L = Z_0 \Rightarrow \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

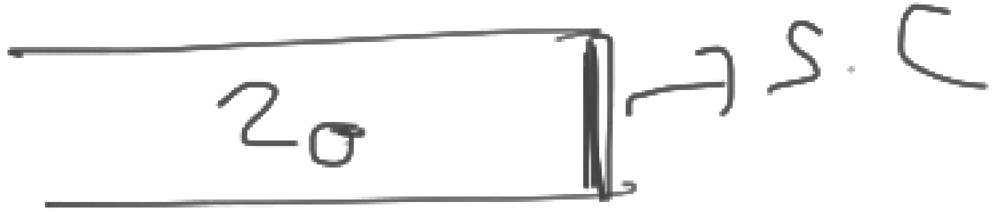
0% reflection



× 2. Short-circuited line

$$Z_L = 0 \Rightarrow \Gamma = -1 = 1\angle\pi = 1\angle 180^\circ$$

100% reflection with change of phase by 180° w.r.t. incident wave



× 3. Open-circuited line

$Z_L = \infty$

$\Rightarrow \Gamma = 1 \rightarrow$ 100% reflection with no change of phase

Ex: 50 Ω line terminated in a 75 Ω load

($Z_0 = 50\,\Omega$)

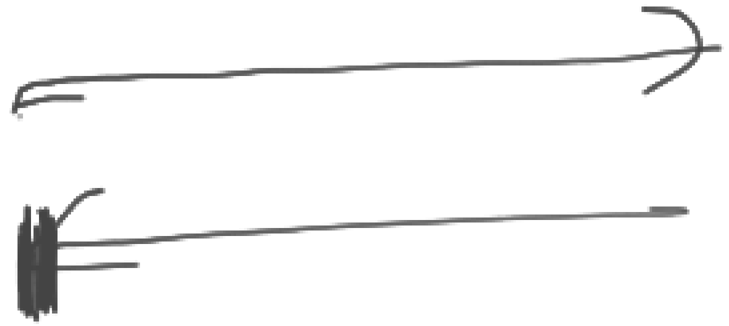
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75-50}{75+50} = \frac{25}{125} = \frac{1}{5} = 0.2$$

$$= 20\%$$

Application: TDR

(Time Domain Reflectometer)

measured reflected signal



$$v = \frac{3\times10^{8}\ \text{m/s}}{\sqrt{\epsilon_r}}$$

t = measured

distance = $v \times t$

$$\Gamma = |\Gamma| e^{j\theta_r} = |\Gamma| \angle \theta_r$$

magnitude of ref. coeff — $|\Gamma|$

angle of reflection coefficient — θ_r

$$\boxed{-1 \le \Gamma \le 1}$$

$$\boxed{0 \le |\Gamma| \le 1}$$

0% to 100%

complex nos.

$x + jy$ or $R\angle\theta$

$x = R\cos\theta$ | $R = \sqrt{x^2+y^2}$

$y = R\sin\theta$ | $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Example: A 100 Ω transmission line is connected to a load consisting of 50 Ω resistor in series with a 10 pF capacitor. Find reflection coefficient for 100 MHz signal.

Sol:

$Z_0 = 100\,\Omega$

$f = 100\,MHz \Rightarrow \omega = 2\pi f = 2\pi \times 10^8\,Hz$

$Z_L = 50\,\Omega$ resistor in series with a capacitor of $10\,pF$ $(10 \times 10^{-12}F)$

$Z_L = R_L + jX_C$

$= R_L - \dfrac{j}{\omega C} = 50 - \dfrac{j}{2\pi \times 10^8 \ldots}$

$= 50 - j159\,\Omega$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j159 - 50}{50 - j159 + 50}$$

$$= x + jy$$

$$\Downarrow$$

$$R \angle \theta$$

$$R = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= -0.76\, e^{+j119.3^\circ}$$

$$= 0.76\, e^{+j119.3^\circ}\, e^{-j180^\circ}$$

$$= +0.76\, e^{-j60.7^\circ}$$

$$= +0.76 \angle -60.7^\circ$$

$$|\Gamma| \angle \theta_r \quad \Downarrow \quad |\Gamma| = 0.76 = 76\%$$

reject

Show that $|\Gamma| = 1$ for purely reactive load

purely reactive $\Rightarrow R = 0$

$X = X_L$ or X_C

inductance / capacitive load

Impedance

$Z = 5 + j40\,\Omega$ ↳ inductive load

$Z = +j40\,\Omega$ ↳ purely inductive load

$Z = 20 - j30\,\Omega$ ↳ capacitive load

$Z = -j30\,\Omega$ ↳ purely capacitive load

Let Z_L be purely inductive load

$$\Rightarrow Z_L = jX_L$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX_L - Z_0}{jX_L + Z_0}$$

$$= \frac{-(Z_0 - jX_L)}{Z_0 + jX_L} = \frac{-\sqrt{Z_0^2 + X_L^2}\, e^{-j\theta}}{\sqrt{Z_0^2 + X_L^2}\, e^{j\theta}},$$

$$\Gamma = -e^{-j2\theta} \quad \text{where } \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{X_L}{Z_0}\right)$$

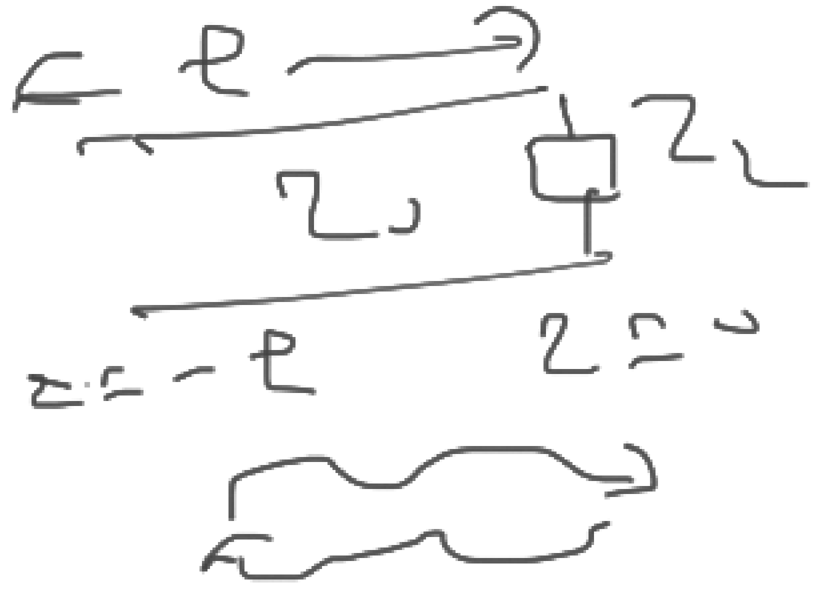
$$\therefore |\Gamma| = \left| e^{-j2\theta} \right| = \left[\left(e^{-j2\theta}\right)\underbrace{\left(e^{-j2\theta}\right)^*}_{\text{complex conjugate}} \right]^{1/2}$$

$$\Rightarrow \boxed{|\Gamma| = 1}$$

$\boxed{\text{STANDING WAVES}}$

Assume line to be lossless, i.e., $\alpha = 0 \Rightarrow \gamma = \alpha + j\beta = \underline{j\beta}$

↳ For this, we recall



(9) or (A) : $V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$

(10) or (B) : $\frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$

Using $\frac{V_0^-}{V_0^+} = \Gamma \Rightarrow \underline{\underline{V_0^- = \Gamma V_0^+}}$

$\therefore$ (A) : $V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z} \right)$

(B) : $I(z) = \frac{V_0^+}{Z_0} \left(e^{-j\beta z} - \Gamma e^{j\beta z} \right)$

$$|V(z)| = \left[V(z) \cdot V^*(z) \right]^{1/2}$$

complex conjugate

$$= \left[\left\{ V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z} \right) \right\} \cdot \left\{ (V_0^+)^* \left(e^{j\beta z} + \Gamma e^{-j\beta z} \right) \right\} \right]^{1/2}$$

$$= \left[\left\{ V_0^+ \left(e^{-j\beta z} + |\Gamma| e^{j\theta_r} e^{j\beta z} \right) \right\}^* \left\{ (V_0^+)^* \left(e^{j\beta z} + |\Gamma| e^{-j\theta_r} e^{-j\beta z} \right) \right\} \right]^{1/2}$$

$$\left[\because \Gamma = |\Gamma| e^{j\theta_r} \right]$$

$$\Rightarrow |V(z)| = |V_0^+| \left[1 + |\Gamma|^2 + |\Gamma| \left(e^{j(2\beta z + \theta_r)} + e^{-j(2\beta z - \theta_r)} \right) \right]^{1/2}$$

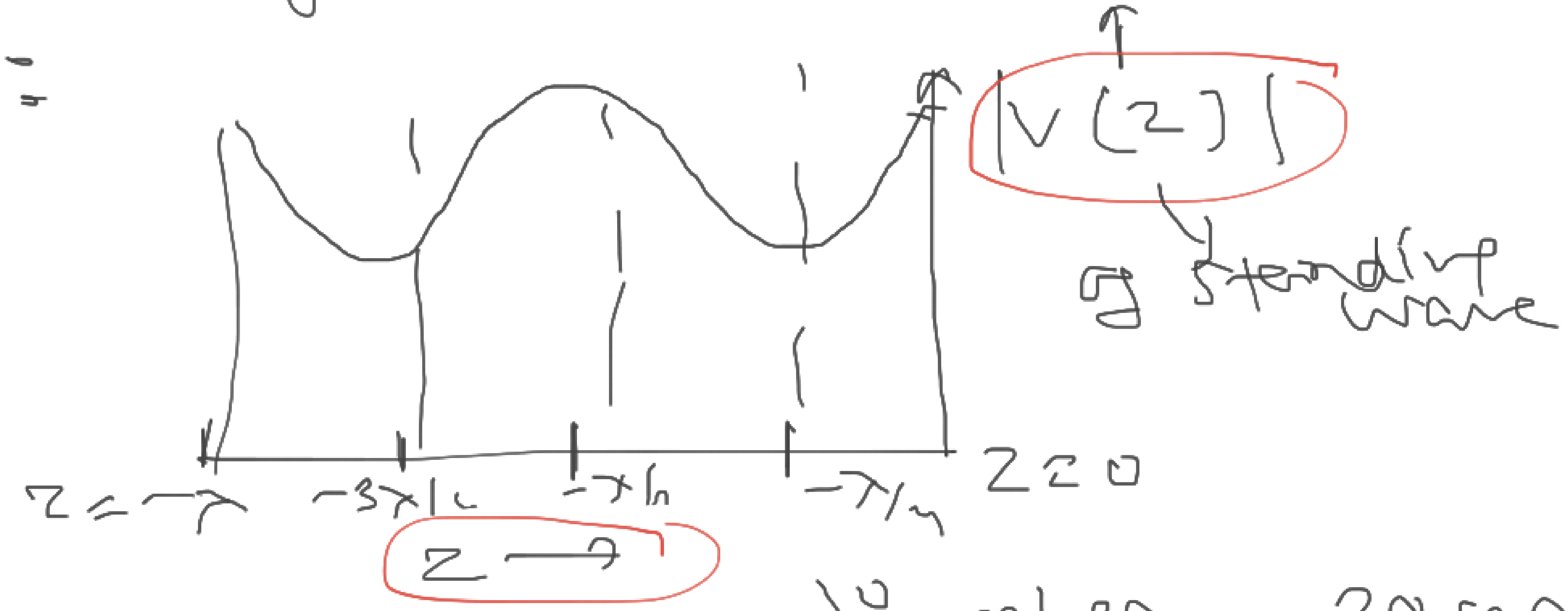
$$\Rightarrow |V(z)| = V_0^+ \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta z + \theta_r) \right]^{1/2} \tag{c}$$

$$\left[\because e^{jx} + e^{-jx} = 2\cos x \right]$$

Similarly, we can derive for $I(z)$

Here, $\beta = \frac{2\pi}{\lambda}$

The variation of $|V(z)|$ & $|I(z)|$ as a function of z (position on transmission line):



Ex: At 1 GHz $(1\times10^{9}\,Hz)$, $\lambda = \frac{c}{f} = \frac{3\times10^{10}\ cm/sec}{1\times10^{9}} = \underline{30\ cm}$

Electrical length = λ at 1 GHz

If length = 15 cm $\Rightarrow$ electrical length = $\frac{\lambda}{2}$ at 1 GHz

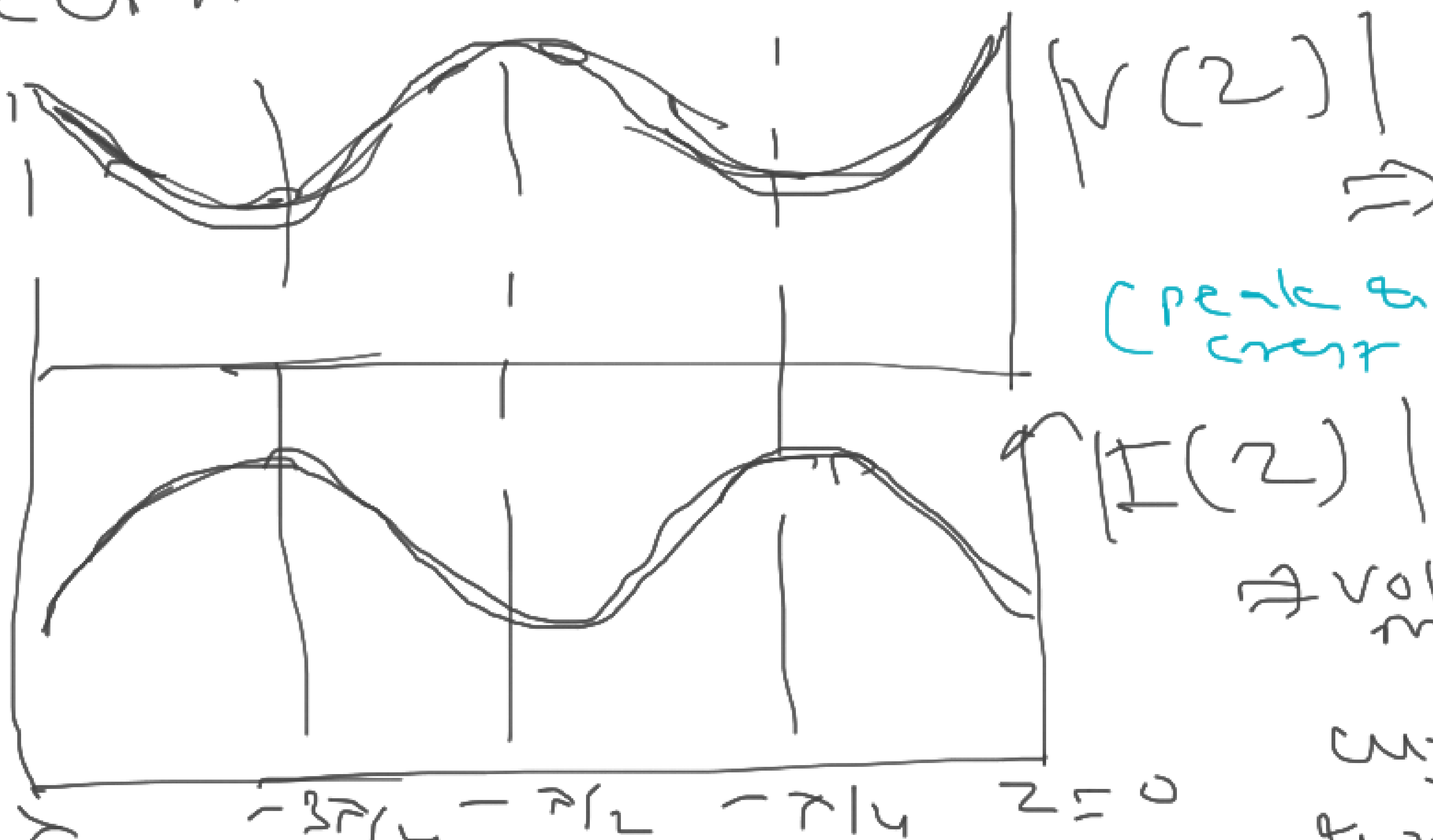
If freq 2 GHz, $\lambda = 15$ cm

$\Rightarrow$ 1 λ @ 2 GHz (for 15 cm)

0.5 λ @ 2 GHz (for 7.5 cm)

$\Downarrow$

electrical length



standing wave (superposition of incident & reflected wave)

↳ travelling wave →

$\Rightarrow$ maxima & minima

(peak & null / crest & trough)

$\Rightarrow$ voltage maxima

$\Downarrow$

current minima & vice-versa

Standing wave : means positions of maxima & minima always remain same for a particular load

Positions of maxima & minima
& amplitude " " " "
for a given load (S.C, O.C, matched load (M.L.) or any other load in general [$Z_L = R_L + jX$])
to be determined

The amplitude of maxima & minima ?

The max. value of standing wave pattern of $|V(z)|$ corresponds to position on line at which incident & reflected waves are in phase.

[incident → forward travelling wave
reflected → backward " "]

i.e., $(2\beta z + \theta_r) = 2n\pi$

$z = -\ell$ $\quad$ $z = 0$

in c

or, $-2\beta\ell_{max} + \theta_r = -2n\pi$,

$n = 0, 1, 2, 3, \ldots$

$0, 2\pi, 4\pi, 6\pi, \ldots$

$$\therefore \; |V(z)|_{max} = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma|(1)\right]^{1/2}$$

from Eqn C

$$= |V_0^+| \left[(1 + |\Gamma|)^2\right]^{1/2}$$

$$\boxed{|V(z)|_{max} = |V_0^+| \left[1 + |\Gamma|\right]}$$

m.L. if $|\Gamma| = 0$, $|V(z)|_{max} = |V_0^+|$

if $|\Gamma| = 1$, $|V(z)|_{max} = 2|V_0^+|$

s.c. o.c. if $|\Gamma| = 0.3$, $|V(z)| = |V_0^+| \left[1 + 0.3\right]$