Source coding: data compression

Channel Coding: data transmission

What is communication?

A communicates with B:
physical acts of A
have induced desired
physical state in B.

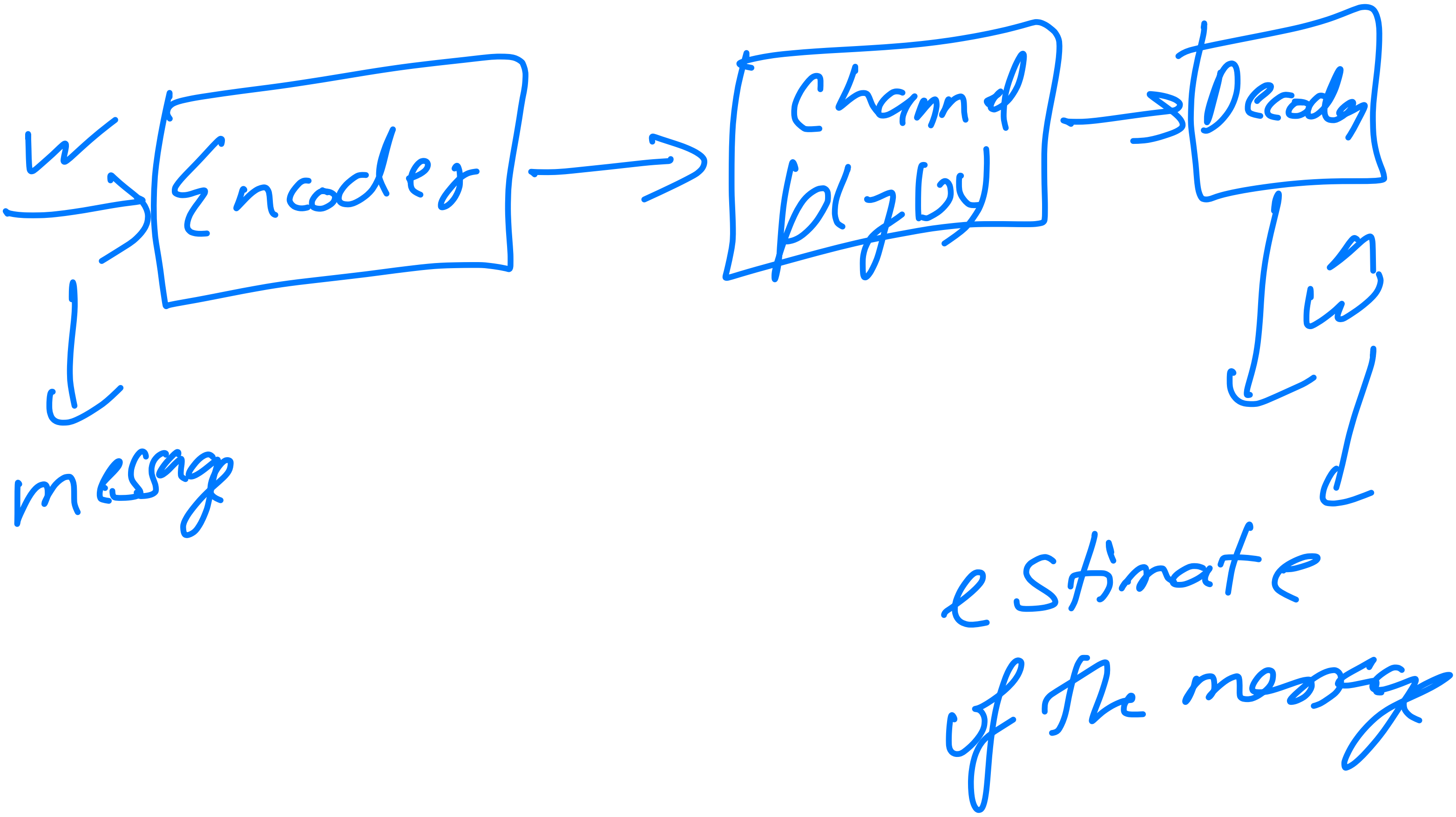
A communication is
successful when receier
B and sender A agree
on what was sant.

Definition: Discrete Channel.

A discrete channel is a system consisting of an input alphabet $\mathcal{X}$ and an output alphabet $\mathcal{Y}$ and a probability transition matrix $p(y|x)$ that expresses the probability of observing the output y when we send x.

The channel is called memoryless if the probability distribution of the output depends only on the input at that time and is conditionally independent of previous channel inputs/outputs.

Communication System



Information Channel Capacity

Defn: The information channel capacity of a discrete memoryless channel (DMC) as

$$C = \max_{p(x)} I(X;Y)$$

Operational channel capacity: highest rate in bits per channel use at which information can be sent with arbitrarily low prob of error.

They are the same.

Channel Capacity

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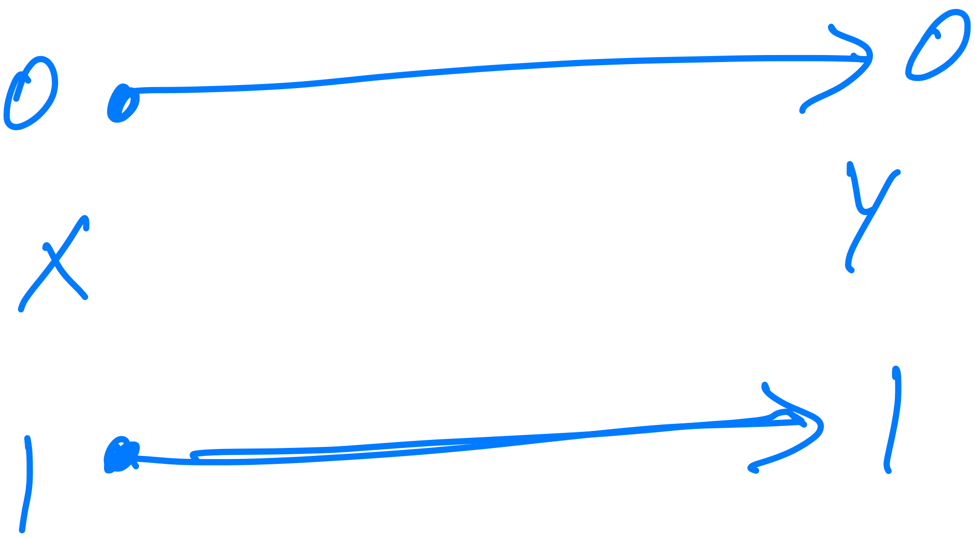
e.g. Noiseless Binary Channel



Operational channel capacity:
$= 1$ bit

Information channel capacity

$$C = \max_{p(x)} I(X;Y)$$

$$I(X;Y) = H(X) - H(X|Y)$$
$$= H(Y) - H(Y|X)$$

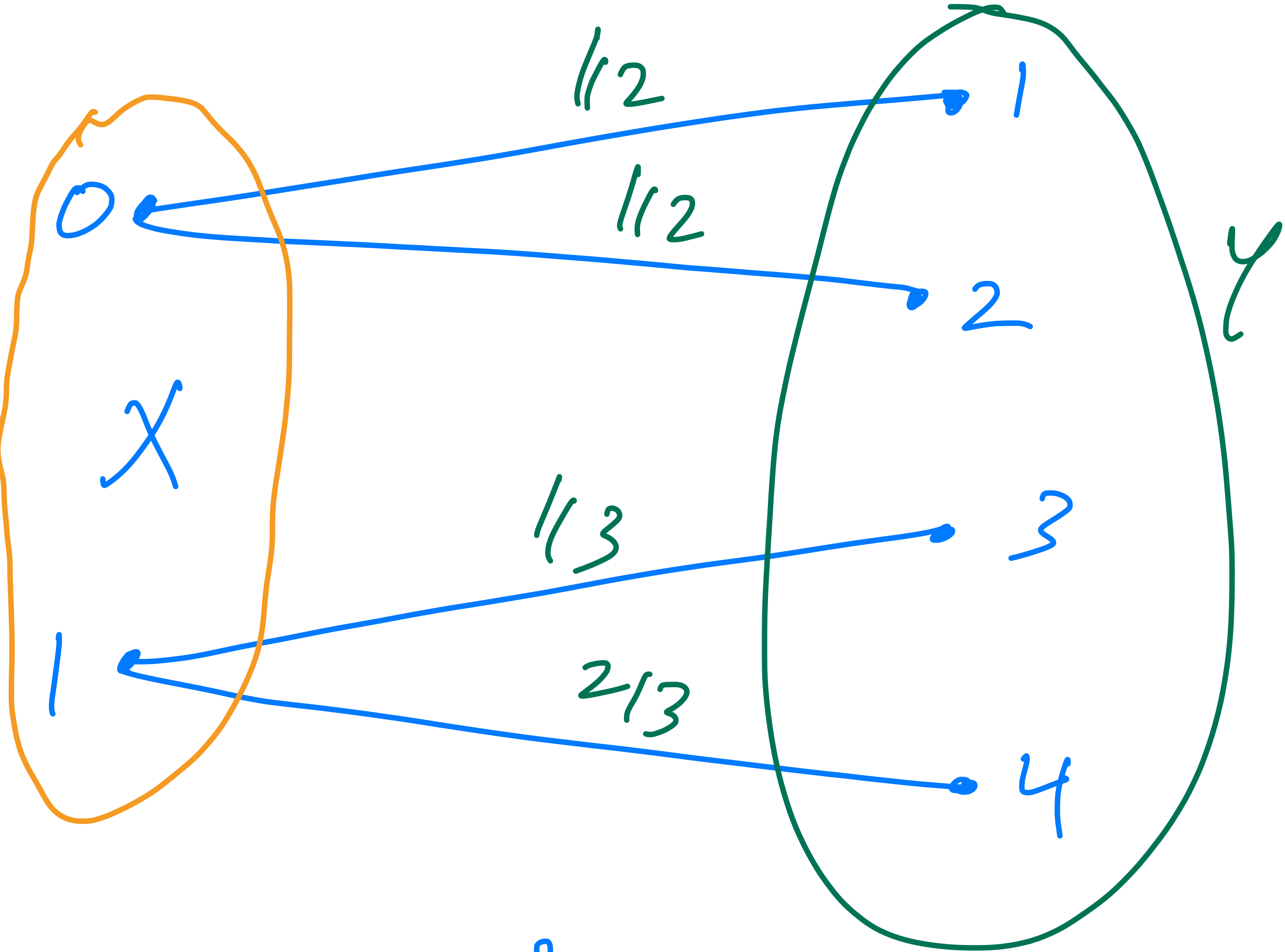
$$\max_{p(x)} \left[\underset{?}{\underset{\parallel}{H(X)}} - \underset{\overset{?}{0}}{\underset{\parallel}{H(X|Y)}} \right]$$

$$= \max_{p(x)} H(X)$$

$$= 1, \text{ for } p(x=0) = p(x=1) = \frac{1}{2}$$

$$p(x=0) = \varepsilon = 1/2$$
$$p(x=1) = 1 - \varepsilon = 1/2$$

e.g: Noisy channel with nonoverlapping outputs



$X = \{0, 1\}$

$Y = \{1, 2, 3, 4\}$

Operational channel capacity:

= 1 bit

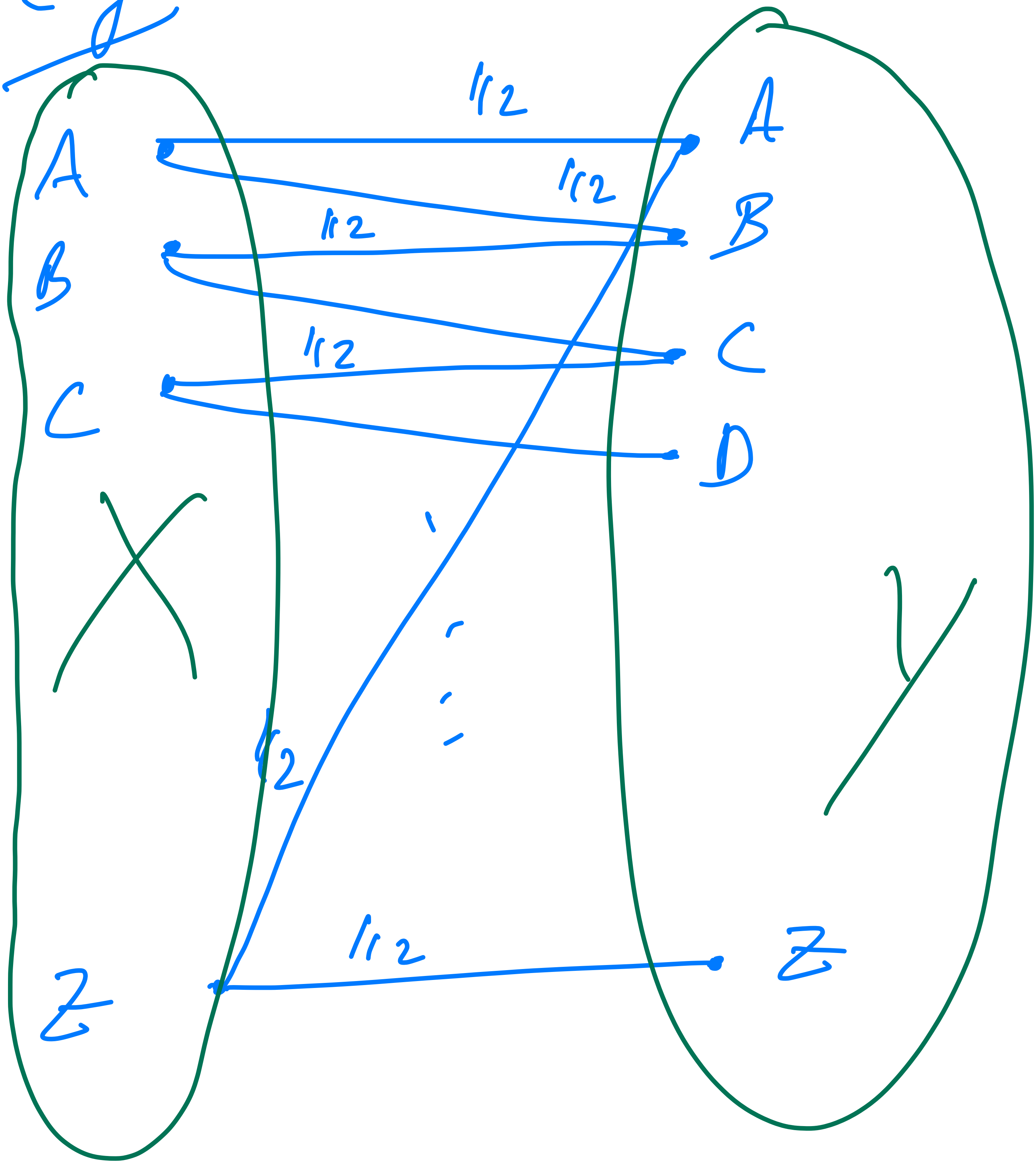
Information channel capacity

$$\max_{p(x)} I(X;Y)$$

$$\max_{p(x)} \left[H(X) - \underbrace{H(X|Y)}_{=0} \right]$$

$$= \max_{p(x)} H(X) = 1 \text{bit}$$

e.g.



$$\max_{p(x)} I(X;Y) =$$

$$\max_{p(x)} \left(H(X) - H(X|Y)\right) =$$

$$\max_{p(x)} \left[H(Y) - H(Y|X) \right]$$

$$H(X|Y) = 1 \text{ bit}$$

$$H(X|Y = c) = H\left(\tfrac{1}{2}, \tfrac{1}{2}\right)$$

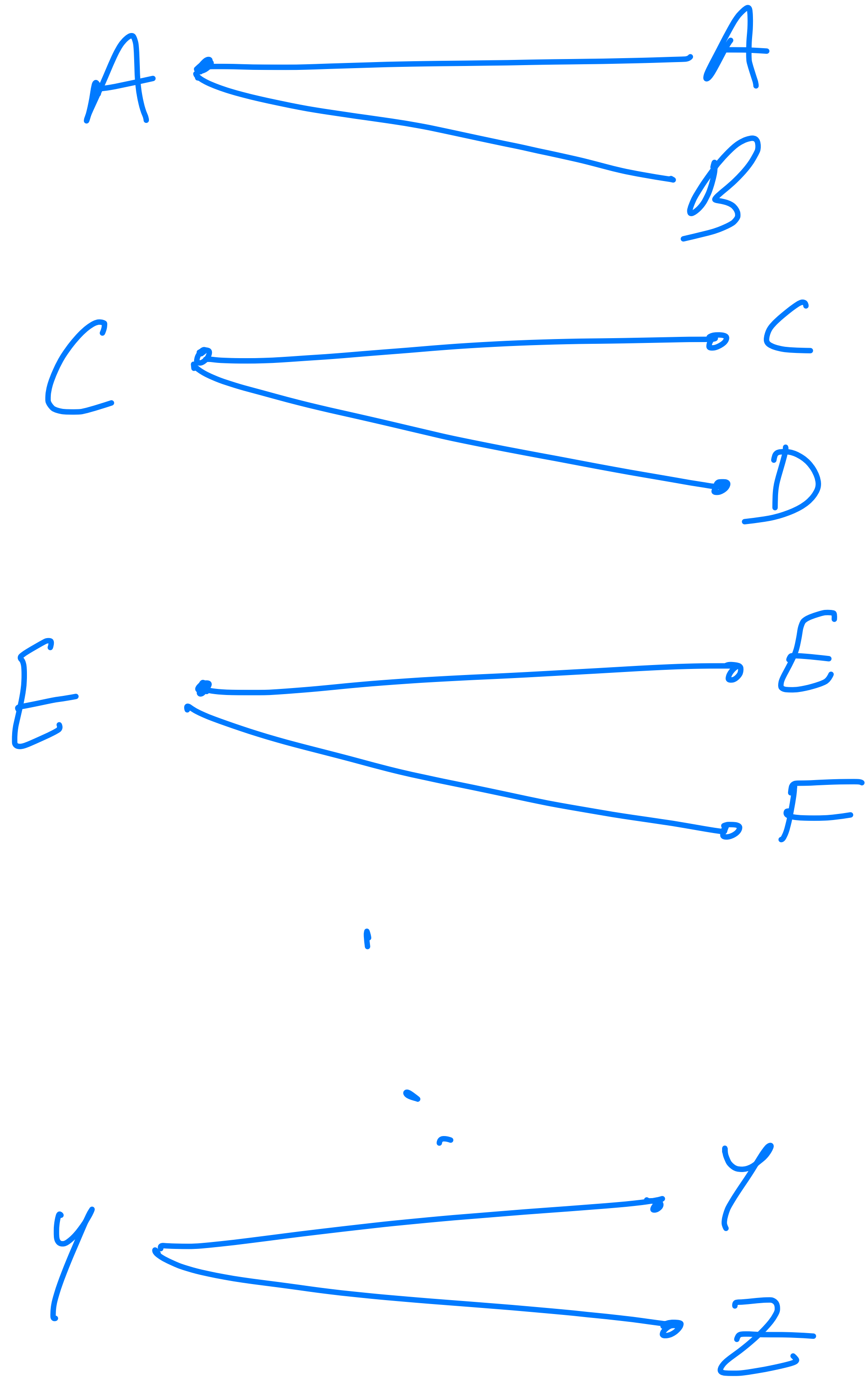
$$= 1 \text{ bit}$$

$$\max_{p(x)} H(X) \quad -1$$

$$= \log_2 26 \quad -1$$

$$= \log_2 13 \quad \text{bits}$$

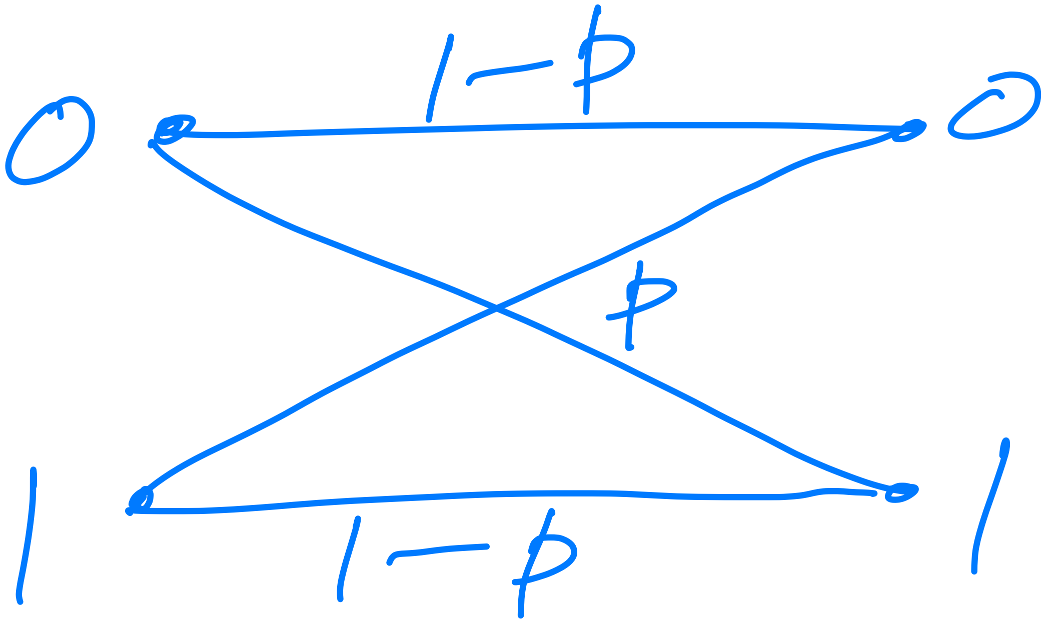
Operational Channel Capacity



$\log_2 13$ bits

e.g. Binary Symmetric Channel

BSC



Information channel capacity

$$I(X;Y) = H(Y) - H(Y|X)$$

$$= H(Y) - \sum p(x) H(Y|X=x)$$

$$= H(Y) - \sum p(x) H(p)$$

$$= H(Y) - H(p)$$

$$\leq 1 - H(p)$$

with equality when input distribution X is uniform.

$$\boxed{C = 1 - H(p)} \text{ bits}$$

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e.g. transition matrix

$$p(y|x) = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.5 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.3 \end{bmatrix}$$

x^{th} row, y^{th} column

$p(y|x)$. $p(1|3) = 0.2$

$C = \log_2 3 - H(0.5, 0.3, 0.2)$

Symmetric channel: all the rows are permutations of each other, same for columns.

Weakly symmetric: rows are permutations of each other. Column sums are equal.

$$\sum_x p(y|x)$$

e.g. $\begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$

Let γ be a row of the transition matrix

$$I(X;Y) = H(Y) - H(Y|X)$$

$$= H(Y) - H(\gamma)$$

$$\leq \log|Y| - H(\gamma)$$

maximized for uniform distribution over X.

For weakly symmetric or symmetric channel

$$C = \log|Y| - H(\text{row})$$

$C \geq 0$ as $I(X;Y) \geq 0$

$C \leq \log |X|$

$C \leq \log |Y|$

Channel Codes

Repetition Code

$1 \rightarrow 11111$

$0 \rightarrow 00000$

Receiver

# of 1's	decode
5	1
4	1
3	1
2	0
1	0
0	0