E.g.:

$n = 20$

$\bar{x} = 10.8$ | $\mu_0 = 10$

$\bar{x} = 7.8$

T.S.

$$\frac{\sqrt{n}}{\sigma} |\bar{x} - \mu_0|$$

$$= \frac{\sqrt{20}}{4} \left| \begin{matrix} 10.8 \\ 7.8 \end{matrix} - 10 \right|$$

$$= \frac{\sqrt{20}}{4} \times \begin{matrix} 0.8 \\ 2.2 \end{matrix}$$

$$= \begin{matrix} 0.894 \\ 2.46 \end{matrix}$$

$P\{|Z| \geq 0.894\}$

2.46

$= 2P\{Z \geq 0.894\}$

$= 0.371$

$0.014 = 1.4\%$

p-value is 0.371

0.014

Not reject the

Null Hypothesis

Rejected

e.g.

Assume $\alpha = 0.05$

$\mu_0 = 10$

$H_0: \mu = 10$

What is the probability that the null Hypothesis will not be rejected when the actual signal value is 9.2

Soln: 2 steps

1st step: $\sigma = 4$, $n = 20$

$\alpha = 0.05$

$H_0: \mu = 10$

$H_1: \mu \neq 10$

Reject H_0 if

$$\frac{\sqrt{20}}{4} \left| \bar{X} - 10 \right| \geq Z_{0.025}$$

$$\left| \bar{X} - 10 \right| \geq \frac{4 \, Z_{0.025}}{\sqrt{20}}$$

$$\left| \bar{X} - 10 \right| \geq 1.753$$

Reject H_0

$\bar{x} \geq 10 + 1.753$

$\bar{x} \leq 10 - 1.753$

$\bar{x} \geq 11.753$ or

$\bar{x} \leq 8.247$

Step 2 : compute the probability

Assume that the population mean is 9.2

$\bar{x}$ is normal, with mean 9.2, std. dev. $\frac{4}{\sqrt{20}} = 0.894$

$$Z = \frac{\bar{x} - 9.2}{0.894}$$

is std. normal.

P(H_0 is rejected)

$$= P(\bar{x} \geq 11.753) + P(\bar{x} \leq 8.247)$$

$$= P\left(\frac{\bar{x} - 9.2}{0.894} \geq \frac{11.753 - 9.2}{0.894}\right)$$

$$+$$

$$P\left(\frac{\bar{x} - 9.2}{0.894} \leq \frac{8.247 - 9.2}{0.894}\right)$$

$$= P(Z \geq 2.856) + P(Z \leq -1.066)$$

$$= 0.0021 + 0.1432$$

$$= 0.1453$$

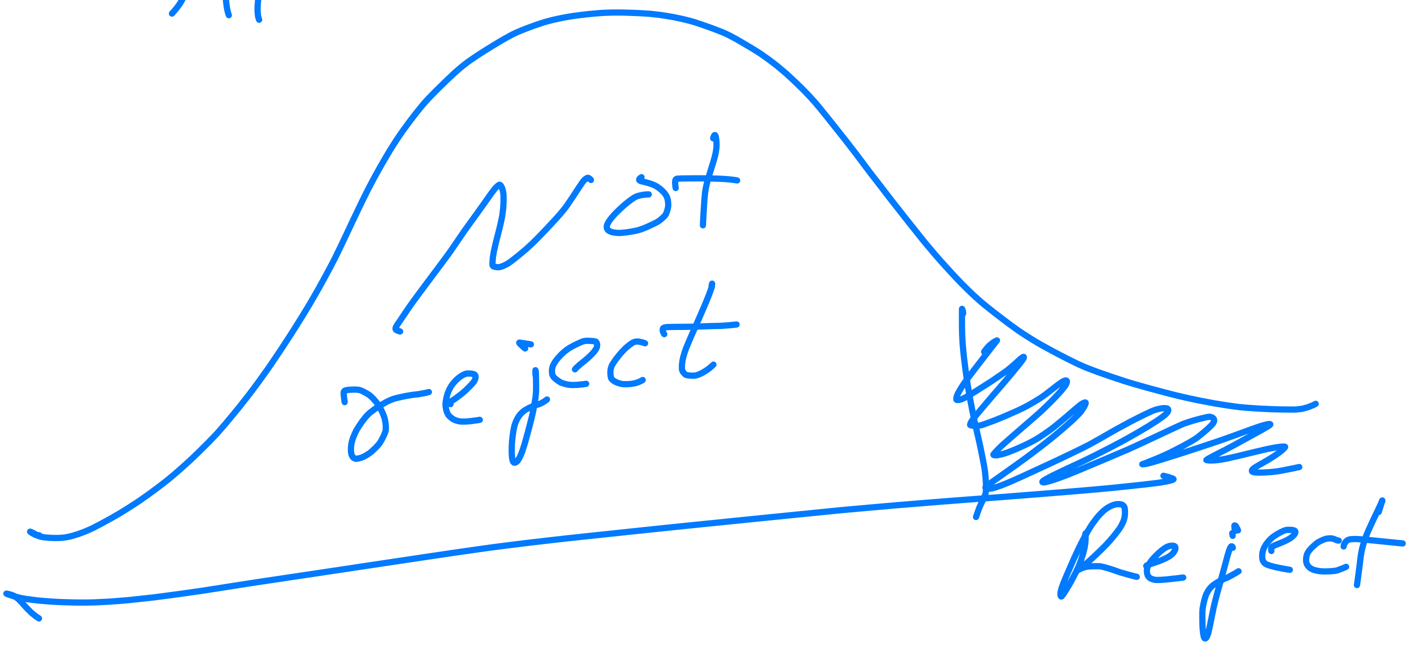
$= P($H_0$ is rejected$)$

$0.8547 = P($H_0$ is not rejected$)$

One-sided tests

$H_0: \mu \leq \mu_0$

$H_1: \mu > \mu_0$



Reject H_0 if

$$\sqrt{n}\,\frac{\bar{X}-\mu_0}{\sigma} > z_\alpha$$

H_0	$\mu = \mu_0$
H_1	$\mu \neq \mu_0$
TS	$\dfrac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma}$
α	Reject H if $\lvert TS \rvert \geq z_{\alpha/2}$
p-value	$2P\{Z \geq \lvert TS \rvert\}$

H_0 : $\mu \leq \mu_0$

H_1 : $\mu > \mu_0$

TS: $\dfrac{\sqrt{n}\,(\bar{X}-\mu_0)}{\sigma}$.

Reject H_0 if $TS > Z_\alpha$

p-value: $P\{Z \geq TS\}$

$H_0: \mu \geq \mu_0$

$H_1: \mu < \mu_0$

TS: $\dfrac{\sqrt{n}\,(\bar{x} - \mu_0)}{\sigma}$

Reject H_0 if $TS \leq -z_\alpha$

p-value: $P\{Z \leq TS\}$

Three mile island

Hyperthyroidism

March 28 1979

December 28 1979

11 babies had

hyperthyroidism

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mean = 3

σ = 2

$H_0: \mu \leq 3$

$H_1: \mu > 3$

$P(X \geq 11)$

$P(X \geq 10.5)$ (continuity correction)

$= P\left(\frac{X-3}{2} \geq \frac{10.5-3}{2} \right)$

$= P(Z \geq 3.75)$

< 0.0001

much less than 1%

e.g.: Prof claim that over half of the adult population is concerned about lack of educational programs on TV.

920 people are interviewed.

478 people agree. (are concerned)

$$\frac{478}{920} \times 100 = 52\%$$

$H_0: p \leq 0.50$

$H_1: p > 0.50$

Binomial distribution

$p = 0.5$

$n = 920$

mean $920 \times 0.5 = 460$

$\sigma = \sqrt{np(1-p)} = \sqrt{230}$

p-value

$P(X \geq 478)$

$= P(X \geq 477.5)$

(continuity correction)

$$= P\left(\frac{X-460}{\sqrt{230}} \geq \frac{477.5-460}{\sqrt{230}} \right)$$

$$= P(Z \geq 1.154)$$

$$= 0.1242$$

very high

$\Rightarrow$ null hypothesis

is not rejected.

e.g. A chip manufacturer claims that at most 2% of the chips are defective. Test this claim.

Sample of 400 chips. 13 are defective.

$\alpha = 0.05$

$$\frac{13}{400} \times 100 = 3.25\%$$

$H_0: \ p \leq 0.02$

$H_1: \ p > 0.02$

$$P\{X \geq 13\}$$

$$P\{X \geq 12.5\}$$ continuity correction

X is binomial

$n = 400$

$p = 0.02$

$$\mu = np = 400 \times 0.02 = 8$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{8 \times 0.98}$$

$$P(X \geq 12.5)$$

$$= P\left(\frac{X-8}{\sqrt{8 \times 0.98}} \geq \frac{12.5-8}{\sqrt{8 \times 0.98}}\right)$$

$$= P(Z \geq 1.607\}$$

= 0.054

This is more than 0.05

This is not strong enough to reject the null hypothesis.

Closing statistics here