Chapter 8: Hypothesis Testing

e.g.: Construction firm

Cables: average breaking strength of the cables is atleast 7000 psi.

You test 10 cables and breaking strength, and you get this data

7500, 6000, 8000, 5000, 4500, 8000, 6500, 7000, 9000, 6000

$\bar{X} = 6750$

EMF radiations cause cancer.

Over the last 3 years, 32 children got cancer.

Average no. of childhood cancer cases over a 3-year period is 16.2 with a std. dev. of 4.7

Definition: A statistical hypothesis is a statement about the nature of a population. It is often stated in terms of population parameter.

e.g.: ITC claims that they have come up with a new method of curing tobacco leaves that will result in the mean nicotine content of a cigarette of 1.5mg or less. There is a researcher who wants to dis-prove this or discredit this.

Null Hypothesis

$H_0: \mu \leq 1.5$

Alternative Hypothesis

$H_1: \mu > 1.5$

μ = mean nicotine content per cigarette

Null hypothesis, H_0, is a statement about the population parameter.

Null hypothesis will be rejected if it appears to be inconsistent with the sample data; otherwise it will not be rejected.

Definition: A test statistic is a statistic whose value is determined from the sample data. Depending upon the value of this test statistic, the null hypothesis will be rejected or not rejected.

In this example, the test statistic TS might be the average nicotine content of the sample of cigarettes.

The statistical test would reject the null hypothesis when this TS is sufficiently larger than 1.5

Def: Critical Region or Rejection Region: the set of values of the TS for which the null hypothesis is rejected.

Reject H_0: if TS is in C

Not reject H_0: if TS is not in C

$\sigma = 0.8$ mg

One possible critical rejection region is

$$C = \left\{ \bar{x} \geq 1.5 + \frac{1.312}{\sqrt{n}} \right\}$$

sample mean

no. of items in the sample

e.g. $n = 100$, $\sqrt{n} = 10$

$$C = \left\{ \bar{x} \geq 1.5 + 0.1312 \right\}$$

$$= \left\{ \bar{x} \geq 1.6312 \right\}$$

if $\bar{x} = 1.65$ mg, then the null hypothesis H_0 is rejected.

if $\bar{x} = 1.6$ mg, then the null hypothesis is not rejected.

$\bar{x} = 1.65$ mg

$n = 36$

$$C = \left\{ \bar{x} \geq 1.5 + \frac{1.312}{6} \right\}$$

$$= \left\{ \bar{x} \geq 1.5 + 0.218 \right\}$$

$= \{ \bar{x} \geq 1.718 \}$

Null Hypothesis H_0 is not rejected.

Rejection of a null hypothesis H_0 is a strong statement, i.e., that H_0 does not appear to be consistent with the observed data. The result that H_0 is not rejected is a weak statement that should be interpreted to mean that H_0 is consistent with the data.

Type 1 error: test rejects H_0 when H_0 is True. This should be very less

Type 2 error: test does not reject H_0 when H_0 is False.

H_0 should be rejected only if sample data is very unlikely when H_0 is True.

Specify a small value α, the test has the property that whenever H_0 is True, the probability of H_0 being rejected is $\leq \alpha$.

α = level of significance

0.1, 0.05, 0.01

If you are trying to establish a certain hypothesis, then that should be alternative hypothesis H_1.

If you are trying to discredit a hypothesis, it should be null hypothesis H_0.