new vaccine

Pharma company claims that it acts (creates immunity) is $\leq$ 45 days.

H_0: vaccine takes > 45 days

H_1: vaccine takes $\leq$ 45 days

Pharma company wants to prove their claim to FDA e.g.:

$H_0: \mu = 0$

$H_1: \mu \neq 0$

result in rejection of H_0 at 5% level of significance

a) μ is significantly far from 0

b) data is significantly strong to show that $\mu \neq 0$

c) Probability ($\mu = 0$) ≤ 0.05

d) hypothesis that $\mu = 0$ was rejected by a procedure that would have resulted

in rejection only 5%
of the time when $\mu = 0$

Mean of a normal population, case of known variance

$X_1, X_2, \ldots, X_n$ are sample from a normal distribution having an unknown mean μ and known variance σ^2

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

for some specific value μ_0

Compute the point estimator of μ = sample mean

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Critical region: when H_0 is rejected, when $\bar{X}$ is far away from μ_0

$$C = \{X_1, X_2, \cdots, X_n \mid |\bar{X} - \mu_0| \geq c\}$$

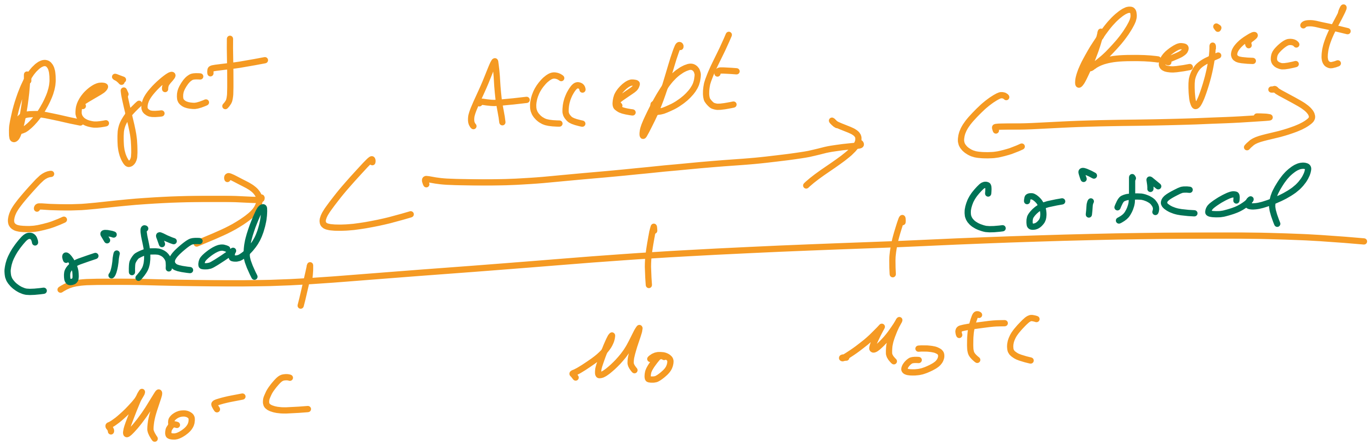
for some suitable value c

$$|\bar{X} - \mu_0| \geq c$$

sample mean

testing

specified



Significance level α

$\alpha =$ probability of type I error

$$P\{|\bar{X} - \mu_0| \geq c\} = \alpha$$

when $\mu = \mu_0$

Probability of type I error

When μ is equal to μ_0,

$\bar{X}$ is normally distributed

with mean μ_0 and

std. dev. $\frac{\sigma}{\sqrt{n}}$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{\sqrt{n}(\bar{X}-\mu_0)}{\sigma}$$

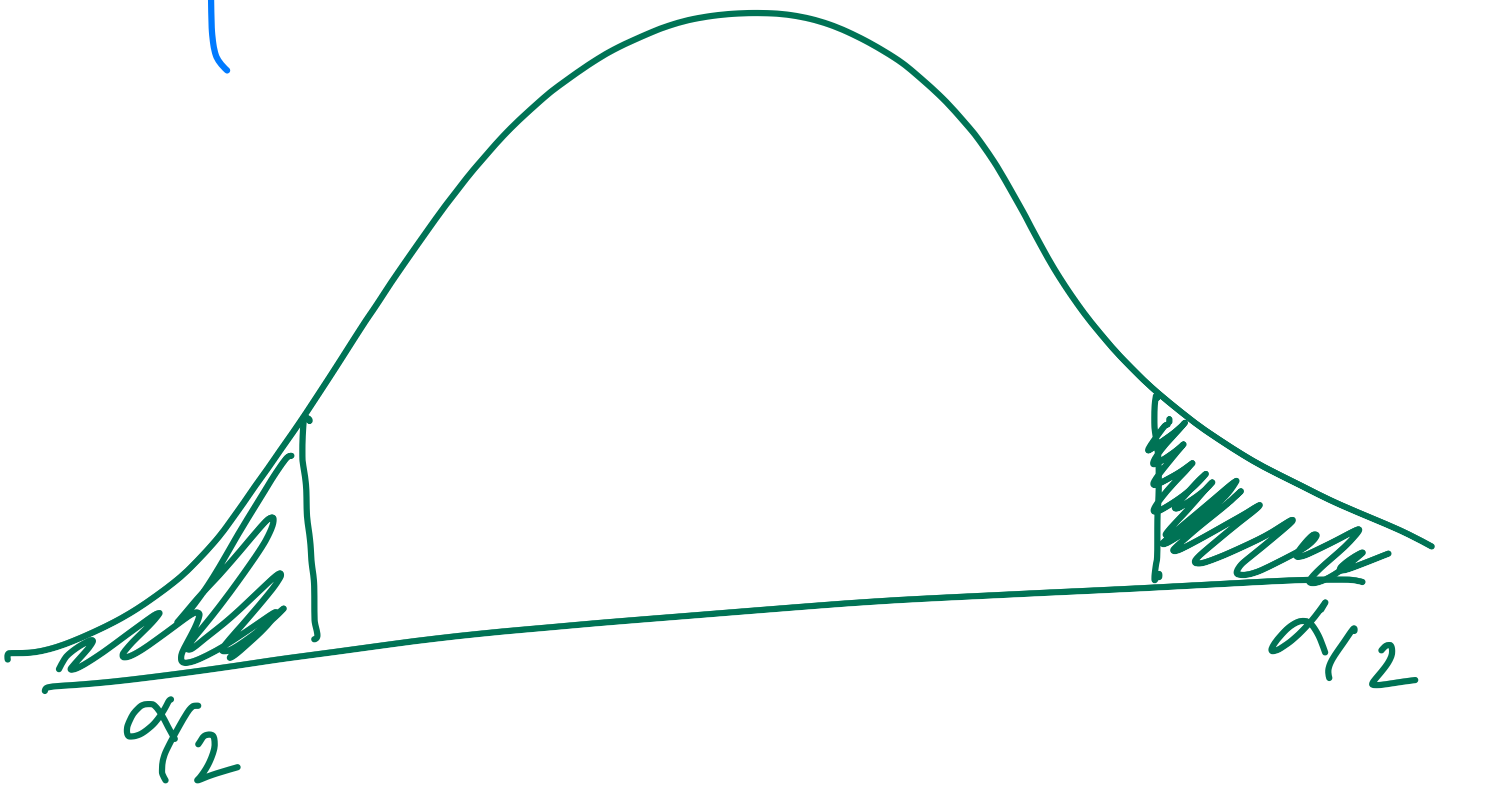
is a std. normal variable

$$|\bar{X} - \mu_0| \geq c$$

$$\frac{\sqrt{n}}{\sigma}\left|\bar{x}-\mu_0\right| \geq \frac{\sqrt{n}}{\sigma}c$$

$$P\Big(\overbrace{\frac{\sqrt{n}}{\sigma}\left|\bar{x}-\mu_0\right|}^{Z} \geq \frac{\sqrt{n}}{\sigma}c\Big)$$

$$= P\left\{|Z| \geq \frac{\sqrt{n}\,c}{\sigma}\right\} = \alpha$$



$$2P\left\{Z \geq \frac{\sqrt{n}\,c}{\sigma}\right\} = \alpha$$

$$P\left\{Z \geq \frac{\sqrt{n}\,c}{\sigma}\right\} = \frac{\alpha}{2}$$

$$P\left\{Z \geq z_{\alpha/2}\right\} = \alpha/2$$

$$\frac{\sqrt{n}\,c}{\sigma} = z_{\alpha/2}$$

$$c = \frac{\sigma\, z_{\alpha/2}}{\sqrt{n}}$$

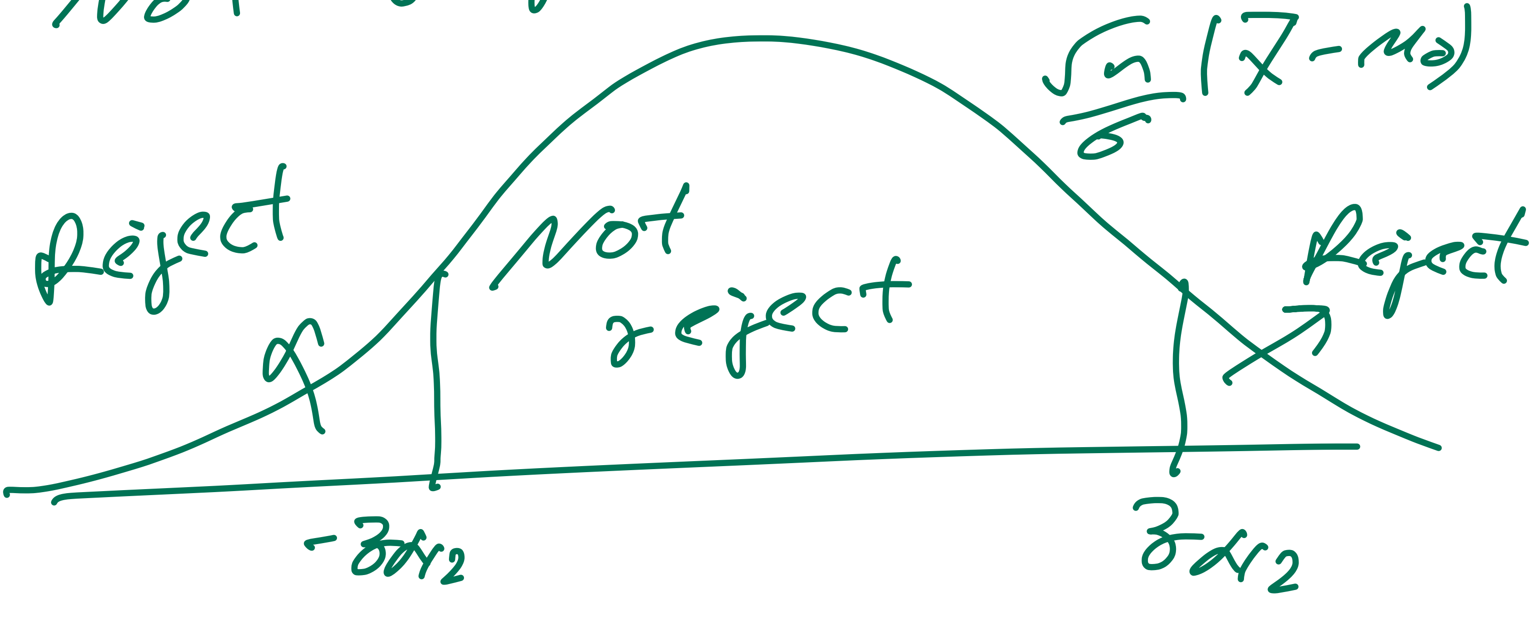
Reject the null hypothesis if

$$|\bar{X} - \mu_0| \geq \frac{z_{\alpha/2}\,\sigma}{\sqrt{n}}$$

Reject H_0 if

$$\frac{\sqrt{n}}{\sigma}|\bar{X} - \mu_0| \geq z_{\alpha/2}$$

Not reject H_0 otherwise



e.g. signal of intensity μ

mean μ

std. dev. 4

It is suspected that the value of the signal intensity is 10.

Test the null hypothesis that $\mu = 10$ if 20 observations are made with mean 11.6. Use 5% level of significance.

10% $\alpha = 0.05$ $\alpha/2 = 0.025$

Soln: $H_0: \mu = 10$

$H_1: \mu \neq 10$

$z_{\alpha/2} = z_{0.025} = \boxed{1.96}$

$$\frac{\sqrt{n}}{\sigma}\left|\bar{x} - \mu_0\right|$$

$$= \frac{\sqrt{20}}{4}\left|11.6 - 10\right|$$

$$= \frac{\sqrt{20}}{4} * 1.6 = \boxed{1.79}$$

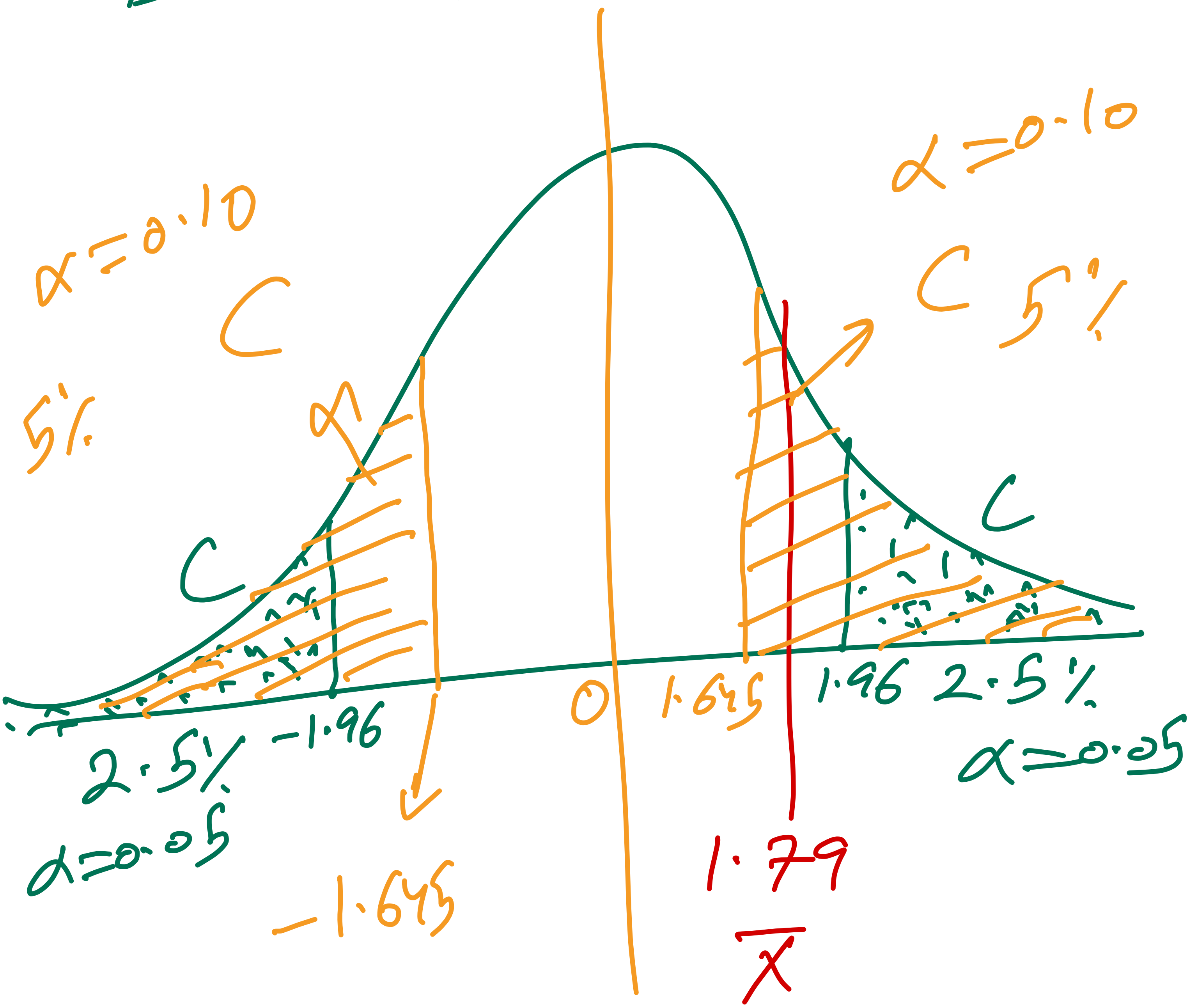
$\Rightarrow$ the null hypothesis is not rejected

Same values, $\alpha = 0.10$

$\alpha/2 = 0.05$

$z_{0.05} = \boxed{1.645}$

$\Rightarrow$ the null hypothesis is rejected



p value

The p-value is the smallest significance level at which the data lead to rejection of the null hypothesis. It gives the probability that data as unsupportive of H_0 as those observed will occur when H_0 is true.

A small p-value ($e.g. \leq 0.05$) is a strong indicator that the null hypothesis is not true. The smaller the p-value, the greater the evidence for the the falsity of H_0.