Graph Summarization

[Summer Research Internship]

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Problem Definition

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- In the most popular group based approach, summarized graph $\overline{G}$ encoded as **supernodes** and **superedges**.
- Graph G is **reconstructed**, as per a fixed scheme, from its summarized graph $\overline{G}$ either perfectly **(lossless summarization)** or with some loss in information **(lossy summarization)**.

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- It's obvious that these uniform scheme may lead to worst accuracy scenario in the **highly degree - skewed** graph.
- And mostly real - world graphs consist of highly degree skewed nodes, i.e., linkage graph of social media platform. Thus this uniform reconstruction scheme is not suitable for real - world graphs.
- So the solution is **Degree Preserving Graph Summarization (DPGS)** which considers degrees of nodes while reconstructing the original graph.

Solution - DPGS

- While relaxing a superedge, assign weights of reconstructed edges proportional to multiplication of degrees of endpoint nodes.
- Uses **Minimum Description Length (MDL)** principle to minimize the **cost of summary graph** and **reconstruction error**.
- Uses **Locality Sensitive Hashing (LSH)** to group candidate nodes and it merges nodes greedily within the groups.
- It's lossy summarization method.

How to find candidate pair of nodes to be merged?

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- The solution to this is **Locality Sensitive Hashing (LSH)**. With this LSH time reduces to $O(n)$.

Minimizing The Description Length

- For minimizing the total description (encoding) length, DPGS uses **Minimum Description Length (MDL)** principle.
- So formulating our objective in mathematical terms,

Minimum Description Length (MDL)

$$\text{Minimize } L(G, \overline{G}) = L(\overline{G}) + L(G|\overline{G})$$

$$\text{where, } L(\overline{G}) = \text{Description length of summarized graph.}$$

$$L(G|\overline{G}) = \text{Description length of errors.}$$

DPGS Algorithm

Input: $G = (V, E)$, iteration T
Output: $\overline{G} = (\overline{V}, \overline{E}, \overline{A})$

$\overline{G} \leftarrow G, \overline{V} \leftarrow V, \overline{E} \leftarrow E$
$t \leftarrow 0$
while $t < T$ do
    $t \leftarrow t + 1$
    Update LSH
    Divide supernodes into disjoint groups by LSH
    for each group g do
        MergeGroup(g)
    end for
end while
return $\overline{G}$

Merge Group Algorithm

Input: $g \subset \overline{V}$

$times \leftarrow log_2|g|$
$nskip \leftarrow 0$
while $nskip < times$ and $|g| >= 1$ do
    $pairs \leftarrow$ Sample $log_2|g|$ node pairs from g
    $(u, v) \leftarrow argmax_{(i,j) \in pairs}\ gain(i, j)$
    if $gain(u, v) > 0$ then
        Merge u and v
        $nskip \leftarrow 0$
    else
        $nskip \leftarrow nskip + 1$
    end if
end while

Novel Reconstruction Scheme(CR Scheme)

- So DPGS uses new scheme for reconstruction, **configuration based reconstruction scheme (CR Scheme)**.

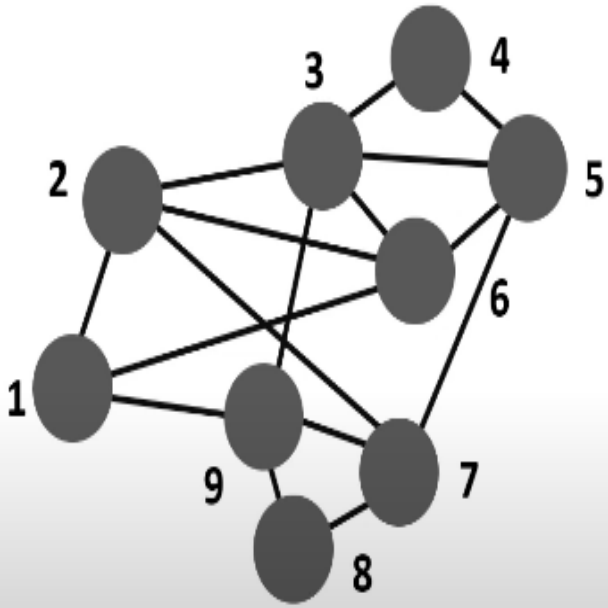
  Let $\overline{A}$ and A' be adjacency matrices of summarized and reconstructed graph respectively,

$$A'(i,j) = \frac{d_i}{D_k}\overline{A}(k,l)\frac{d_j}{D_l}$$

  where, d_i & d_j are degrees of nodes i and j;

  D_k & D_l are degrees of supernodes k and l.

Input Graph

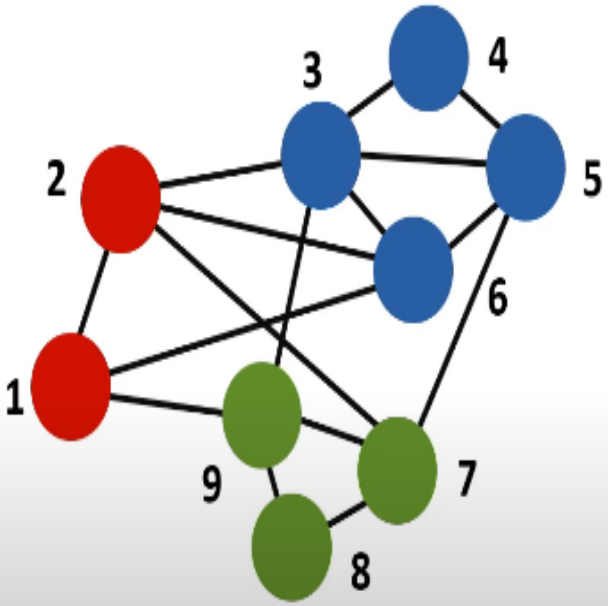


Input Graph

	1	2	3	4	5	6	7	8	9
1	0	1	0	0	0	1	0	0	1
2	1	0	1	0	0	1	1	0	0
3	0	1	0	1	1	1	0	0	1
4	0	0	1	0	1	0	0	0	0
5	0	0	1	1	0	1	1	0	0
6	1	1	1	0	1	0	0	0	0
7	0	1	0	0	1	0	0	1	1
8	0	0	0	0	0	0	1	0	1
9	1	0	1	0	0	0	1	1	0

Adjacency Matrix

Input Candidate Grouped Graph

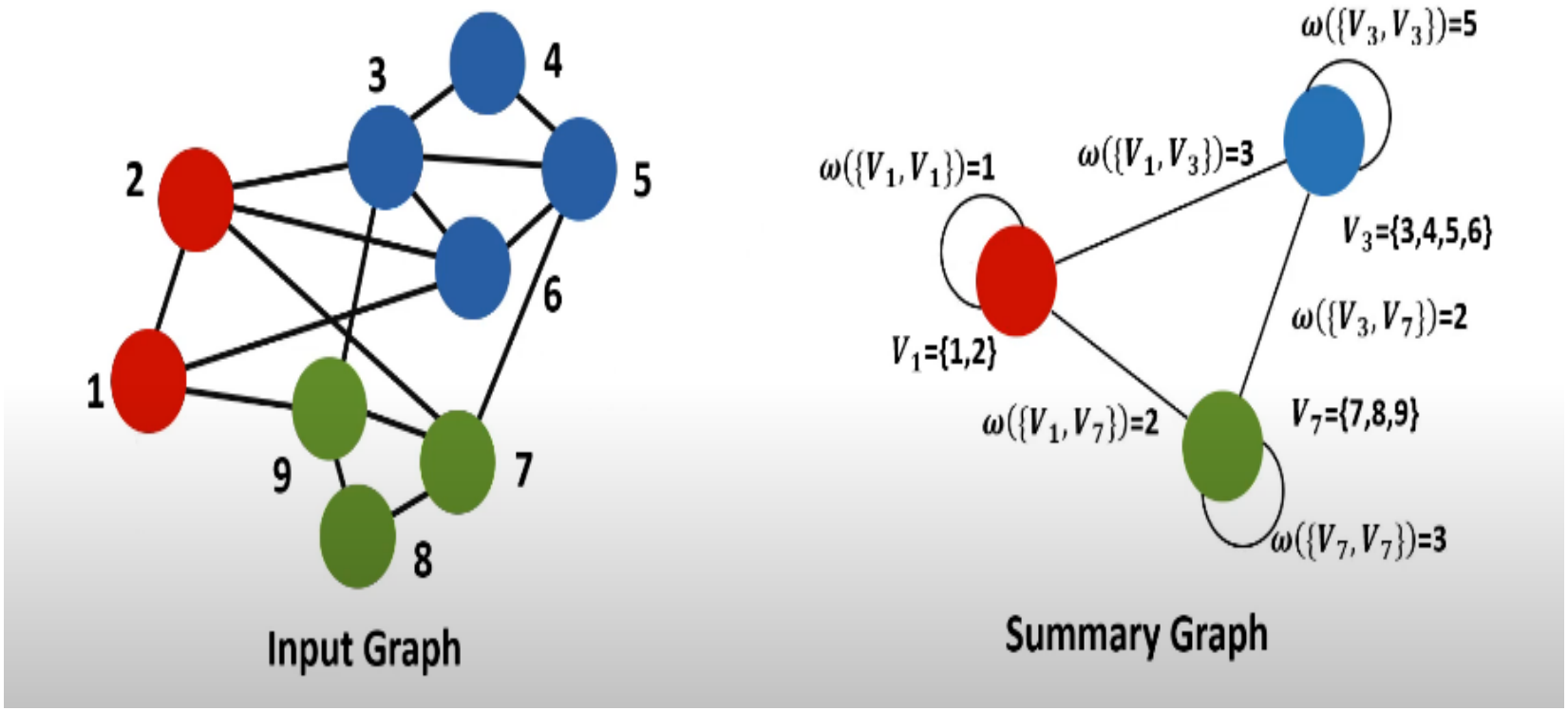


Input Graph

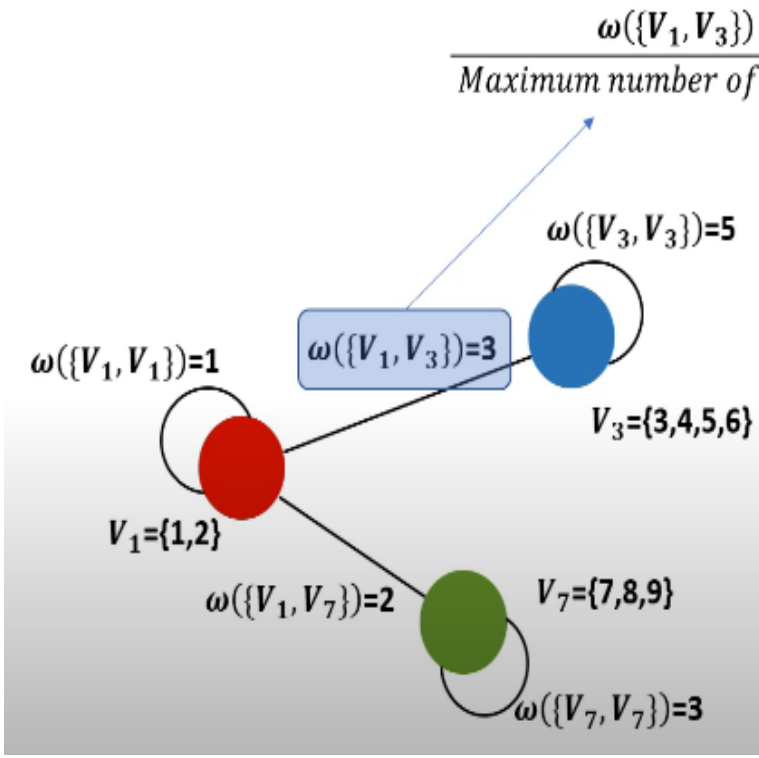
	1	2	3	4	5	6	7	8	9
1	0	1	0	0	0	1	0	0	1
2	1	0	1	0	0	1	1	0	0
3	0	1	0	1	1	1	0	0	1
4	0	0	1	0	1	0	0	0	0
5	0	0	1	1	0	1	1	0	0
6	1	1	1	0	1	0	0	0	0
7	0	1	0	0	1	0	0	1	1
8	0	0	0	0	0	0	1	0	1
9	1	0	1	0	0	0	1	1	0

Adjacency Matrix

Visualizing Summarized Graph from Input Graph



Summarized Graph



	1	2	3	4	5	6	7	8	9
1	0	1	3/8	3/8	3/8	3/8	1/3	1/3	1/3
2	1	0	3/8	3/8	3/8	3/8	1/3	1/3	1/3
3	3/8	3/8	0	5/6	5/6	5/6	0	0	0
4	3/8	3/8	5/6	0	5/6	5/6	0	0	0
5	3/8	3/8	5/6	5/6	0	5/6	0	0	0
6	3/8	3/8	5/6	5/6	5/6	0	0	0	0
7	1/3	1/3	0	0	0	0	0	1	1
8	1/3	1/3	0	0	0	0	1	0	1
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What if CR Scheme is used instead of uniform scheme for reconstruction?

- As mentioned in the formulation, it incorporates degrees of nodes.
- For e.g., let's assign weight of edge $(1, 4)$ using CR scheme. Node 1 and 4 belongs to supernodes *Red* and *Blue* respectively.

$$\begin{aligned} A'(1,4) &= \frac{d(1)}{D(Red)} \overline{A}(Red, Blue) \frac{d(4)}{D(Blue)} \\ &= \frac{3}{7} 3 \frac{2}{14} \\ &= 0.1836 \end{aligned}$$

But previously with uniform scheme, value of $A'(1,4)$ was $\frac{3}{8} = 0.375$ which is unnecessarily high as 4 have less degree.

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- Out of which **Correction Set - Based Approach** is commonly used.
- Here as output, summarization method gives correction set in addition to summarized graph.
- **Correction set:** Set of edges we need to add or remove to reconstruct the graph perfectly or with less error.
- The current state of the art correction set based graph summarization algorithm is **SWeG**.

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- Despite the impressive performance of SWeG compared to other algorithms, there are several steps in the algorithm which bottleneck its performance.

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  1. Merge the nodes into supernodes.
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  3. Drop some edges from superedges and correction set to make reconstruction more compact.
- SWeG is faster and elegant than all of its competitors, yields better compression than other methods, and can also run in a distributed setting.
- Despite the impressive performance of SWeG compared to other algorithms, there are several steps in the algorithm which bottleneck its performance.
- Still there are major bottlenecks in SWeG, i.e., finding the candidate pair to be merged and the merging algorithm itself.

Correction Set Based Approach...

So the solution to this problem is **Locality Sensitive Hashing Divide Merge Encode [LDME]**.

Learning

- First of all we have researched the field of graph summarization.
- Then we have focused on impact of degree skewed nodes, which is our main area of research.
- Then we have compared analytical results provided by DPGS method.
- We also have looked upon the vast topic of ML for grouping similar objects, LSH.

References

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