


# Center for Professional Enhancement

# Mensuration

**Mensuration** is the branch of mathematics which deals with the study of Geometric shapes, their area, volume and related parameters.

# Learning Outcomes

**Through this Topic Candidates should be able to**

•understand the basic concepts of mensuration

•observe the data given and interpret it from the given problem

•apply the Concepts to solve Company Specific Aptitude tests

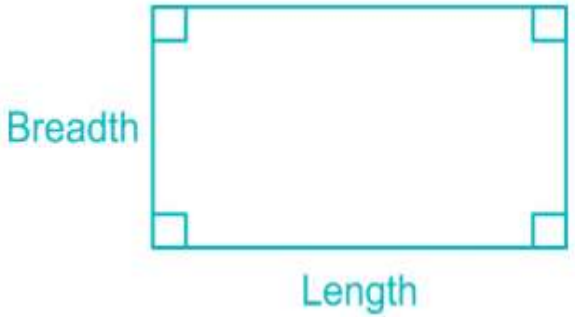
•apply the Concepts to solve questions asked in various government exams like SSC CGL, Banking Sector etc.

# Contents

- **2D Figures: Area, Perimeter.**
- **3D Figures: Volume, Curved surface Area and Total surface Area.**

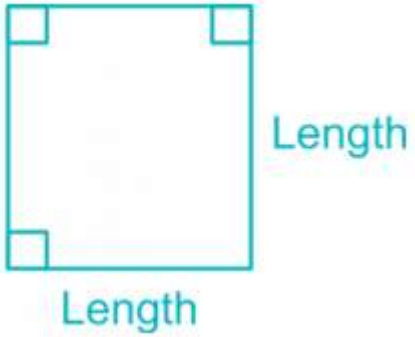
| 2D Figures | 3D Figures |
|---|---|
| Rectangle | Cuboid |
| Square | Cube |
| Triangle | Cylinder |
| Circle | Cone |
| Parallelogram | Sphere |
| Trapezium | Hemisphere |

# 2D Figures: Rectangle



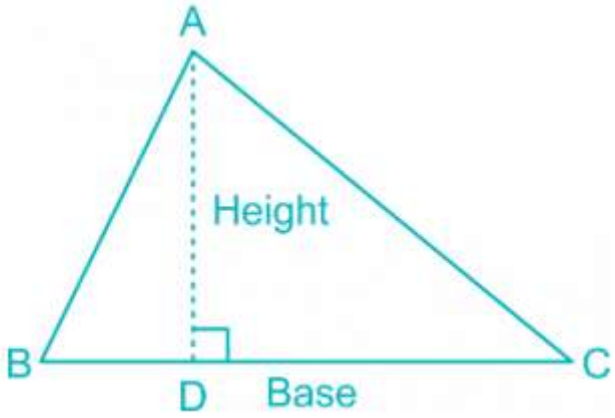

- Area of Rectangle = Length × Breadth.
- Perimeter of a Rectangle = 2 × (Length + Breadth)
- Length of the Diagonal = $\sqrt{(Length^2 + Breadth^2)}$

# Square




- Area of a Square = Length × Length = $(\text{Length})^2$
- Perimeter of a square = 4 × Length
- Length of the Diagonal = $\sqrt{2}$ × Length

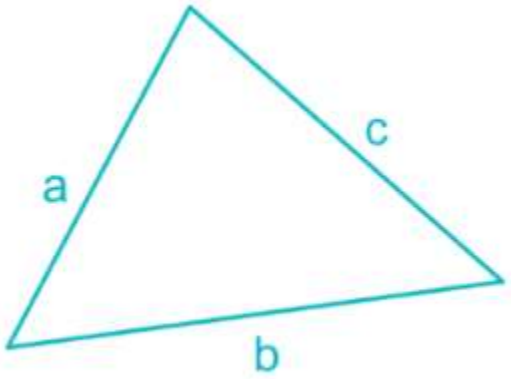
# Triangle




- Area of a triangle=(1/2)(Base × Height)=(1/2)(BC×AD)

# Triangle

For a triangle with sides measuring a, b and c, respectively:

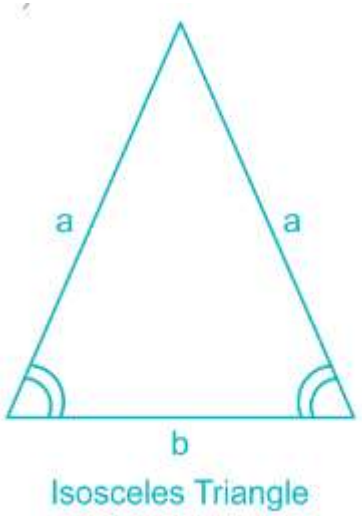



- Perimeter = a + b + c
- s = semi perimeter = perimeter/2 = (a+b+c)/2
- Area of Triangle, A=$\sqrt{s(s-a)(s-b)(s-c)}$
  (This is also known as "Heron's formula")

# Isosceles Triangle

- Area of Isosceles Triangle: $\frac{b}{4}\sqrt{4a^2 - b^2}$

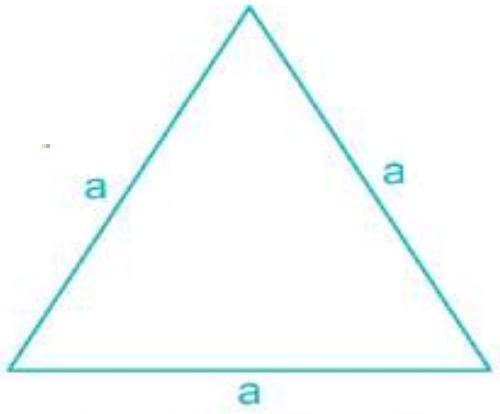
(Where a = length of two equal side, b = length of base of isosceles triangle.)



Isosceles Triangle

# Equilateral Triangle

- Area of Equilateral Triangle : $\frac{\sqrt{3}}{4} \times a^2$

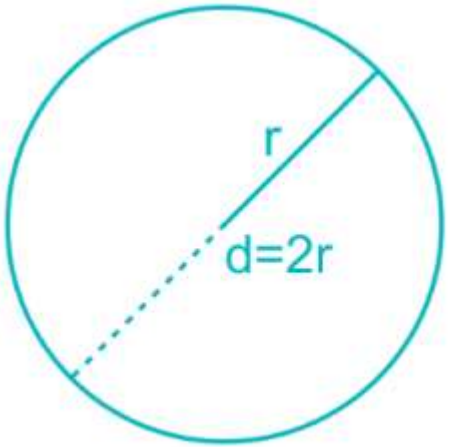


Equilateral Triangle

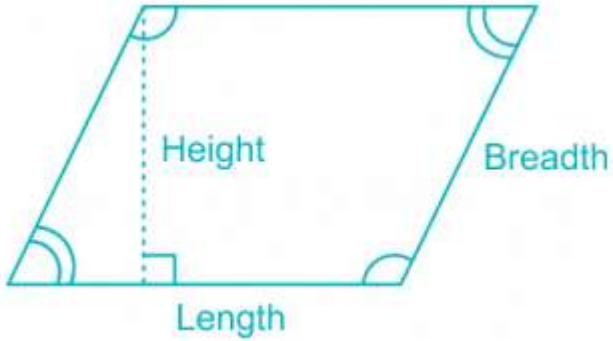
# Circle & Semicircle

- Area of a circle = $\pi r^2 = \pi d^2/4$
- Circumference of a circle = $2\pi r = \pi d$
- Circumference of a semicircle = $\pi r$
- Area of semicircle = $\pi r^2/2$

(In the following formulae, r = radius and d = diameter of the circle)

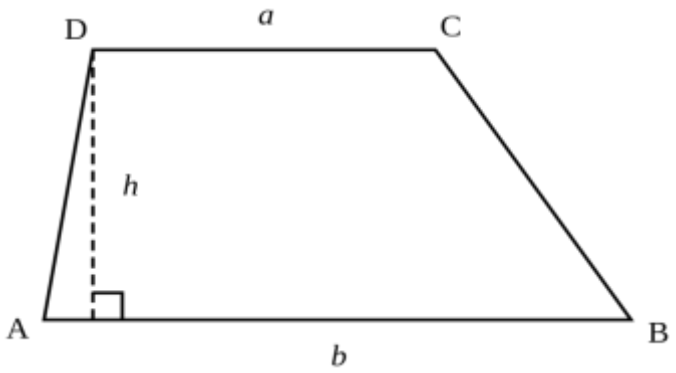


# Parallelogram

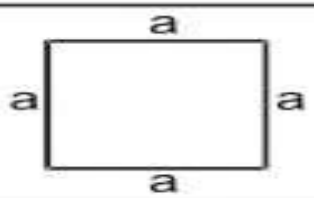
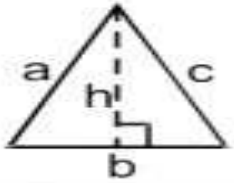
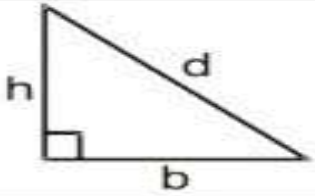
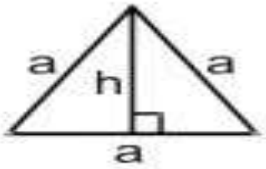
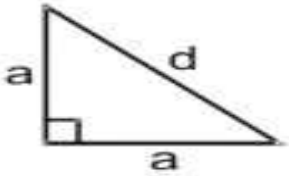


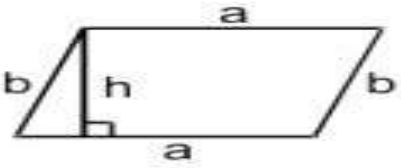
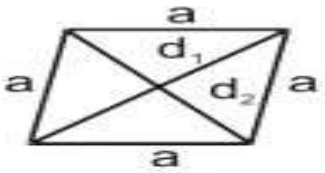
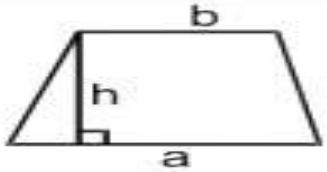
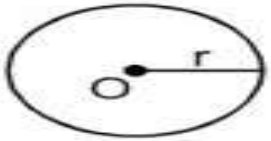
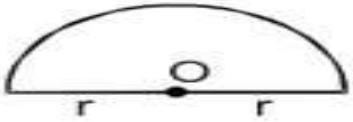

- Area of a Parallelogram = Length × Height
- Perimeter of a Parallelogram = 2 × (Length + Breadth)

# Trapezium




- Area of a trapezium =(1/2)× sum of parallel sides × distance between parallel sides

| Name | Figure | Perimeter | Area |
|---|---|---|---|
| Rectangle |   | 2 (a + b) | ab |
| Square |   | 4a | $a^2$ |
| Triangle |   | a + b + c = 2s | $1 = \frac{1}{2} \times b \times h$ <br> $2 = \sqrt{s(s-a)(s-b)(s-c)}$ |
| Right triangle |   | b + h + d | $\frac{1}{2}$ bh |
| Equilateral triangle |   | 3a | 1. $\frac{1}{2}$ ah <br> 2. $\frac{\sqrt{3}}{4} a^2$ |
| Isosceles right triangle |   | 2a + d | $\frac{1}{2} a^2$ |

| | | | |
|---|---|---|---|
| Parallelogram |  | 2 (a + b) | ah |
| Rhombus |  | 4a | $\frac{1}{2} d_1 d_2$ |
| Trapezium |  | Sum of its four sides | $\frac{1}{2}$ h (a + b) |
| Circle |  | $2\pi r$ | $\pi r^2$ |
| Semicircle |  | $\pi r + 2r$ | $\frac{1}{2} \pi r^2$ |
| Ring (shaded region) |  | ---- | $\pi (R^2 - r^2)$ |
| Sector of a circle |  | l + 2r<br>where<br>$l = (\theta/360) \times 2\pi r$ | $\theta/360° \times \pi r^2$ |

# Practice Questions:

1. The area of a square is 4096 sq. cm. Find the ratio of the breadth and length of a rectangle whose length is twice the side of the square and breadth is 24 cm less than the side of the square

A. 5:32

B. 7:16

C. 5:16

D. None of these

2. A wire in the form of a circle of radius 3.5 m is bent in the form of a rectangle whose length and breadth in the ratio of 6:5. What is the area of rectangle.

A.30

B.60

C.120

D.None of these

3. The circumference of two circles are 264 m and 352 m. Find the difference between area of the larger and smaller circles.

A.4123

B.8642

C.4312

D.2612

4. What would be the cost of building 7 m wide garden around a circular field with diameter equal to 280m, if the cost per sq. m for building the garden is Rs. 21

A.Rs. 156242

B.Rs. 248521

C.Rs. 132594

D.None of these

5. A cow is tied on the corner of a rectangular field of size 30m*20m by a 14 m long rope. The area of the region that she can graze is

A.350 sq. m

B.196 sq. m

C.154 sq. m

D.22 sq. m

6. At each corner of a triangular field of side 26m, 28m and 30m a cow is tethered by a rope of length 7m. The area un-grazed by the cow is

A.336 sq. m

B.259 sq. m

C.154 sq. m

D.77 sq. m

7. The radius of a circular field is equal to the side of a square field. If the difference between the perimeter of the circular field and that of the square field is 32m, what is the perimeter of the square field?

A.84m

B.95m

C.56m

D.28m

8. Two equal maximum sized circular plates are cut-off from a circular paper-sheet of circumference 352 cm. The circumference of each circular plate is.

A.176 cm

B.180 cm

C.165 cm

D.150 cm

9. A wire, when bent in the form of a square, encloses a region having area 121 cm2. If the same wire is bent into the form of a circle, then the area of the circle is?

A.144 sq. cm

B.180 sq. cm

C.154 sq. cm

D.176 sq. cm

10. The area of a triangle is 216 $cm^2$ and its sides are in the ratio 3 : 4 : 5. The perimeter of the triangle is

A.6 cm

B.12 cm

C.36 cm

D.72 cm

11. The ratio between the perimeter and the breadth of a rectangle is 5 : 1. If the area of the rectangle is 216 sq. cm, what is the length of the rectangle?

A)16

B) 18

C) 20

D) 22

12. Four circles having equal radii are drawn with centres at the four corners of a square. Each circle touches the other two adjacent circle. If remaining area of the square is 168 cm square, what is the size of the radius of the circle?

A. 1.4 cm

B. 14 cm

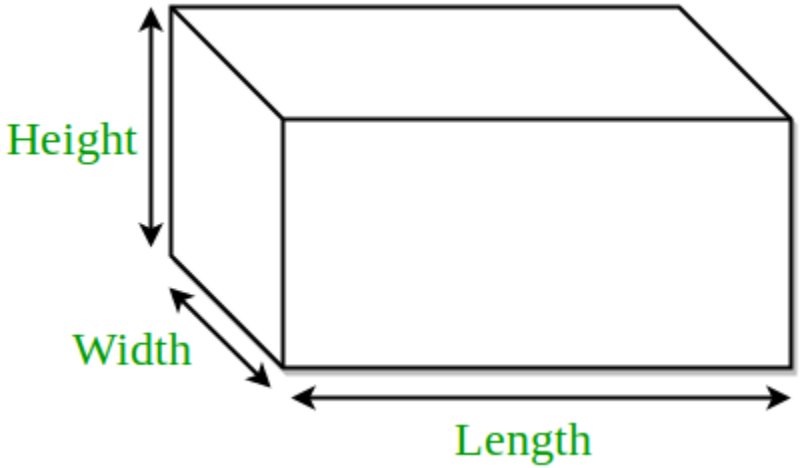
C. 35 cm

D. 21 cm

# Mensuration- 3D

# 3D Figures: Cuboid

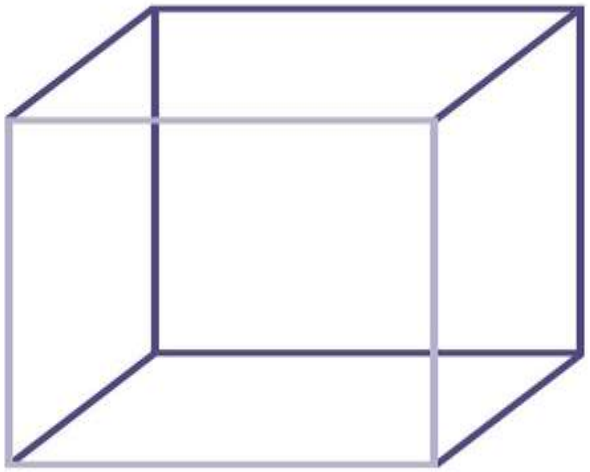
Let
l= length
b= breadth
h= height. Then,




- **Volume** = ($l$ x $b$ x $h$) cubic units.
- **Lateral Surface area** = 2( $b$ + $l$) h sq. units.
- **Surface area** = 2($lb$ + $bh$ + $lh$) sq. units.
- **Diagonal** = $\sqrt{(l^2 + b^2 + h^2)}$ units.

# Cube

Let each edge of a cube be of length *a*. Then,

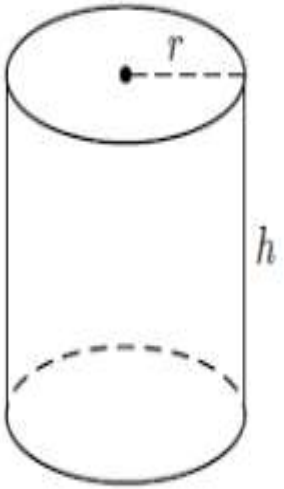


- **Volume** = $a^3$ cubic units.
- **Lateral Surface area** = $4a^2$ sq. units
- **Surface area** = $6a^2$ sq. units.
- **Diagonal** = $\sqrt{3}a$ units.

# Cylinder

Let radius of base = $r$ and Height (or length) = $h$. Then,

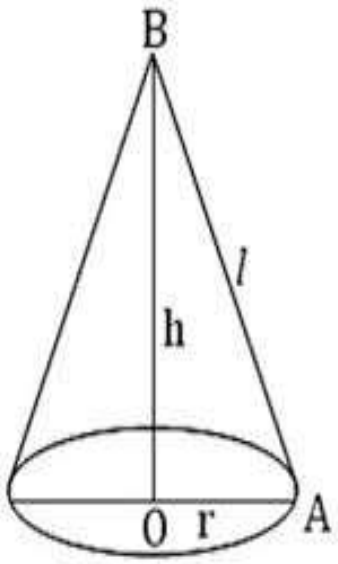
- **Volume** = ($\pi r^2 h$) cubic units.
- **Curved surface area =** ($2\pi r\, h$) sq. units.
- **Total surface area** = $2\ \pi\ r(h + r)$ sq. units.



# Cone

Let radius of base = $r$ and Height = $h$. Then,

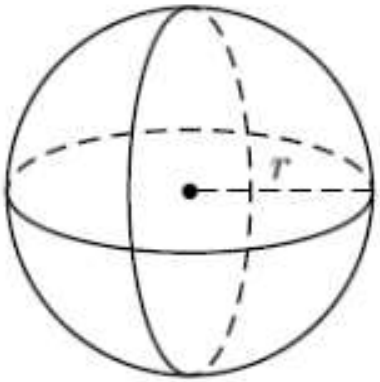
- **Slant height,** $l = \sqrt{h^2 + r^2}$ units.
- **Volume** = (1/3) π $r^2h$ cubic units.
- **Curved surface area** = (π $rl$) sq. units.
- **Total surface area** = (π $rl$ + π $r^2$) sq. units.



# Sphere

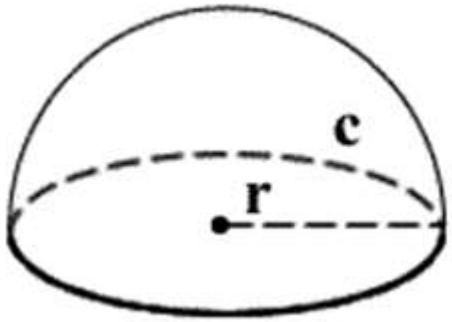
Let the radius of the sphere be $r$. Then,

- **Volume** = (4/3) π $r^3$ cubic units.
- **Surface area** = (4 π $r^2$) sq. units.



# Hemisphere

Let the radius of a hemisphere be *r*. Then,




- **Volume** = (2/3) π $r^3$ cubic units.
- **Curved surface area** = (2 π $r^2$) sq. units.
- **Total surface area** = (3 π $r^2$) sq. units.

# Practice Questions:

1. If the areas of the three adjacent faces of a cubical box are 120 cm square, 72 cm square and 60 cm square respectively, then the volume of the box is:

A. 800 cm cube

B. 680 cm cube

C. 700 cm cube

D. 720 cm cube ✓

2. A wooden box measures 20 cm × 12 cm × 10 cm. the thickness of the wood is 1 cm. The volume of the wood required to make the box is:

A. 960 cm cube

B. 900 cm cube

C. 1000 cm cube

D. 1100 cm cube

3. The size of a wooden block is (15 cm × 12 cm × 20 cm). How many such blocks will be required to construct a solid wooden cube of minimum size?

A. 50

B. 40

C. 60

D. 55

4. Two solid cylinders of radii 4 cm and 5 cm and lengths 6 cm and 4 cm, respectively are recast into cylindrical disc of thickness 1 cm. The radius of the disc is

A. 7 cm

B. 14 cm

C. 21 cm

D. 28 cm

5. The radius of cross-section of a solid cylindrical rod of iron is 50 cm. The cylinder is melted down and formed into 6 solid spherical balls of the same radius as that of the cylinder. The length of the rod (in m) is

A. 0.8

B. 2

C. 3

D. 4

6. A cone, a hemisphere and a cylinder stand on equal bases of radius R and have equal heights H. Their whole surfaces are in the ratio:

A. $(\sqrt{3}+1) : 3 : 4$

B. $(\sqrt{2}+1) : 7 : 8$

C. $(\sqrt{2}+1) : 3 : 4$

D. None of these

7. A large solid sphere of diameter 15 m is melted and recast into several small spheres of diameter 3 m. What is the percentage increase in the surface area of the smaller spheres over that of the large sphere?

A. 200%

B. 400%

C. 500%

D. can't be determined

8. A hemispherical basin 150 cm in diameter holds water one hundred and twenty times as much a cylindrical tube. If the height of the tube is 15 cm, then the diameter of the tube (in cm) is:

A. 23

B. 24

C. 25

D. 26

9. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the Centre of one end and the other end as its base. The volumes of the cylinder, hemisphere and the cone are respectively in the ratio of:

A. 3 : √3 : 2

B. 3 : 2 : 1

C. 1 : 2 : 3

D. 2 : 3 : 1

10. A cuboidal block 6cm x 9cm x 12cm is cut up into an exact number of equal cubes. The least possible number of equal cubes will be

A)6

B) 9

C) 24

D) 30

11. The height of a cone is 40 cm. The cone is cut parallel to its base such that the volume of the small cone is 1/64 of the cone. Find at which height from base the cone is cut?

A. 20 cm

B. 30 cm

C. 25 cm

D. 22.5 cm


Thank you

# HEIGHT AND DISTANCE

# Content

***1) Introduction***
***i. Trigonometry***
***ii. Trigonometry identities***
***iii. Values of T ratio***
***iv. Angle of elevation and depression***

***2) Problems***
***i. Two of the sides given***
***ii. One angle and one side given***
***iii. Two heights and one angle***
***iv. Two angles and one height***
***v. Two angles and two heights***
***vi. Calculating time and distance***

***3)Practice problems***

# 1) Introduction

This chapter deals with finding the heights, distances and angles using trigonometric values

# 1.i) Trigonometry:

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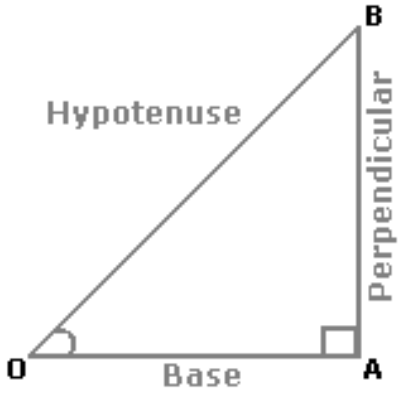
**Trigonometry:**

In a right angled $\Delta$ OAB, where $\angle BOA = \theta$,




i. $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{AB}{OB}$;

ii. $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{OA}{OB}$;

iii. $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{AB}{OA}$;

iv. $\text{cosec}\,\theta = \dfrac{1}{\sin\theta} = \dfrac{OB}{AB}$;

v. $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{OB}{OA}$;

vi. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{OA}{AB}$;

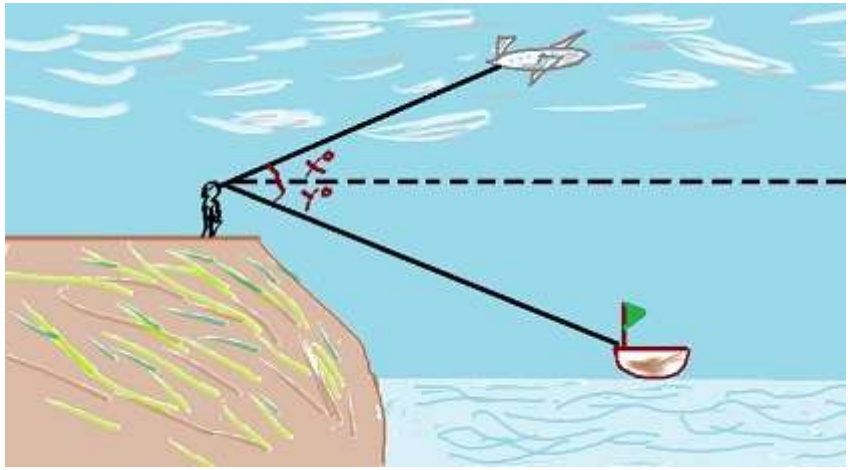
## 1.ii) Trigonometry Identities:

$\sin^2 \theta + \cos^2 \theta = 1.$
$1 + \tan^2 \theta = \sec^2 \theta.$
$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

## 1.iii) Values of T-ratios:

| | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| sin θ | 0 | 1/2 | $1/\sqrt{2}$ | $\sqrt{3}/2$ | 1 |
| cos θ | 1 | $\sqrt{3}/2$ | $1/\sqrt{2}$ | 1/2 | 0 |
| tan θ | 0 | $1/\sqrt{3}$ | 1 | $\sqrt{3}$ | Not defined |

# 1.iv) Angle of elevation and depression



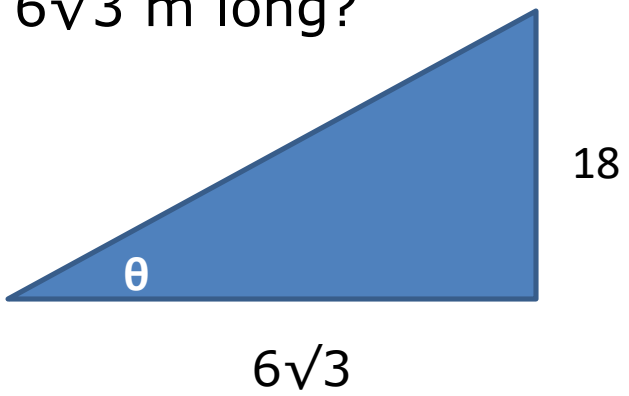

x – angle of elevation
y – angle of depression

***Note:*** *The base line for angle of elevation and angle of depression will always be the horizontal line.*

# 2) Problems

## 2.i) Two of the sides given

**Example:** Find the angle of elevation of the sun when the shadow of a pole of 18 m height is $6\sqrt{3}$ m long?




**Given:**

Perpendicular = 18 m

Base = $6\sqrt{3}$ m

Angle = ?

**Solution:**

**$\tan\theta$ = perpendicular/base**

$= 18/ 6\sqrt{3}$

$= 3/\sqrt{3}$

$= \sqrt{3}$

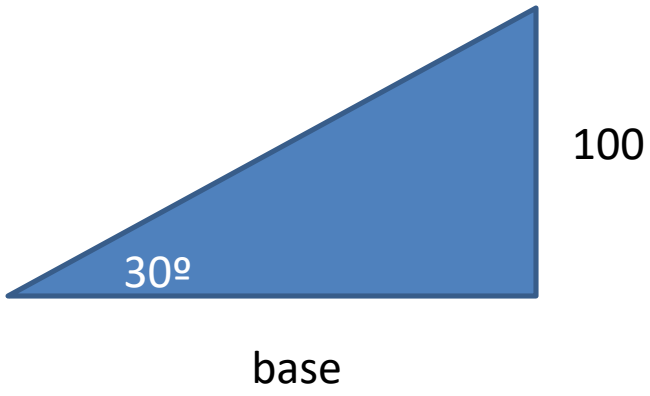
**$\theta$ = 60º**

# 2.ii) One angle and one of the sides given

**Example:** From a point P on a level ground, the angle of elevation of the top tower is 30º. If the tower is 100 m high, then what is the distance of point P from the foot of the tower.

**Given:**

Perpendicular = 100 m
Angle = 30º
Base = ?

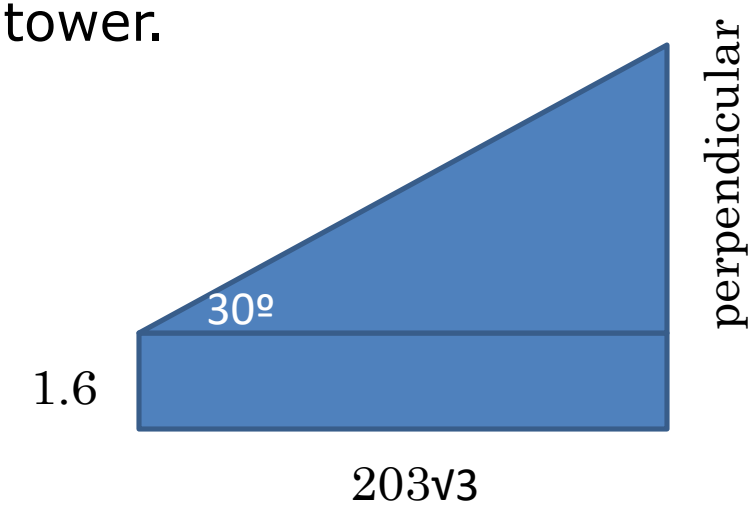



**Solution:**

**tan θ = perpendicular/base**
tan 30º = 100/base
1/ √3 = 100/base
Base = 100 √3
**= 173 m**

# 2.iii) Two heights and one angle

**Example:** An observer 1.6 m tall is 203√3 m away from a tower. The angle of elevation from his eye to the top of the tower is 30º. Find the height of the tower.




**Given:**
Base = 203√3 m
Angle = 30º

Height = perpendicular + 1.6 = ?

**Solution:**

**tan θ = perpendicular / base**
tan 30º = perpendicular / 203√3
1/ √3 = perpendicular / 203√3
Perpendicular = 203√3 / √3
**Perpendicular = 203 m**

**Height of the tower = perpendicular + 1.6**

= 203 + 1.6

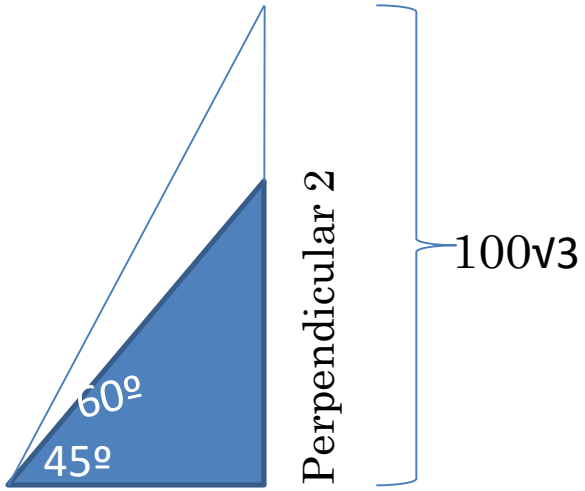
**= 204.6m**

# 2.iv) Two angles and one height

**Example:** An aeroplane when 100√3 m high passes vertically above another aeroplane at an instant when their angles of elevation at same observing point are 60° and 45° respectively. Approximately, how many meters higher is the one than the other?

**Given:**

Perpendicular 1 = 100√3 m
Angle 1 = 60°
Angle 2 = 45°




Difference b/w the heights =Perpendicular 1 - Perpendicular 2 = ?

**Solution:**

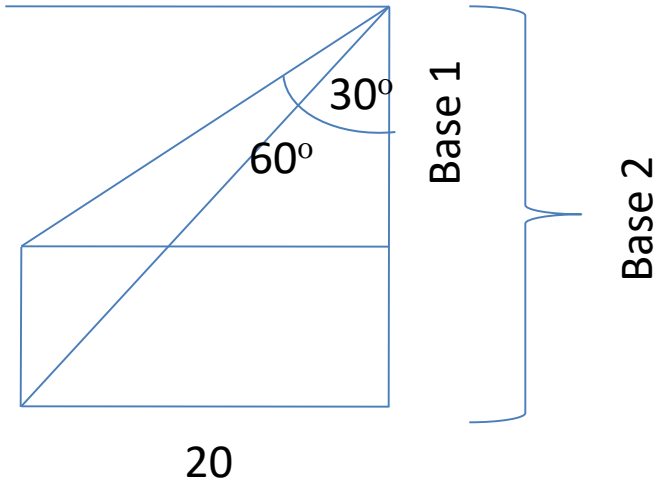
**Angle 1 = Perpendicular 1/ Base**
tan 60° = 100√3 / base
√3 = 100√3 /base
Base = 100√3 / √3
**Base = 100**

**Angle 2 = Perpendicular 2/ Base**
tan 45° = Perpendicular 2 / 100
1 = Perpendicular 2 / 100
**Perpendicular 2 = 100**

Difference b/w the heights =Perpendicular 1 - Perpendicular 2
= 100√3 – 100
= 173 – 100
**= 73 m**

# 2.v) Two angles and two heights

**Example:** Two towers face each other separated by a distance $d$ = 20 m. As seen from the top of the first tower, the angle of depression of the second tower's base is $60^o$ and that of the top is $30^o$. What is the height of the second tower?




**Given:**

Angle 1 = $90^o - 30^o = 60^o$

Angle 2 = $90^o - 60^o = 30^o$

Perpendicular = 20m

Height of the second tower = Base 2 – Base 1

**Solution:**

**Angle 2 = perpendicular/base 2**

$\tan 30^o = 20 / \text{base } 2$

$1/\sqrt{3} = 20 / \text{base } 2$

**Base 2 $= 20\sqrt{3}$**

**Angle 1 = perpendicular / base 1**

$\tan 60^o = 20 / \text{base } 1$

$\sqrt{3} = 20 / \text{base } 1$

**Base 1 $= 20 / \sqrt{3}$**

Height of the second tower = Base 2 – Base 1

$= 20\sqrt{3} - 20 / \sqrt{3}$

$= 20\sqrt{3} - 20\sqrt{3} / 3$

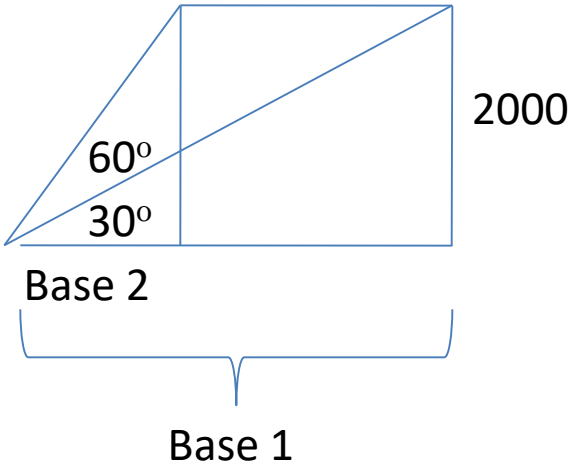
$= 20\sqrt{3} ( 1 - 1/3)$

$= 20\sqrt{3} (2/3)$

**$= 40\sqrt{3} / 3$**

# 2.vi) Calculating time and speed

**Example:** You are stationed at a radar base and you observe an unidentified plane at an altitude $h$ = 2000 m flying towards your radar base at an angle of elevation = 30$^o$. After exactly one minute, your radar sweep reveals that the plane is now at an angle of elevation = 60$^o$ maintaining the same altitude. What is the speed (in m/s) of the plane?




**Given:**

Angle 1 = 30$^o$
Angle 2 = 60$^o$
Perpendicular = 2000 m
Time = 60 sec
Speed = distance / time = (base 1 – base 2) / time

**Solution:**

**Angle 1 = perpendicular / base 1**
tan $30^o$ = 2000 / base 1
$1/\sqrt{3}$ = 2000 / base 1
**Base 1 = $2000\sqrt{3}$**

**Angle 2 = perpendicular / base 2**
Angle $60^o$ = 2000 / base 2
$\sqrt{3}$ = 2000 / base 2
**Base 2 = 2000 / $\sqrt{3}$**

**Speed = distance / time**
= (base 1 – base 2) / time
= ($2000\sqrt{3}$ - 2000 / $\sqrt{3}$) / 60
**= $200\sqrt{3}$ / 9 m/s**

# 3) Practice problems

1. Find the angle of elevation of the sun when the shadow of a pole of 18 m height is $6\sqrt{3}$ m long?

A. 30°

B. 60°

C. 45°

D. None of these

2. The angle of elevation of the sun, when the length of the shadow of a tree is √3 times the height of tree, is :

A. 30 degree

B. 45 degree

C. 60 degree

D. 9 degree

3. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is:

A. 2.3 m

B. 4.6 m

C. 7.8 m

D. 9.2 m

4. An observer 1.6 m tall is 20√3 away from a tower. The angle of elevation from his eye to the top of the tower is 30°. The heights of the tower is:

A. 21.6 m

B. 23.2 m

C. 24.72 m

D. None of these

5. From a tower of 80 m high, the angle of depression of a bus is 30°. How far is the bus from the tower?

A. 80 m
B. 80√3 m
C. 80/√3m
D. 240 m

6. From a point P on a level ground, the angle of elevation of the top of a tower is $30^0$ If the tower is 100 m high, the distance of point P from the foot of the tower is:

A. 149 m

B. 156 m

C. 173 m

D. 200 m

7. The thread of a kite is 120 m long and it is making 30° angular elevation with the ground .What is the height of the kite?

A. 60 m

B. 20 m

C. 40 m

D. 10 m

8. When the sun's altitude changes from 30° to 60°, the length of the shadow of a tower decreases by 70m. What is the height of the tower?

A. 55.6 m

B. 60.6 m

C. 65.6 m

D. 70.6 m

9. The length of the shadow of a vertical tower on level ground increases by 10 metres when the altitude of the sun changes from 45° to 30°. Then the height of the tower is:

A. $5\sqrt{3}$ m

B. $10(\sqrt{3} + 1)$ m

C. $5(\sqrt{3} + 1)$ m

D. $10\sqrt{3}$ m

10. Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse is observed from the ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is:

A. 173 m
B. 200 m
C. 273 m
D. 300 m

11. A vertical post 15 ft. high is broken at a certain height and its upper part, not completely separated meets the ground angle of $30^0$. Find the height at which the post is broken.

A. 10 ft.

B. 5 ft.

C. $15\sqrt{3}\ (2-\sqrt{3})$ ft.

D. $5\sqrt{3}$ ft.

12. The angle of elevation of the top of a tower from a point on the ground is 30° and moving 70 meters towards the tower it becomes 60°. The height of the tower is:

A. 10 meter
B. $10/\sqrt{3}$ meter
C. $10\sqrt{3}$ meter
D. $35\sqrt{3}$ meter

13. The top of a 15 m high tower makes an angle of elevation of 60 degree with the bottom of an electric pole and an angle of 30 degree with the top of the pole. What is the height of the pole?

A. 12 m

B. 10 m

C. 11 m

D. 5 m

14. Two pillars of equal height are on either side of a road, which is 120m wide. The angles of elevation of the top of the pillars are $60^0$ and $30^0$ at a point on the road between the pillars. Find the height of the pillars.

A. $10\sqrt{3}$ m

B. $30\sqrt{3}$ m

C. $20\sqrt{3}$ m

D. None of these

THANKS
FOR
LISTENING