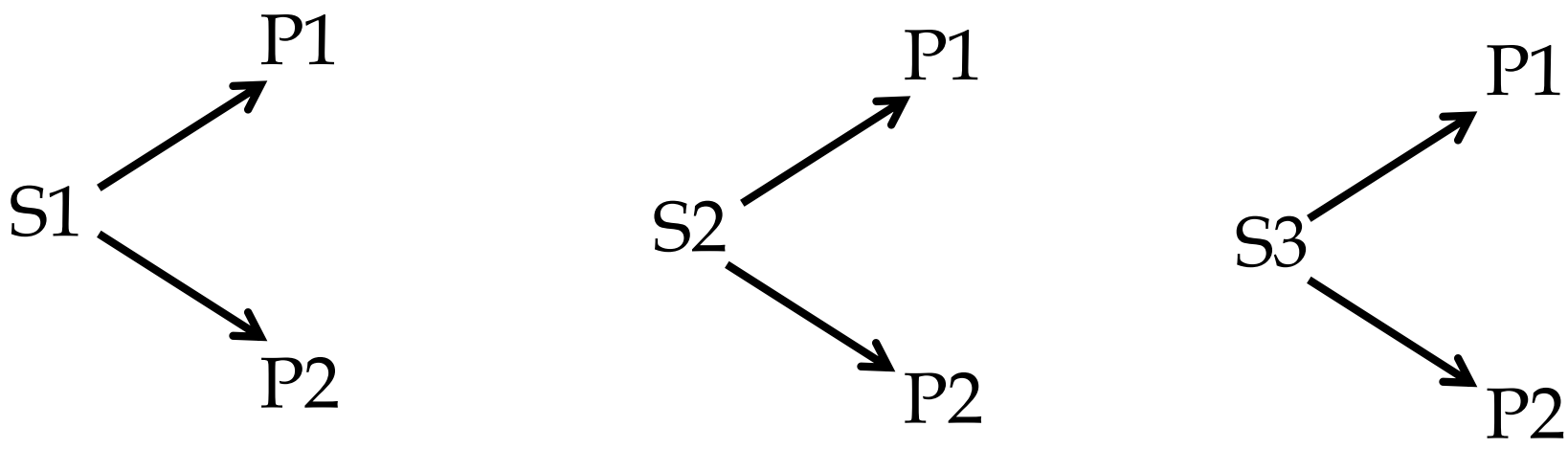
Permutation
&
Combination

Fundamental Principal of counting

- **Rule of product :** If there are **'m'** ways to do a process and there are **'n'** ways to do another, then total number of ways of doing both process is given by **'m x n'**

• If there are 3 shirts and 2 pants then in how many ways a person can dress up for a seminar?



Directly, Total ways = 3 * 2 = 6 ways

- **Rule of addition :** If there are **'m'** ways to do a process and there are **'n'** ways to do another and we can not do both at the same time, then there are '**m + n**' ways to choose one of the actions.

• If there are 3 formal shoes and 2 casual shoes then in how many ways we can choose a footwear for a party.

F1, F2, F3, C1, C2

F1 or F2 or F3 or C1 or C2

Total ways = 3 + 2 = 5 ways

Question: Let us assume you have 3 shirts, 4 pants, 3 shoes and 2 sandals to wear. Find in how many ways you can decide an outfit.

1. **Shirt – Pants – Shoes**
2. **Shirt – Pants – Sandal**

Note : Multiplication ------- **" AND"** (Stages)

Addition---------- **" OR"** (Choice)

Difference between Permutation and Combination

Permutation : Arrangement
: Order matters

Combination : Selection
: Order doesn't matters

Permutations and Combinations

Number of permutations (order matters) of *n* things taken *r* at a time:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Number of combinations (order does not matter) of *n* things taken *r* at a time:

$$C(n,r) = \frac{n!}{(n-r)!r!}$$

Question: If suppose we have 3 objects A, B, C then find no. of ways in which any 2 items can be selected.

Answer:

1. AB (BA)
2. BC (CB)
3. CA (AC)

Question: If suppose we have 3 objects A, B, C then find no. of ways to arrange any 2 items.

Answer:

1. AB
2. BA
3. BC
4. CB
5. CA
6. AC

Practice Question

1. In how many ways can we select a team of 4 players out of 15 eligible players.

[A] 1365

[B] 1455

[C] 1295

[D] 1525

2. In a class there are 6 boys and 5 girls. In how many ways can a group of 5 members to be formed by selecting 3 boys and 2 girls.

[A] 350

[B] 300

[C] 250

[D] 200

3. In how many ways 3 VIPs can be seated in 3 seats of first row of a function.

[A] 3

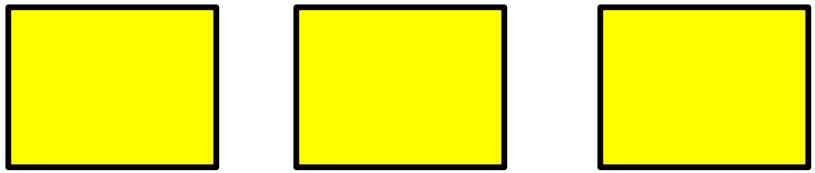
[B] 4

[C] 5

[D] 6

Note : Number of ways of arranging ‘n’ different items in a row = n !

In previous question,



3 x 2 x 1 = 3! = 6 ways

4. In how many ways 5 medals of different games can be arranged in a shelf.

[A] 100

[B] 110

[C] 120

[D] 150

5. Suppose you have to choose a 3 letter password. First letter is an alphabet, followed by a number and last one is an special character. There are 5 special character available. Find no. of ways to choose password.

[A] 1050

[B] 1200

[C] 1300

[D] 1560

Problems on Numbers

6. How many 2 digit numbers can be made from the digits 1, 2, 3 and 4 without repetition?

[A] 24

[B] 18

[C] 12

[D] 6

7. How many 4 digit numbers are possible with the digits 1, 2, 3, 6, 7, 8 and 9 without repetition?

[A] 720

[B] 480

[C] 840

[D] 320

8. How many 4 digit numbers are possible with the digits 1, 2, 3, 6, 7, 8 and 9 if repetition is allowed?

[A] 2401

[B] 820

[C] 343

[D] 729

9. How many 4 digit numbers can be made from the digits 7, 8, 5, 0, and 4 without repetition?

[A] 70

[B] 96

[C] 84

[D] 48

10. How many 3 digit numbers greater than 400 can be made with the digits 2, 3, 4, 0, 5, 6 (digits cannot be repeated)?

[A] 119

[B] 59

[C] 120

[D] 60

11. How many 3 digit numbers between 200 and 700 can be made with the digits 1, 3, 4, 0, 5, 6 (digits cannot be repeated) ?

[A] 80

[B] 120

[C] 60

[D] None of these

12. How many 3 digit number can be formed with the digits 5, 6, 2, 3, 7 and 9 which are divisible by 5 and none of its digit is repeated?

[A] 12

[B] 16

[C] 20

[D] 24

13. How many 4 digit number can be formed with the digits 0, 1, 2, 3, 4, 5, 6 which are divisible by 5 and none of its digit is repeated?

[A] 120

[B] 100

[C] 220

[D] 320

14. How many 4 digit odd number can be formed with the digits 0, 1, 2, 3, 4, 5, 6 if none of its digit is repeated?

[A] 120

[B] 100

[C] 220

[D] 300

15. How many 4 digit even number can be formed with the digits 0, 1, 2, 3, 4, 5, 6 if none of its digit is repeated?

[A] 120

[B] 420

[C] 220

[D] 200

16. Find the no of 3 digit numbers such that at least one of the digit is 6 (with repetitions)?

[A] 252

[B] 345

[C] 648

[D] 560

Problems on Words:

17. In How many different ways the letters of the word EQUATION can be arranged ?

[A] 7!

[B] 8!

[C] 9!

[D] 6!

18. In How many different ways the letters of the word EQUATION can be arranged, if it starts with letter Q ?

[A] 7!

[B] 8!

[C] 9!

[D] 6!

19. In How many different ways the letters of the word EQUATION can be arranged, if it starts with consonants?

[A] 7!

[B] 8!

[C] 2*7!

[D] 3*7!

20. In How many ways the word OPTICAL be arranged such that all vowels are together?

[A] 720

[B] 820

[C] 2160

[D] 1000

21. In How many ways the word OPTICAL be arranged such that all vowels are never together?

[A] 720

[B] 1000

[C] 2160

[D] 4320

22. In How many ways the word MANPOWER be arranged such that all vowels are together?

[A] 3! 6!

[B] 2! 7!

[C] 3! 5!

[D] 4! 4!

23. In How many ways letters of word PRAISE be arranged such that all consonants are together?

[A] 3! 4!

[B] 4! 4!

[C] 3! 5!

[D] 4! 5!

24. In How many ways letters of word PREVIOUS be arranged such that all vowels always come together?

[A] 1440

[B] 2880

[C] 4320

[D] 840

25. In how many ways can the letters of word FLEECED be arranged?

[A] 410

[B] 880

[C] 840

[D] 1260

26. Find the total arrangement of the letters of the word “MISSISSIPPI?

[A] 34650

[B] 32540

[C] 28450

[D] 24560

27. In how many different ways can the letter of the word “ELEPHANT” be arranged so that E’s are never together?

[A] 5040

[B] 15120

[C] 20160

[D] 35280

28. Find the total arrangement of the letters of the word “INVISIBILITY” such that all ‘I’ always come together.

[A] 8!

[B] 8!*5!

[C] 8!*5

[D] 7!*5!

29. In how many ways can the letters of the word "MACHINE" be arranged so that the vowels may occupy only odd positions?

[A] 4*7!

[B] 576

[C] 288

[D] 4 * 4!

30. Find the rank of the word “CHASM” if all the words can be formed by permuting the letters of this word without repetition are arranged in dictionary order.

[A] 24

[B] 31

[C] 32

[D] 30

31. Find the rank of the word “JAIPUR” if all the words can be formed by permuting the letters of this word without repetition are arranged in dictionary order.

[A] 241

[B] 122

[C] 123

[D] 242

31. Find the rank of the word “INDIA” if all the words can be formed by permuting the letters of this word without repetition are arranged in dictionary order.

[A] 41

[B] 42

[C] 45

[D] 46

32. Find the rank of the word “GOOGLE” if all the words can be formed by permuting the letters of this word without repetition are arranged in dictionary order.

[A] 78

[B] 84

[C] 85

[D] 88

Problems on Combination (Group Formation)

33. In how many ways a group of 4 men and 3 women be made out of a total of 8 men and 5 women?

[A] 720

[B] 700

[C] 120

[D] 360

34. There are 8 men and 7 women. In how many ways a group of 5 people can be made such that the particular woman is always to be included?

[A] 860

[B] 1262

[C] 1001

[D] 1768

35. There are 4 men and 3 women. In how many ways a group of three people can be formed such that there is at least 1 women in the group.

[A] 40

[B] 20

[C] 34

[D] 31

36. In a group of 6 boys and 5 girls, 5 students have to be selected. In how many ways it can be done so that at least 2 boys are included.

a) 1524

b) 1526

c) 1540

d) 1560

37. A box contains ten balls out of which 3 are red and rest blue. In how many ways can a random sample of six balls be drawn so that at most 2 red balls are included.

[A] 105

[B] 189

[C] 168

[D] 175

38. In a party there are 12 persons. How many handshakes are possible if every person handshake with every other person?

[A] 66

[B] 24

[C] 72

[D] 68

Circular arrangements

n distinct objects --------- Linear---------n!
n distinct objects---------- Circular----- (n-1)!

Note: In circle there is symmetry and hence there is no starting and end point, so when we need to arrange n distinct objects around a circle 1st object will break the symmetry (specify the position) and it can be done in 1 way and rest (n-1) objects can be arranged in (n-1)! Ways

Circular arrangement of n objects= 1 x (n-1)!= (n-1)!

If there is a difference between Clockwise and anti-Clockwise arrangement , and if

1. We need to arrange r objects out of n objects then **= nPr/r**

2. We need to arrange all n distinct objects **=nPn/n = n!/n = (n-1)!**

If there is no difference between Clockwise and anti-Clockwise arrangement (like in case of Garlands, Bead and Necklace etc.) , and if

1. We need to arrange r objects out of n objects then = **nPr/2r**

2. We need to arrange all n distinct objects = **nPn/2n = n!/2n = (n-1)!/2**

39. In how many ways 5 Americans and 5 Indians be seated along a circular table, so that they occupy alternative positions

[A] 5! 5!

[B] 6! 4!

[C] 4! 5!

[D] 4! 4!

40. A meeting of 20 delegates is to be held in a hotel. In how many ways these delegates can be seated around a circular table if 3 particular delegates always seat together.

[A] 17! 3!

[B] 18! 3!

[C] 17! 4!

[D] None

41. How many triangles can be formed by joining the vertices of hexagon?

[A] 20

[B] 12

[C] 24

[D] 10

42. How many diagonals can be formed by joining the vertices of hexagon?

[A] 10

[B] 12

[C] 9

[D] 8

Probability

Probability

Probability is a measure of likelihood that an event will occur.

Example: **Tossing a coin:** When a coin is tossed, there are two possible outcomes : either heads (H) or tails (T). We say that the probability of the coin landing H is ½. And the probability of the coin landing T is ½

TERMINOLOGY

Random Experiment: Experiments whose outcomes are unpredictable is known as Random Experiments. For example: Tossing a coin

Sample Space(S): It is the collection of all possible outcome of an experiment. Example: In tossing a coin one time S={H,T}

Event: The outcome of an experiment is known as Event. Mathematically we can say that event is a subset of sample space.

Example: Getting a head while tossing a coin one time is an event.

Types of Events:

1. Complementary Event: It is denoted by E^c *or* E'. It contains all the outcomes of sample space that is not in E.

For example If S={1,2,3,4,5,6} and E=[odd no.]={1,3,5}
Then E'=[Even number]={2,4,6}

2. Equally likely Events: If E and F are two events such that P(E)= P(F) then these are called Equally Likely events. For e.g. In Tossing a coin probability of coming up of head and tail is Equal.

3. Mutually Exclusive Event: Two or more events are said to be mutually exclusive if both cannot occur simultaneously in the same experiment.

Example: In a throw of single coin, either head can come or tail can come. There will be no common outcome in those events. It means $E \cap F = \Phi$ (null Set)

4. **Collective Exhaustive Events**: If E and F are two events and both events gives complete sample space then these are called Exhaustive events.
E **U** F = S

5. Independent Events: Two events are said to be independent of each other when the happening of one event does not affect the happening of other event and vice versa. Here sample spaces are different for both cases.

When two events A and B are independent, the probability of both occurring is:
P(A and B)= P(A). P(B)

Probability: The probability of an event is defined as the ratio of no. of ways an event can happen to the no. of ways sample space can happen.

Let S be the sample space and let E be the event.
PROBABILITY= n(E)/n(S)

6. Dependent Events: Two **events** are **dependent** if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.

When two events, A and B, are dependent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

where $P(B \mid A)$ is the **conditional probability** of an event B in relationship to an event A is the Probability that event B occurs given that event A has already occurred.

Example: The probability of choosing a jack on the second pick given that a queen was chosen on the first pick (without replacement) is called a ***conditional probability*.**

Rules of Probability

1. $0 \leq P(E) \leq 1$
2. $P(S) = 1$ (Definite event)
3. $P(\Phi) = 0$ (Impossible event)
4. If A′ denotes (not-A), then $P(A') = 1 - P(A)$.
5. For any events A and B we have :

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

But for mutually exclusive event **$P(A \cap B) = 0$**

For mutually exclusive events

$P(A \cup B) = P(A) + P(B)$

6. $P(A \cap B) = P(A).\ P(B/A) = P(B).\ P(A/B)$

7. $P(A \cup B \cup C) = P(A)+P(B)+P(C)-P(A \cap B)-P(B \cap C)$

$-P(C \cap A)+P(A \cap B \cap C)$

Coins

One coin {H,T}= 2

Two coins={HH,HT,TH,TT}= 4

Three Coin={HHH,TTT,HHT,TTH,HTH,THT,THH,HTT}= 8

Four Coins={HHHH, TTTT

HHHT, TTTH

HHTH, TTHT

HTHH, THTT

THHH, HTTT,

HHTT, HTTT,

HTTH, HTHT

TTHH, THTH} =16

Question 1: In a simultaneous toss of 2 coins find the probability of 2 Tails?

[A] 1/2

[B] 1/4

[C] 2/3

[D] 3/4

Question 2: In a simultaneous toss of 2 coins find the probability of Exactly 1 Tail?

[A] 1/3

[B] 2/3

[C] 3/4

[D] 1/2

Question 3: In a simultaneous toss of 2 coins find the probability of No Tail?

[A] 1/3

[B] 1/4

[C] 1/2

[D] 2/3

Question 4: In a simultaneous toss of 2 coins find the probability of No head?

[A] 1/2

[B] 1/4

[C] 1/3

[D] 2/3

Question 5: Three coins are tossed simultaneously. Find the probability of all are heads.

[A] 1/4

[B] 1/8

[C] 2/3

[D] 5/8

Question 6: Three coins are tossed simultaneously. Find the probability of exactly two heads.

[A] 1/4

[B] 1/8

[C] 3/8

[D] 5/8

Question 7: Three coins are tossed simultaneously. Find the probability of at least two heads.

[A] 1/4

[B] 1/2

[C] 1/8

[D] 3/8

Question 8: Three coins are tossed simultaneously. Find the probability of no heads.

[A] 1/8

[B] 1/4

[C] 1/2

[D] 5/8

Question 9: Three coins are tossed simultaneously. Find the probability of at least 1 head and 1 tail.

[A] 1/8

[B] 1/4

[C] 3/4

[D] 5/8

Question 10: 4 coins are tossed simultaneously. Find the probability exactly 3 tails

[A] 1/8

[B] 1/4

[C] 1/2

[D] 5/8

Question 11: 4 coins are tossed simultaneously. Find the probability at least 1 tail.

[A] 1/16

[B] 1/4

[C] 1/2

[D] 15/16

DICE

When a dice is thrown= {1, 2 , 3, 4, 5, 6} =6

When two dice are thrown, S =

{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} = 36

Question 1: In a single throw of 2 Dice, What is the probability of a doublet?

[A] 1/3

[B] 1/36

[C] 1/6

[D] 1/12

Question 2: In a single throw of 2 Dice, What is the probability of getting sum equals to:

I. 5
II. Multiple of 5
III. 7
IV. Multiple of 3
V. Greater than 9

Short Cut For Two Dice

SUM	2	3	4	5	6	7	8	9	10	11	12
Fav.	1	2	3	4	5	6	5	4	3	2	1

Question 3: In a simultaneous throw of 3 Dice find the probability of getting a total of 5.

[A] 1/6

[B] 1/36

[C] 1/216

[D] 5/216

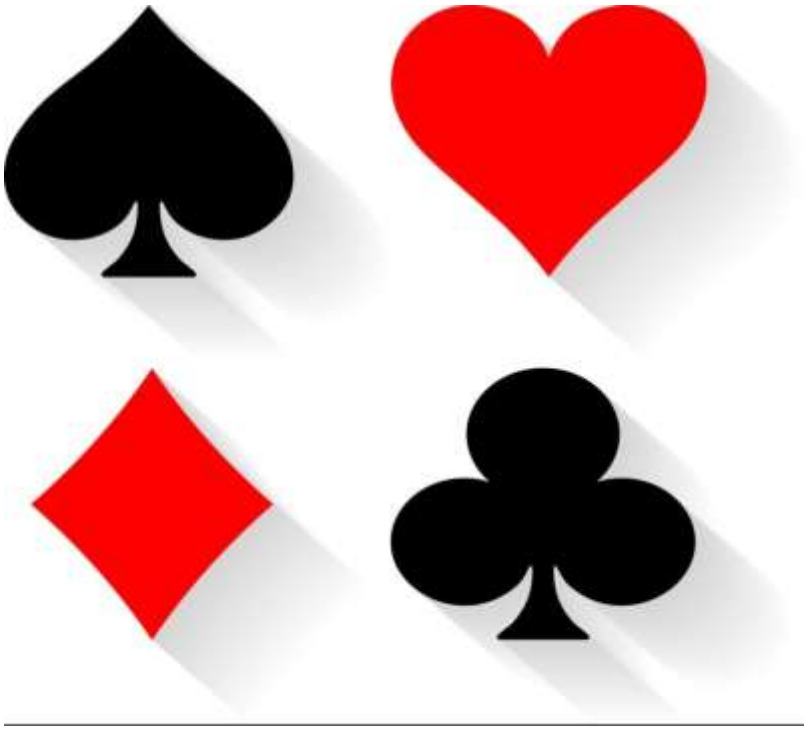
Cards

1. There are four Suits in a deck of Card viz. **Spade**, **Diamond**, **Heart**, **Club**.

2. **Red card=26** (13 Diamonds+ 13 Hearts)

3. 3. **Black Card= 26** (13 Clubs+13 Spades)

4. Every suit contains 13 cards viz. **Ace**, **2 to 10**, **Jack**, **Queen**, **King**



Face cards and Honored Card

Note: When ace is included in Face cards the combination is called Honored card.

Question 1: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability of picking a black card?

[A] 1/3

[B] 1/2

[C] 1/4

[D] 1/13

Question 2: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability of picking a Ace of spades or the jack of diamonds?

[A] 1/52

[B] 1/26

[C] 1/13

[D] 1/4

Question 3: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability of picking an ace?

[A] 1/13

[B] 1/52

[C] 1/26

[D] 1/4

Question 4: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability that the card is either a red card or a King?

[A] 5/13

[B] 7/13

[C] 9/13

[D] 1/52

Question 5: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability that it is neither club nor queen?

[A] 4/13

[B] 5/13

[C] 7/13

[D] 9/13

Balls in a Bag OR marbles in a Bag

Questions 1: .A bag contains 5 red balls and 7 blue balls. Two balls are drawn at random without replacement, and then find the probability of that one is red and other is blue.

[A] 33/65

[B] 35/66

[C] 37/66

[D] 41/65

Questions 2: . A urn contains 4 red balls, 5 green balls and 6 white balls, if one ball is drawn at random, find the probability that it is neither red nor white.

[A] 1/3

[B] 1/4

[C] 1/5

[D] 2/3

Questions 3: . A bag contains 6 red balls and 7 white balls. Another bag contains 5 red balls and 3 white balls. One ball is selected from each. Find the probability that one ball is red and one is white?

[A] 53/104

[B] 47/104

[C] 63/104

[D] 51/104

Questions 4: In a bag there are 4 white, 4 red and 2 green balls. Two balls are drawn at random. What is the probability that at least one ball is of red color?

[A] 4/3

[B] 7/3

[C] 1/3

[D] 2/3

Questions 5: . A bag contains 2 red caps, 4 blue caps, 3 yellow caps and 5 green caps. If three caps are picked at random, what is the probability that none is green?

[A] 2/13

[B] 3/13

[C] 1/13

[D] 5/13

Questions 6: A bag contains 5 red and 7 white balls. Four balls are drawn out one by one and not replaced. What is the probability that they are alternatively of different colors?

[A] 7/99

[B] 11/99

[C] 14/99

[D] 19/99

Questions 7: A basket contains 5 red 4 blue 3 green marbles. If three marbles picked up random, What is the probability that either all are green or all are red?

[A] 1/20

[B] 7/20

[C] 3/20

[D] 9/20

Questions 8: A bag contains 3 red balls and 8 blacks ball and another bag contains 5 red balls and 7 blacks balls, one ball is drawn at random from either of the bag, find the probability that the ball is red.

[A] 93/264

[B] 95/264

[C] 91/264

[D] 97/264

Miscellaneous Questions:

Questions 1: A fair dice is rolled twice. The probability that an odd number will follow on even number is? **GATE-2005**

[A] 1/2

[B] 1/6

[C] 1/3

[D] 1/4

Questions 2: An examination consists of two papers, Paper1 and 2. The probability of failing in Paper1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper2, the probability of failing in paper 1 is 0.6. the probability of failing in both the papers is? **GATE-2007**

[A] 0.5

[B] 0.18

[C] 0.12

[D] 0.06

Bayes' Theorem:

Let S be the sample space and let E1,E2,E3,……En be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E1 or E2 or E3…or En. then,

$$P(Ei/A) = \frac{P(Ei)P(A/Ei)}{\sum_{i=1}^{n} P(Ei)P(A/Ei)}$$

Question : A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts. Find the probability of the lost card being a heart?

[A] 12/50

[B] 8/50

[C] 11/50

[D] 9/50

Binomial Distribution:

If **p** is the probability of success of any event and **q** is the probability of failure of that event, then probability of event success **x times** in **n trials**(i.e. x success and n-x failure) is given by:

$$P(X=x) = {}_{n}C_{x}\ p^{x}\ q^{n-x}$$

Where X= Random variable, x= no. of success in n trials
P= probability of success, q= 1-p= probability of failure

Question: Find the probability of getting sum 9 exactly two in three times with a pair of dice.

Answer= 8/243

thank you!