

Softmax exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](http://vision.stanford.edu/teaching/cs231n/assignments.html) (<http://vision.stanford.edu/teaching/cs231n/assignments.html>) on the course website.

This exercise is analogous to the SVM exercise. You will:

- implement a fully-vectorized **loss function** for the Softmax classifier
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** with numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

In [2]:

```
import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

#from __future__ import print_function

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading extenrnal modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

In [3]:

```
def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000, num_dev=500):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the linear classifier. These are the same steps as we used for the
    SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'
    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # subsample the data
    mask = list(range(num_training, num_training + num_validation))
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = list(range(num_training))
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = list(range(num_test))
    X_test = X_test[mask]
    y_test = y_test[mask]
    mask = np.random.choice(num_training, num_dev, replace=False)
    X_dev = X_train[mask]
    y_dev = y_train[mask]

    # Preprocessing: reshape the image data into rows
    X_train = np.reshape(X_train, (X_train.shape[0], -1))
    X_val = np.reshape(X_val, (X_val.shape[0], -1))
    X_test = np.reshape(X_test, (X_test.shape[0], -1))
    X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis = 0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image
    X_dev -= mean_image

    # add bias dimension and transform into columns
    X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
    X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
    X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
    X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

    return X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev

# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('dev data shape: ', X_dev.shape)
print('dev labels shape: ', y_dev.shape)
```

Train data shape: (49000, 3073)

Train labels shape: (49000,)

```
Validation data shape: (1000, 3073)
Validation labels shape: (1000,)
Test data shape: (1000, 3073)
Test labels shape: (1000,)
dev data shape: (500, 3073)
dev labels shape: (500,)
```

Softmax Classifier

Your code for this section will all be written inside **cs231n/classifiers/softmax.py**.

In [4]:

```
# First implement the naive softmax loss function with nested loops.
# Open the file cs231n/classifiers/softmax.py and implement the
# softmax_loss_naive function.

from cs231n.classifiers.softmax import softmax_loss_naive
import time

# Generate a random softmax weight matrix and use it to compute the loss.
W = np.random.randn(3073, 10) * 0.0001
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As a rough sanity check, our loss should be something close to -log(0.1).
print('loss: %f' % loss)
print('sanity check: %f' % (-np.log(0.1)))
```

```
loss: 2.384431
sanity check: 2.302585
```

Inline Question 1:

Why do we expect our loss to be close to $-\log(0.1)$? Explain briefly.**

Your answer: *Fill this in*

In [5]:

```

# Complete the implementation of softmax_loss_naive and implement a (naive)
# version of the gradient that uses nested loops.
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As we did for the SVM, use numeric gradient checking as a debugging tool.
# The numeric gradient should be close to the analytic gradient.
from cs231n.gradient_check import grad_check_sparse
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)

# similar to SVM case, do another gradient check with regularization
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)

```

```

numerical: -0.213898 analytic: -0.213898, relative error: 3.954537e-07
numerical: 0.696629 analytic: 0.696629, relative error: 4.684757e-08
numerical: -0.172587 analytic: -0.172587, relative error: 1.590728e-07
numerical: 2.560807 analytic: 2.560807, relative error: 3.053302e-08
numerical: 1.783102 analytic: 1.783102, relative error: 1.201710e-08
numerical: -0.434282 analytic: -0.434282, relative error: 1.161133e-07
numerical: 1.191474 analytic: 1.191474, relative error: 2.190725e-08
numerical: 2.404192 analytic: 2.404192, relative error: 3.938666e-09
numerical: 2.768147 analytic: 2.768147, relative error: 2.050756e-08
numerical: -4.684141 analytic: -4.684141, relative error: 8.244356e-09
numerical: -2.103165 analytic: -2.103165, relative error: 8.171253e-09
numerical: -0.818012 analytic: -0.818012, relative error: 4.973498e-08
numerical: -4.402197 analytic: -4.402197, relative error: 9.524552e-09
numerical: 0.082125 analytic: 0.082125, relative error: 3.778729e-07
numerical: 0.910623 analytic: 0.910623, relative error: 9.127808e-08
numerical: -4.511888 analytic: -4.511888, relative error: 1.734890e-09
numerical: 1.680068 analytic: 1.680068, relative error: 5.561678e-09
numerical: -1.973945 analytic: -1.973945, relative error: 3.133355e-09
numerical: 3.006867 analytic: 3.006867, relative error: 1.763728e-08
numerical: 0.737572 analytic: 0.737571, relative error: 1.581659e-07

```

In [6]:

```
# Now that we have a naive implementation of the softmax loss function and its gradient,
# implement a vectorized version in softmax_loss_vectorized.
# The two versions should compute the same results, but the vectorized version should be
# much faster.
tic = time.time()
loss_naive, grad_naive = softmax_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('naive loss: %e computed in %fs' % (loss_naive, toc - tic))

from cs231n.classifiers.softmax import softmax_loss_vectorized
tic = time.time()
loss_vectorized, grad_vectorized = softmax_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# As we did for the SVM, we use the Frobenius norm to compare the two versions
# of the gradient.
grad_difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('Loss difference: %f' % np.abs(loss_naive - loss_vectorized))
print('Gradient difference: %f' % grad_difference)
```

```
naive loss: 2.384431e+00 computed in 0.129931s
vectorized loss: 2.384431e+00 computed in 0.150601s
Loss difference: 0.000000
Gradient difference: 0.000000
```

In [26]:

```

# Use the validation set to tune hyperparameters (regularization strength and
# Learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of over 0.35 on the validation set.
from cs231n.classifiers import Softmax
results = {}
best_val = -1
best_softmax = None
learning_rates = [1e-8, 1e-7, 5e-7, 1e-6]
regularization_strengths = [5e2, 1e3, 1e4, 5e4]

#####
# TODO:
# Use the validation set to set the learning rate and regularization strength. #
# This should be identical to the validation that you did for the SVM; save   #
# the best trained softmax classifier in best_softmax.                         #
#####

softmax = Softmax()

for lr in learning_rates:
    for reg in regularization_strengths:
        loss_hist = softmax.train(X_train, y_train, learning_rate=lr, reg=reg, num_iters=15000)

        y_train_pred = softmax.predict(X_train)
        acc_train = np.mean(y_train == y_train_pred)

        y_val_pred = softmax.predict(X_val)
        acc_val = np.mean(y_val == y_val_pred)

        results[(lr, reg)] = (acc_train, acc_val)

        if acc_val > best_val:
            best_val = acc_val
            best_softmax = softmax

#####
#                                     END OF YOUR CODE                         #
#####

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_val)

lr 1.000000e-08 reg 5.000000e+02 train accuracy: 0.139653 val accuracy: 0.15
2000
lr 1.000000e-08 reg 1.000000e+03 train accuracy: 0.169000 val accuracy: 0.17
2000
lr 1.000000e-08 reg 1.000000e+04 train accuracy: 0.193367 val accuracy: 0.19
5000
lr 1.000000e-08 reg 5.000000e+04 train accuracy: 0.259469 val accuracy: 0.27
3000
lr 1.000000e-07 reg 5.000000e+02 train accuracy: 0.373612 val accuracy: 0.38
7000

```

```
lr 1.000000e-07 reg 1.000000e+03 train accuracy: 0.391224 val accuracy: 0.407000
lr 1.000000e-07 reg 1.000000e+04 train accuracy: 0.358469 val accuracy: 0.379000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.309204 val accuracy: 0.325000
lr 5.000000e-07 reg 5.000000e+02 train accuracy: 0.407612 val accuracy: 0.402000
lr 5.000000e-07 reg 1.000000e+03 train accuracy: 0.405184 val accuracy: 0.409000
lr 5.000000e-07 reg 1.000000e+04 train accuracy: 0.354204 val accuracy: 0.373000
lr 5.000000e-07 reg 5.000000e+04 train accuracy: 0.291796 val accuracy: 0.310000
lr 1.000000e-06 reg 5.000000e+02 train accuracy: 0.410367 val accuracy: 0.404000
lr 1.000000e-06 reg 1.000000e+03 train accuracy: 0.395673 val accuracy: 0.403000
lr 1.000000e-06 reg 1.000000e+04 train accuracy: 0.344122 val accuracy: 0.351000
lr 1.000000e-06 reg 5.000000e+04 train accuracy: 0.287347 val accuracy: 0.305000
best validation accuracy achieved during cross-validation: 0.409000
```

In [27]:

```
# evaluate on test set
# Evaluate the best softmax on test set
y_test_pred = best_softmax.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('softmax on raw pixels final test set accuracy: %f' % (test_accuracy, ))
```

softmax on raw pixels final test set accuracy: 0.306000

In [28]:

```
# Visualize the learned weights for each class
w = best_softmax.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)

w_min, w_max = np.min(w), np.max(w)

classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```

