Homework 4

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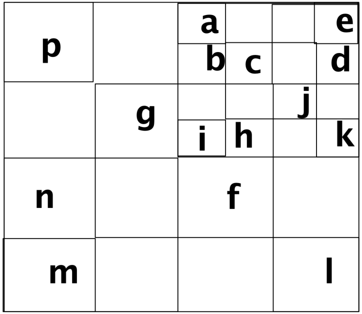
October 14, 2020

Problem 1



Each letter in the picture above denotes a charge. The squares are obtained in the process of building a quadtree. Assume that the multipole coefficients have been already determined for each square in the multipole construction phase. State how $\phi(p), \phi(g), \phi(b), \phi(m)$ get calculated in the potential evaluation phase. You only need to describe how the effect due to the other charges gets added into each potential.

Example: Calculation of $\phi(f)$: You merely need to state the following.

- Multipole expansions of the square containing each set of the following charges: {a,b,c}, {d,e}, {j}, {k}, {m}, {n}, {p}
- Direct calculation of the potential due to each of the following charges: g,h,i,l

Problem 2

Suppose we have the following bus routes

Route	Origin	Destination	Travel time	Expected waiting time
100	IIT	Dadar	50 min	20 min
101	IIT	Dadar	65 min	10 min
200	Dadar	Colaba	30 min	30 min
201	Dadar	Colaba	65 min	5 min
300	IIT	Colaba	70 min	60 min

I wish to travel from IIT to Colaba minimizing total time in expectation. What is the best plan as per the model in the lecture on Mumbai Navigator? Also work out and state the expected travel time as per your plan.

Problem 3

DPV 5.16: Prove the following two properties of the Huffman encoding scheme.

1. If some character occurs with frequency more than $2/5$, then there is guaranteed to be a codeword of length 1.
2. If all characters occur with frequency less than $1/3$, then there is guaranteed to be no codeword of length 1.

Problem 4

A matching in a graph is a subset of the edges such that no two edges have any endpoint in common. A matching having the maximum number of edges is called a maximum matching.

Give a greedy algorithm to find a maximum matching in a tree. It should run in time polnomial in n where n is the number of vertices in the tree. (Linear time is possible, but do not worry about it and keep your answer simple.)

Your answer should be structured as greedy choice + optimal substructure. Assuming that the graph is represented as an adjacency list, give as simple an analysis of the running time as possible – you only need to get polynomial – so do not sweat.

Problem 5

The input to this problem consists of arrays of not necessarily distinct positive integers: D[1..n], P[1..n]. D[i], P[i] resepctively are the the deadline and profit of job i. Time is divided into slots 1, 2, ... and each job must run for a single slot, and in a slot only one job can run. If job i runs during one of the slots 1,...,D[i] then we get a profit P[i]. If job i is run after slot D[i] there is no profit. Our goal is to maximize the profit.
In the greedy setting, we operate in n steps: in each step we pick one of the jobs and schedule it in one of the slots (within or beyond its deadline). Both of these operations should have a greedy solution, perhaps something like: pick the job with the least/largest deadline/profit and schedule it at the earliest/latest possible time.
Give your greedy choice for each step and prove that there is an optimal (max profit) solution in which the selected job is at the selected time.
Then show optimal substructure: show that you can create a new instance such that the schedule for the new instance will give you a schedule for the original instance. Note that you may modify the deadlines and profits to get the new instance if you wish. The ony requirement is that overall the time must be polynomial in n.
You must answer in the format above.