Energie ON

E = R +

D: ExE > E: (n.y) -> n & y = n.y

(RxE → E: (x,x) → x @n = nx

Hontrons que (E, D, &) est un R-e. V 2

=> M outros que (E, E) et un groye abelien:

· Darrociative:

(n⊕y)⊕ 3 = n⊕ (y⊕ 8)

{ (x ⊕y) ⊕ z = (x-y). z = x.y.z.

n⊕(y⊕3)= n.(y-3)= n..y-3

d'où @ est associative.)

· @ Commutative:

{ n & y = n.y.

Lyon=y-x

d'où Dest commatative.

. l'element neutre de 1

ne= eon= n

x ⊕ e= x => n.e= n.

e = 1.

•
$$\forall \alpha, \beta \in \mathbb{R}, \forall \chi \in E : (\alpha + \beta) \oplus \pi \stackrel{?}{=} (\alpha \oplus \chi) \oplus (\beta \oplus \chi)$$

$$\begin{cases} (\alpha + \beta) \oplus \pi = \chi^{\alpha + \beta} \\ (\alpha \oplus \chi) \oplus (\beta \oplus \chi) = (\chi^{\alpha}) \cdot (\chi^{\beta}) = \chi^{\alpha + \beta} \end{cases}$$

· ∀x ∈ R, ∀x, y ∈ E: x ⊕ (x ⊕ y)° = (x ⊕ 2) ⊕ (x ⊕ y).

$$\begin{cases} & \propto \otimes (n \otimes y) = \alpha \otimes (n \cdot y) = (n \cdot y)^{\alpha} \\ & (\alpha \otimes u) \oplus (\alpha \otimes y) = u^{\alpha} \cdot y^{\alpha} = (u \cdot y)^{\alpha} \end{cases}$$

d'en
$$(x \oplus (n \oplus y) = (x \oplus x) \oplus (x \oplus y).$$

· V x, B & R, YXEE: (x · B) @ x = x @ (B @ x)

E= IR

B: E x E → E: n ⊕ y = n + y + 1

●: RxE → E: × ⊕ n = x n+x+1

versions que E est un R-e.V

=> (E, 1) un groupe abelien:

· 1 associative:

(x⊕y)⊕ ? = n⊕ (y⊕3)

 $\left\{ (x \oplus y) \oplus 3 = (n+y+1) \oplus 3 = n+y+1 + 3+1 = n+y+3+2. \\ n \oplus (y \oplus 3) = n \oplus (y+3+1) = n+y+3+1 = n+y+3+2. \right.$

d'où @ est associative.

· D commutative:

 $(n \oplus y) \stackrel{?}{=} (y \oplus n)$

 $\begin{cases} \mathcal{A} \oplus \mathcal{Y} = \mathcal{N} + \mathcal{Y} + \Lambda. \\ \mathcal{Y} \oplus \mathcal{N} = \mathcal{Y} + \mathcal{N} + \Lambda. \end{cases}$

d'où @ est commutative.

· l'elevet neutre de D:

n⊕e=e⊕n=n

x ⊕ e = x => x+e+1= x.

(e=-1)

· l'elevent Synctrique de 1 2.

x ⊕ n'= n' ⊕ x = e = -1

NEN'= -1 => N+n'+1=-1

n'= - 2-n.)

Alos (E. 0) est un groupe abelien

· Ya, BER, Yne E: (x+B)@n= (x@x) (B@x). $\begin{cases} (\alpha + \beta) & \Rightarrow n = (\alpha + \beta)n + \alpha + \beta - \Lambda = \alpha n + \beta n + \alpha + \beta - \Lambda \\ (\alpha \otimes n) & \Rightarrow (\beta \otimes n) = (\alpha n + \alpha - \Lambda) & \Rightarrow (\beta n + \beta - \Lambda) = \alpha n + \beta n + \alpha + \beta - \Lambda \end{cases}$

d'où ((x+B) @ n = (x@ n) ⊕ (B@ n).)

. V x ∈ R, Vny ef: x (noy) = (x @ n) ⊕ (x @ y)

 $\begin{cases} & \propto \bigoplus (n \oplus y) = \propto \bigoplus (n + y + 1) = \alpha (n + y + 1) + \alpha - 1 = \alpha n + \alpha y + 2\alpha - 1. \\ & (\alpha \bigoplus n) \bigoplus (\alpha \bigoplus y) = (\alpha n + \alpha - 1) \bigoplus (\alpha y + \alpha - 1) = (\alpha n + \alpha - 1) + (\alpha y + \alpha - 1) + 1 = \alpha n + \alpha y + 2\alpha - 1. \end{cases}$

d'où (x⊕(n⊕y) = (x@n)⊕(x@y).)

· ∀ x, B ∈ R, ∀ x ∈ E: (x. B) @ n = & @ (B@n).

 $\left\{ \begin{array}{l} (\alpha \cdot \beta) \otimes \varkappa = \alpha \cdot \beta \cdot \varkappa + \alpha \cdot \beta - \Lambda \\ \\ \angle \otimes (\beta \otimes \varkappa) = \alpha \otimes (\beta \varkappa + \beta - \Lambda) = \alpha \beta \varkappa + \alpha \beta - \alpha + \alpha - \Lambda = \alpha \beta \varkappa + \alpha \beta - \Lambda \end{array} \right.$

d'où ((x. B) @ n = x@ (B +n).)

Alors ((E, D, 0) et un R-e.v.)

E l'enseille des matrices 2x 2 (Hexe (R))

E= { (a b) / a,b,c,d eR}.

Montros que E est un R-e.v

=> Hatras que (E1+) est un groge abelier =

· + associative:

$$\left\{ \begin{bmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} \right\} + \begin{pmatrix} a'' & b'' \\ c'' & d'' \end{pmatrix} = \begin{pmatrix} a+a' & b+b' \\ c+a' & d+d' \end{pmatrix} + \begin{pmatrix} a'' & b'' \\ c'' & d'' \end{pmatrix} = \begin{pmatrix} a+a'+a'' & b+b'+b'' \\ c+c'+c'' & d+d'+d'' \end{pmatrix}.$$

$$\left(\begin{pmatrix} a & b \\ c & d' \end{pmatrix} + \begin{pmatrix} a'' & b'' \\ c' & d'' \end{pmatrix} \right) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a' + a'' & b' + b'' \\ c' + c'' & d' + d'' \end{pmatrix}_{o} = \begin{pmatrix} a + a' + a'' & b + b' + b'' \\ c + c' + c'' & d + d' + d'' \end{pmatrix}.$$

d'où (+ est associative.)

· + commutative:

$$\begin{pmatrix}
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} + \begin{pmatrix}
a' & b' \\
c' & d'
\end{pmatrix} = \begin{pmatrix}
a+a' & b+b' \\
c+c' & d+d'
\end{pmatrix}.$$

$$\begin{pmatrix}
a' & b' \\
c' & d'
\end{pmatrix} + \begin{pmatrix}
a & b \\
c' & d'
\end{pmatrix} = \begin{pmatrix}
a'+a & b'+b \\
c'+c & d'+d
\end{pmatrix}.$$

d'où (+ est commutative.)

$$\begin{pmatrix} ab \\ cd \end{pmatrix} + e = e + \begin{pmatrix} a & b \\ cd \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

$$e = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

· l'elevet Synetique de +:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} = \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} = \begin{pmatrix} o & 0 \\ 0 & o \end{pmatrix}.$$

$$\left(\begin{array}{cc} a' b' \\ c' d' \end{array}\right) = \left(\begin{array}{cc} -a - b \\ -c - d \end{array}\right).$$

$$(\lambda + \lambda') \cdot X = \lambda + \lambda' \begin{pmatrix} \alpha & b \\ c & d \end{pmatrix} = \begin{pmatrix} (\lambda + \lambda') \alpha & (\lambda + \lambda') b \\ (\lambda + \lambda') c & (\lambda + \lambda') d \end{pmatrix}$$

$$\left\{ \begin{array}{l} (\lambda + \lambda') \cdot X = \lambda + \lambda' \begin{pmatrix} \alpha & b \\ c & d \end{pmatrix} = \begin{pmatrix} (\lambda + \lambda') \alpha & (\lambda + \lambda') b \\ (\lambda + \lambda') c & (\lambda + \lambda') d \end{pmatrix} \right.$$

$$\left(\lambda \cdot X \right) + \begin{pmatrix} \lambda' \cdot X \end{pmatrix} = \begin{pmatrix} \lambda \cdot \alpha & \lambda \cdot b \\ \lambda \cdot c & \lambda \cdot d \end{pmatrix} + \begin{pmatrix} \lambda' \cdot \alpha & \lambda' \cdot b \\ \lambda' \cdot c & \lambda' \cdot d \end{pmatrix} = \begin{pmatrix} (\lambda + \lambda') \alpha & (\lambda + \lambda') b \\ (\lambda + \lambda') c & (\lambda + \lambda') d \end{pmatrix} .$$

$$A'$$
 on $(\lambda + \lambda') - X = (\lambda \cdot X) + (\lambda' \cdot X)$

$$\begin{cases} \lambda \left(x + Y \right) = \lambda \cdot \begin{bmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} \end{bmatrix} = \lambda \begin{pmatrix} a + a' & b + b' \\ c + c' & d + d' \end{pmatrix} = \begin{pmatrix} \lambda \left(a + a' \right) & \lambda \left(b + b' \right) \\ \lambda \left(c + c' \right) & \lambda \left(d + d' \right) \end{pmatrix}$$

$$\begin{cases} \lambda\left(X+Y\right) = \lambda \cdot \begin{bmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} \end{bmatrix} = \lambda \begin{pmatrix} a+a' & b+b' \\ c+c' & d+d' \end{pmatrix} = \begin{pmatrix} \lambda(a+a') & \lambda(b+b') \\ \lambda(c+c') & \lambda(d+d') \end{pmatrix} \\ \begin{pmatrix} \lambda,X \end{pmatrix} + \begin{pmatrix} \lambda,Y \end{pmatrix} = \lambda \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \lambda \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} = \begin{pmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{pmatrix} + \begin{pmatrix} \lambda a' & \lambda b' \\ \lambda c' & \lambda d' \end{pmatrix} = \begin{pmatrix} \lambda(a+a') & \lambda(b+b') \\ \lambda(c+c') & \lambda(d+d') \end{pmatrix}$$

$$\begin{array}{lll}
\cdot \forall \lambda, \lambda' \in \mathbb{R}, \forall x \in \mathbb{E} : (\lambda, \lambda'), x = \lambda, (\lambda', x). \\
(\lambda, \lambda'), \chi = \lambda \lambda' \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \lambda \lambda' a & \lambda \lambda' b \\ \lambda \lambda' c & \lambda \lambda' d \end{pmatrix}, \\
\lambda \cdot \begin{pmatrix} \lambda', x \end{pmatrix} = \lambda, \begin{pmatrix} \lambda' a & \lambda' b \\ \lambda' c & \lambda d \end{pmatrix} = \begin{pmatrix} \lambda \lambda' a & \lambda \lambda' b \\ \lambda \lambda' c & \lambda \lambda' d \end{pmatrix}, \\
\lambda' \circ \alpha : (\lambda, \lambda') \times = \lambda, (\lambda', x)
\end{array}$$

$$\begin{array}{ll}
\lambda \cdot \begin{pmatrix} \lambda' \cdot x \end{pmatrix} = \lambda \cdot (\lambda', x)
\end{array}$$

Enerale O4:

$$E = \mathbb{R}^{\varrho}.$$

$$2 - (n,y) + (n',y') = (n+n',y+y'), \lambda(n,y) = (\lambda n,y)$$

$$= \lambda(\mathbb{R}^{\varrho},+) \text{ in gauge abelien}$$

· + associative:

$$\begin{split} & \left[\left(x_{1} y_{2} \right) + \left(x_{1} '', y'' \right) = \left(x_{1} + x_{1} '', y_{2} + y'' \right) + \left(x_{1} '', y'' \right) = \left(x_{1} + x_{1} '' + x'' + y'' + y'' \right) \\ & \left(x_{1} y_{2} \right) + \left(x_{1} '', y'' \right) + \left(x_{1} '', y'' \right) = \left(x_{1} + x_{1} '' + x'' + y'' + y'' \right) \\ & \left(x_{1} y_{2} \right) + \left(x_{1} '', y'' \right) + \left(x_{1} '', y'' \right) = \left(x_{1} + x_{1} '' + x'' + y'' + y'' \right) \\ & \left(x_{1} y_{2} \right) + \left(x_{1} '', y'' \right) + \left(x_{1} '', y'' \right) = \left(x_{1} + x'' + x'' + y'' + y'' + y'' \right) \\ & \left(x_{1} y_{2} \right) + \left(x_{1} '', y'' \right) + \left(x_{1} '', y'' \right) = \left(x_{1} + x'' + x'' + y'' + y'' + y'' + y'' \right) \\ & \left(x_{1} y_{2} \right) + \left(x_{1} '', y'' \right) + \left(x_{1} '', y'' \right) = \left(x_{1} + x'' + x'' + y'' + y''$$

. + Commutative:

d'où (+ est associatie)

· + commutatif: (n,y) + (n',y') = (n',y') + (n,y).

(n,y)+(n,y)= (n+n, 4+A).

(n',y')+(n,y) = (n'+n,y'+y).

d'où (+ est connutatie)

· l'eleut rente de +:

(n,y)+e=e+(n,y)=(n,y)

e= (0,0).

· l'elevet synetique de +:

(n,y)+ (n',y')= (n',y')+ (n,y)= (0,0).

(n, y) + (n, 'A,) = (0'0')

(n+n',y+y')=(0,0).

(n',y') = (-n,-y).

Alas (E,+) et un groupe abelier

· + x ek, + x, y e E: x (x+y) = x x+ xy.

X(x+y) = X (n+n', 4+y') = (X (4+y'), X (x+x')).

 $\lambda.X + \lambda.Y = (\lambda y, \lambda z) + (\lambda y', \lambda x') = (\lambda (y+y'), \lambda (x+z')).$

d'où () (x+Y) = 1 x + 24.)

A Y'Y, EK' A XEE: (Y+Y,) X= XX+Y, X $(\lambda + \lambda') \cdot X = (\lambda + \lambda') (n,y) = ((\lambda + \lambda') y, (\lambda + \lambda') x)$ 1.x+1,x=(1,4,1)+(1,4,1,x)=((1+1,1)+(1+1,1)) d'où $(\lambda + \lambda') X = \lambda X + \lambda' X$. Y X, X'EK, Y XEE: (X, X). X = X. (X'. X). (x.x). X= (x.x)y, (x.x)x) 1 - (x'x) = 1 - (x'y, x'x) = ((x.x')y, (x.x') x). d'on (\(\lambda . \(\rangle '\) \(\tau \) Alors (E est in R-e.v) { - (niy) + (n'iy') = (nn', yy'), , \(\niy) = (\lambda niy) -> (E,+) un groupe commitative. [(n,y)+(n',y')]+(n',y'') = (n,y)+[(n',y')+(n'',y'')]. [(n,y)+(n',y")]+(n',y")=(nn',yy')+(n",y")=(nn'n",yy'y). (n,y) + [(n,1)) + (n,1)] = (n'A) + (n,n,1,1,2,3,1) = (xx, n, 1,2,2,2,1) d'où + associative. · + commutative: (nig)+ (n'ig') = (n'ig')+ (nig). (r,y) + (n',y') = (xn', yy') (n',y')+(n,y) = (n'or ,y'y) d'où (+ commutatie)